Advanced Microwave Guided-Structures and Analysis
Professor Bratin Ghosh
Department of Electronic and Electrical Communication
Indian Institute of Technology, Kharagpur
Lecture 27
Radiation from a Magnetic Current Source (Contd.)

Welcome to this session of lecture on the radiation from a magnetic current source in a homogeneous medium, we will be computing the spherical wave components due to the magnetic current source.

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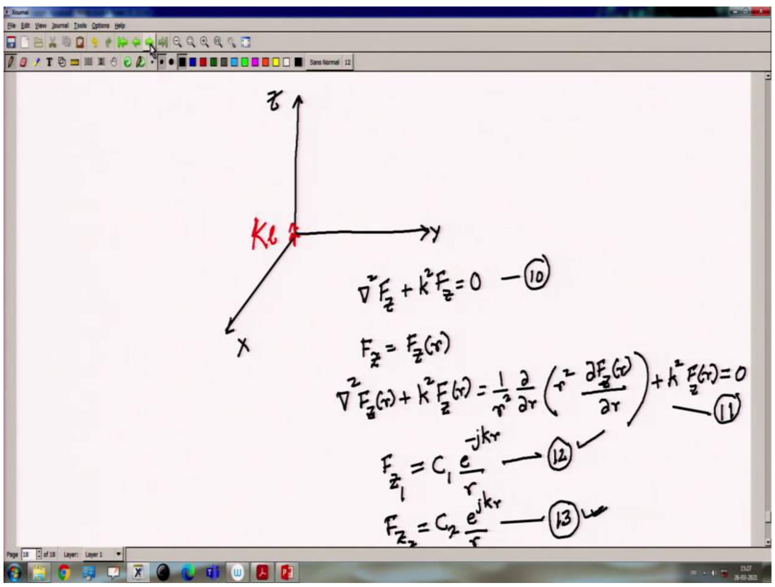

So, let us draw the x, y, z coordinates for this problem, this is x, that is y, that is z, and we have a magnetic current source, the magnetic current source has a magnetic dipole moment Kl, this K is capital, not to be confused with the propagation constant which is small k and it is placed at the origin. So, because the current is z directed the magnetic current source is z directed, **F** will also have z component only, and therefore the Helmholtz equation can be written as $\nabla^2 F_z + k^2 F_z = 0$.

So, we call this equation 10, also due to the spherical symmetry of the problem, also the fields will be spherically symmetric due to the nature of the problem, so let us express Fz as a function of the radial coordinates Fz(r). And therefore equation 10 can be rewritten as, in the nomenclature of the spherical wave components $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial F_z(r)}{\partial r}\right) + k^2 F_z(r) = 0$.

Again this differential equation has two independent solutions given by Fz1 equal to $c_1 \frac{e^{-jkr}}{r}$, and Fz2 equal to $c_2 \frac{e^{jkr}}{r}$, we call this equation 11, we call this equation 12, and we call this equation 13. So, this is the equation of a forward going wave or an outward going wave, so this is the equation of outward traveling wave, while this is the equation for an inward traveling wave, so because the dipole is radiating and it has to obey the radiation condition we will take the solution to be Fz1 the solution Fz1 also vanishes at r tends to infinity.

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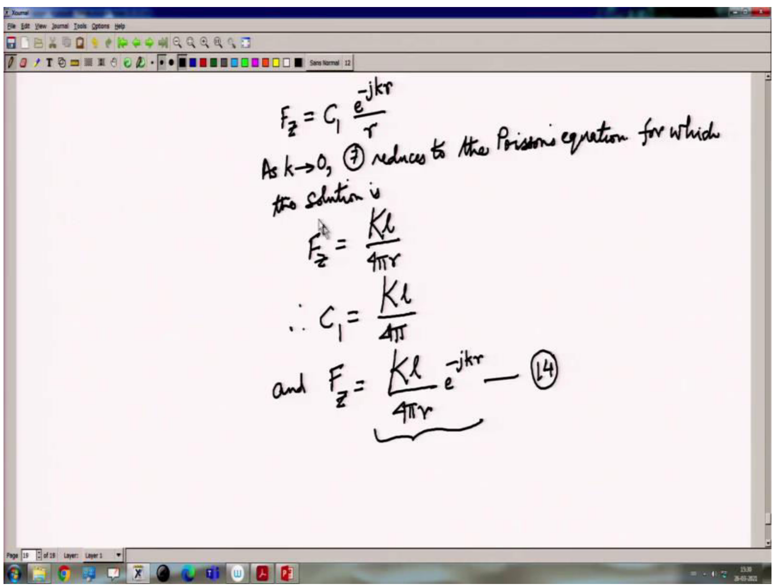

Therefore, we choose Fz as $c_1 \frac{e^{-jkr}}{r}$. Now, as k tends to 0 equation 7 reduces to the poisson's equation for which the solution is Fz is equal to $\frac{Kl}{4\pi r}$, this K is a capital K therefore our constant C1 must be equal to $\frac{Kl}{4\pi}$. And therefore, Fz will be equal to $\frac{Kl}{4\pi} \frac{e^{-jkr}}{r}$, this is equation number 14.

So, this is again the equation for a spherical wave, since the surfaces of constant phase are spheres. Then we compute the electromagnetic field radiated by the magnetic current source by substituting equation 14, the expression for Fz into the original equations which enable me to calculate the electric and magnetic fields due to a magnetic current source which are equations 8 and 9.

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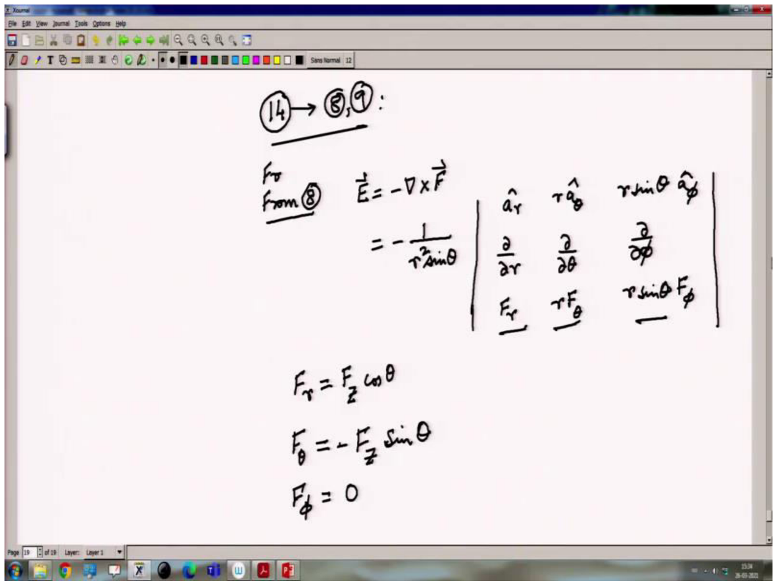

So, by substituting 14 into equations 8 and 9, we can calculate the electric and magnetic fields radiated by the magnetic current source. So, first from 8 we have **E** equal to minus curl of **F** that is equal $\frac{-1}{r^2 \sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ F_r & F_\theta & r\sin\theta F_\phi \end{vmatrix}$.

Again referring to the diagram we drew before for the relationship between the rectangular coordinates and the spherical coordinates for the case of the electric current source you refer back to the diagram, we will have Fr equal to Fz cos theta F theta will be equal to minus Fz sin theta and F phi will be 0. Now, all we have to do is to substitute these here and calculate the value of the electric field.

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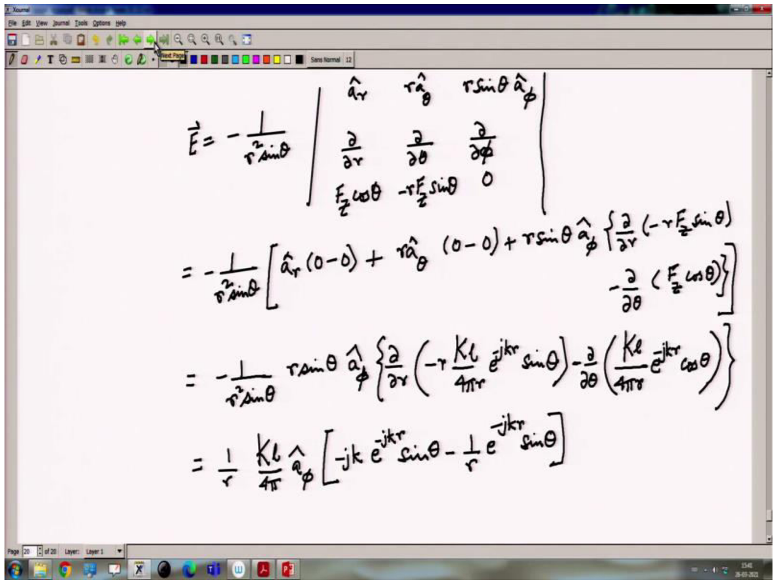

So, that electric field will be equal to $\vec{E} = \frac{1}{r}\frac{kl}{4\pi}\hat{a}_{\phi}(-jke^{-jkr}\sin(\theta) - \frac{1}{r}e^{-jkr}\sin(\theta))$

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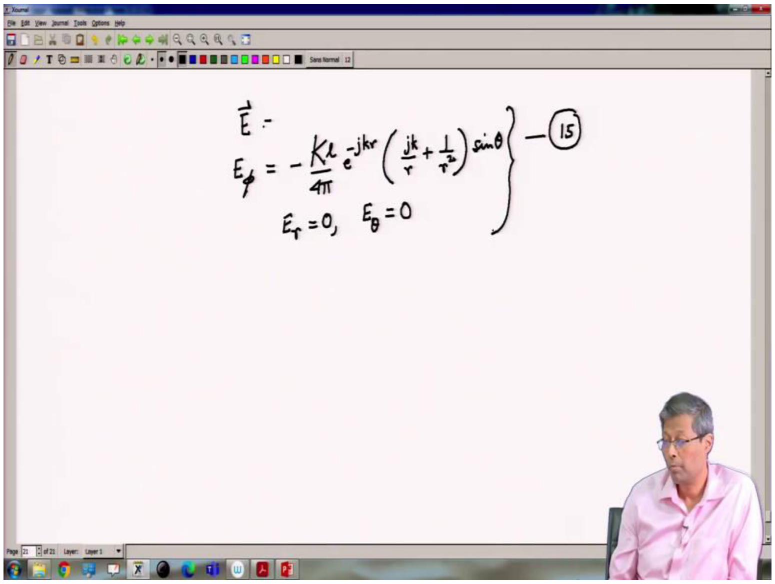

And that eventually works out to be Ephi with Er to be 0 and E theta to be 0. So, therefore the radiated electric field will only have a phi component with the radial component of the electric field 0 and with the elevation component or the theta component of the electric field also to be equal to 0. So we can club all of them into equation 15. So, let us stop here, we will next calculate the radiated magnetic field components by the magnetic current source. Thank you.