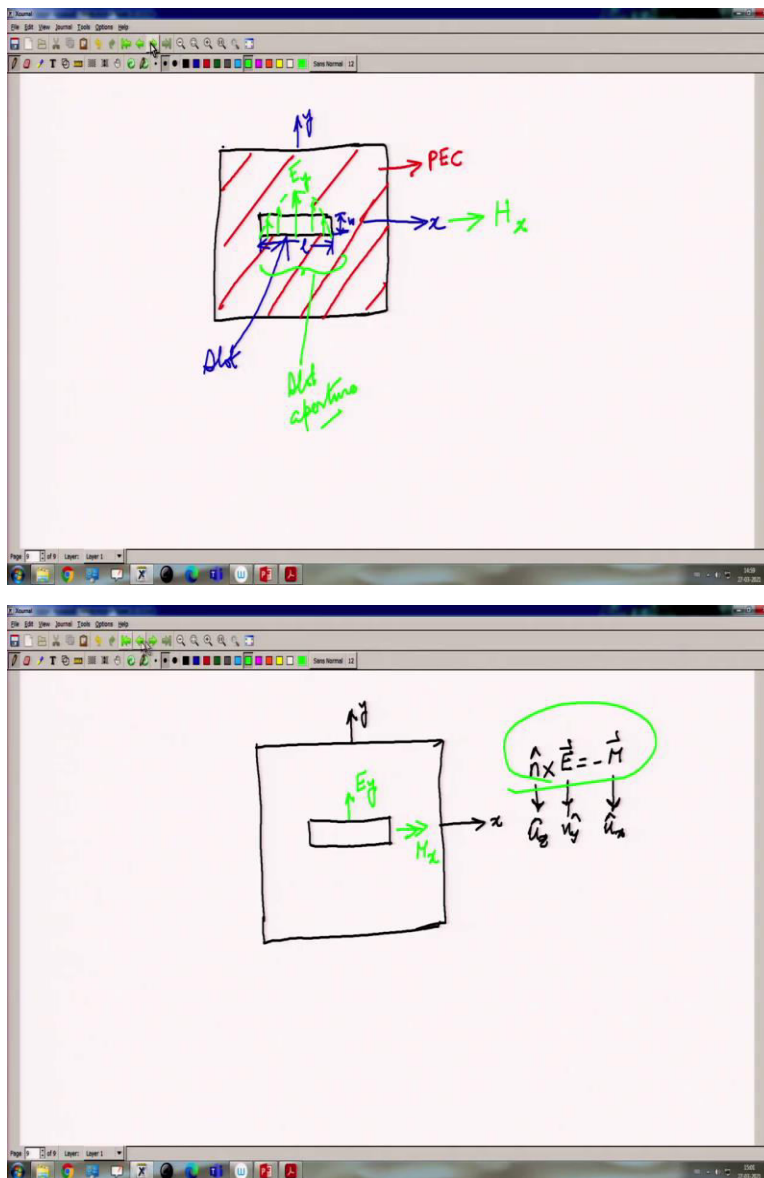


Advanced Microwave Guided- Structures and Analysis
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Lecture 29
Radiation from a Magnetic Current Source (Contd.)

Welcome to this lecture on the application of the magnetic current source to the field computation for electromagnetic field problems, particularly the Green's function computation for electromagnetic field problems. So, let us go to the lecture.

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So, we will illustrate the use of the magnetic current for the slot coupled problem. So, let us see the configuration of a slot in a ground plane. So, this is a ground plane and this is a slot so, this part is PEC or Perfect Electric Conductor and this is the slot, for the slot the electric field component is in this direction this is the width of the slot and this is the length of the slot.

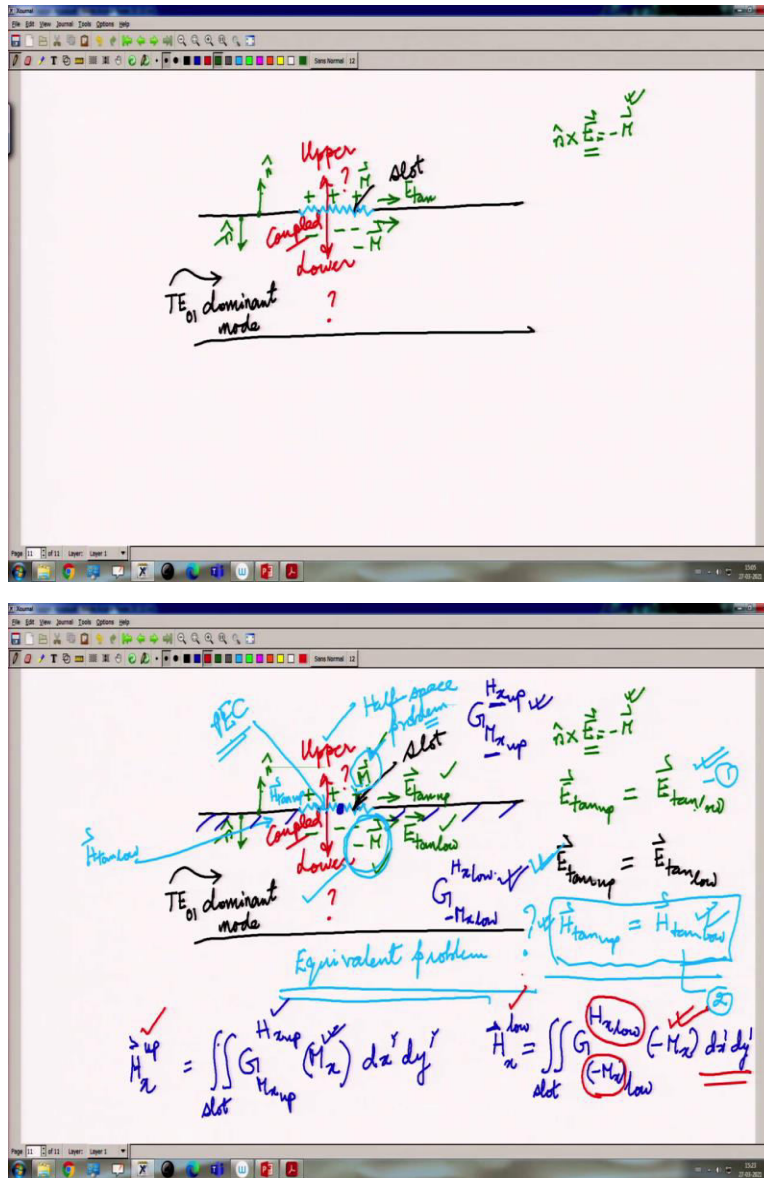
So, the electric field is along the width of the slot, the width of the slot is also much smaller than the length because otherwise a component of the electric field will also develop along the length and will contribute to the cross pole radiation which we do not want. So, the width of the slot is much smaller than the length. The maximum electric field in the slot is in the central part of the slot and the electric field distribution is of this form, it is 0 on the two sides of the slot.

So, this part is also called the slot aperture. So, this is the configuration of the slot. So, this is the electric field and if this is the x direction, if this is the y direction the electric field is along the y direction the magnetic field of the slot is along the x direction. So, this is the direction of the magnetic field in the slot. Now, this electric field can also be replaced by a magnetic current directed along the length of the slot.

So, again consider this as the ground plane and this is the slot and this is the x direction, this is the y direction. So, because $\mathbf{n} \times \mathbf{E} = -\mathbf{M}$ and \mathbf{n} is directed along the z axis which is normal upwards from the screen coming towards me, \mathbf{E} is along the y axis. So, \mathbf{M} will be directed along the so $\mathbf{z} \times \mathbf{y}$. So, it will be directed around the minus x axis.

So, \mathbf{M} will be directed because it is minus \mathbf{M} . So, \mathbf{M} will be directed along the x axis. So, therefore, \mathbf{M} will be directed in this fashion. So, the equivalent magnetic current will be directed in this direction. So, therefore, the slot electric field which is directed along the y direction with the maximum along the y direction can also be replaced by the equivalent magnetic current through this relationship which is directed along the length of the slot.

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With this picture, let us go or let us consider a slot coupled rectangular waveguide problem, let us consider a slot resident on the broad wall of a rectangular waveguide with the slot radiating in free space. So, the configuration is like this. So, this is a rectangular waveguide and from here the dominant mode, the TE 01 dominant mode; we are going to come to the description of the dominant mode is incident and this is the slot. Now, you see that these two problems, the upper problem which is the free space and the lower problem, they are coupled to this slot. Across the slot this problem is coupled. So, we cannot solve this problem we cannot write down the fields in this region or in this region, because the two problems are coupled with each other.

So, in order to remove this coupling, what we do is that we close this surface by a conductor. So, this is a conductor and we place magnetic currents plus M and minus M on the upper surface and on the lower surface of the conductor. So, this is the vertical unit normal on the upper surface and unit normal on the lower surface.

So, again because $\mathbf{n} \times \mathbf{E}$ equal to minus \mathbf{M} and because the unit normal on the upper surface is directed upwards unit normal on the lower surface is directed downwards the electric field on this surface over the slot and the electric field on the lower surface of the slot they are matched because the magnetic currents are equal and opposite signs on the upper surface and the lower surface.

So, because of the reversal in sign of the normal the value of the electric field or the tangential electric field just on the upper surface and on the lower surface, they are the same. So, if I call this $E_{\text{tan up}}$ this slot little bit above. So, we call this $E_{\text{tan up}}$ and the electric field on the lower surface as $E_{\text{tan low}}$. So, placement of the magnetic current plus M and minus M on the upper and the lower surfaces of the slot matches these two electric fields.

Therefore, we will have $E_{\text{tan up}}$ equal to $E_{\text{tan low}}$. Therefore, we will have $E_{\text{tan up}}$ equal to $E_{\text{tan low}}$ but that is not the only boundary condition which is present across the slot across. The original slot with this conductor absent that tangential magnetic fields if I write that which is $H_{\text{tan up}}$ here and at this location $H_{\text{tan low}}$ they would also match.

So, in the original problem $H_{\text{tan up}}$ will also be $H_{\text{tan low}}$ that also has to be obeyed, but we have not obeyed that as yet we have obeyed this, but we have not obeyed this one as yet. So, this condition needs to be obeyed. So, this is called construction of an equivalent problem by constructing this equivalent problem, we have closed these gap or slot by a conductor and decoupled the upper problem and the lower problem.

So, for the lower problem the situation is that the waveguide is excited by the TE 01 dominant mode and this magnetic current minus M resides over the slot region and radiates inside the waveguide this radiation from the magnetic current generates all the scattered modes in the waveguide.

Similarly, the magnetic current plus M resides on the upper surface of the slot and radiates in free space. This is also called the half space problem, where the magnetic current resides on an

infinite ground plane which is coincident with the broad wall of the waveguide or which is the broad wall of the waveguide and radiates in free space.

So, for the upper problem, we have the half space problems, with the magnetic current source radiating in free space and for the lower problem the minus M magnetic current generates the scattered fields or the scattered modes inside the waveguide. But all that said and done we have to obey this continuity equation as well.

Because in the construction of the equivalent problem the most important thing is obeying the original boundary conditions which existed across the slot. When I construct something equivalent, it means that I have change certain circumstance or inserted some kind of a boundary condition like for example the perfect electric conductor here.

So, this is the perfect electric conductor PEC, but introduction of the PEC and a consequent decoupling of the upper and lower problem has to be done by respecting the original boundary conditions which is this one number 1 and these two and this one number 2. This boundary condition I have already obeyed by placing the magnetic currents plus M and minus M on the upper surface and the lower surface of the slot.

Remember these magnetic currents are unknown to me, all I know that they should be of opposite signs on the upper surface and the lower surface and that they should be directed along the length of the slot but what is their magnitude I do not know. So, I do not know two things or I have not done two things either do not know or I have not done two things, one I do not know the magnitude of the magnetic current.

Secondly, I have not obeyed this boundary condition which existed across the slot previously. So, what do I do? I try to do both of the same things together. So, I compute the magnetic field radiated by this magnetic current residing on the upper surface of the slot. But, because the magnetic current is not known, the situation is similar to the case of the Pocklington situation or the Pocklington integral equation where the electric current was not known.

So, this magnetic current is not known, but what I have accomplished till now is that when I compute the radiated fields due to these magnetic current residing on the upper surface of the conductor, I completely forget what is below the conductor that problem is completely irrelevant to me that is decoupled that is irrelevant to me.

Similarly, when I calculate the radiated field due to this magnetic current minus M residing on the lower surface of the conductor, the upper part of the problem is completely irrelevant to me. So, therefore, a complete decoupling is achieved. Now, I need to find out therefore, the radiated magnetic field on the upper part for I need to find out the radiated magnetic field for the upper part of the problem, but I do not know the magnitude of the magnetic current.

So, therefore, similar to the case of the electric current source and the Pocklington integral equation, I will assume a point source to be located here a point magnetic current source to be located here and find the radiated field for this point magnetic source in the upper part of the problem.

Similarly, I would assume a point magnetic source located here, which is corresponding to this magnetic current minus M and find out the radiated field inside the waveguide due to the magnetic current the unit impulse magnetic current resident on the lower surface of the conductor. Therefore, my complete attention shifts from the amplitude of the magnetic current source to the boundary condition of the problem.

This is the heart of the Green's function, we when we talk of the Green's function we completely forget who is exciting the problem the source of excitation we take that to be an impulse source. We did that for the electric current source we are now also going to do for the magnetic current source, we find out the response of the system under all given boundary conditions, which is excited with a unit impulse source.

So, therefore, our only concern is the satisfaction of the boundary additions we have to satisfy all the boundary conditions of the problem, because my source is given to me. So, therefore, that information will be contained in the Green's function, because for the upper part of the problem, the satisfaction of the boundary condition due to this impulse source.

And finding out the electromagnetic fields due to this impulse source is nothing but the computation of the Green's function for the upper problem. Similarly, the computation of the electromagnetic fields due to the unit impulse force corresponding to minus M resident on the lower part of the conductor and finding out the scattered field inside the waveguide will correspond to the Green's function of the lower part of the problem.

In the Green's function of the lower part of the problem the boundary conditions of the rectangular waveguide are of supreme importance they have to be obeyed. Similarly, for the Green's function for the upper part of the problem the boundary conditions of this conducting plate is of supreme importance is a conducting plate. So, there is a conductors, so, boundary conditions of this conducting plate is of supreme importance.

So, let us therefore, write down the fields radiated in the upper part that will be double integral slot. So, after the Green's functions for the upper problem and the lower problem are computed, let us call the Green's function for the upper problem as G due to M_x up and because we are interested in matching the tangential components of the magnetic field, we will be computing H_x for the upper part of the problem.

Because we have to match the x component of the magnetic field which is along the length of the slot and the magnetic current also is directed along the x direction which is along the length of the slot. Similarly, we are going to calculate the Green's function due to the minus M_x component which is in the lower part and we will be calculating H_x for the lower part.

So, the source of the Green's function is minus M_x for the lower part and we are going to calculate the tangential magnetic field H_x for the lower part. After these two Green's functions are computed, we are not going to go to the computations of these Green's function in fact, these Green's functions are pretty much well known Green's functions, it has been investigated by many authors quite many years back.

So, we are not going to go to the derivation of this Green's function, but we will be assuming that after these Green's functions are known, how is the magnetic current relevant to me. So, after these Green's functions are known, we are going to calculate the field radiated in the upper part which is the convolution of the Green's function M_x for the upper part of the problem with the magnetic current M_x resident over dx prime dy prime over the slot.

So, this is the magnetic field the tangential magnetic field x directed which is radiated by the x directed magnetic current residing on the upper part of the conducting surface the magnetic current being M_x . So, this is the Green's function corresponding to the magnetic current M_x resident on the upper part of the slot or the upper part of the conductor which is now shutting of the slot and radiating a current and radiating a field H_x up in the upper part that is, that Green's

function is convolved with this magnetic current and integrated over the area of the slot these two primes they denote the location of the source because the magnetic current is located over the entire slot.

So, this is integrated over the slot, this will give me the total magnetic field which is radiated for the upper part. Similarly, that total magnetic field radiated for the lower part will be given by double integral Green's function minus M_x low, H_x low minus M_x dx prime dy prime over the area of the slot because the current resident on the lower surface is minus M_x .

This M is x directed as we saw in the previous discussion, the magnetic current is along the length of the slot which is assumed along the x axis. So, it is minus M_x and this is the tangential magnetic field in the slot which is also x directed it is convolved with the magnetic current minus x , it is convolved with the tangential magnetic current minus M_x .

It is convolved with the tangential magnetic current minus M_x and integrated over the surface area of the slot over which the minus M_x exists. This gives me the tangential magnetic field radiated up in the, for the upper problem and this is the tangential magnetic field radiated for the lower problem. So, now, we have to match the total tangential magnetic fields for the lower problem and that for the upper problem.

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$$\begin{aligned}
 \vec{H}_{inc} + \iint_{slot} G_{H_{xlow}} (-M_x) dx' dy' \\
 = \iint_{slot} G_{H_{xup}} M_x dx' dy'
 \end{aligned}$$

\downarrow
H-field integral equation

So, for that we write down the integral equation which is H incident plus over the slot G we have minus M_x low H_x low times minus M_x times dx prime dy prime. This is the total magnetic field on the lower surface of the slot. This is the incident magnetic field due to the dominant T_{10} mode. This is the dominant magnetic field, this is the magnetic field corresponding to the dominant T_{01} mode of the waveguide.

This is the magnetic field the x component of the magnetic field radiated by due to the current minus M_x inside the waveguide and that should be equal to double integral over the slot again the Green's function M_x up H_x up for the upper problem times M_x times M_x times dx prime dy prime. So, this is the magnetic field the x component of the tangential magnetic field radiated due to the magnetic current M_x resident on the upper surface of the conducting plate that is shutting up the slot.

So, now, by application of the method of moment, the unknown magnetic current M_x is obtained because the Green's function is known, the H incident is known this Green's function is also known, the Green's function for the upper problem is known, the Green's function for the lower problem is known, the H incident for the dominant mode is known the dominant TE_{01} mod is known. So, we are only left to calculate the unknown magnetic current M_x which will be calculated using the method of moments this equation is also called the H field integral equation.

So, now, you see how the magnetic current is relevant for any slot coupled problem in electromagnetics without the introduction of the magnetic current we will not be able to construct the equivalent solution or equivalent problem corresponding to the slot coupled problem. This is not the only situations where the magnetic current is important. There are many other situations where equivalence principle is invoked and the magnetic current is introduced to construct an equivalent problem. So, at least you have an insight into a very important need for the magnetic current in the solution of the slot coupled electromagnetic field problem. Thank you.