Advanced Microwave Guided-Structures and Analysis
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Lecture No. 25
Radiation from an Electric Current Source Tutorials

Hello every1, let us solve some few more numerical problems on relation between wave numbers.

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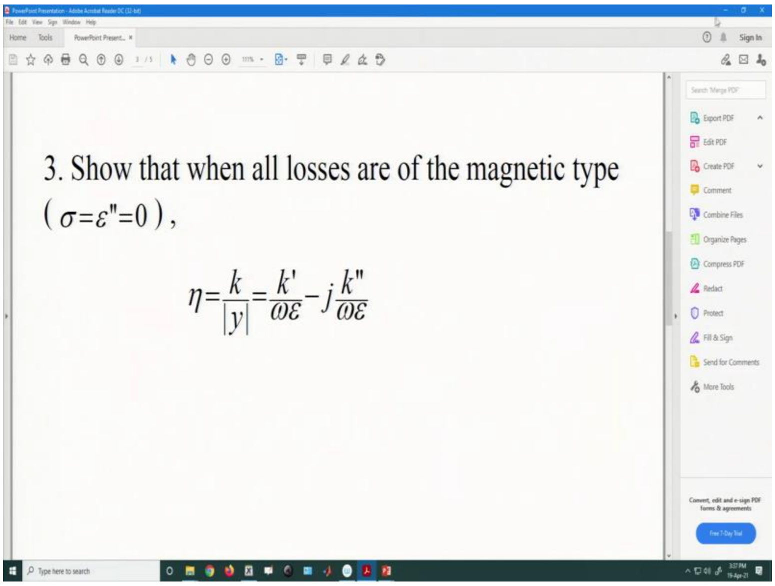
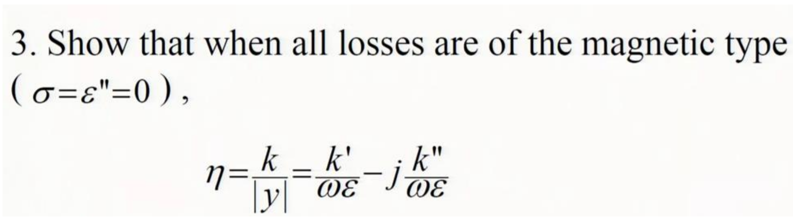


So, to start with the first problem states that



So, let us start solving this.

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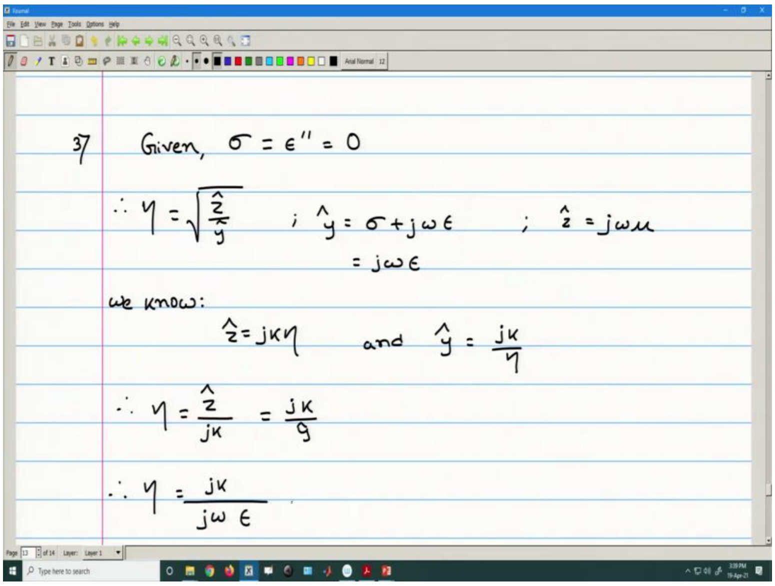

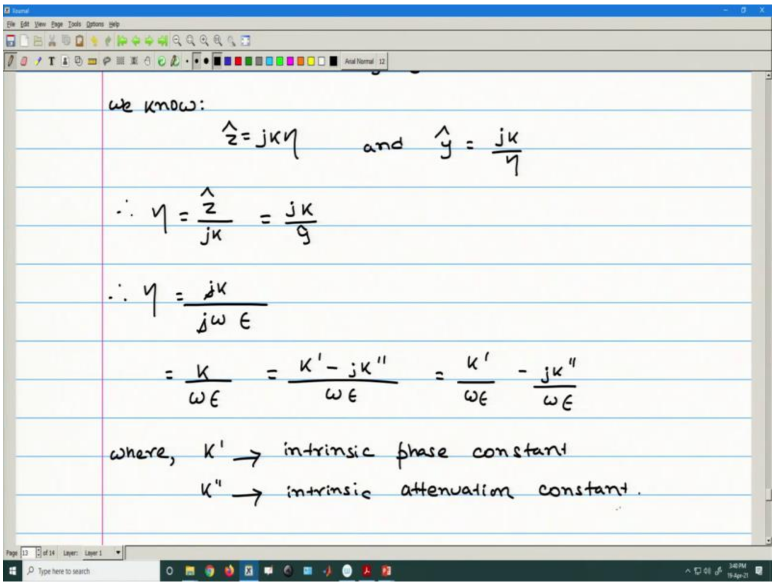

So, we are given that

37 Given, $\sigma = \epsilon'' = 0$

$$\therefore \eta = \sqrt{\frac{\hat{z}}{\hat{y}}} \quad ; \quad \hat{y} = \sigma + j\omega\epsilon \quad ; \quad \hat{z} = j\omega\mu$$

$$= j\omega\epsilon$$

we know:

$$\hat{z} = jK\eta \quad \text{and} \quad \hat{y} = \frac{jK}{\eta}$$

$$\therefore \eta = \frac{\hat{z}}{jK} = \frac{jK}{y}$$

$$\therefore \eta = \frac{jK}{j\omega\epsilon}$$

we know:

$$\hat{z} = jK\eta \quad \text{and} \quad \hat{y} = \frac{jK}{\eta}$$

$$\therefore \eta = \frac{\hat{z}}{jK} = \frac{jK}{y}$$

$$\therefore \eta = \frac{\cancel{j}K}{\cancel{j}\omega\epsilon}$$

$$= \frac{K}{\omega\epsilon} = \frac{K' - jK''}{\omega\epsilon} = \frac{K'}{\omega\epsilon} - \frac{jK''}{\omega\epsilon}$$

where, $K' \rightarrow$ intrinsic phase constant

$K'' \rightarrow$ intrinsic attenuation constant.

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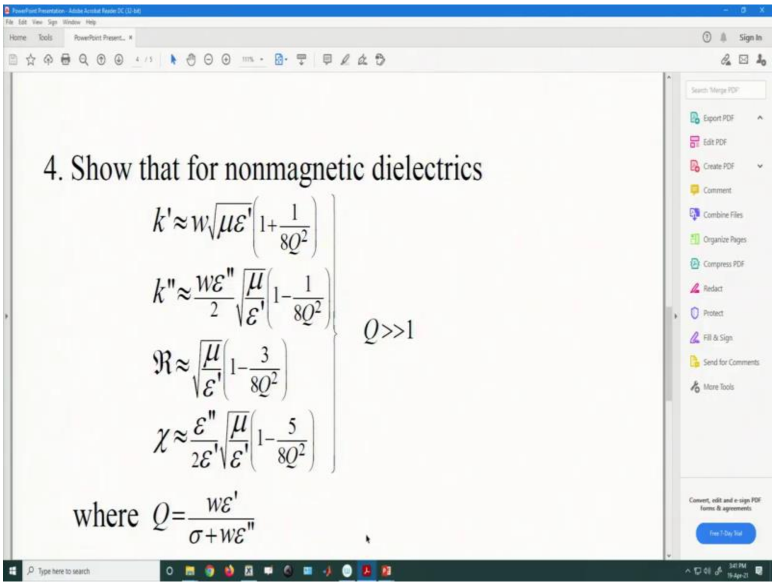

The next problem is that

4. Show that for nonmagnetic dielectrics

$$\left.\begin{aligned}
k' &\approx w\sqrt{\mu\varepsilon'}\left(1+\frac{1}{8Q^2}\right)\\
k'' &\approx \frac{w\varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}}\left(1-\frac{1}{8Q^2}\right)\\
\Re &\approx \sqrt{\frac{\mu}{\varepsilon'}}\left(1-\frac{3}{8Q^2}\right)\\
\chi &\approx \frac{\varepsilon''}{2\varepsilon'}\sqrt{\frac{\mu}{\varepsilon'}}\left(1-\frac{5}{8Q^2}\right)
\end{aligned}\right\}\quad Q>>1$$

where $Q=\dfrac{w\varepsilon'}{\sigma+w\varepsilon''}$

So again, we will start solving.

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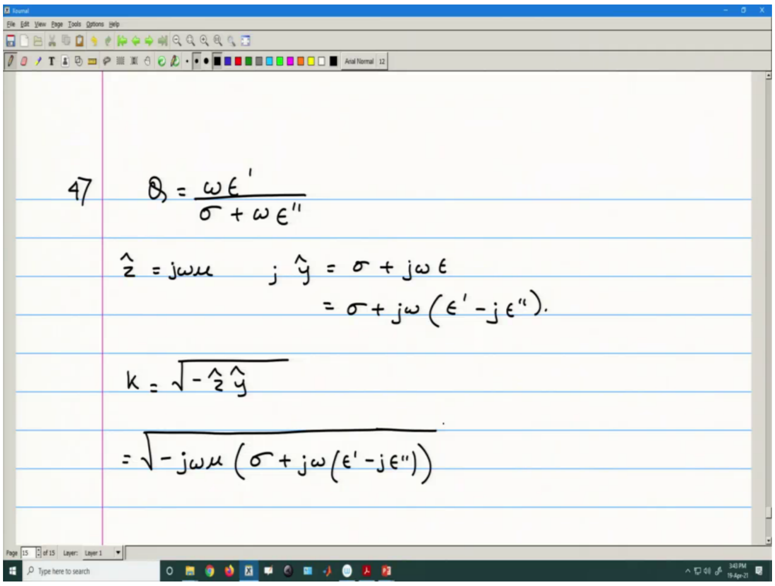
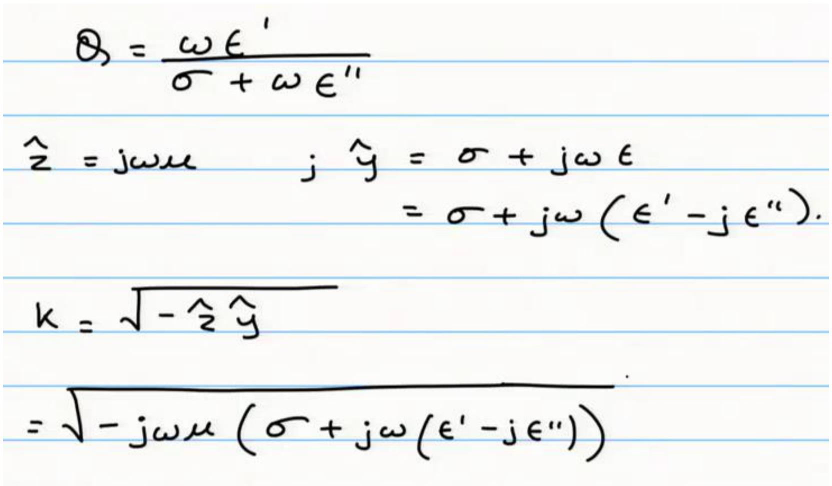


So, we it is given that quality factor is written as



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$$k = \sqrt{-\hat{z}\hat{y}}$$

$$= \sqrt{-j\omega\mu\left(\sigma + j\omega\left(\epsilon' - j\epsilon''\right)\right)}$$

$$= \sqrt{-j\omega\mu\left(\sigma + j\omega\epsilon' + \omega\epsilon''\right)}$$

$$= \sqrt{-j\omega\mu\sigma + \omega^2\mu\epsilon' - j\omega^2\mu\epsilon''}$$

$$= \sqrt{-j\omega\mu\left(\sigma + \omega\epsilon''\right) + \omega^2\mu\epsilon'}$$

$$= \sqrt{\omega^2\mu\epsilon'\left(1 - \frac{j\omega\mu\left(\sigma + \omega\epsilon''\right)}{\omega^2\mu\epsilon'}\right.}$$

Now, we will simplify this just open up the braces, we will get

$$k = \sqrt{-\hat{z}\hat{y}}$$

$$= \sqrt{-j\omega\mu\left(\sigma + j\omega\left(\epsilon' - j\epsilon''\right)\right)}$$

$$= \sqrt{-j\omega\mu\left(\sigma + j\omega\epsilon' + \omega\epsilon''\right)}$$

$$= \sqrt{-j\omega\mu\sigma + \omega^2\mu\epsilon' - j\omega^2\mu\epsilon''}$$

$$= \sqrt{-j\omega\mu\left(\sigma + \omega\epsilon''\right) + \omega^2\mu\epsilon'}$$

$$= \sqrt{\omega^2\mu\epsilon'\left(1 - \frac{j\omega\mu\left(\sigma + \omega\epsilon''\right)}{\omega^2\mu\epsilon'}\right.}$$

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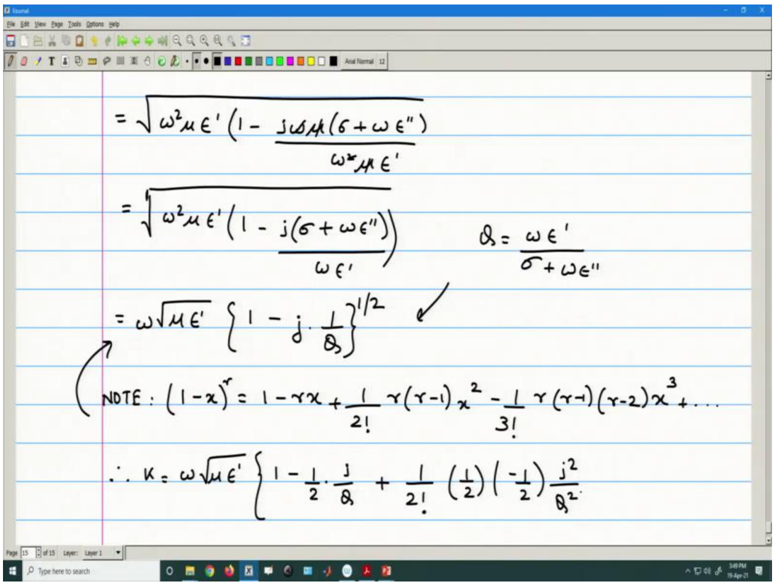
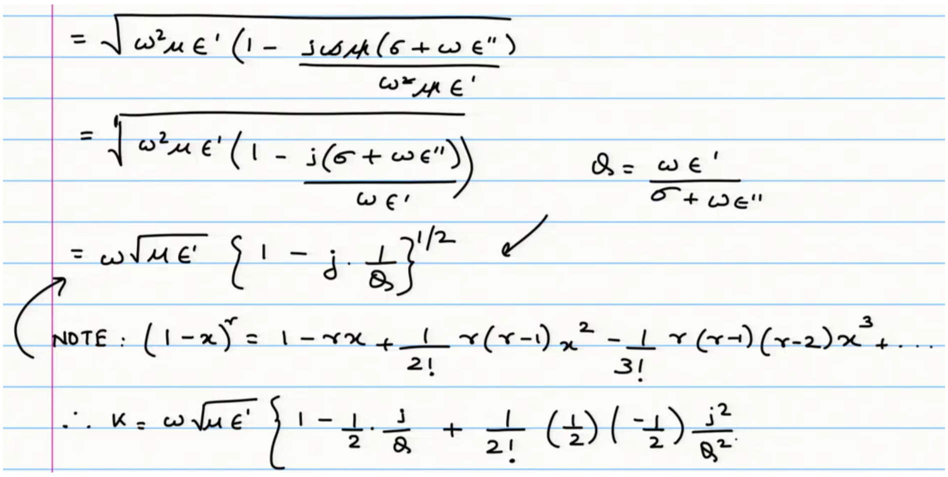
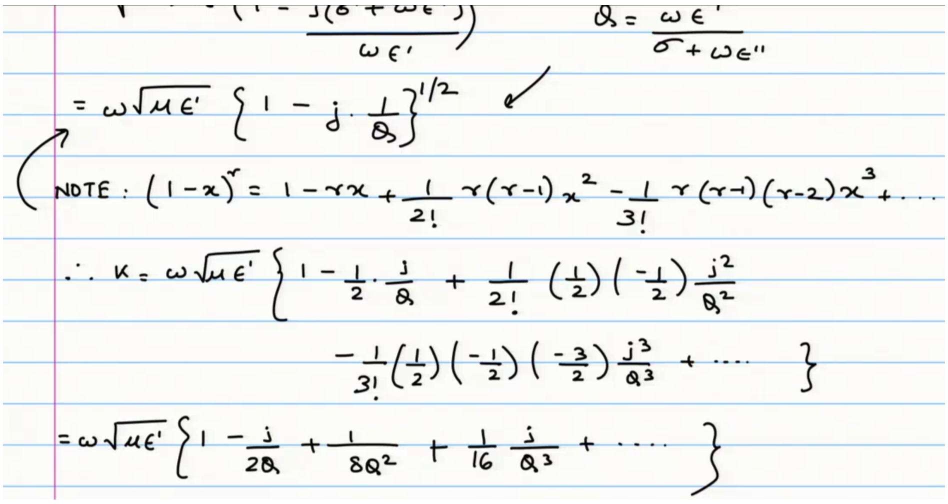


Then we can write this





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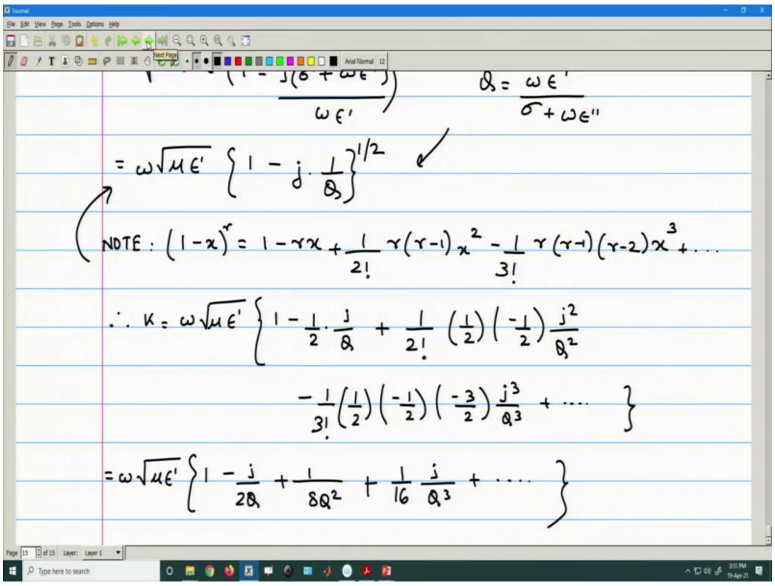

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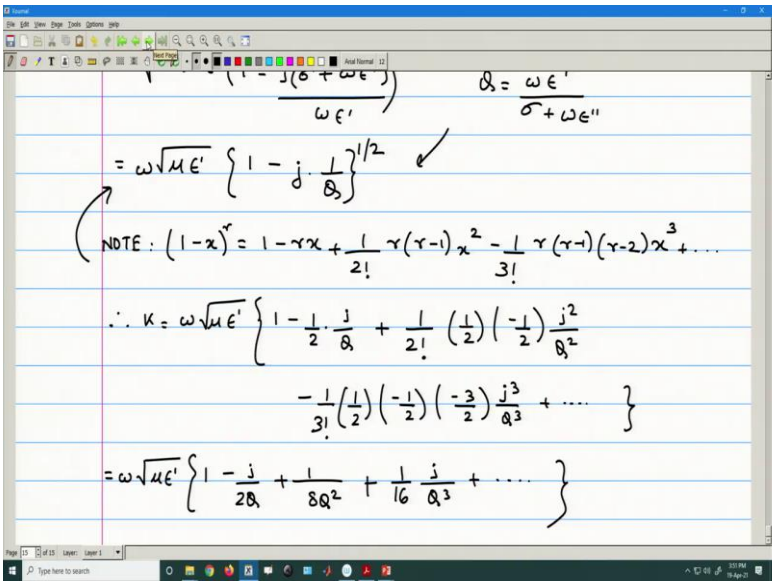
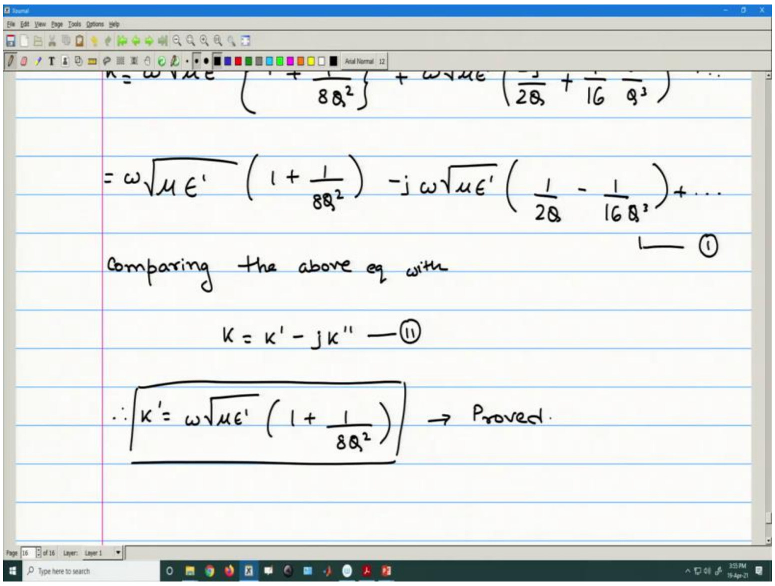

$Q = \frac{\omega \epsilon'}{\sigma + \omega\epsilon''}$
$= \omega\sqrt{\mu\epsilon'}\left\{1 - j\cdot\frac{1}{Q}\right\}^{1/2}$
NOTE : $(1-x)^r = 1 - rx + \frac{1}{2!}r(r-1)x^2 - \frac{1}{3!}r(r-1)(r-2)x^3 + \dots$
$\therefore K = \omega\sqrt{\mu\epsilon'}\left\{1 - \frac{1}{2}\cdot\frac{j}{Q} + \frac{1}{2!}\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\frac{j^2}{Q^2} - \frac{1}{3!}\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)\frac{j^3}{Q^3} + \dots\right\}$
$= \omega\sqrt{\mu\epsilon'}\left\{1 - \frac{j}{2Q} + \frac{1}{8Q^2} + \frac{1}{16}\frac{j}{Q^3} + \dots\right\}$



So, let us write it as separating real and imaginary parts, separating the real and imaginary parts,

$$k = \omega\sqrt{\mu\epsilon'}\left(1 + \frac{1}{8Q^2}\right) + \omega\sqrt{\mu\epsilon'}\left(\frac{-j}{2Q} + \frac{j}{16}\frac{1}{Q^3}\right) \cdots$$

$$= \omega\sqrt{\mu\epsilon'}\left(1 + \frac{1}{8Q^2}\right) - j\,\omega\sqrt{\mu\epsilon'}\left(\frac{1}{2Q} - \frac{1}{16Q^3}\right) + \cdots \quad \text{— (I)}$$

Comparing the above eq with

$$k = k' - jk'' \quad \text{— (II)}$$

$$\therefore \boxed{k' = \omega\sqrt{\mu\epsilon'}\left(1 + \frac{1}{8Q^2}\right)} \rightarrow \text{Proved.}$$

So, this we got this is proved.

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$$k'' = \omega\sqrt{\mu\epsilon'}\left(\frac{1}{2Q} - \frac{1}{16Q^3}\right)$$

$$= \frac{\omega\sqrt{\mu\epsilon'}}{2Q}\left(1 - \frac{1}{8Q^2}\right)$$

$$= \frac{\cancel{\omega}\sqrt{\mu\epsilon'}\,(\sigma + \omega\epsilon'')}{2 \times \cancel{\omega}\epsilon'}\left(1 - \frac{1}{8Q^2}\right)$$

$$\boxed{k'' = \sqrt{\frac{\mu}{\epsilon'}}\,\frac{\omega\epsilon''}{2}\left(1 - \frac{1}{8Q^2}\right)} \quad \text{(Proved).}$$

And for k double prime we will take the imaginary part.

$$k'' = \omega\sqrt{\mu\epsilon'}\left(\frac{1}{2Q} - \frac{1}{16Q^3}\right)$$

$$= \frac{\omega\sqrt{\mu\epsilon'}}{2Q}\left(1 - \frac{1}{8Q^2}\right)$$

$$= \frac{\cancel{\omega}\sqrt{\mu\epsilon'}\,(\sigma + \omega\epsilon'')}{2 \times \cancel{\omega}\epsilon'}\left(1 - \frac{1}{8Q^2}\right)$$

$$\boxed{k'' = \sqrt{\frac{\mu}{\epsilon'}}\,\frac{\omega\epsilon''}{2}\left(1 - \frac{1}{8Q^2}\right)} \qquad (\text{Proved}).$$

So, this is also proved. Now further resistance and the reactants. So, for that we will start with the eta.

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$$= \frac{\cancel{\omega}\sqrt{\mu\epsilon'}\;(\sigma + \omega\epsilon'')}{2 \times \cancel{\omega}\epsilon'}\left(1 - \frac{1}{8Q^2}\right)$$

$$\boxed{K'' = \sqrt{\frac{\mu}{\epsilon'}}\;\frac{\omega\epsilon''}{2}\left(1 - \frac{1}{8Q^2}\right)} \qquad (\text{Proved}).$$

$$\rightarrow \quad \eta = \sqrt{\frac{\hat{z}}{\hat{y}}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega(\epsilon' - j\epsilon'')}}$$

$$= \sqrt{\frac{j\omega\mu}{j\omega\epsilon' + \omega\epsilon''}}$$

So, first and second done, for third and fourth we will start with we know eta is given by

$$\rightarrow \quad \eta = \sqrt{\frac{\hat{z}}{\hat{y}}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega(\epsilon' - j\epsilon'')}}$$

$$= \sqrt{\frac{j\omega\mu}{j\omega\epsilon' + \omega\epsilon''}}$$

$$= \sqrt{\frac{\mu}{\epsilon'}}\left[\frac{j\omega}{j\omega + \frac{\omega\epsilon''}{\epsilon'}}\right]^{1/2}$$

$$= \sqrt{\frac{\mu}{\epsilon'}}\left\{\frac{j\omega\epsilon'}{j\omega\epsilon' + \omega\epsilon''}\right\}^{1/2}$$

$$= \sqrt{\frac{\mu}{\epsilon'}}\left\{\frac{1}{1 + \frac{\omega\epsilon''}{j\omega\epsilon'}}\right\}^{1/2}$$

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$$= \sqrt{\frac{\mu}{\epsilon'}} \left[\frac{j\omega}{j\omega + \frac{\omega \epsilon''}{\epsilon'}} \right]^{1/2}$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{j\omega\epsilon'}{j\omega\epsilon' + \omega\epsilon''} \right\}^{1/2}$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{1}{1 + \frac{\omega\epsilon''}{j\omega\epsilon'}} \right\}^{1/2}$$

That is we can write it as

$$= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{j\omega\epsilon'}{j\omega\epsilon' + \omega\epsilon''} \right\}^{1/2}$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{1}{1 + \frac{\omega\epsilon''}{j\omega\epsilon'}} \right\}^{1/2}$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \left[\frac{1}{1 + \frac{1}{jQ}} \right]^{1/2}$$

$$\therefore \eta = \sqrt{\frac{\mu}{\epsilon'}} \left(1 + \frac{1}{jQ} \right)^{-1/2}$$

$$\therefore \eta = \sqrt{\frac{\mu}{\epsilon'}} \left(1 + \frac{1}{jQ} \right)^{-1/2}$$

NOTE :

$$(1+x)^r = 1 + rx + \frac{1}{2!} r(r-1)x^2 + \frac{1}{3!} r(r-1)(r-2)x^3 + \cdots$$

$$\Rightarrow \eta = \sqrt{\frac{\mu}{\epsilon'}} \left[1 + \left(-\frac{1}{2}\right)\left(\frac{1}{jQ}\right) + \frac{1}{2!}\left(-\frac{1}{2}\right)\left(\frac{-3}{2}\right)\left(\frac{1}{jQ}\right)^2 + \frac{1}{3!}\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)\left(\frac{-5}{2}\right)\left(\frac{1}{jQ}\right)^3 + \cdots \right]$$

$$\Rightarrow \eta = \sqrt{\frac{\mu}{\epsilon'}} \left[1 + \left(-\frac{1}{2}\right)\left(\frac{1}{jQ}\right) + \frac{1}{2!}\left(-\frac{1}{2}\right)\left(\frac{-3}{2}\right)\left(\frac{1}{jQ}\right)^2 + \frac{1}{3!}\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)\left(\frac{-5}{2}\right)\left(\frac{1}{jQ}\right)^3 + \cdots \right]$$

$$\Rightarrow \eta = \sqrt{\frac{\mu}{\epsilon'}} \left[1 - \frac{1}{2jQ} - \frac{3}{8Q^2} + \frac{15}{48} \frac{1}{jQ^3} \cdots \right]$$

Separating the real and imaginary parts :

$$\eta = \sqrt{\frac{\mu}{\epsilon'}} \left[1 - \frac{3}{8Q^2} \right] + \sqrt{\frac{\mu}{\epsilon'}} \left[\frac{-1}{2jQ} + \frac{15}{48} \frac{1}{jQ^3} \cdots \right]$$

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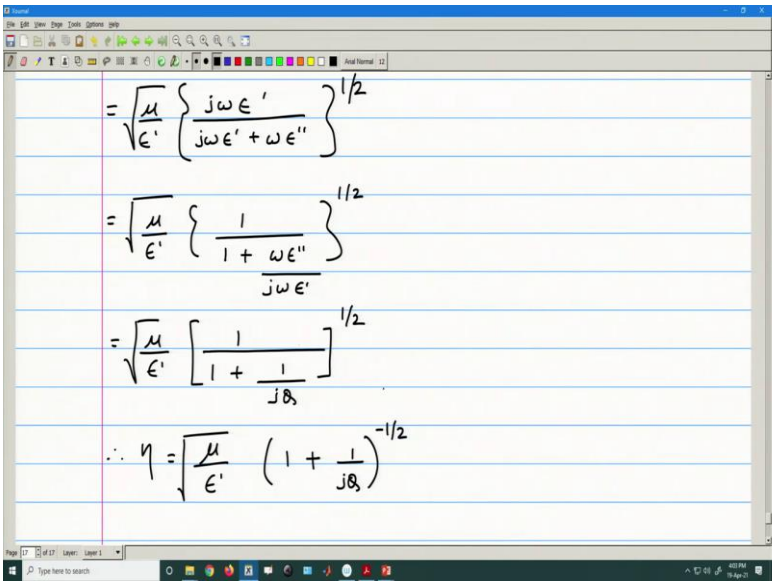

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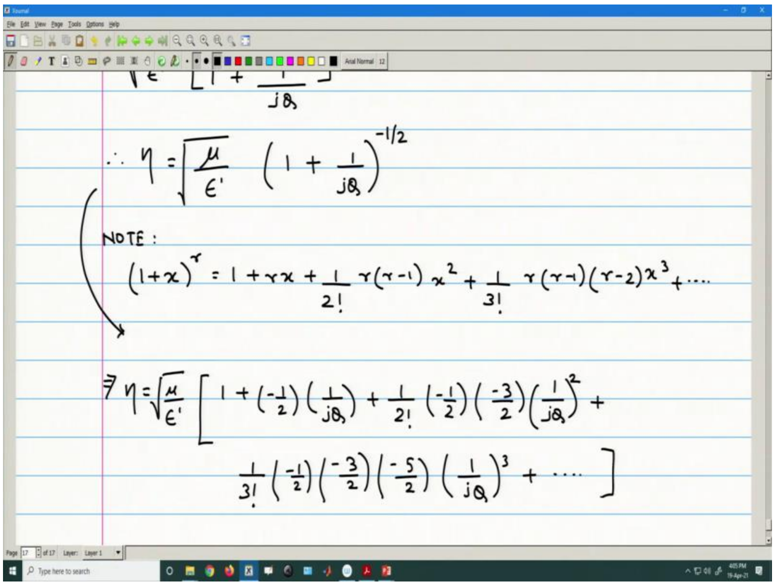

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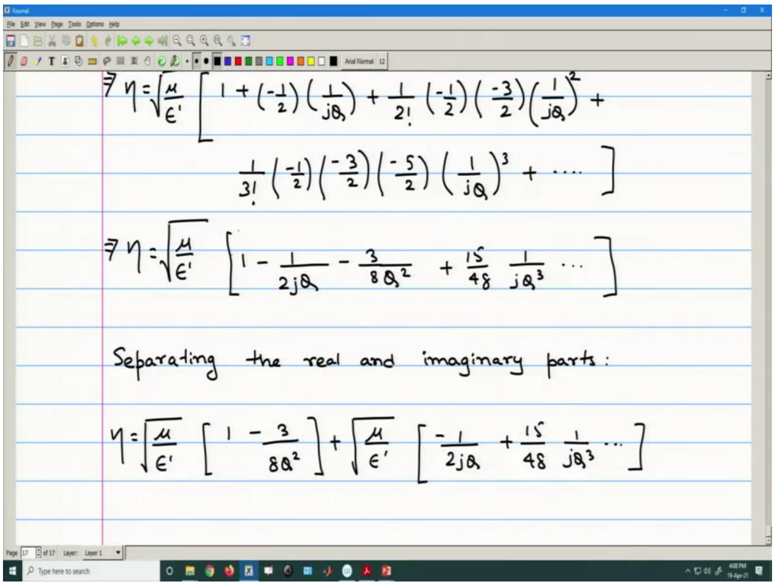

So, separating the real and imaginary parts, separating the real and imaginary parts as we will get

$$= \sqrt{\frac{\mu}{\epsilon'}}\left[1 - \frac{3}{8Q^2}\right] + \sqrt{\frac{\mu}{\epsilon'}}\left(\frac{j}{2Q} - \frac{5}{16}\frac{j}{Q^3}\right)$$

$$\therefore \boxed{R = \sqrt{\frac{\mu}{\epsilon'}}\left(1 - \frac{3}{8Q^2}\right)} \rightarrow (\text{Proved})$$

$$X = \sqrt{\frac{\mu}{\epsilon'}}\left(\frac{1}{2Q} - \frac{5}{16Q^3}\right)$$

$$= \sqrt{\frac{\mu}{\epsilon'}}\,\frac{1}{2Q}\left[1 - \frac{5}{8Q^2}\right]$$

$$= \sqrt{\frac{\mu}{\epsilon'}}\,\frac{1\,\omega\epsilon''}{2\cdot\omega\epsilon'}\left[1 - \frac{5}{8Q^2}\right]$$

$$\boxed{X = \sqrt{\frac{\mu}{\epsilon'}}\,\frac{\epsilon''}{2\epsilon'}\left(1 - \frac{5}{8Q^2}\right)} \rightarrow (\text{Proved})$$

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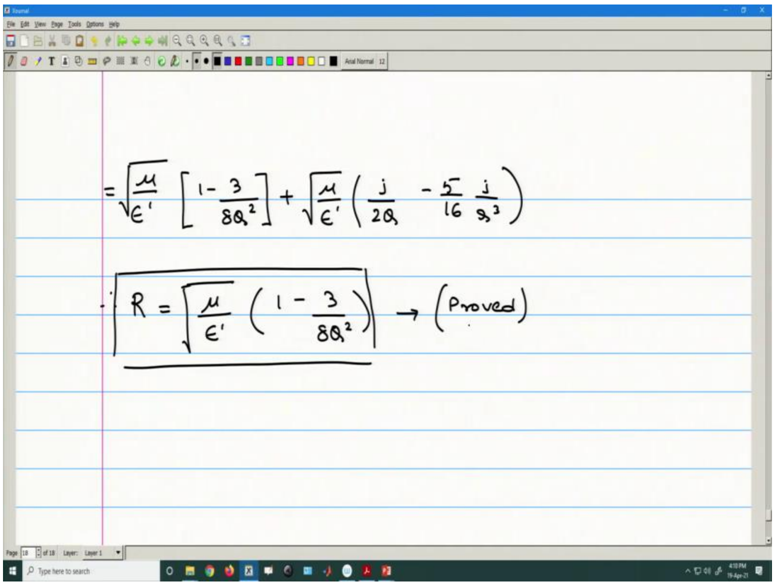

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So, our next class we will also solve some few more problems on wave numbers and then we will move into electric current source. Thank you.