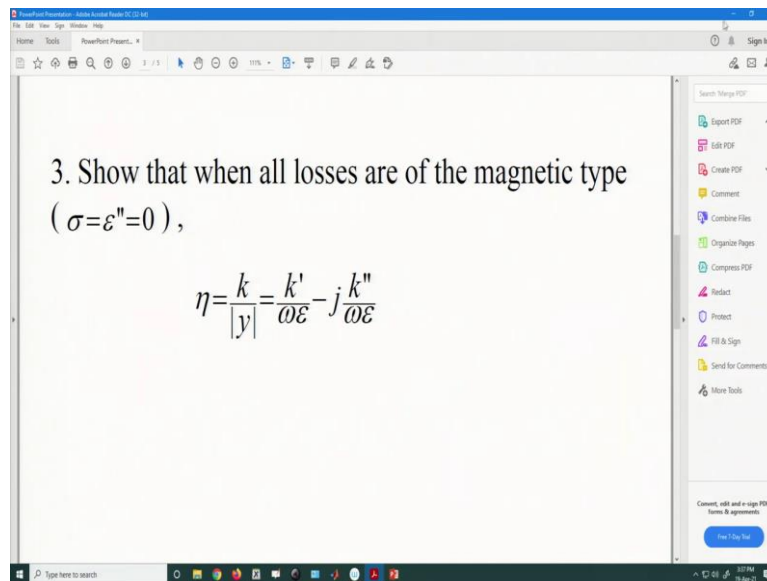


Advanced Microwave Guided-Structures and Analysis
Professor. Bratin Ghosh
Department of E & ECE
Indian Institute of Technology, Kharagpur
Lecture No. 25
Radiation from an Electric Current Source Tutorials

Hello everyone, let us solve some few more numerical problems on relation between wave numbers.

(Refer Slide Time: 0:26)



3. Show that when all losses are of the magnetic type ($\sigma = \epsilon'' = 0$),

$$\eta = \frac{k}{|y|} = \frac{k'}{\omega\epsilon} - j \frac{k''}{\omega\epsilon}$$

So, to start with the first problem states that

3. Show that when all losses are of the magnetic type ($\sigma = \epsilon'' = 0$),

$$\eta = \frac{k}{|y|} = \frac{k'}{\omega\epsilon} - j \frac{k''}{\omega\epsilon}$$

So, let us start solving this.

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37 Given, $\sigma = \epsilon'' = 0$

$$\therefore \eta = \sqrt{\frac{\hat{z}}{y}} \quad ; \quad \hat{y} = \sigma + j\omega\epsilon \quad ; \quad \hat{z} = j\omega\mu$$
$$= j\omega\epsilon$$

we know:

$$\hat{z} = jk\eta \quad \text{and} \quad \hat{y} = \frac{jk}{\eta}$$
$$\therefore \eta = \frac{\hat{z}}{jk} = \frac{j\omega\mu}{jk}$$
$$\therefore \eta = \frac{j\omega\mu}{j\omega\epsilon}$$

we know:

$$\hat{z} = jk\eta \quad \text{and} \quad \hat{y} = \frac{jk}{\eta}$$
$$\therefore \eta = \frac{\hat{z}}{jk} = \frac{j\omega\mu}{jk}$$
$$\therefore \eta = \frac{j\omega\mu}{j\omega\epsilon}$$
$$= \frac{\omega\mu}{\omega\epsilon} = \frac{k' - jk''}{\omega\epsilon} = \frac{k'}{\omega\epsilon} - \frac{jk''}{\omega\epsilon}$$

where, $k' \rightarrow$ intrinsic phase constant
 $k'' \rightarrow$ intrinsic attenuation constant.

So, we are given that

37 Given, $\sigma = \epsilon'' = 0$

$$\therefore \eta = \sqrt{\frac{\hat{z}}{y}} \quad ; \quad \hat{y} = \sigma + j\omega\epsilon \quad ; \quad \hat{z} = j\omega\mu \\ = j\omega\epsilon$$

we know:

$$\hat{z} = jk\eta \quad \text{and} \quad \hat{y} = \frac{jk}{\eta}$$

$$\therefore \eta = \frac{\hat{z}}{jk} = \frac{jk}{g}$$

$$\therefore \eta = \frac{jk}{j\omega\epsilon}$$

we know:

$$\hat{z} = jk\eta \quad \text{and} \quad \hat{y} = \frac{jk}{\eta}$$

$$\therefore \eta = \frac{\hat{z}}{jk} = \frac{jk}{g}$$

$$\therefore \eta = \frac{jk}{j\omega\epsilon}$$

$$= \frac{k}{\omega\epsilon} = \frac{k' - jk''}{\omega\epsilon} = \frac{k'}{\omega\epsilon} - \frac{jk''}{\omega\epsilon}$$

where, $k' \rightarrow$ intrinsic phase constant

$k'' \rightarrow$ intrinsic attenuation constant.

(Refer Slide Time: 4:14)

4. Show that for nonmagnetic dielectrics

$$\left. \begin{aligned} k' &\approx w\sqrt{\mu\epsilon'}\left(1+\frac{1}{8Q^2}\right) \\ k'' &\approx \frac{w\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}}\left(1-\frac{1}{8Q^2}\right) \\ \mathcal{R} &\approx \sqrt{\frac{\mu}{\epsilon'}}\left(1-\frac{3}{8Q^2}\right) \\ \chi &\approx \frac{\epsilon''}{2\epsilon'}\sqrt{\frac{\mu}{\epsilon'}}\left(1-\frac{5}{8Q^2}\right) \end{aligned} \right\} Q \gg 1$$

where $Q = \frac{w\epsilon'}{\sigma + w\epsilon''}$

The next problem is that

4. Show that for nonmagnetic dielectrics

$$\left. \begin{aligned} k' &\approx w\sqrt{\mu\epsilon'}\left(1+\frac{1}{8Q^2}\right) \\ k'' &\approx \frac{w\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}}\left(1-\frac{1}{8Q^2}\right) \\ \mathcal{R} &\approx \sqrt{\frac{\mu}{\epsilon'}}\left(1-\frac{3}{8Q^2}\right) \\ \chi &\approx \frac{\epsilon''}{2\epsilon'}\sqrt{\frac{\mu}{\epsilon'}}\left(1-\frac{5}{8Q^2}\right) \end{aligned} \right\} Q \gg 1$$

where $Q = \frac{w\epsilon'}{\sigma + w\epsilon''}$

So again, we will start solving.

(Refer Slide Time: 5:32)

47 $Q_s = \frac{\omega \epsilon'}{\sigma + \omega \epsilon''}$

$$\hat{z} = j\omega\mu \quad ; \quad \hat{y} = \sigma + j\omega\epsilon$$
$$= \sigma + j\omega(\epsilon' - j\epsilon'')$$
$$k = \sqrt{-\hat{z}\hat{y}}$$
$$= \sqrt{-j\omega\mu(\sigma + j\omega(\epsilon' - j\epsilon''))}$$

So, we it is given that quality factor is written as

$$Q_s = \frac{\omega \epsilon'}{\sigma + \omega \epsilon''}$$
$$\hat{z} = j\omega\mu \quad ; \quad \hat{y} = \sigma + j\omega\epsilon$$
$$= \sigma + j\omega(\epsilon' - j\epsilon'')$$
$$k = \sqrt{-\hat{z}\hat{y}}$$
$$= \sqrt{-j\omega\mu(\sigma + j\omega(\epsilon' - j\epsilon''))}$$

(Refer Slide Time: 7:05)

$$\begin{aligned}
 k &= \sqrt{-\hat{z} \hat{y}} \\
 &= \sqrt{-j\omega\mu (\sigma + j\omega(\epsilon' - j\epsilon''))} \\
 &= \sqrt{-j\omega\mu (\sigma + j\omega\epsilon' + \omega\epsilon'')} \\
 &= \sqrt{-j\omega\mu\sigma + \omega^2\mu\epsilon' - j\omega^2\mu\epsilon''} \\
 &= \sqrt{-j\omega\mu (\sigma + \omega\epsilon'') + \omega^2\mu\epsilon'} \\
 &= \sqrt{\omega^2\mu\epsilon' \left(1 - \frac{j\omega\mu(\sigma + \omega\epsilon'')}{\omega^2\mu\epsilon'}\right)}
 \end{aligned}$$

Now, we will simplify this just open the braces, we will get

$$\begin{aligned}
 k &= \sqrt{-\hat{z} \hat{y}} \\
 &= \sqrt{-j\omega\mu (\sigma + j\omega(\epsilon' - j\epsilon''))} \\
 &= \sqrt{-j\omega\mu (\sigma + j\omega\epsilon' + \omega\epsilon'')} \\
 &= \sqrt{-j\omega\mu\sigma + \omega^2\mu\epsilon' - j\omega^2\mu\epsilon''} \\
 &= \sqrt{-j\omega\mu (\sigma + \omega\epsilon'') + \omega^2\mu\epsilon'} \\
 &= \sqrt{\omega^2\mu\epsilon' \left(1 - \frac{j\omega\mu(\sigma + \omega\epsilon'')}{\omega^2\mu\epsilon'}\right)}
 \end{aligned}$$

(Refer Slide Time: 9:21)

$$\begin{aligned}
 &= \sqrt{\omega^2 \mu \epsilon' \left(1 - \frac{j\omega \mu (\sigma + \omega \epsilon'')}{\omega^2 \mu \epsilon'}\right)} \\
 &= \sqrt{\omega^2 \mu \epsilon' \left(1 - \frac{j(\sigma + \omega \epsilon'')}{\omega \epsilon'}\right)} \quad \alpha = \frac{\omega \epsilon'}{\sigma + \omega \epsilon''} \\
 &= \omega \sqrt{\mu \epsilon'} \left\{1 - j \frac{1}{\alpha}\right\}^{1/2} \quad \checkmark \\
 \text{NOTE: } (1-x)^r &= 1 - rx + \frac{1}{2!} r(r-1)x^2 - \frac{1}{3!} r(r-1)(r-2)x^3 + \dots \\
 \therefore k &= \omega \sqrt{\mu \epsilon'} \left\{1 - \frac{1}{2} \cdot \frac{j}{\alpha} + \frac{1}{2!} \left(\frac{1}{2}\right) \left(-\frac{1}{2}\right) \frac{j^2}{\alpha^2}\right.
 \end{aligned}$$

Then we can write this

$$\begin{aligned}
 &= \sqrt{\omega^2 \mu \epsilon' \left(1 - \frac{j\omega \mu (\sigma + \omega \epsilon'')}{\omega^2 \mu \epsilon'}\right)} \\
 &= \sqrt{\omega^2 \mu \epsilon' \left(1 - \frac{j(\sigma + \omega \epsilon'')}{\omega \epsilon'}\right)} \quad \alpha = \frac{\omega \epsilon'}{\sigma + \omega \epsilon''} \\
 &= \omega \sqrt{\mu \epsilon'} \left\{1 - j \frac{1}{\alpha}\right\}^{1/2} \quad \checkmark \\
 \text{NOTE: } (1-x)^r &= 1 - rx + \frac{1}{2!} r(r-1)x^2 - \frac{1}{3!} r(r-1)(r-2)x^3 + \dots \\
 \therefore k &= \omega \sqrt{\mu \epsilon'} \left\{1 - \frac{1}{2} \cdot \frac{j}{\alpha} + \frac{1}{2!} \left(\frac{1}{2}\right) \left(-\frac{1}{2}\right) \frac{j^2}{\alpha^2}\right.
 \end{aligned}$$

$$\begin{aligned}
 &= \omega \sqrt{\mu \epsilon'} \left\{1 - j \frac{1}{\alpha}\right\}^{1/2} \quad \checkmark \\
 \text{NOTE: } (1-x)^r &= 1 - rx + \frac{1}{2!} r(r-1)x^2 - \frac{1}{3!} r(r-1)(r-2)x^3 + \dots \\
 \therefore k &= \omega \sqrt{\mu \epsilon'} \left\{1 - \frac{1}{2} \cdot \frac{j}{\alpha} + \frac{1}{2!} \left(\frac{1}{2}\right) \left(-\frac{1}{2}\right) \frac{j^2}{\alpha^2}\right. \\
 &\quad \left. - \frac{1}{3!} \left(\frac{1}{2}\right) \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \frac{j^3}{\alpha^3} + \dots \right\} \\
 &= \omega \sqrt{\mu \epsilon'} \left\{1 - \frac{j}{2\alpha} + \frac{1}{8\alpha^2} + \frac{j}{16\alpha^3} + \dots \right\}
 \end{aligned}$$

(Refer Slide Time: 13:05)

$$= \omega \sqrt{\mu \epsilon'} \left\{ 1 - j \frac{1}{Q} \right\}^{1/2}$$

NOTE: $(1-x)^r = 1 - rx + \frac{1}{2!} r(r-1)x^2 - \frac{1}{3!} r(r-1)(r-2)x^3 + \dots$

$$\therefore K = \omega \sqrt{\mu \epsilon'} \left\{ 1 - \frac{1}{2} \cdot \frac{1}{Q} + \frac{1}{2!} \left(\frac{1}{2} \right) \left(\frac{1}{2} - 1 \right) \frac{j^2}{Q^2} - \frac{1}{3!} \left(\frac{1}{2} \right) \left(\frac{1}{2} - 1 \right) \left(\frac{1}{2} - 2 \right) \frac{j^3}{Q^3} + \dots \right\}$$

$$= \omega \sqrt{\mu \epsilon'} \left\{ 1 - \frac{j}{2Q} + \frac{1}{8Q^2} + \frac{j}{16Q^3} + \dots \right\}$$

(Refer Slide Time: 15:33)

$$= \omega \sqrt{\mu \epsilon'} \left\{ 1 - j \frac{1}{Q} \right\}^{1/2}$$

NOTE: $(1-x)^r = 1 - rx + \frac{1}{2!} r(r-1)x^2 - \frac{1}{3!} r(r-1)(r-2)x^3 + \dots$

$$\therefore K = \omega \sqrt{\mu \epsilon'} \left\{ 1 - \frac{1}{2} \cdot \frac{1}{Q} + \frac{1}{2!} \left(\frac{1}{2} \right) \left(\frac{1}{2} - 1 \right) \frac{j^2}{Q^2} - \frac{1}{3!} \left(\frac{1}{2} \right) \left(\frac{1}{2} - 1 \right) \left(\frac{1}{2} - 2 \right) \frac{j^3}{Q^3} + \dots \right\}$$

$$= \omega \sqrt{\mu \epsilon'} \left\{ 1 - \frac{j}{2Q} + \frac{1}{8Q^2} + \frac{j}{16Q^3} + \dots \right\}$$

$$K = \omega \sqrt{\mu \epsilon'} \left(1 + \frac{1}{8Q^2} \right) - j \omega \sqrt{\mu \epsilon'} \left(\frac{1}{2Q} - \frac{1}{16Q^3} \right) + \dots$$

Comparing the above eq with

$$K = K' - jK'' \quad \text{--- (1)}$$

$$\therefore \boxed{K' = \omega \sqrt{\mu \epsilon'} \left(1 + \frac{1}{8Q^2} \right)} \rightarrow \text{Proved.}$$

So, let us write it as separating real and imaginary parts, separating the real and imaginary parts,

$$\kappa = \omega \sqrt{\mu \epsilon} \left(1 + \frac{1}{8\sigma^2} \right) + \omega \sqrt{\mu \epsilon} \left(\frac{j}{2\sigma} + \frac{j}{16\sigma^3} \right)$$

$$= \omega \sqrt{\mu \epsilon'} \left(1 + \frac{1}{8\sigma^2} \right) - j \omega \sqrt{\mu \epsilon'} \left(\frac{1}{2\sigma} - \frac{1}{16\sigma^3} \right) + \dots$$

Comparing the above eq with

$$\kappa = \kappa' - j \kappa'' \quad \text{--- (1)}$$

$$\therefore \boxed{\kappa' = \omega \sqrt{\mu \epsilon'} \left(1 + \frac{1}{8\sigma^2} \right)} \rightarrow \text{Proved.}$$

So, this we got this is proved.

(Refer Slide Time: 19:06)

The screenshot shows a digital whiteboard with the following handwritten derivation for κ'' :

$$\kappa'' = \omega \sqrt{\mu \epsilon'} \left(\frac{1}{2\sigma} - \frac{1}{16\sigma^3} \right)$$

$$= \frac{\omega \sqrt{\mu \epsilon'}}{2\sigma} \left(1 - \frac{1}{8\sigma^2} \right)$$

$$= \frac{\omega \sqrt{\mu \epsilon'} (\sigma + \omega \epsilon'')}{2 \times \omega \epsilon'} \left(1 - \frac{1}{8\sigma^2} \right)$$

$$\boxed{\kappa'' = \frac{\mu}{\epsilon'} \frac{\omega \epsilon''}{2} \left(1 - \frac{1}{8\sigma^2} \right)} \quad (\text{Proved.})$$

And for κ'' we will take the imaginary part.

$$k'' = \omega \sqrt{\mu \epsilon'} \left(\frac{1}{2Q} - \frac{1}{16Q^3} \right)$$

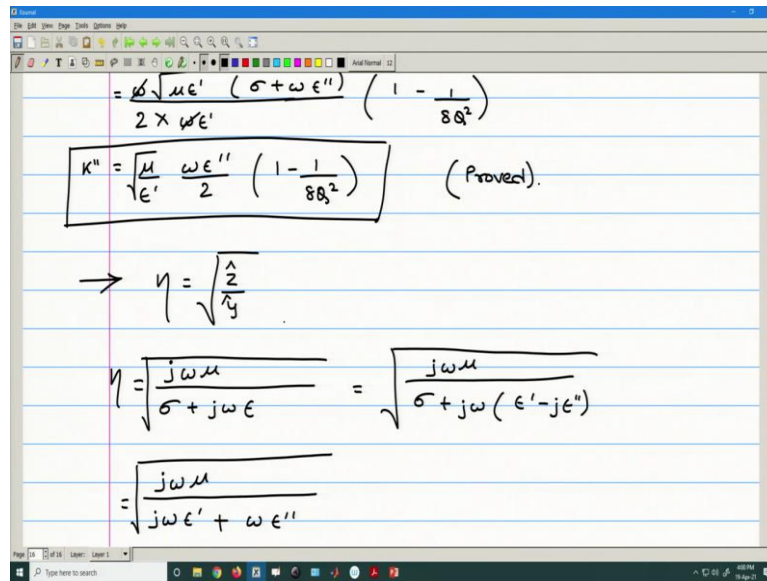
$$= \frac{\omega \sqrt{\mu \epsilon'}}{2Q} \left(1 - \frac{1}{8Q^2} \right)$$

$$= \frac{\omega \sqrt{\mu \epsilon'} (\sigma + \omega \epsilon'')}{2 \times \omega \epsilon'} \left(1 - \frac{1}{8Q^2} \right)$$

$$\boxed{k'' = \sqrt{\frac{\mu}{\epsilon'}} \frac{\omega \epsilon''}{2} \left(1 - \frac{1}{8Q^2} \right)} \quad (\text{Proved}):$$

So, this is also proved. Now further resistance and the reactants. So, for that we will start with the eta.

(Refer Slide Time: 21:56)



$$= \frac{\mu \epsilon' (\sigma + \omega \epsilon'')}{2 \times \omega \epsilon'} \left(1 - \frac{1}{8\delta^2} \right)$$

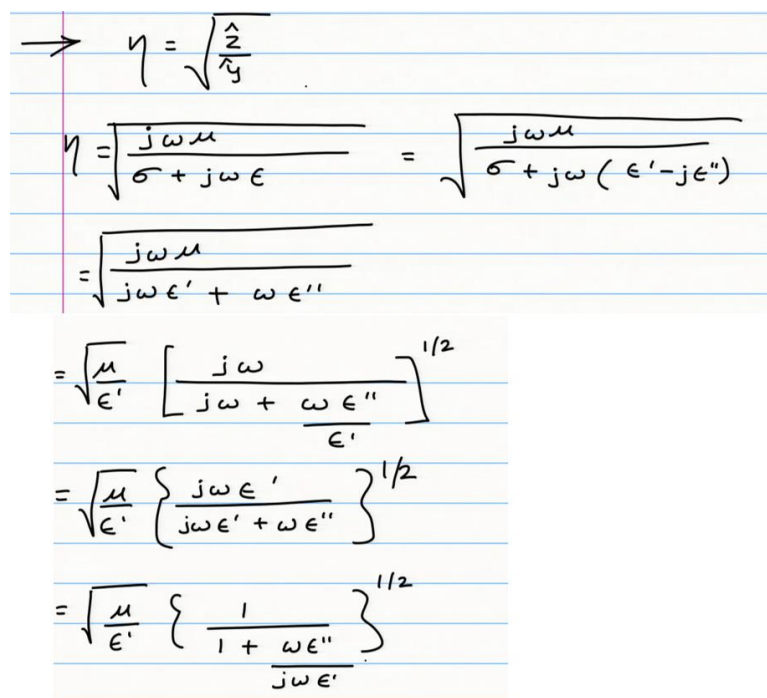
$$\boxed{K'' = \frac{\mu}{\epsilon'} \frac{\omega \epsilon''}{2} \left(1 - \frac{1}{8\delta^2} \right)} \quad (\text{Prove it}).$$

$$\rightarrow \eta = \sqrt{\frac{Z}{Y}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega(\epsilon' - j\epsilon'')}}$$

$$= \sqrt{\frac{j\omega\mu}{j\omega\epsilon' + \omega\epsilon''}}$$

So, first and second done, for third and fourth we will start with we know eta is given by



$$\rightarrow \eta = \sqrt{\frac{Z}{Y}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega(\epsilon' - j\epsilon'')}}$$

$$= \sqrt{\frac{j\omega\mu}{j\omega\epsilon' + \omega\epsilon''}}$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \left[\frac{j\omega}{j\omega + \frac{\omega\epsilon''}{\epsilon'}} \right]^{1/2}$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{j\omega\epsilon'}{j\omega\epsilon' + \omega\epsilon''} \right\}^{1/2}$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{1}{1 + \frac{\omega\epsilon''}{j\omega\epsilon'}} \right\}^{1/2}$$

(Refer Slide Time: 24:11)

$$\begin{aligned}
 &= \sqrt{\frac{\mu}{\epsilon'}} \left[\frac{j\omega}{j\omega + \frac{\omega \epsilon''}{\epsilon'}} \right]^{1/2} \\
 &= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{j\omega \epsilon'}{j\omega \epsilon' + \omega \epsilon''} \right\}^{1/2} \\
 &= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{1}{1 + \frac{\omega \epsilon''}{j\omega \epsilon'}} \right\}^{1/2}
 \end{aligned}$$

That is we can write it as

$$\begin{aligned}
 &= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{j\omega \epsilon'}{j\omega \epsilon' + \omega \epsilon''} \right\}^{1/2} \\
 &= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{1}{1 + \frac{\omega \epsilon''}{j\omega \epsilon'}} \right\}^{1/2} \\
 &= \sqrt{\frac{\mu}{\epsilon'}} \left[\frac{1}{1 + \frac{1}{j\omega \epsilon'}} \right]^{1/2} \\
 \therefore \eta &= \sqrt{\frac{\mu}{\epsilon'}} \left(1 + \frac{1}{j\omega \epsilon'} \right)^{-1/2}
 \end{aligned}$$

$$\therefore \eta = \sqrt{\frac{\mu}{\epsilon'}} \left(1 + \frac{1}{j\omega \epsilon'} \right)^{-1/2}$$

NOTE:

$$(1+x)^r = 1 + rx + \frac{r(r-1)}{2!} x^2 + \frac{r(r-1)(r-2)}{3!} x^3 + \dots$$

$$\Rightarrow \eta = \sqrt{\frac{\mu}{\epsilon'}} \left[1 + \left(-\frac{1}{2}\right) \left(\frac{1}{j\omega \epsilon'}\right) + \frac{1}{2!} \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \left(\frac{1}{j\omega \epsilon'}\right)^2 + \frac{1}{3!} \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \left(-\frac{5}{2}\right) \left(\frac{1}{j\omega \epsilon'}\right)^3 + \dots \right]$$

$$\Rightarrow \eta = \sqrt{\frac{\mu}{\epsilon'}} \left[1 + \left(-\frac{1}{2}\right) \left(\frac{1}{j\omega \epsilon'}\right) + \frac{1}{2!} \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \left(\frac{1}{j\omega \epsilon'}\right)^2 + \frac{1}{3!} \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \left(-\frac{5}{2}\right) \left(\frac{1}{j\omega \epsilon'}\right)^3 + \dots \right]$$

$$\Rightarrow \eta = \sqrt{\frac{\mu}{\epsilon'}} \left[1 - \frac{1}{2j\omega \epsilon'} - \frac{3}{8\omega^2 \epsilon'^2} + \frac{15}{48} \frac{1}{j\omega^3 \epsilon'^3} \dots \right]$$

Separating the real and imaginary parts:

$$\eta = \sqrt{\frac{\mu}{\epsilon'}} \left[1 - \frac{3}{8\omega^2 \epsilon'^2} \right] + \sqrt{\frac{\mu}{\epsilon'}} \left[-\frac{1}{2j\omega \epsilon'} + \frac{15}{48} \frac{1}{j\omega^3 \epsilon'^3} \dots \right]$$

(Refer Slide Time: 25:58)

$$= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{j\omega\epsilon'}{j\omega\epsilon' + \omega\epsilon''} \right\}^{1/2}$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{1}{1 + \frac{\omega\epsilon''}{j\omega\epsilon'}} \right\}^{1/2}$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \left[\frac{1}{1 + \frac{1}{j\beta}} \right]^{1/2}$$

$$\therefore \eta = \sqrt{\frac{\mu}{\epsilon'}} \left(1 + \frac{1}{j\beta} \right)^{-1/2}$$

(Refer Slide Time: 27:04)

$$\sqrt{\frac{\mu}{\epsilon'}} \left[1 + \frac{1}{j\beta} \right]^{-1/2}$$

$$\therefore \eta = \sqrt{\frac{\mu}{\epsilon'}} \left(1 + \frac{1}{j\beta} \right)^{-1/2}$$

NOTE:

$$(1+x)^r = 1 + rx + \frac{1}{2!} r(r-1)x^2 + \frac{1}{3!} r(r-1)(r-2)x^3 + \dots$$

$$\Rightarrow \eta = \sqrt{\frac{\mu}{\epsilon'}} \left[1 + \left(-\frac{1}{2}\right)\left(\frac{1}{j\beta}\right) + \frac{1}{2!} \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{1}{j\beta}\right)^2 + \frac{1}{3!} \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(\frac{1}{j\beta}\right)^3 + \dots \right]$$

(Refer Slide Time: 29:50)

Handwritten mathematical derivation on a digital whiteboard:

$$\eta = \sqrt{\frac{\mu}{\epsilon'}} \left[1 + \left(-\frac{1}{2}\right) \left(\frac{1}{j\alpha}\right) + \frac{1}{2!} \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \left(\frac{1}{j\alpha}\right)^2 + \frac{1}{3!} \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \left(-\frac{5}{2}\right) \left(\frac{1}{j\alpha}\right)^3 + \dots \right]$$

$$\eta = \sqrt{\frac{\mu}{\epsilon'}} \left[1 - \frac{1}{2j\alpha} - \frac{3}{8\alpha^2} + \frac{15}{48} \frac{1}{j\alpha^3} \dots \right]$$

Separating the real and imaginary parts:

$$\eta = \sqrt{\frac{\mu}{\epsilon'}} \left[1 - \frac{3}{8\alpha^2} \right] + \sqrt{\frac{\mu}{\epsilon'}} \left[-\frac{1}{2j\alpha} + \frac{15}{48} \frac{1}{j\alpha^3} \dots \right]$$

So, separating the real and imaginary parts, separating the real and imaginary parts as we will get

Handwritten mathematical derivation on a digital whiteboard:

$$= \sqrt{\frac{\mu}{\epsilon'}} \left[1 - \frac{3}{8\alpha^2} \right] + \sqrt{\frac{\mu}{\epsilon'}} \left(\frac{j}{2\alpha} - \frac{5}{16} \frac{j}{\alpha^3} \right)$$

$$\boxed{R = \sqrt{\frac{\mu}{\epsilon'}} \left(1 - \frac{3}{8\alpha^2} \right)} \rightarrow (\text{Proved})$$

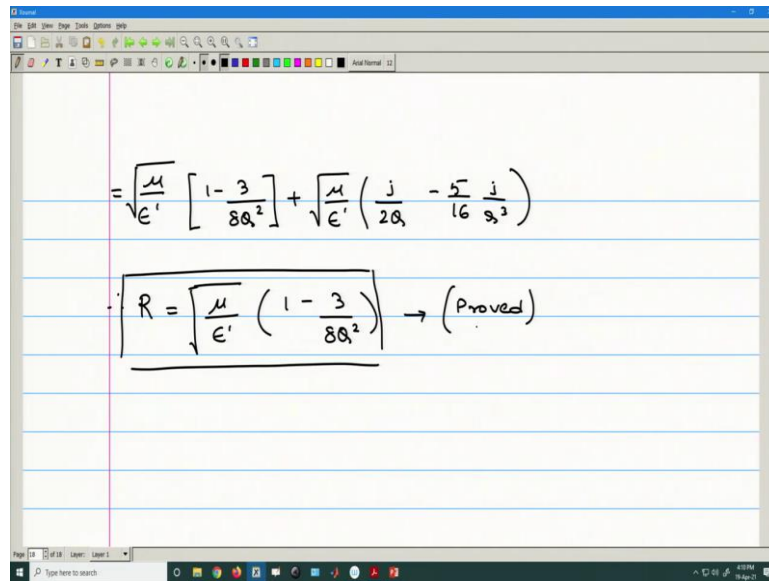
$$X = \sqrt{\frac{\mu}{\epsilon'}} \left(\frac{1}{2\alpha} - \frac{5}{16\alpha^3} \right)$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \frac{1}{2\alpha} \left[1 - \frac{5}{8\alpha^2} \right]$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \frac{1 \cdot \omega \epsilon''}{2 \cdot \omega \epsilon'} \left[1 - \frac{5}{8\alpha^2} \right]$$

$$\boxed{X = \sqrt{\frac{\mu}{\epsilon'}} \frac{\epsilon''}{2\epsilon'} \left(1 - \frac{5}{8\alpha^2} \right)} \rightarrow (\text{Proved})$$

(Refer Slide Time: 32:33)



The image shows a digital whiteboard with a blue border and a toolbar at the top. The whiteboard contains two lines of handwritten mathematical equations. The first line is an equality between two square root terms. The second line shows a boxed equation for R, followed by an arrow pointing to the word 'Proved'.

$$= \sqrt{\frac{\mu}{\epsilon'}} \left[\frac{1-3}{80^2} \right] + \sqrt{\frac{\mu}{\epsilon'}} \left(\frac{j}{20} - \frac{5-j}{16} \frac{j}{80^2} \right)$$
$$\therefore \boxed{R = \sqrt{\frac{\mu}{\epsilon'}} \left(\frac{1-3}{80^2} \right)} \rightarrow \text{(Proved)}$$

(Refer Slide Time: 33:55)

So, our next class we will also solve some few more problems on wave numbers and then we will move into electric current source. Thank you.