Advanced Microwave Guided-Structures and Analysis

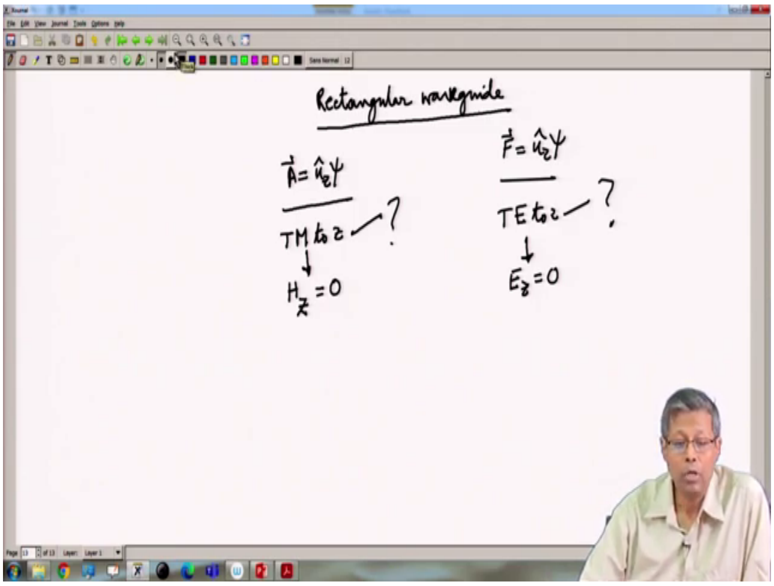
Professor Bratin Ghosh
Department of Electronics & Electrical Communication Engineering
Indian Institute of Technology Kharagpur

Lecture 32

Rectangular Waveguide - I

Welcome to the first session of the lecture on the Rectangular Waveguide, called the Rectangular Waveguide I. Here, we will investigate the the construction of the potential functions for the rectangular waveguide, for the rectangular waveguide modes. We will investigate the dispersion equation for the rectangular waveguide. The cut off frequency and the dominant mode and the propagation characteristics of the TE and TM modes in the rectangular waveguide. Let us go to the lecture.

(Refer Slide Time: 00:59)



We had previously constructed the electromagnetic field solutions for two cases, one when the magnetic vector potential was **A** equal to **uz** Psi, and another when the electric vector potential was **F** is equal to **uz** Psi, these are the two cases for which we had constructed the, the field solutions. And we found that for this case where the magnetic vector potential was **z -** directed it gave rise to a transverse magnetic to z mode that is the mode for which Hz equal to 0.

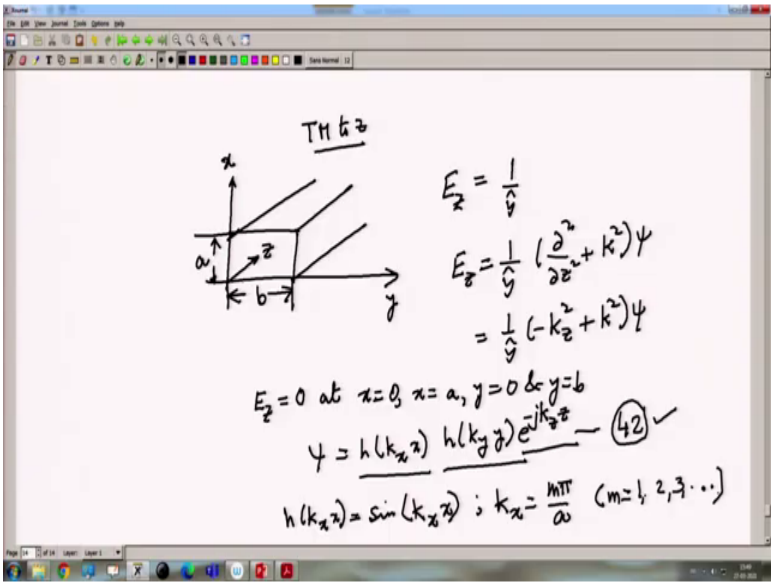
The z directed magnetic field was 0, and for this case the solution came out be TE to z for which the z directed electric field was 0. When we now, apply this knowledge to any kind of analysis of any guided mode behavior in a waveguide, the first thing we need to understand is that, whether these are valid mode sets in the rectangular waveguide. Whether these modes are indeed valid modes for the rectangular waveguide? These modes whether they are supported in the rectangular waveguide?

So, for that purpose, we need to take the field expressions for these modes, the TM to z and the TE to z and we need to satisfy the boundary conditions of the waveguide. If the boundary conditions are properly and adequately satisfied by the TE to z or the TM to z modes, we simply conclude that those modes are valid mode sets for the waveguide. If they are not, these modes sets are not valid modes sets for the waveguide. And we have to continue to hunt for alternative mode sets.

And in the context of the rectangular waveguide, the only alternative mode sets are two other mode sets where A will be equal to uy Psi and F will be equal to uy Psi or A equal to ux Psi and F is equal ux Psi. We already discussed those things before. But, let us assume that we are lucky and these kinds of a potential functions which is A equal to uz Psi, and F is equal uz Psi, will indeed be valid mode sets in the waveguide. We think they would be but, we are not sure, in order to better substantiate our claim and be sure that, the TE to z and TM to z mode sets are valid mode sets in the waveguide.

We now, need to satisfy the boundary conditions of the waveguide. We also know that, in a rectangular waveguide the transverse electromagnetic mode that is TEM mode does not exist. Because, the waveguide does not have a central conductor so, the TEM mode does not exist in the rectangular waveguide. So, now let us see, whether the TM to z mode is a valid mode?

(Refer Slide Time: 05:00)



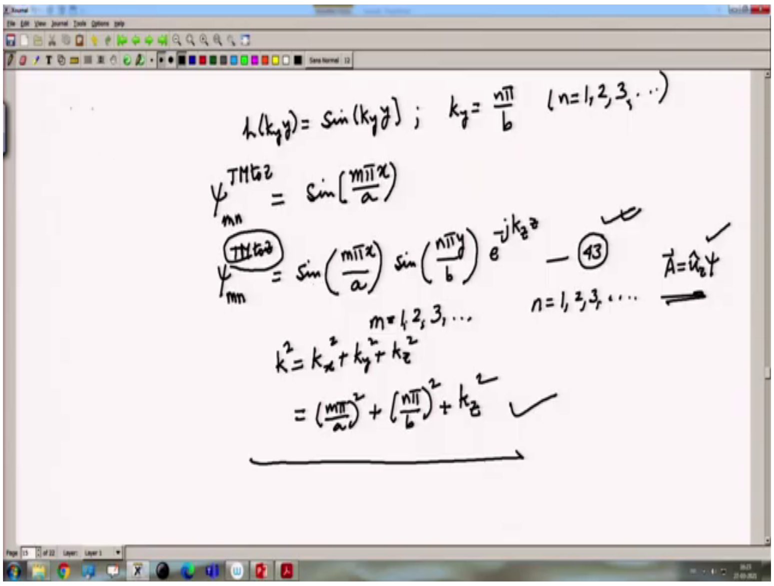
The waveguide is bounded by conducting walls on all four sides so, this is the waveguide let us called this direction x, this direction y, so this direction will be z and let this dimension be a, and let that dimension be b. If we would look at the z component of the electric field, for the TM to z mode does not have Hz component.

Because, the magnetic field will only have Hx and Hy so, the TM to z mode has the Ez component along the z direction only. So, if I write down the expression for the electric field for the TM to z mode, it will be Ez equal to $\frac{1}{\hat{y}}(k^2 - k_z^2)\psi$.

This electric field will have to vanish at the four walls of the rectangular waveguide. So, we will have to satisfy the boundary conditions that is 0 at x equal to 0, x equal to a, y equal to 0 and y equal to b. because, of the nature of the boundary conditions along x and along y, this can only happen if the variation along x and y is given by a sinusoid. So, we have if the potential function is expressed by $h(k_x x)$, $h(k_y y)$ with the propagation term along the z direction being given by $e^{-jk_z z}$ let us call this equation number 42.

Maintaining continuity with our previous equation numbers there was one gap with respect to the magnetic current sum of the equation numbers for the magnetic current but, that would not create a confusion because those equations are already given different equation number in another slide. So, you would be able to refer to those equations whenever you need them. So, because of that there is a gap in equation number here but that should not cause any confusion. So, this refers to the x variation of the potential function, this refers to the y variation of the potential function, and this is the z variation of the potential function. So, $h(k_x x)$ has to be given by $\sin(k_x x)$ in this case, with kx given by $\frac{m\pi}{a}$, m equal to 1, 2, 3.

(Refer Slide Time: 10:04)

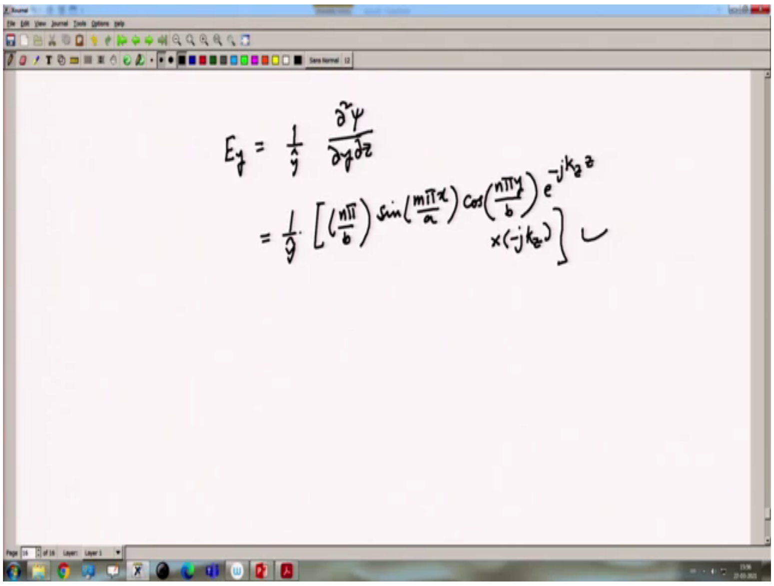


Similarly, $h(k_y y)$ is given by $\sin(k_y y)$ where ky equal to $\frac{n\pi}{b}$ where, n equal to 1, 2, 3 and so on. So, therefore the Psi function for the TM to z mode, that is transverse magnetic to z mode, will be given by ψ_{mn} TM to z as $\sin(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)e^{-jk_z z}$ that is 43. Where m is 1, 2, 3 and n is 1, 2, 3.

Also, the separation equation which we wrote down before in this case, becomes $k^2 = k_x^2 + k_y^2 + k_z^2$ and that is given by $(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2 + k_z^2$. So, if we substitute this Psi function, into the equations for the electric and magnetic field we derived, corresponding to **A** equal to uz Psi, which corresponds to the TM to z mode. We will obtain all the electric and magnetic field components corresponding to the TM to z mode.

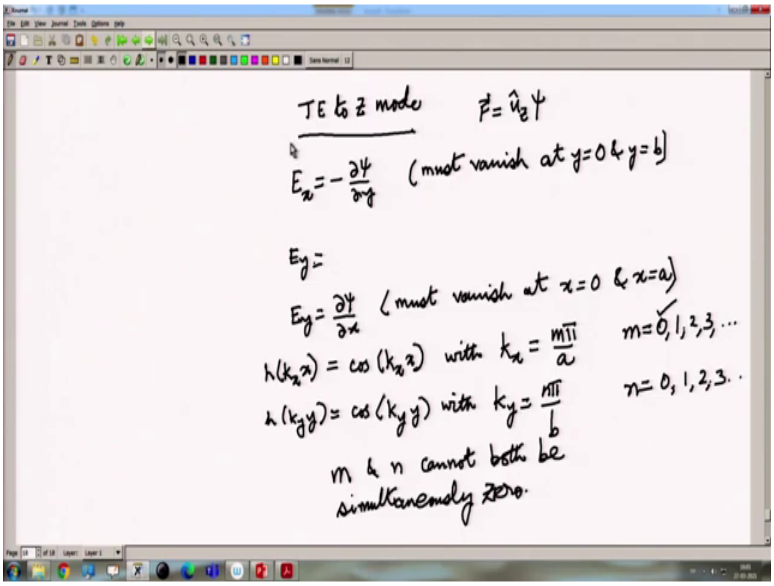
So, we just substitute the Psi function in those equations, we will get all the electric and magnetic fields corresponding to the TM to z mode for rectangular waveguide.

(Refer Slide Time: 14:56)



For instance, if we need to find out the Ey component, which is given by $\frac{1}{\hat{y}}\frac{\partial^2\psi}{\partial y\partial z}$ for the TM to z mode for the substitution **A** equal to uz Psi, we need to just perform this two differentiations so, it will be $\frac{n\pi}{b}$ for the differentiation with respect to y $\sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)e^{-jk_z z}$, multiplied by minus jkz, whatever that is. So, this is an example, of how the Ey component of the electric field for the TM to z mode for the rectangular waveguide can be obtained. So, we will substitute 1 by y cap the value of y cap and we will just simplify. Now, let us consider the fields that are transverse electric to z.

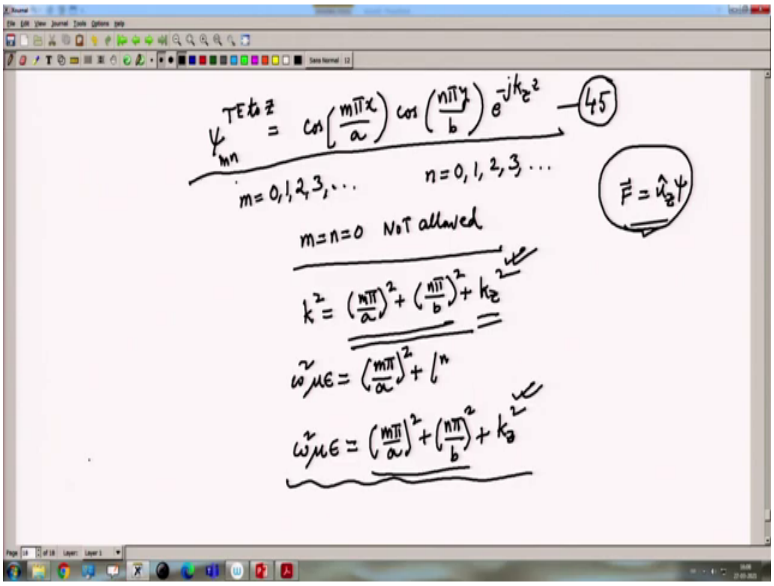
(Refer Slide Time: 17:00)



We got or we obtained the expressions for the TE to z mode, by the substitution **F** equal to uz Psi. There we found that Ez was equal to 0 which corresponds to the TE to z mode. So, we pick up those fields, and we substitute and we apply the boundary conditions of the waveguide and see, whether it is possible to satisfy the boundary conditions of the waveguide using those field expressions. Because, there is no z component of the electric field, we have to use the Ex and the Ey components of the fields.

So, writing down the Ex and Ey components of the fields, we have Ex equal to minus $\frac{\partial \psi}{\partial y}$ which must vanish. This must vanish at y equal 0 and y equal to b, similarly we have Ey as $\frac{\partial \psi}{\partial x}$ which must vanish at x equal to 0 and x equal to a. This can only happen if $h(k_x x)$ which we defined before, the x variation of the Psi function that is equal to $\cos(k_x x)$ with kx as $\frac{m\pi}{a}$, m is equal to 0 and 1 and 2, because we are dealing, m equal to 0, 1, 2, 3, and so on. Because, we are dealing with the Cosine function where m equal to 0 also as a choice. And $h(k_y y)$ will be given by $\cos(k_y y)$ with ky as $\frac{n\pi}{b}$ with n equal to 0, 1, 2, 3 and so on. However, the field vanish if m and n are both 0, so therefore, we cannot have m and n both 0, m and n cannot both be simultaneously 0. M and n cannot both be simultaneously 0. Otherwise, the fields vanish.

(Refer Slide Time: 21:39)



So therefore the Psi function for mnth mode corresponding to the transverse electric to z field is given by $cos(\frac{m\pi}{a}x)cos(\frac{n\pi}{b}y)e^{-jk_z z}$, we called this equation as 45. Where, m equal to 0, 1, 2, 3 etc and n equal to 0, 1, 2, 3 etce but, m equal to n equal to 0 is not allowed.

Now, the separation equation which we wrote down for the TM to z mode which is k square equal to m Pi by a whole square plus n Pi by b whole square plus kz square. It is the same in this case, so the same separation equation k square is m Pi by a whole square, plus n Pi by b whole square plus kz square, whole is good.

So, therefore note that, for both the TM and TE modes kx depends on m and n that means the propagation constant along the z direction depends on the mode number. And of course the frequency because, k will depend on frequency, k is given Omega root Mu Epsilon. So, it will depend on the mode number and the frequency. Also, if the frequency is very high, so k is very very high in that case, so that kz will be very high.

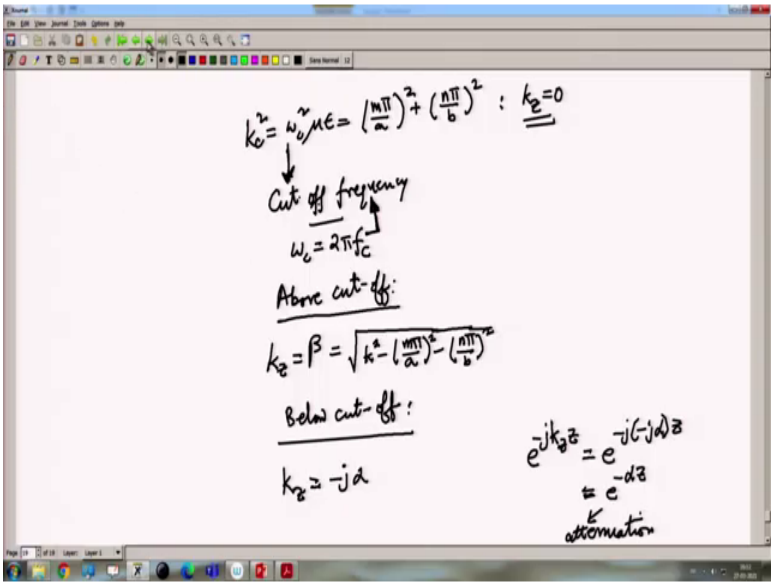
And it may be these two terms m Pi by a whole square, plus n Pi by b whole square, for a sufficiently high frequency may be neglected or may be small, for a sufficiently high frequency. So, as the frequency becomes very high, kz approaches the propagation constant in free space which is k, so the propagation constant in waveguide approaches to that of free space or the propagation constant in a homogeneous medium, filled with the same media parameters as that of the waveguide for a sufficiently high frequency.

So, similar to the TM to z mode the expressions for the electric and magnetic fields for the TE to z mode can be obtained by substituting this Psi function, into the equations for the electric and magnetic fields that we have derived or that we had given before for the case F is equal to uz Psi.

Because, this choice generates the TE to z mode so, substituting this Psi function, into the electric and magnetic fields corresponding to F equal to uz Psi, we obtained the electric and magnetic fields in the waveguide, for the TE to z mode. So, let us write this equation as the explicitly shown is dependence on frequency, Omega square Mu Epsilon, plus m Pi by a whole square, so let us write this equation in order to show its explicit dependence on the frequency, as Omega square Mu Epsilon, equal to m Pi by a whole square, plus n Pi by b whole square, plus kz square.

So, we now see also that, corresponding to the frequency Omega c square Mu Epsilon equal to m Pi by a whole square, plus n Pi by b whole square, kz becomes equal to 0. So, kz becomes equal to 0, means the propagation along the z stops, there is no propagation along the z direction. So, Omega equal Omega c, for which Omega c square Mu Epsilon equal to m Pi by a whole square, plus n Pi by b whole square, the frequency Omega c is called the cut-off frequency, below which the propagation constant becomes complex.

(Refer Slide Time: 28:33)

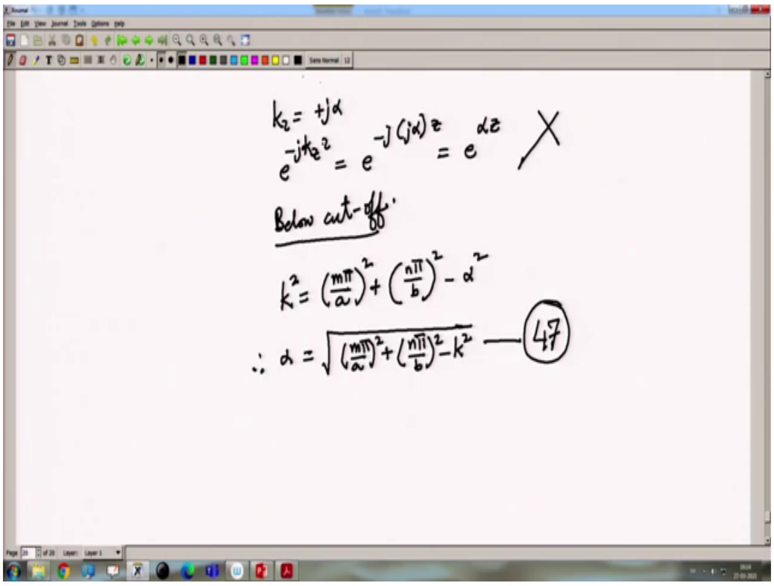


So, we write, kc square equal to Omega c square Mu Epsilon that is equal to m Pi by a whole square, plus n Pi by b whole square, corresponding to case kz equal to 0 defines the cut-off frequency or the cut-off condition for the waveguide. So, when Omega is less than Omega c, kz becomes complex and the wave is attenuated or the wave is called evanescent.

So, Omega c is twice Pi fc so, which corresponds this is the radian cut-off frequency, this is the cut-off frequency in Hertz or Giga Hertz. So, as we said that, below the cut-off frequency the mode is evanescent and above the cut-off frequency the mod propagates.

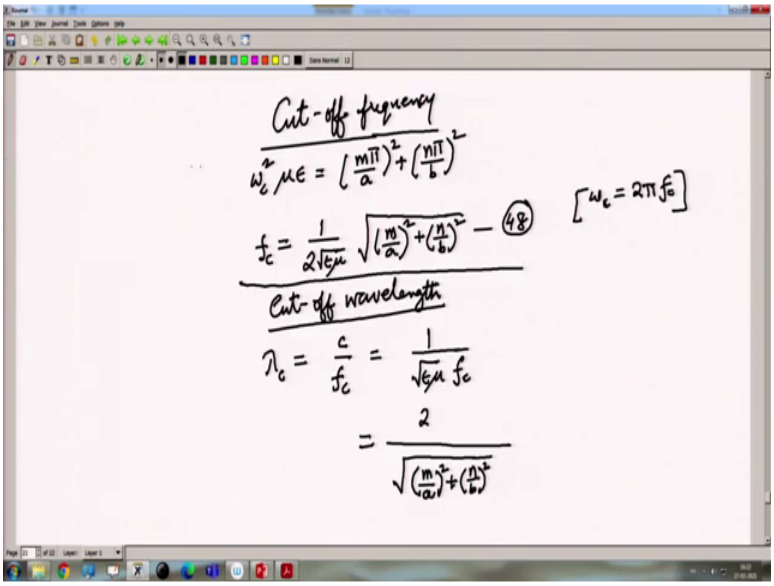
Particularly above cut-off kz is real it is Beta and that is given by root of k square minus m Pi by a whole square, minus n Pi by b whole square. And below cut-off, kz is minus j Alpha, we have to choose minus j Alpha, and not plus j Alpha because, the variation along the z is e to the power minus jkz z, so if we choose kz equal to minus j Alpha, it becomes minus j times minus j Alpha z and that is equal to e to the power minus Alpha z, which represents attenuation so, this represents attenuation. So, had we made the other choice?

(Refer Slide Time: 32:02)



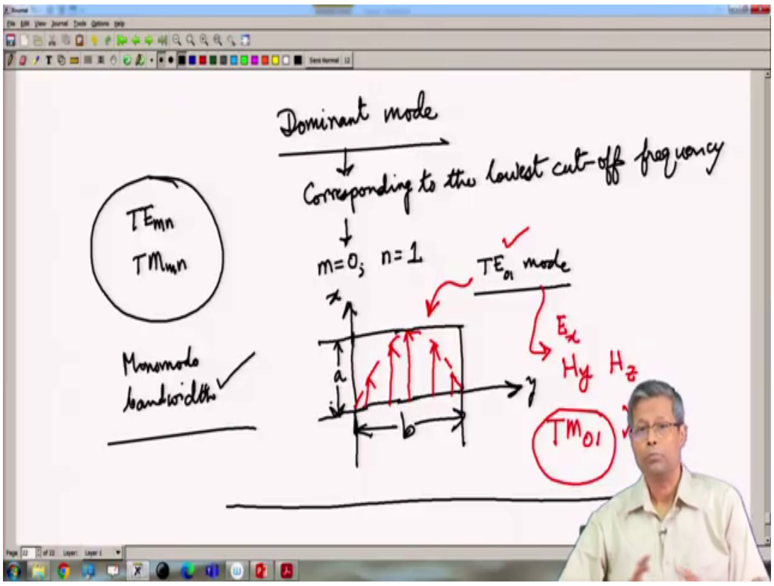
Had we made the choice kz equal to plus j Alpha, this way e to the power minus jkz z, would have corresponded to minus j into plus j Alpha, that is e to the power Alpha z. So, which represents wave increase along the z direction, which is not possible. So, we forced this choice, so below cut-off we have, k square as m Pi by a whole square, plus n Pi by b whole square, minus Alpha square. So, therefore Alpha is given by for that attenuation constant root of m Pi by a whole square, plus n Pi by b whole square, minus k square. So, we called this equation 47.

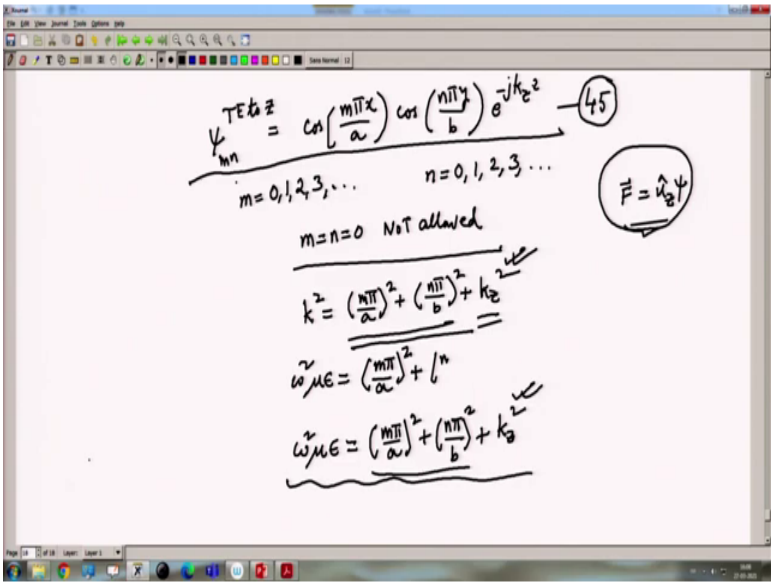
(Refer Slide Time: 34:10)



As we also wrote before, the cut-off frequency, is given by Omega c square Mu Epsilon is m Pi by a whole square, plus n Pi by b whole square, and from which fc is given by 1 by 2 root Epsilon Mu, root of m by a whole square, plus n by b whole square, we called this 48. Where, Omega c equal to twice Pi fc So, the cut-off wavelength is given by Lambda c equal c by fc is equal to 1 by root of Epsilon Mu fc and that is given 2 by root of m by a whole square, plus n by b whole square.

(Refer Slide Time: 38:06)





The dominant mode, which is the mode corresponding to the lowest cut-off frequency. It corresponds to the lowest cut-off frequency, is given by m equal to 0, n equal to 1, so if m equal to 0, it has a uniform if you derive the equations for the fields for the electric and magnetic fields for the TE to z or in this case, the TE 0, 1 mode which is the dominant mode. If I again draw the cross section of the waveguide, so this is the x axis, this is the y axis, this is a and that is b, that is b so, the electric field, will only have a x- component. And the magnetic field will only have a y component and z component.

For the TE 0,1 mode the electric field is going to be maximum here. So, the electric field is going to be maximum here, and it is going to be 0 at the two edges of the waveguide. Because, the tangential fields at y equal to 0 and y equal to b they will be 0. So, this is the electric field distribution of the TE 01 mode. The TM 01 mode will be degenerate to the TE 01 mode, in the sense, that it is going to have the same cut-off frequency as the TE 01 mode. But, however the fields, of the TM 01 mode are 0, so therefore, the TM 01 mode do not exist.

That is, fortunate for us, because had it existed its filed components would have been very different from the TE 01 mode because, the fields of the TE mn mode, is very different from the fields of the TM mn mode. But, it must be remembered that, the TE mn mode, is degenerate with the TM mn mode. It means that, the TE mn mode, and the TE it means so the TE mn mode, and the TM mn mode, had the same cut-off frequencies. Why? Because this equation is independent of whether the mode is TE or TM.

Also, its propagation characteristics which is kz, the value of kz as defined through here, or for the TM mode, or as defined through here for the TM mode, they are the same. That so, kz will be also the same for the TE mn and the TM mn mode so, we called this degenerate. Okay, but because the TM 01 mode does not exist, the TM 01 mode does not exist, so the dominant mode is only the TE 01 mode. The field distribution again of the TE 01 mode, has a uniform field, along the x direction because, the field strength is invariant along the x because, the variation along the x is 0.

So, we always want to maximize what we called the mono mode bandwidth of the wavelength. For any guided structure, we want to maximize the Mono mode bandwidth that is the frequency gap between the dominant mode and the own set of the next higher mode, because that is where we like to have, because that is where we want to have propagation. Because after the onset of the next higher order mode, we will have two modes propagating so, the two modes are going to share power.

Therefore, we want the propagation to happen in what we called the Mono mode region that is between the cut-off frequency of the dominant mode and the cut-off frequency of the next higher order mode. So, this concludes our first slot of the Waveguide Characteristics. We will continue with other characteristics of the Waveguide, thank you.