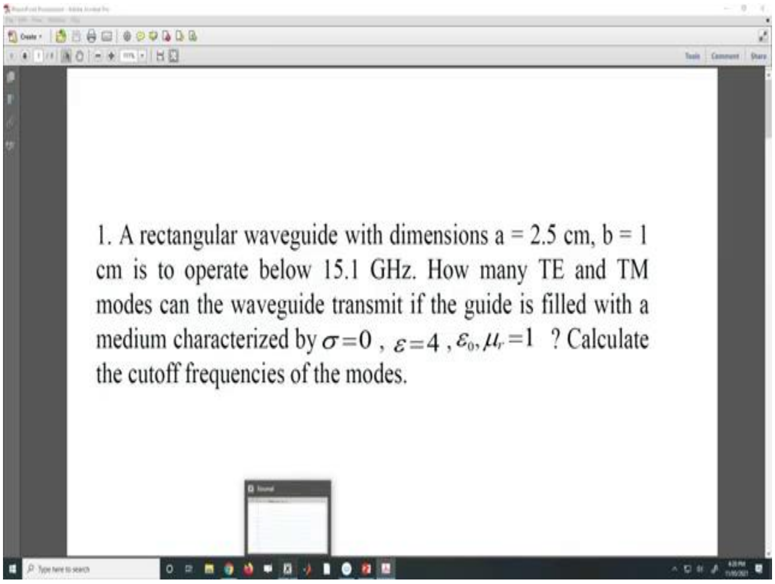
Advanced Microwave Guided-Structures and Analysis

Professor Bratin Ghosh
Department of Electronics & Electrical Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture 33

Rectangular Waveguide – I Tutorials

(Refer Slide Time: 00:18)



Hello everyone, today we will solve numerical problems based on Rectangular Waveguide. So, our first problem states that, a rectangular waveguide with dimension a equals to 2.5 centimeter, b equals 1 centimeter is to operate below 15.1 Gig Hertz. How many TE and TM modes can the waveguide transmit if the guide is filled with a medium characterize by sigma equals to 0, Epsilon equals to 4, Epsilon Naught and Mu are equals to 1. Calculate the cut-off frequencies of the modes.

(Refer Slide Time: 00:55)

The cut off frequency is given by:

$$f_{c_{mn}} = \frac{u'}{2}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$$

Given $a = 2.5$ cm ; $b = 1$cm $\Rightarrow \frac{a}{b} = 2.5$

$$u' = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{\sqrt{\mu_r\epsilon_r}} = \frac{c}{2}$$

$$f_{c_{mn}} = \frac{u'}{2}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$$

Given $a = 2.5$ cm ; $b = 1$cm $\Rightarrow \frac{a}{b} = 2.5$

$$u' = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{\sqrt{\mu_r\epsilon_r}} = \frac{c}{2}$$

Hence,

$$f_{c_{mn}} = \frac{c}{4}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$$

Hence,

$$f_{c_{mn}} = \frac{c}{4}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$$

$$= \frac{c}{4a}\sqrt{m^2+n^2\cdot\frac{a^2}{b^2}}$$

$$= \frac{c}{4a}\sqrt{m^2+n^2\cdot(2.5)^2}$$

$$= \frac{3\times10^8}{4\times2.5\times10^{-2}}\sqrt{m^2+6.25n^2}$$

$$= 3\sqrt{m^2+6.25n^2}\ \text{GHz}$$

So, to start with we know, the cut-off frequency is given by

$$f_{c_{mn}} = \frac{\mu'}{2}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$$

Where, m and n are the indices for the modes.

So, we had given that a is 2.5 centimeter and b is 1 centimeter. So, we can write

$$f_{c_{mn}} = \frac{\mu'}{2}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$$

Given $a = 2.5\text{ cm}$; $b = 1\text{cm}$ $\Rightarrow \frac{a}{b} = 2.5$

$$\mu' = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{c}{2}$$

Hence,

$$f_{c_{mn}} = \frac{c}{4}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$$

Now, substituting the values we will get

Hence,

$$f_{c_{mn}} = \frac{c}{4}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$$

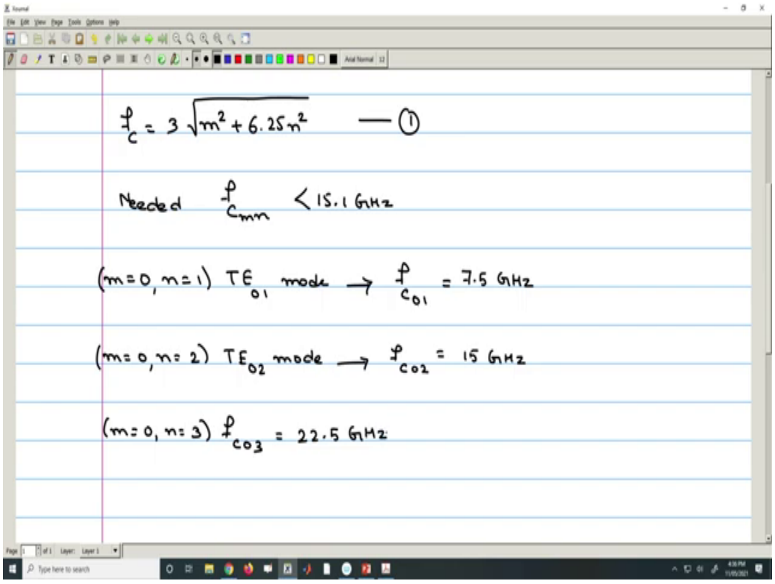
$$= \frac{c}{4a}\sqrt{m^2 + n^2 \cdot \frac{a^2}{b^2}}$$

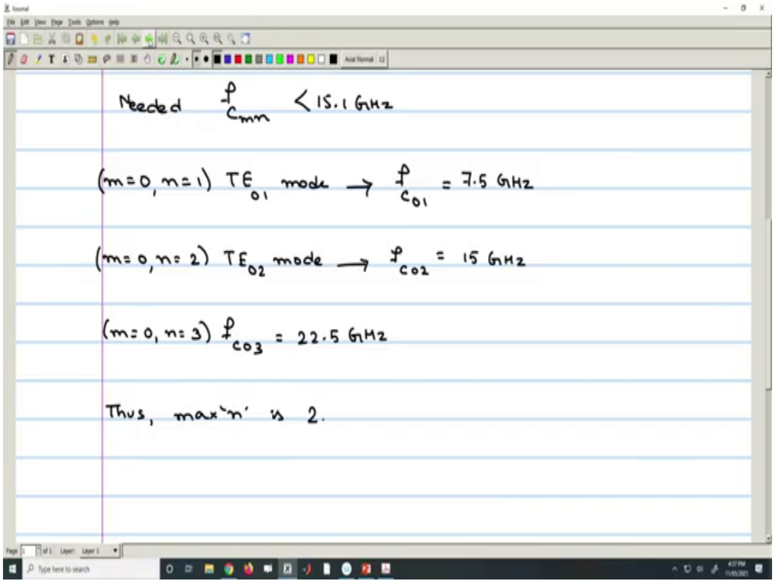
$$= \frac{c}{4a}\sqrt{m^2 + n^2 \cdot (2.5)^2}$$

$$= \frac{3\times 10^8}{4\times 2.5\times 10^{-2}}\sqrt{m^2 + 6.25n^2}$$

$$= 3\sqrt{m^2 + 6.25n^2}\ \text{GHz}$$

(Refer Slide Time: 05:37)



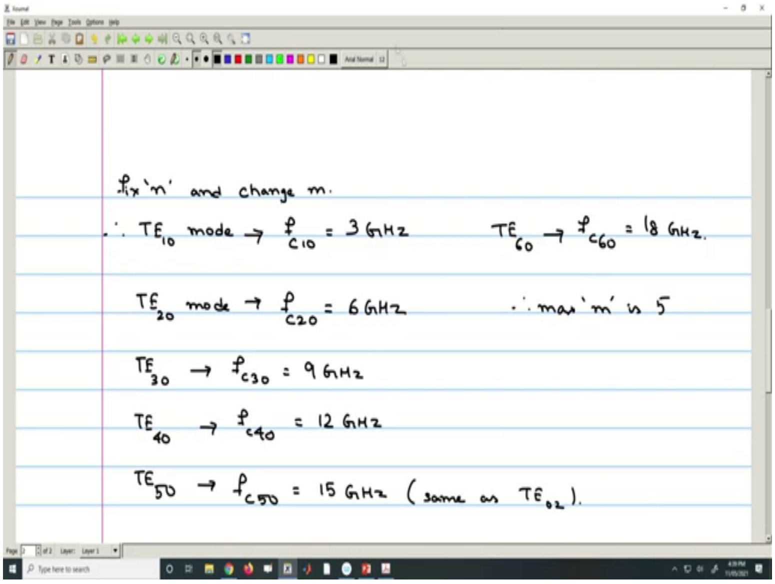


here it is given that, we need to operate the waveguide below 15.1 Giga Hertz. So, it is needed to be that fc mn should be less than 15.1 Gig Hertz. Now, a systematic way of doing this is to fix m or n and then increase the other until fc mn is greater than 15.1 Giga Hertz.

So, what we will do, we will first fix m and then we will fix n so, first fixing m we get like for m equals to 0 and n equals to 1 suppose, we are getting TE_{01} for TE_{01} mode we will get the cut-off frequency as fc_{01}. So, from equation number 1 we will substitute the values of indices m and n in equation 1, and then it will yield us fc 1 as 7.5 Giga Hertz.

So, in equation 1, we are substituting first m=0 and n=1 and therefore for TE_{01} mode we are getting fc_{01} as 7.5 Giga Hertz. So, we are now fixing m so, now the next will be m is 0 n is 2 therefore, for TE_{02} mode we will get f of c 02 as 15 Giga Hertz. Similarly, for m=0 n=3 we will get f, we will get $f_{c\,03}$ as 22.5 Giga Hertz. So, 22.5 is coming greater than 15.1 so, we can say that maximum n can be 2 so, to conclude we can say thus, maximum n is 2.

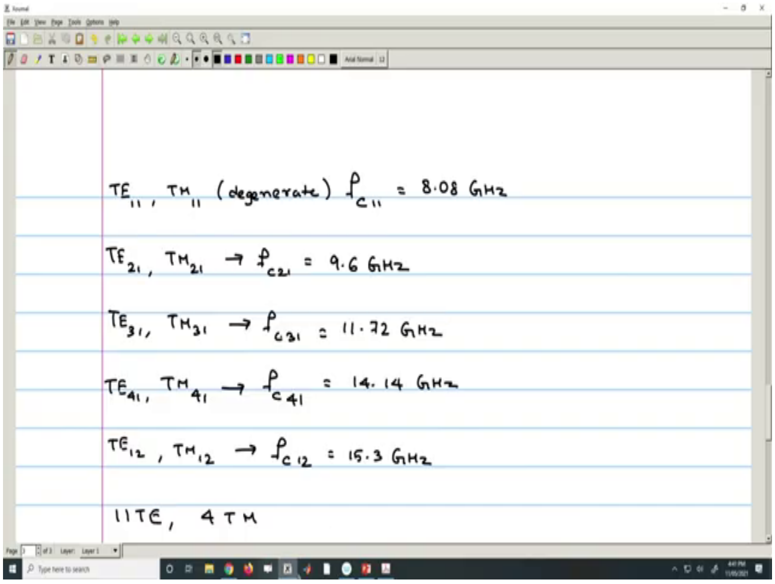
(Refer Slide Time: 08:59)



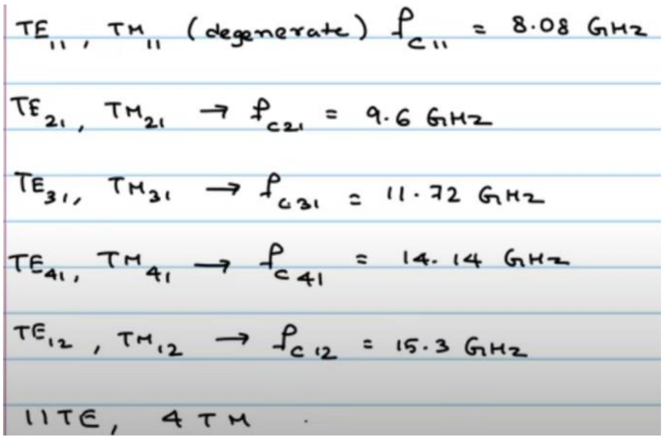
Now, now what will do? We will now fix n and then change m so, now we will, now we will fix n and change m therefore, for TE_{10} mode we will have f of c_{10} as 3 Giga Hertz again this we are getting from equation number 1, for the indices m=1 and n=0. So, now we will again change the next value. That is for TE_{20} so, for TE_{20} mode f will be f of c_{20} that is 6 Giga Hertz similarly we can find out, for TE_{30} we will have 9 Giga Hertz, for TE_{40} we will have 12 Giga Hertz, TE_{50} we will have 15 Giga Hertz and then for TE_{60} we will have 18 Giga Hertz.

Now, this TE 5 0 where fc 5 0 is 15 Giga Hertz this is same as TE 02 therefore from above we can conclude that maximum m is 5 because, 18 Giga Hertz is again greater than 15.1. Now, what we know that, we know the maximum m and n now, we will try the other possible combinations in between these maximum values. So, since we know the maximum value of m is 5 and n is 2 we will try some other possible conditions in between these maximum values.

(Refer Slide Time: 11:46)

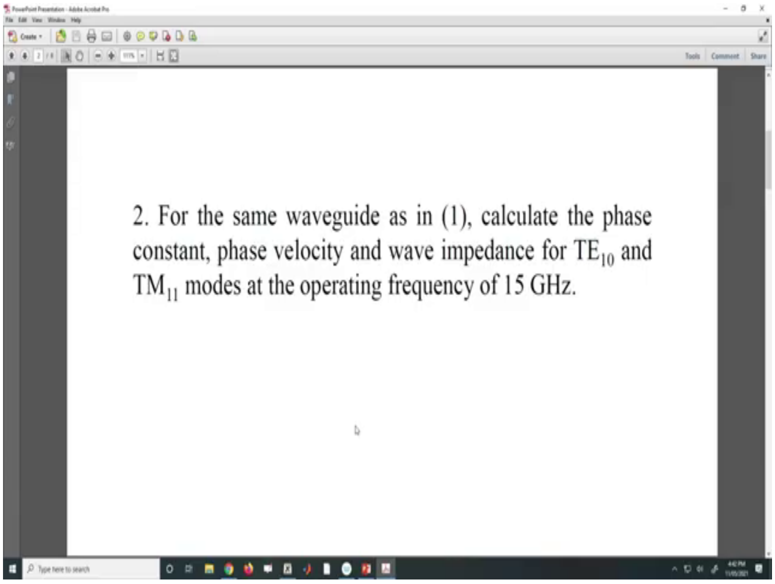


So, we will have



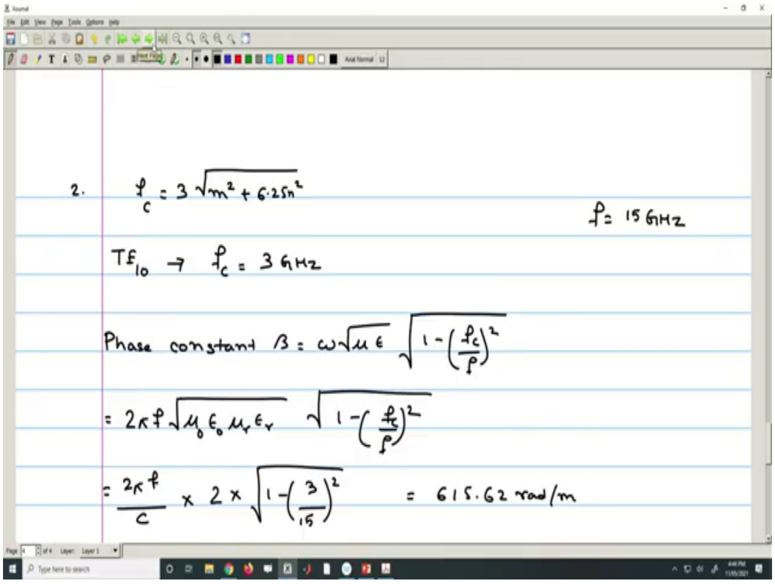
So, now those modes who's cut-off frequency unless than or equal to 15.1 Gig Hertz, will be transmitted so, among all the possible conditions we will see, we can see that 11 TE and 4 TM modes are possible, so, the next problem.

(Refer Slide Time: 13:37)



So, for this same waveguide as an question 1, we need to calculate the phase constant, phase velocity and wave impedance for TE_{10} and TM_{11} modes at the operating frequency of 15 Giga Hertz.

(Refer Slide Time: 14:04)



Now, first we need to calculate phase constant so, phase constant is given by,

$$f_c = 3\sqrt{m^2 + 6.25n^2}$$

$f = 15\ GHz$

$TE_{10} \rightarrow f_c = 3\ GHz$

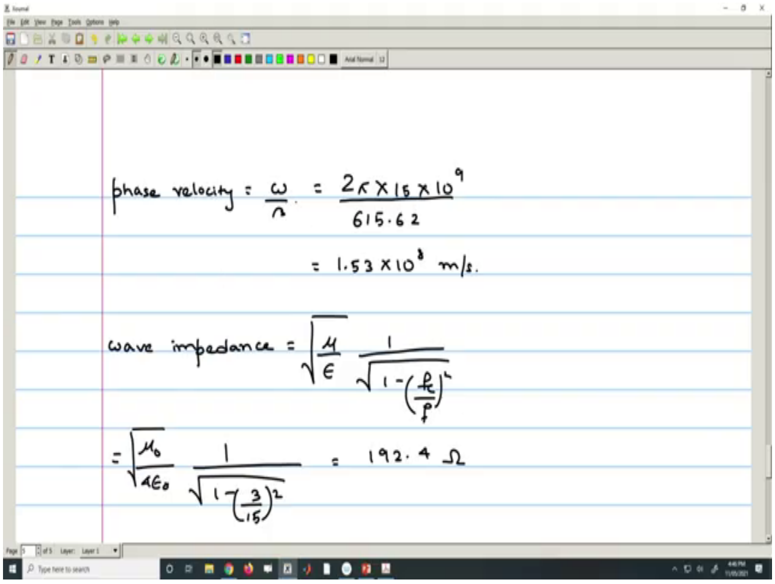
Phase constant $\beta = \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$= 2\pi f\sqrt{\mu_0\epsilon_0\mu_r\epsilon_r}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$= \frac{2\pi f}{c} \times 2 \times \sqrt{1-\left(\frac{3}{15}\right)^2} = 615.62\ rad/m$

So, substituting the values will give us 615.62 Rad per meter.

(Refer Slide Time: 16:35)



Next is phase velocity :

$$\text{phase velocity} = \frac{\omega}{\beta} = \frac{2\pi \times 15 \times 10^{9}}{615.62}$$

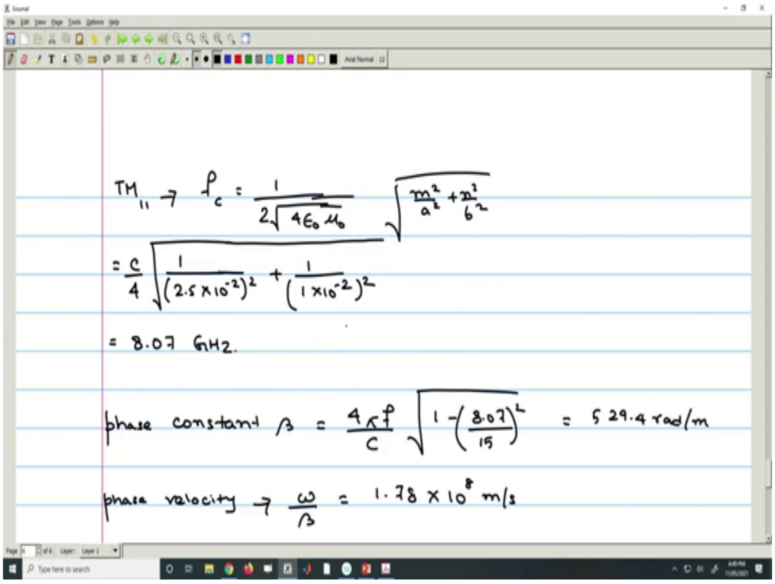
$$= 1.53 \times 10^{8} \text{ m/s.}$$

$$\text{wave impedance} = \sqrt{\frac{\mu}{\epsilon}}\, \frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \sqrt{\frac{\mu_0}{4\epsilon_0}}\, \frac{1}{\sqrt{1-\left(\frac{3}{15}\right)^2}} = 192.4\ \Omega.$$

similar thing we need to repeat for TM 11.

(Refer Slide Time: 18:26)



So, for TM 11,

$$TM_{11} \rightarrow f_c = \frac{1}{2\sqrt{4\epsilon_0 \mu_0}} \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

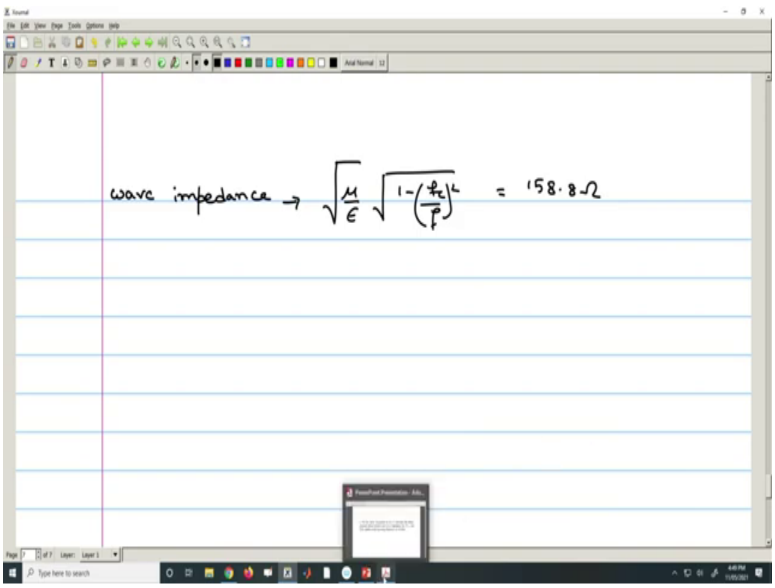
$$= \frac{c}{4}\sqrt{\frac{1}{(2.5\times10^{-2})^2} + \frac{1}{(1\times10^{-2})^2}}$$

$$= 8.07 \text{ GHz.}$$

phase constant $\beta = \frac{4\pi f}{c}\sqrt{1-\left(\frac{8.07}{15}\right)^2} = 529.4$ rad/m

phase velocity $\rightarrow \frac{\omega}{\beta} = 1.78 \times 10^8$ m/s.

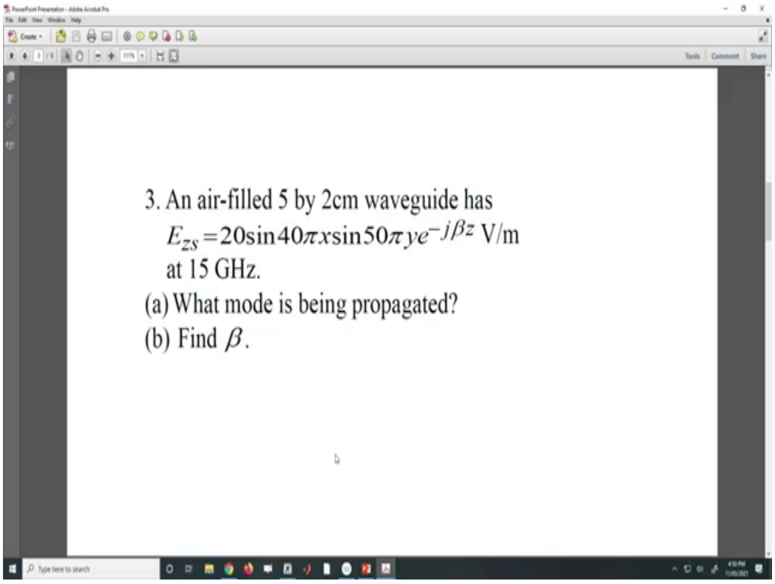
(Refer Slide Time: 21:02)



substituting we will get 158.8 Ohm.

Now, the next question.

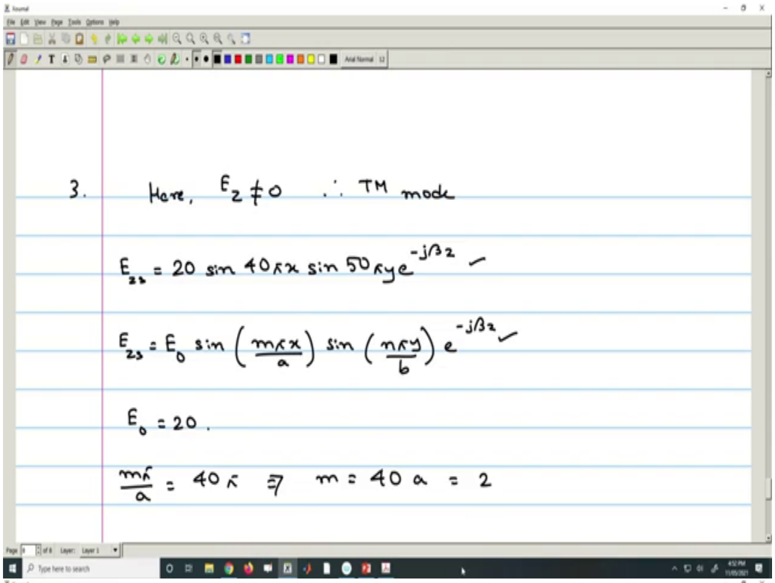
(Refer Slide Time: 21:34)



The third question says

3. An air-filled 5 by 2cm waveguide has
$E_{zs} = 20\sin 40\pi x \sin 50\pi y e^{-j\beta z}$ V/m
at 15 GHz.
(a) What mode is being propagated?
(b) Find β.

(Refer Slide Time: 22:10)



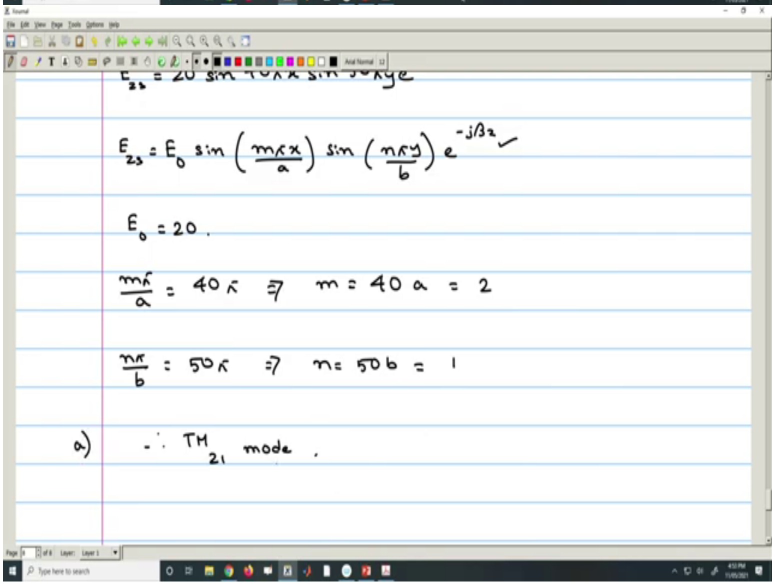
3. Here, $E_z \neq 0$ $\therefore$ TM mode

$$E_{zs} = 20 \sin 40\pi x \sin 50\pi y e^{-j\beta z}$$

$$E_{zs} = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

$$E_0 = 20.$$

$$\frac{m\pi}{a} = 40\pi \Rightarrow m = 40a = 2$$



$$E_{zs} = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

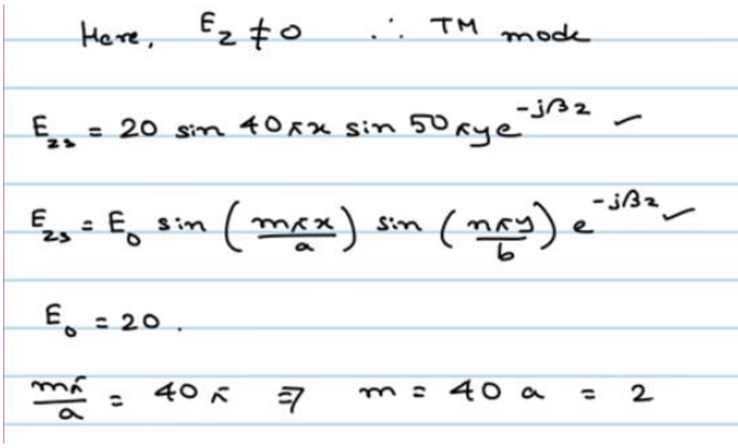
$$E_0 = 20.$$

$$\frac{m\pi}{a} = 40\pi \Rightarrow m = 40a = 2$$

$$\frac{n\pi}{b} = 50\pi \Rightarrow n = 50b = 1$$

a) $\therefore$ TM_{21} mode.

So, here Ezs is nonzero therefore, it is a TM mode now,



Here, $E_z \neq 0$ $\therefore$ TM mode

$$E_{zs} = 20 \sin 40\pi x \sin 50\pi y e^{-j\beta z}$$

$$E_{zs} = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

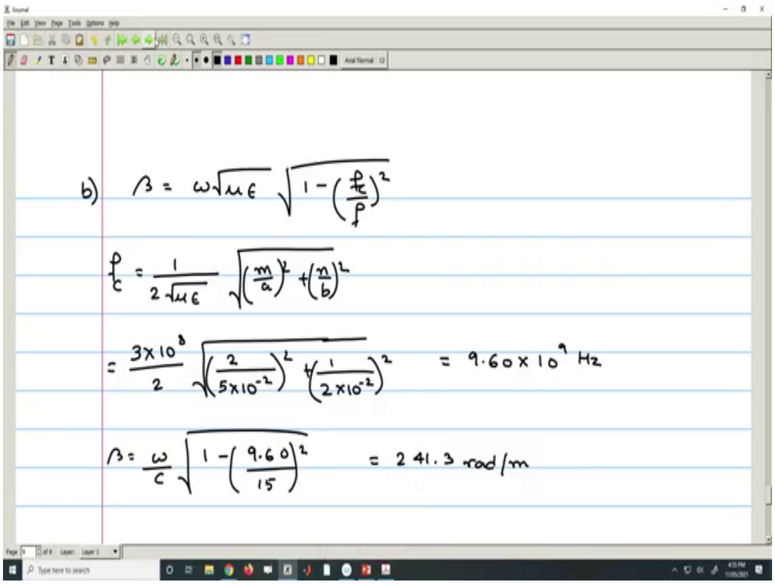
$$E_0 = 20.$$

$$\frac{m\pi}{a} = 40\pi \Rightarrow m = 40a = 2$$

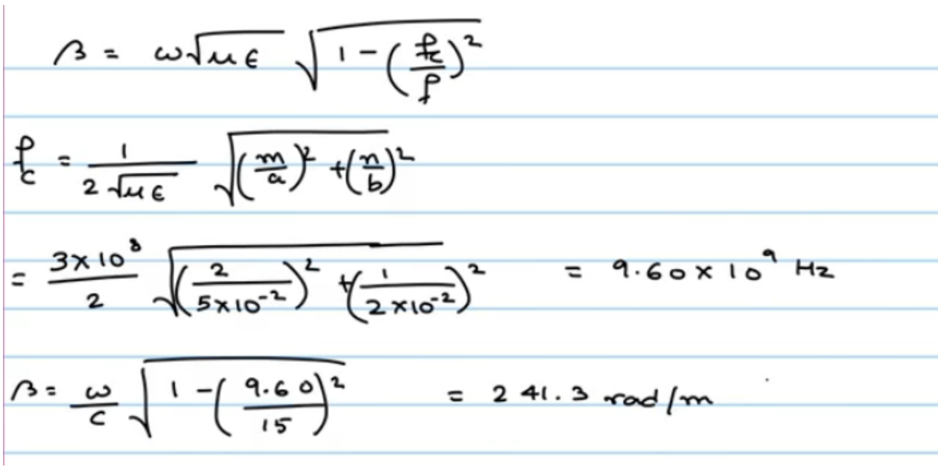
m comes out as 2.

And again, comparing the above two equations we get n as 1. Therefore, the indices are 2 and 1 so, TM_{21} mode is being propagated.

(Refer Slide Time: 25:03)

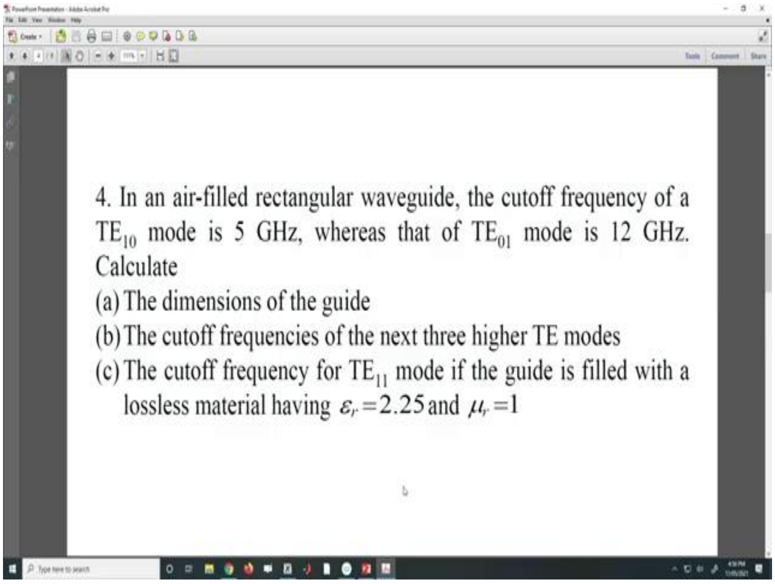


And again, we need to find Beta so, Beta we know,



So, we will get Beta as 241.3.

(Refer Slide Time: 27:26)



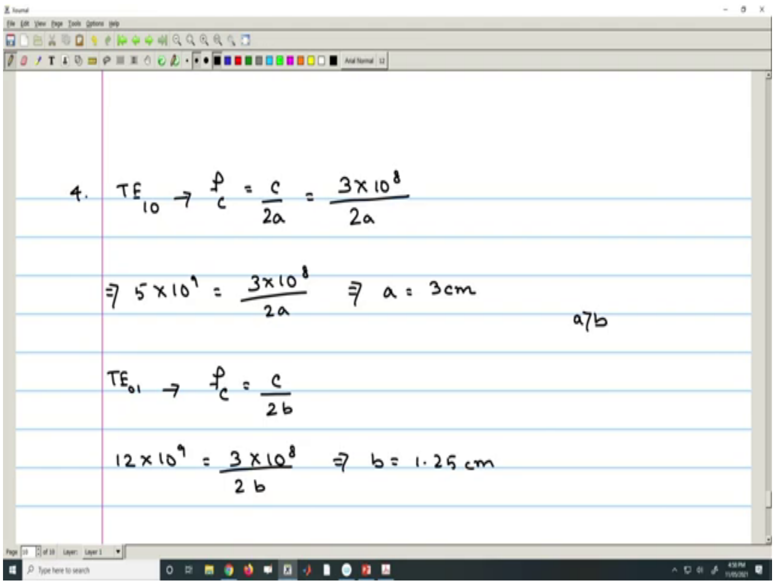
4. In an air-filled rectangular waveguide, the cutoff frequency of a TE_{10} mode is 5 GHz, whereas that of TE_{01} mode is 12 GHz. Calculate

(a) The dimensions of the guide

(b) The cutoff frequencies of the next three higher TE modes

(c) The cutoff frequency for TE_{11} mode if the guide is filled with a lossless material having $\varepsilon_r = 2.25$ and $\mu_r = 1$

(Refer Slide Time: 28:08)



So,

$$TE_{10} \rightarrow f_c = \frac{c}{2a} = \frac{3\times10^8}{2a}$$

$$\Rightarrow 5\times10^9 = \frac{3\times10^8}{2a} \Rightarrow a = 3\,cm$$

$a > b$

$$TE_{01} \rightarrow f_c = \frac{c}{2b}$$

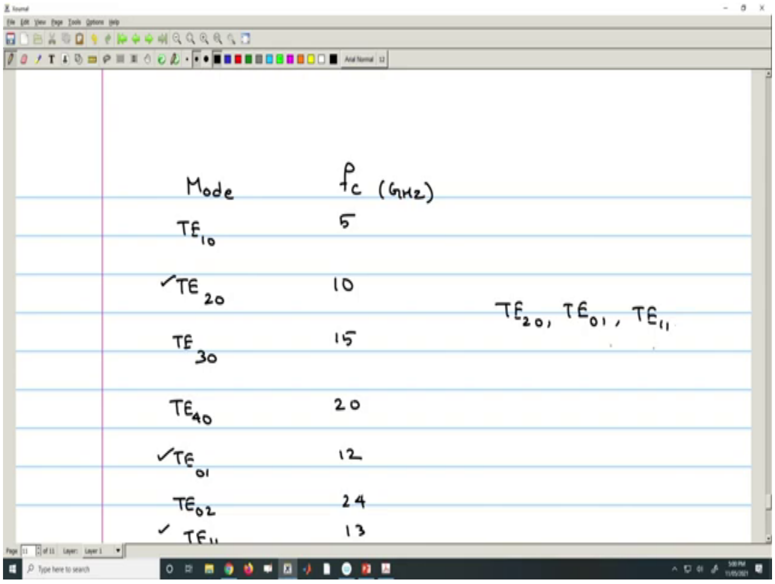
$$12\times10^9 = \frac{3\times10^8}{2b} \Rightarrow b = 1.25\,cm$$

Now, since a is greater than b, since a is greater than b then next higher order modes are calculated as.

Mode	f_c (GHz)
TE_{10}	5
✓ TE_{20}	10
TE_{30}	15
TE_{40}	20
✓ TE_{01}	12
TE_{02}	24
✓ TE_{11}	13

TE_{20}, TE_{01}, TE_{11}

(Refer Slide Time: 30:05)



So, the next part:

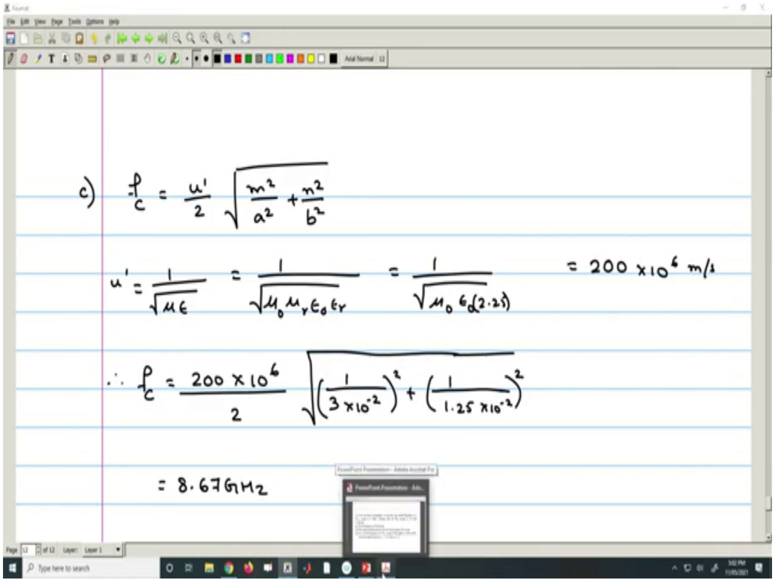
$$f_c = \frac{u'}{2}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$$

$$u' = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}} = \frac{1}{\sqrt{\mu_0\epsilon_0(2.25)}} = 200\times10^6 \text{ m/s}$$

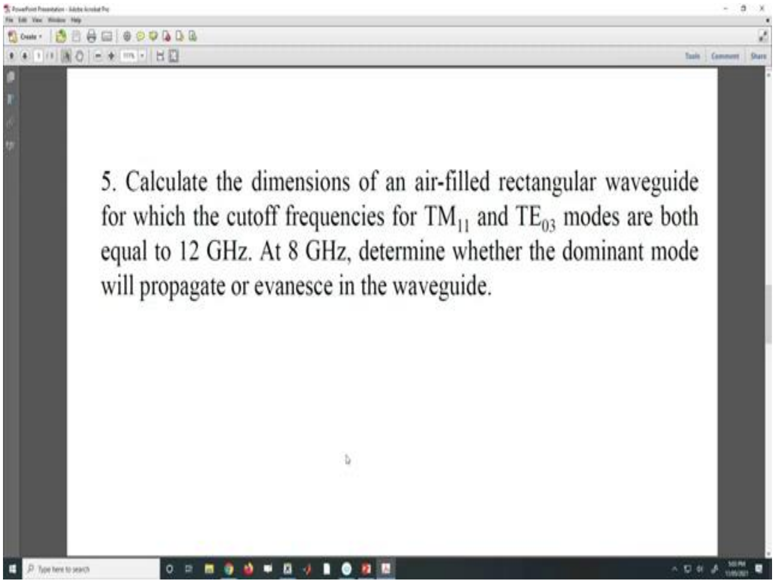
$$\therefore f_c = \frac{200\times10^6}{2}\sqrt{\left(\frac{1}{3\times10^{-2}}\right)^2+\left(\frac{1}{1.25\times10^{-2}}\right)^2}$$

$$= 8.67 \text{ GHz}$$

(Refer Slide Time: 32:22)



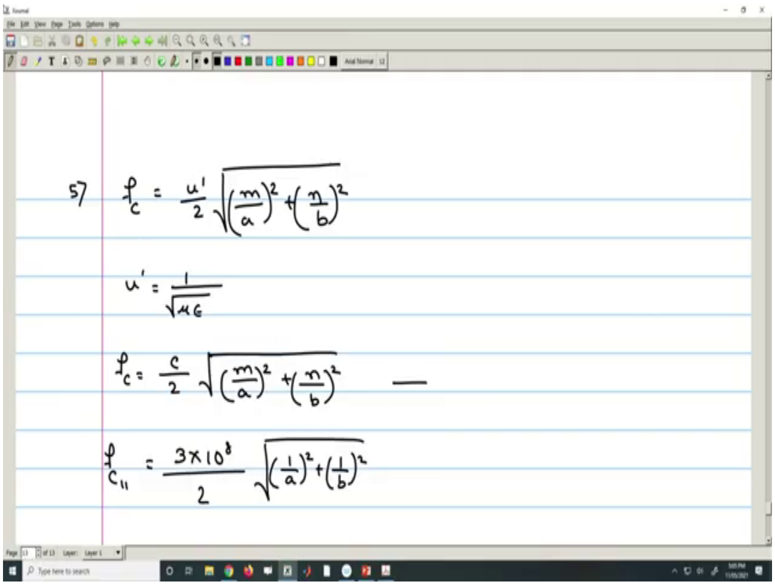
(Refer Slide Time: 34:28)



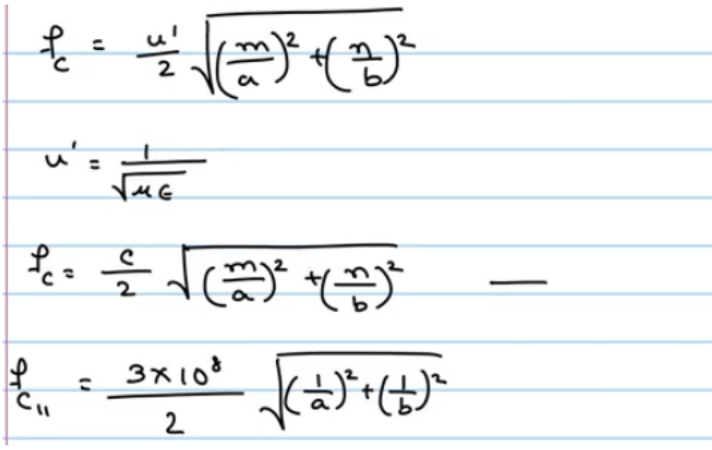
Next question,

5. Calculate the dimensions of an air-filled rectangular waveguide for which the cutoff frequencies for TM_{11} and TE_{03} modes are both equal to 12 GHz. At 8 GHz, determine whether the dominant mode will propagate or evanesce in the waveguide.

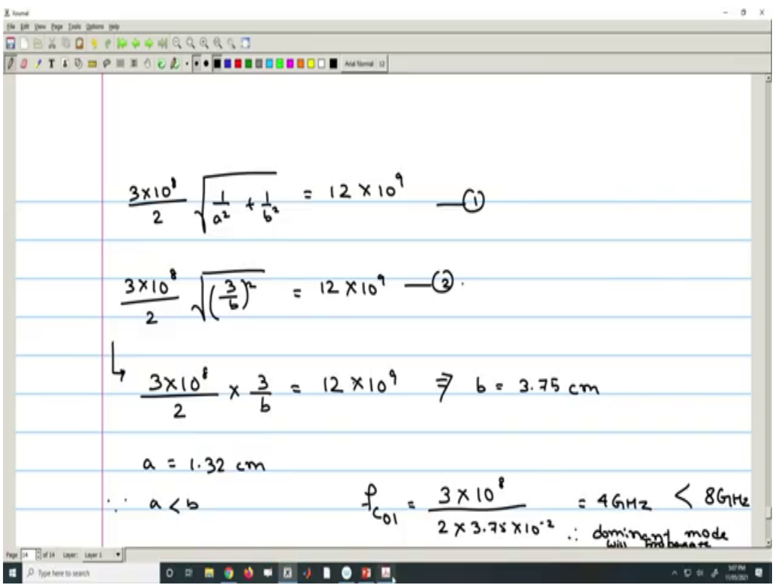
(Refer Slide Time: 34:58)



Fine, so now again to start with we will start from cut-off frequency



(Refer Slide Time: 36:51)



$$\frac{3\times10^8}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = 12\times10^9 \quad \text{— (1)}$$

$$\frac{3\times10^8}{2}\sqrt{\left(\frac{3}{b}\right)^2} = 12\times10^9 \quad \text{— (2)}$$

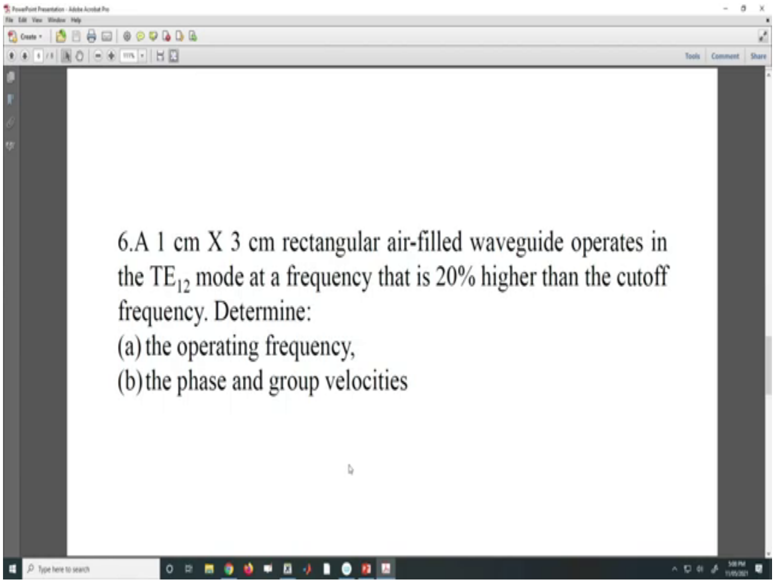
$$\frac{3\times10^8}{2}\times\frac{3}{b} = 12\times10^9 \Rightarrow b = 3.75\ \text{cm}$$

$a = 1.32$ cm

$\because a < b$ $\qquad f_{c_{01}} = \frac{3\times10^8}{2\times3.75\times10^{-2}} = 4\ \text{GHz} < 8\ \text{GHz}$ $\therefore$ dominant mode

so, we can comment like that, a dominant mode will propagate, dominant mode will propagate, next question.

(Refer Slide Time: 39:40)



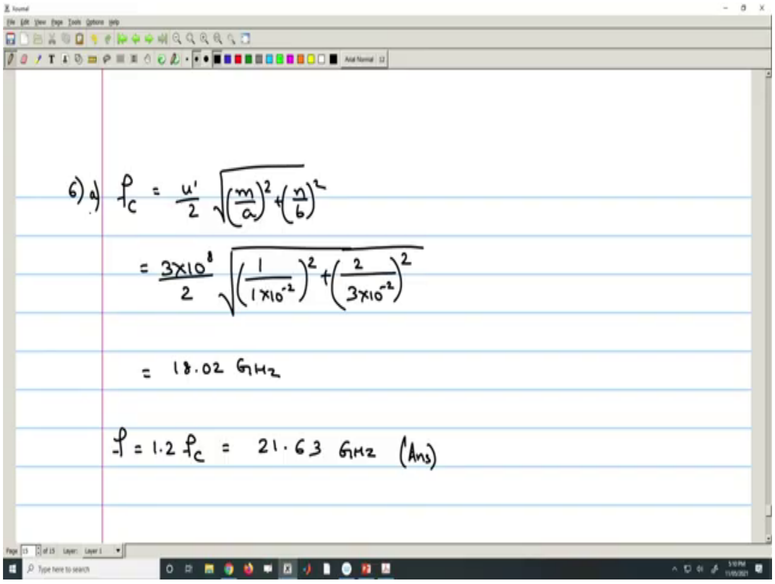
The next question says that

6.A 1 cm X 3 cm rectangular air-filled waveguide operates in the TE_{12} mode at a frequency that is 20% higher than the cutoff frequency. Determine:

(a) the operating frequency,

(b) the phase and group velocities

(Refer Slide Time: 40:10)



So, in the given question, there is 1 centimeter cross 3-centimeter rectangular air-filled waveguide, and it is operating in TE 12 mode.

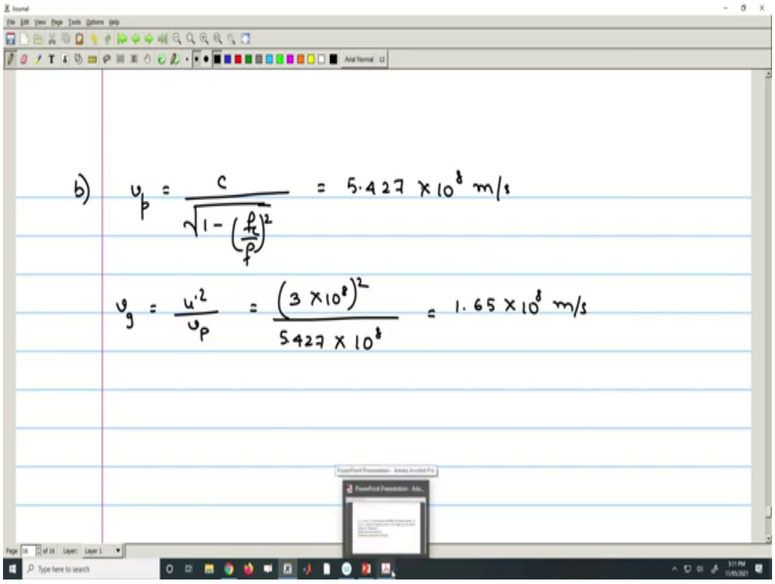
$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$$= \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{1\times10^{-2}}\right)^2+\left(\frac{2}{3\times10^{-2}}\right)^2}$$

$$= 18.02 \text{ GHz}$$

$$f = 1.2 f_c = 21.63 \text{ GHz (Ans)}$$

(Refer Slide Time: 42:11)



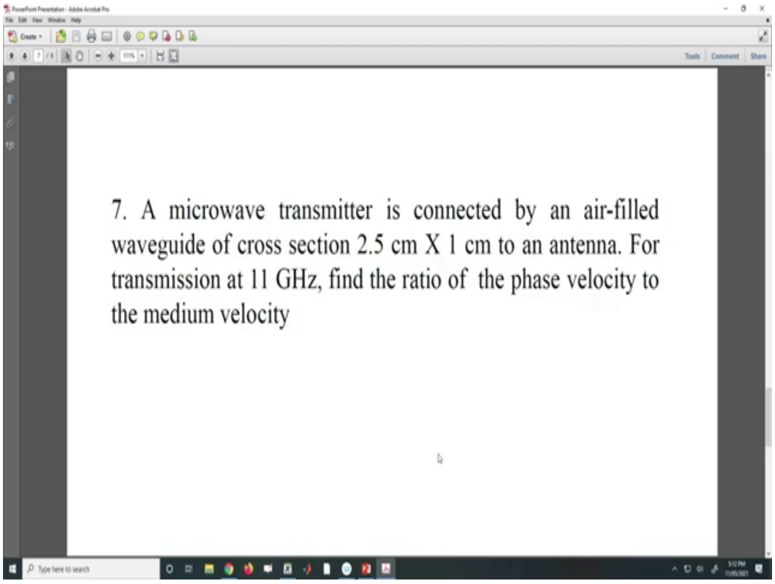
Now, sub part b :

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = 5.427 \times 10^8 \text{ m/s}$$

$$v_g = \frac{u^2}{v_p} = \frac{(3\times10^8)^2}{5.427\times10^8} = 1.65\times10^8 \text{ m/s}$$

So, phase velocity and group velocity is done, next question.

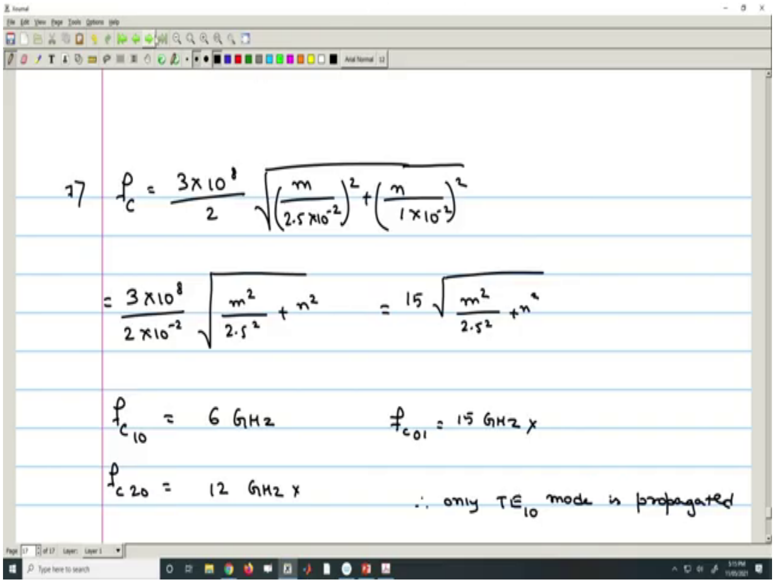
(Refer Slide Time: 43:39)



So, the next question is

7. A microwave transmitter is connected by an air-filled waveguide of cross section 2.5 cm X 1 cm to an antenna. For transmission at 11 GHz, find the ratio of the phase velocity to the medium velocity

(Refer Slide Time: 44:17)



So, a microwave transmitter is connected by an air-filled waveguide, of cross section 2.5 centimeter cross 1 centimeter. So, again we will start with calculating the cut-off frequency

$$f_c = \frac{3\times10^8}{2}\sqrt{\left(\frac{m}{2.5\times10^{-2}}\right)^2+\left(\frac{n}{1\times10^{-2}}\right)^2}$$

$$= \frac{3\times10^8}{2\times10^{-2}}\sqrt{\frac{m^2}{2.5^2}+n^2} \qquad = 15\sqrt{\frac{m^2}{2.5^2}+n^2}$$

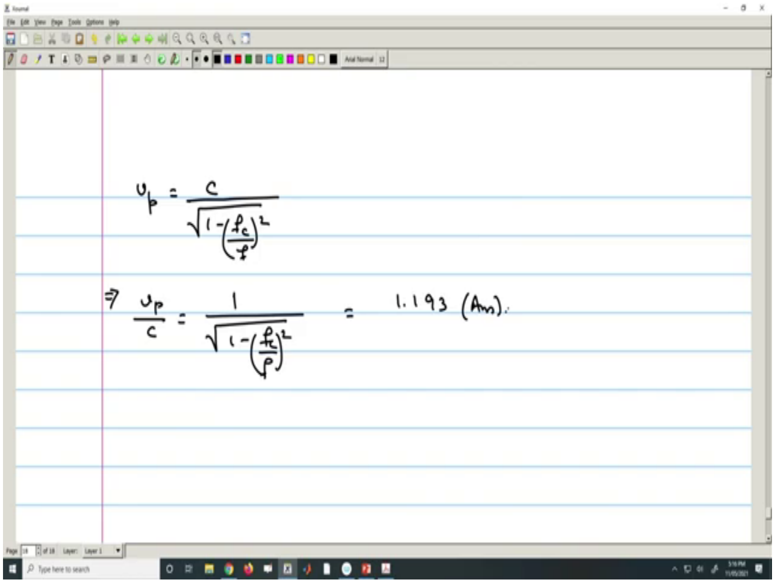
$$f_{c_{10}} = 6 \text{ GHz} \qquad f_{c_{01}} = 15 \text{ GHz} \times$$

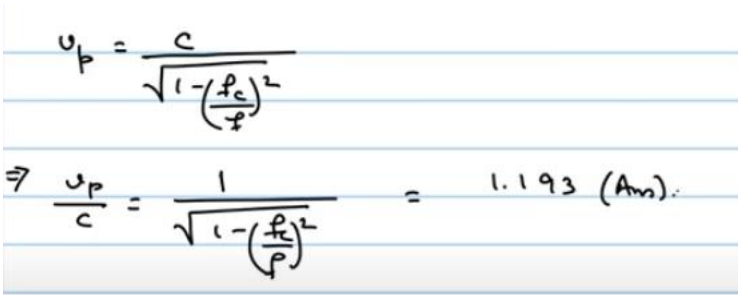
$$f_{c_{20}} = 12 \text{ GHz} \times$$

$\therefore$ only TE_{10} mode is propagated.

And then we will find the ratio of phase velocity to the medium velocity for this mode.

(Refer Slide Time: 47:04)





So, thank you this is all for today.