Advanced Microwave Guided-Structures and Analysis
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Lecture 34
Rectangular Waveguide II

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Welcome to the next session of the lecture on rectangular waveguides. In this session, we are going to investigate the power flow in the waveguide, the characteristic impedance of the TM and TE modes in the waveguide and the computation of power dissipation on the waveguide walls. Let us go to the lecture.

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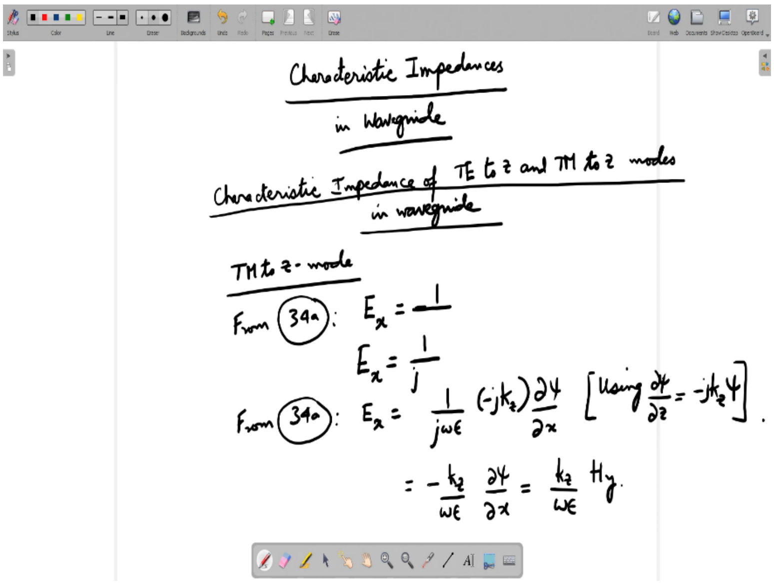

So, we begin with the characteristic impedance of the TE and TM modes in the waveguide. So, let us begin with the characteristic impedances of the TE and TM modes in the waveguide. So, first let us consider the TM to Z mode. So, we know the field equations for the TM to Z mode. So, particularly if you look at equation 34a, we can obtain the electric field component Ex, we can write it as $\frac{1}{j\omega\varepsilon}(-jk_z)\frac{\partial\psi}{\partial x}$ So, this can be written as $\frac{-k_z}{\omega\varepsilon}\frac{\partial\psi}{\partial x}$ and that is $\frac{k_z}{\omega\varepsilon}H_y$ because minus $\frac{\partial\psi}{\partial x}$ is Hy.

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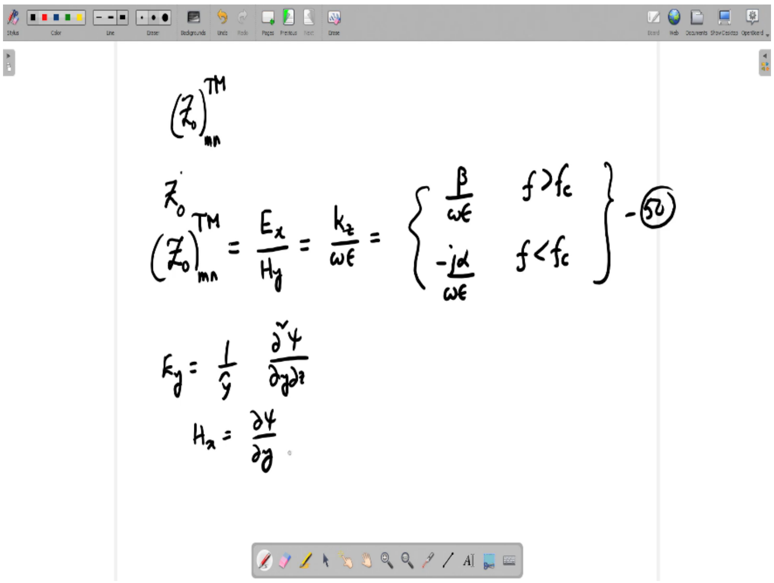

So, therefore, the characteristic impedance Zo_{mn} for the TM mode is given by Ex by Hy that is kz by omega epsilon and that is beta by omega epsilon above the cut-off frequency Fc and that is equal to minus j alpha by omega epsilon for the frequency below the cut-off frequency. So, let us call this set of equation 50 in continuation with our previous set of equations.

Now, we could compute the characteristic impedance in a different way. We could find out the characteristic impedance by considering the ratio of Ey and Hx because Ey and Hx are also the transverse field components. So, let us see what happens if you consider the characteristic impedance as a ratio between Ey and Hx. So, Ey is given by $\frac{1}{\hat{y}} \frac{\partial^2\psi}{\partial y \partial z}$ and Hx is given by $\frac{\partial\psi}{\partial y}$.

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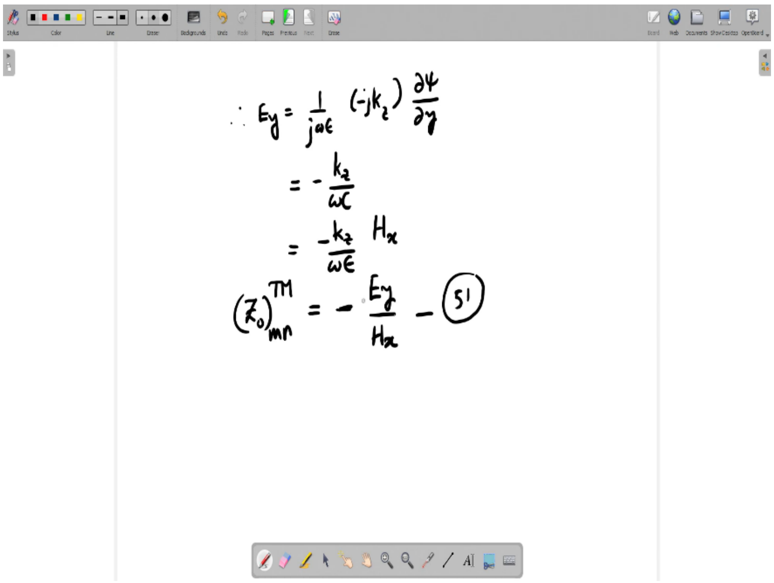

And therefore, we can write Ey as $\frac{1}{j\omega\varepsilon}(-jk_z)\frac{\partial\psi}{\partial y}$ and that is equal to $\frac{-k_z}{\omega c}$ and that is equal to $\frac{-k_z}{\omega\varepsilon}H_x$. So, my Zo_{mn} TM becomes equal to minus Ey by Hx. So, this should be kept in mind that if I consider the different set of transverse components, the characteristic impedance changes its sign.

So, therefore, in that sense the characteristic impedance is not uniquely defined with respect to the transverse components. So, this should be borne in mind for the waveguide structure.

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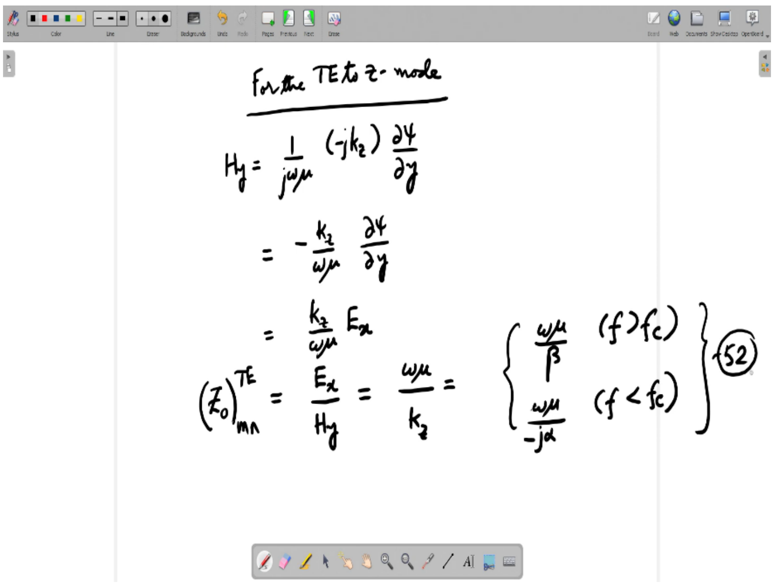

So, let us now go to the TE to Z mode. We have in this case, Hy as $\frac{1}{j\omega\mu}(-jk_z)\frac{\partial\psi}{\partial y}$ and that is equal to $\frac{-k_z}{\omega\mu}\frac{\partial\psi}{\partial y}$ and that is $\frac{k_z}{\omega\mu}E_x$. So, my Zo_{mn} TE is Ex by Hy it is $\frac{\omega\mu}{k_z}$ and that is equal to omega mu by beta for the operating frequency greater than the cut-off frequency Fc and that is equal to omega mu by minus j alpha for the frequency lower than the cut-off frequency. So, we call this set equation 52.

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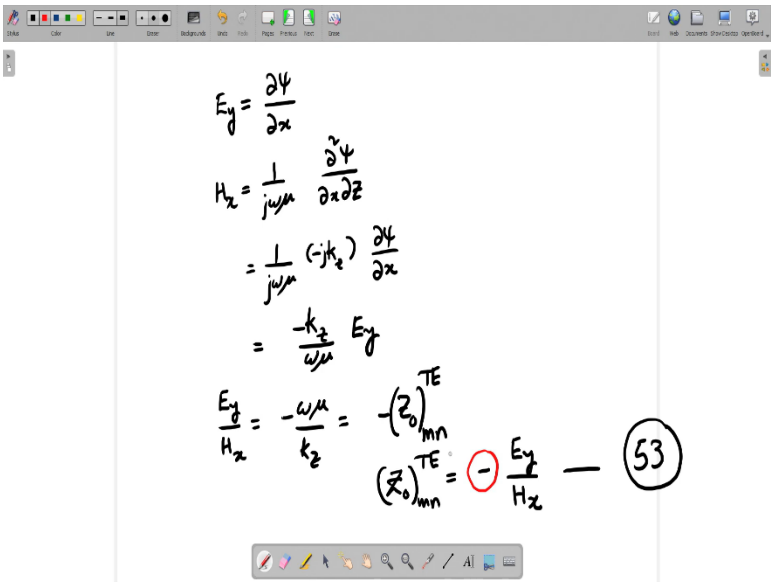

So, similar to the previous case, we can also quickly write that if I consider the ratio between Ey and Hx instead of Ex and Hy, how does the definition of characteristic impedance change or how does the expression for characteristic impedance altered? So, Ey is $\frac{\partial\psi}{\partial x}$, Hx is $\frac{1}{j\omega\mu}\frac{\partial^2\psi}{\partial x\partial z}$ that is 1 by j omega mu minus jkz $\frac{\partial\psi}{\partial x}$ that is minus kz by omega mu Ey because $\frac{\partial\psi}{\partial x}$ is Ey.

So, therefore, Ey by Hx becomes minus omega mu by kz and that is equal to minus Z0 mn TE. So, in other words Z0 mn TE becomes equal to minus Ey by Hx. So, we call this equation 53. So, note gain the change in sign due to taking the ratio between Ey and Hx. This phenomenon is similar to the TM to Z mode.

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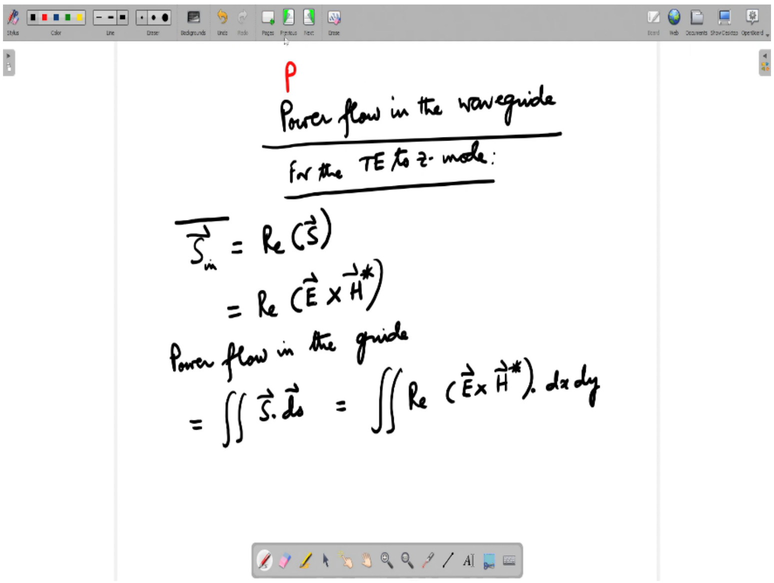

Now, let us go to the concept of power flow in the waveguide. So, when we are discussing the conservation of power, we encountered the power flow term. We encountered the pointing vector and we found that the integration of the pointing vector over a surface gives the power flow to the surface. So, we use the same concept and apply it to the rectangular waveguide.

So, let us consider the TE to Z mode. The concept of the case of the TM to Z mode can be similarly obtained. So, we have S in bar as real part of S, we already interpreted these terms in the power conservation theorem and that is real part of E cross H star and therefore, power flow in the guide is double integral S dot ds that is double integral real part of E cross H star dot dx dy.

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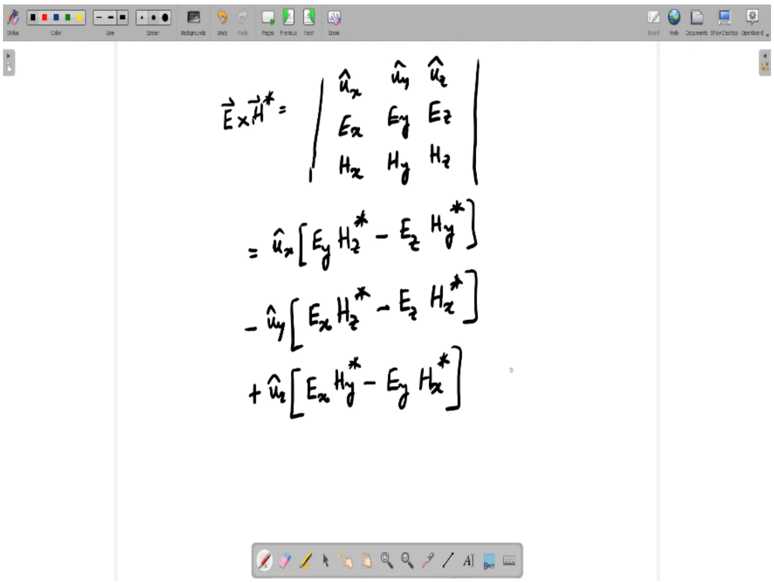

Now, E cross H star is nothing but $\begin{vmatrix} \hat{u}_x & \hat{u}_y & \hat{u}_z \\ E_x & E_y & E_z \\ H_x^* & H_y^* & H_z^* \end{vmatrix}$ expanding this cross product $\hat{u}_x(E_yH_z^* - E_zH_y^*) - \hat{u}_y(E_xH_z^* - E_zH_x^*) + \hat{u}_z(E_xH_y^* - E_yH_x^*)$. Now since for the TE to Z mode we have Ez equal to 0.

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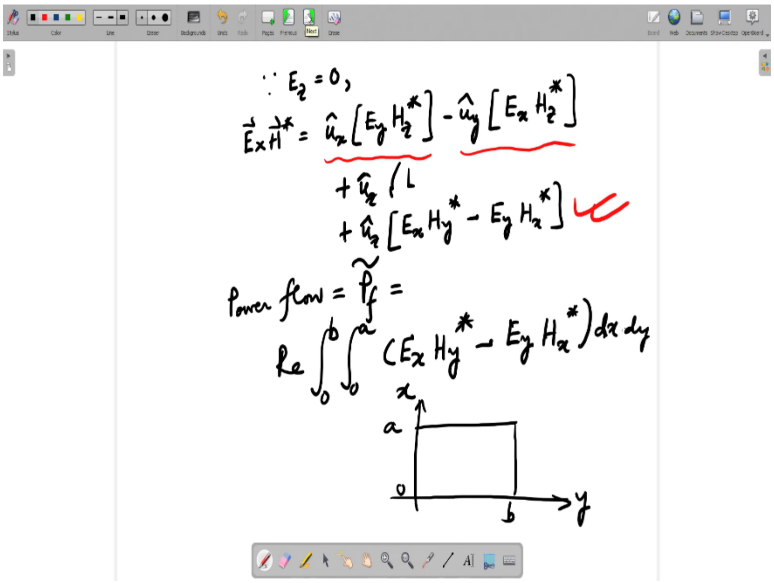

Since Ez equal to 0, we have E cross H star equal to $\hat{u}_x(E_yH_z^*) - \hat{u}_y(E_xH_z^*) + \hat{u}_z(E_xH_y^* - E_yH_x^*)$. So, therefore, for the power flow now, because I have that dot dx dy term or the dot ds term which is pointing along the user direction, these two terms are not going to contribute to the power flow and this term only is going to contribute to the power flow.

So, the power flow comes or the time average power flow is real part of $\int_0^b \int_0^a (E_x H_y^* - E_y H_x^*) dxdy$. So, here we are considering the waveguide cross section as before. So, this is my x axis, this is my y axis, this is my 0 to a, 'a' is along the x direction and the b dimension is along the y direction.

So, if we substitute for Ex Hy and Ey Hx with finally would obtain the expression for the time average power flow performing this integration and taking the real part.

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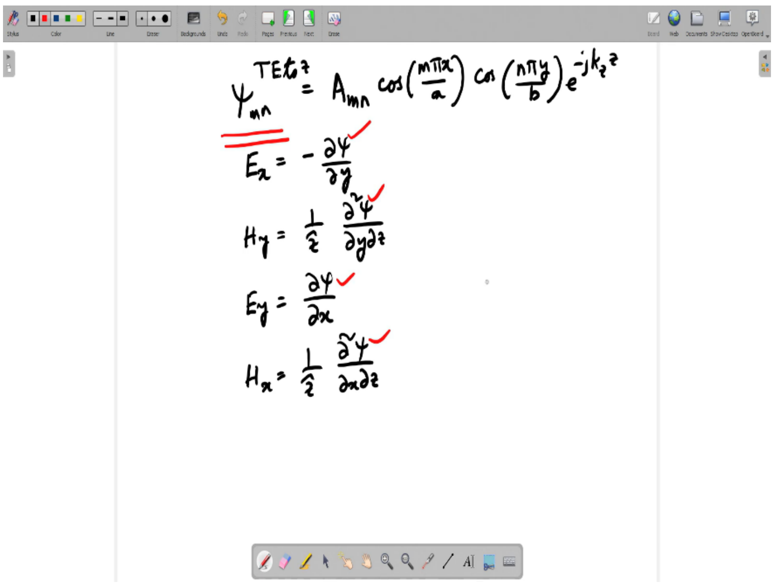

So, for that we consider the psi function for the TE to Z mode. We know already that it is given by $A_{mn} \cos(\frac{m\pi}{a}x) \cos(\frac{n\pi}{b}y) e^{-jk_z z}$. A_{mn} is the amplitude of the mnth mode. So, we know that Ex is minus $\frac{\partial \psi}{\partial y}$, HY is $\frac{1}{\hat{z}}\frac{\partial^2 \psi}{\partial y \partial z}$. Ey is $\frac{\partial \psi}{\partial x}$ and Hx is $\frac{1}{\hat{z}}\frac{\partial^2 \psi}{\partial x \partial z}$. So, therefore, after performing these operations on this psi function.

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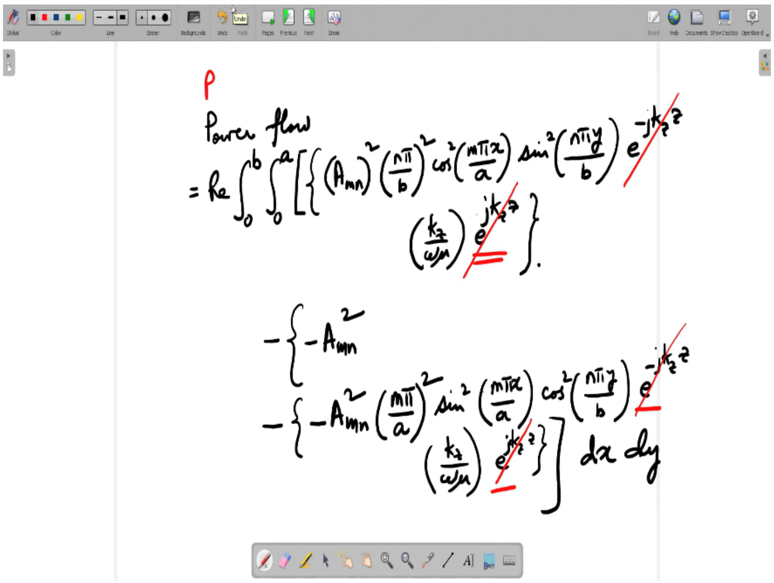

and substituting the psi function here, here, here and here and finally evaluating the integrand Ex Hy star minus Ey Hx star, we will obtain the power flow as a real part of integration from 0 to b, 0 to a, A_{mn} square n pi by b square cos square m pi x by a, sin square n pi y by b, e to the power minus jkzZ kz by omega mu times e to the power plus jkz Z. This is because of the star operation. So, therefore, it changes sign from e to the power minus jkzZ to e to the power plus jkz Z.

So, this term and this term cancel out and then we have the other term is minus and then we have the other term which is minus of minus A mn square m pi by a whole square, sin square m pi x by a, cos square n pi y by b, e to the power minus jkz Z times kz by omega mu e to the power plus jkz Z and then finally dx dy.

So, again these two terms are as a result of change from the normal to the star domain. So, therefore, the e to the power minus jkzZ changes to e to the power plus jkzZ and therefore, these two cancel each other.

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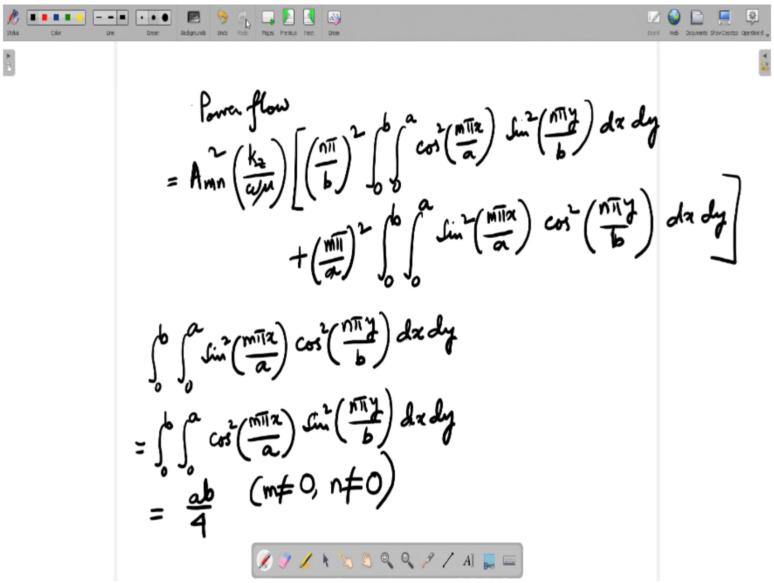

And therefore, the power flow becomes A_{mn} square kz by omega mu n pi by b square integration from 0 to b, 0 to a, cos square m pi x by a, sin square n pi y by b dx dy plus m pi by a whole square integration from 0 to b, 0 to a sin square n pi x by a, cos square n pi y by b dx dy. Now, the value of these integrals are please check yourself, 0 to b, 0 to a sin square m pi x by a, cos square n pi y by b, dx dy is 0 to b, 0 to a cos square m pi x by a, sin square n pi y by b dx dy and that is ab by 4, if m and n are not 0.

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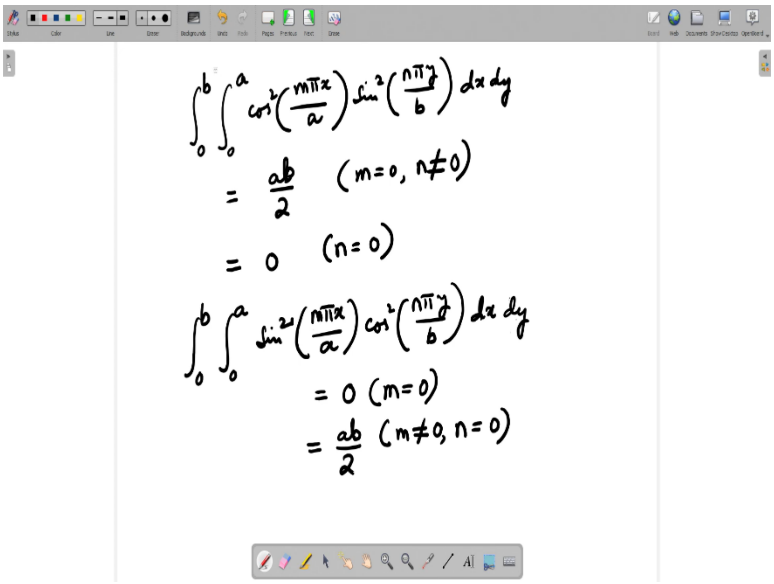

This integral 0 to b, 0 to a, cos square m pi x by a, sin square n pi y by b dx dy is ab by 2, if m equal to 0 and n not equal to 0 and is 0, if n equal to 0. This integral 0 to b, 0 to a, sin square m pi x by a, cos square n pi y by b dx dy is 0 for m equal to 0 and is ab by 2 for m not equal to 0 and n equal to 0.

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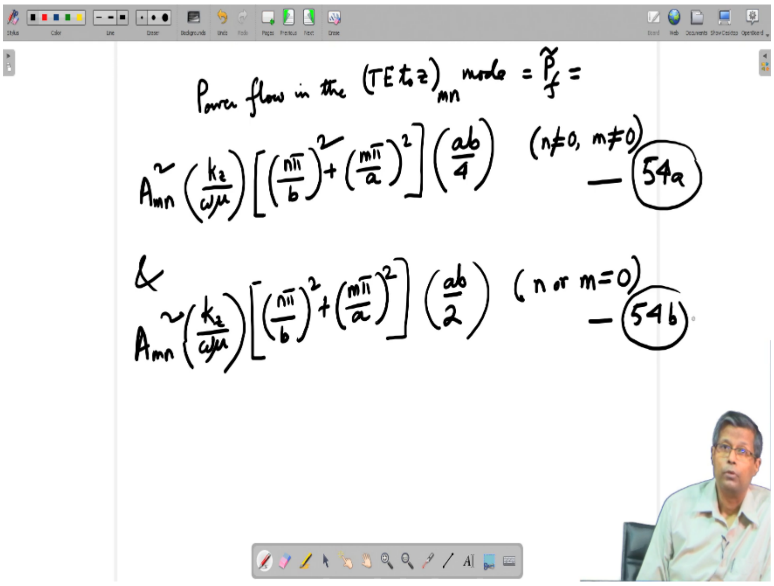

So, therefore, the time average power flow will be given by power flow in the TE to Z mn mode, that is, Pf Tilda that will be given by A_{mn} square kz by omega mu n pi by b whole square plus m pi by a whole square times ab by 4, for n not equal to 0, m not equal to 0. Let us call this equation 54a and it will be given by A_{mn} square kz by omega mu n pi by b whole square plus m pi by a whole square, ab by 2 for n or m equal to 0.

So, let us call this equation 54b. However, it is to be noted that if n modes are propagating simultaneously, the total power is the sum of the powers in the individual modes because of orthogonality. So, n modes are propagating simultaneously in the waveguide, the total power is going to be the sum of the powers in the individual modes. So, let us stop here. We will continue