Advanced Microwave Guided-Structures and Analysis
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Lecture 35
Rectangular Waveguide II (contd.)

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Hello, welcome to this session on the continuation for the rectangular waveguide. In this session we are going to discuss the power loss on the waveguide walls and the attenuation as a result of power loss, the expression for that relation.

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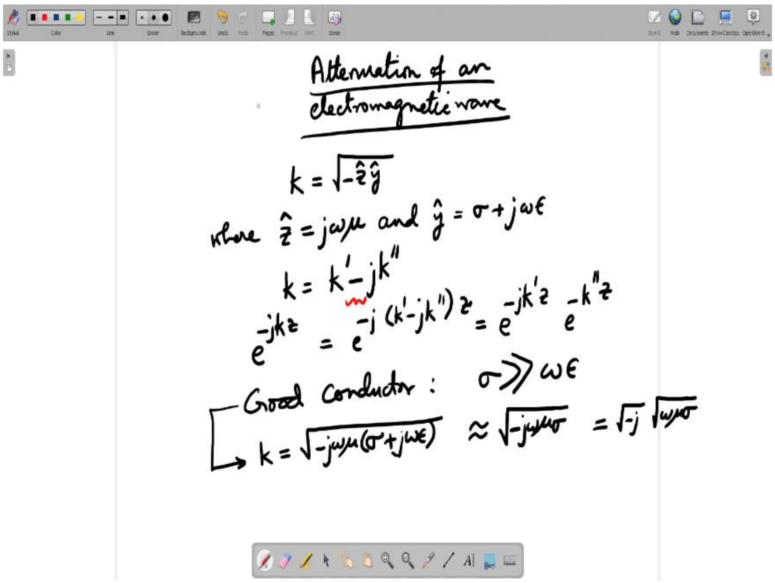

So, first of all we consider the attenuation of an electromagnetic wave in a homogeneous medium. So, we know that the propagation constant is given

Attenuation of an electromagnetic wave

$$k = \sqrt{-\hat{z}\hat{y}}$$

where $\hat{z} = j\omega\mu$ and $\hat{y} = \sigma + j\omega\epsilon$

$$k = k' - jk''$$

So, this minus sign is dictated by the fact that the electromagnetic wave will experience the attenuation and not get amplified as it traverses through the medium. So, that means

$$e^{-jkz} = e^{-j(k'-jk'')z} = e^{-jk'z}\, e^{-k''z}$$

So, a good conductor is characterized by sigma is very much greater than omega epsilon. It is characterized by the fact that sigma is very much greater than omega epsilon. So, under that condition for a good conductor, so, for a good conductor

Good conductor : $\sigma \gg \omega\epsilon$

$$k = \sqrt{-j\omega\mu(\sigma + j\omega\epsilon)} \approx \sqrt{-j\omega\mu\sigma} = \sqrt{-j}\,\sqrt{\omega\mu\sigma}$$

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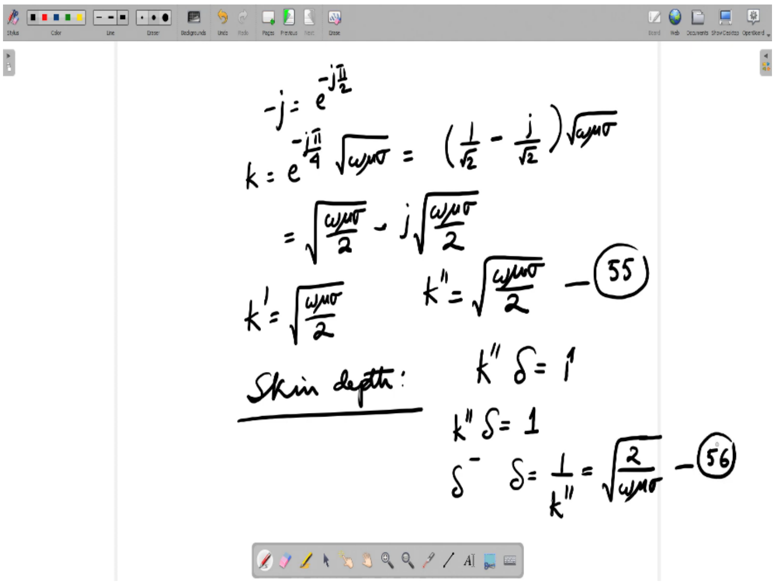

Now, minus j can be written as

$$-j = e^{-j\frac{\pi}{2}}$$

$$k = e^{-j\frac{\pi}{4}}\sqrt{\omega\mu\sigma} = \left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right)\sqrt{\omega\mu\sigma}$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}} - j\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$k' = \sqrt{\frac{\omega\mu\sigma}{2}} \qquad k'' = \sqrt{\frac{\omega\mu\sigma}{2}} \quad \text{— (55)}$$

Now, we know that the skin depth is the depth for which the field amplitude becomes one by E times its initial value or 36.8% of its initial value. So, that is corresponding to the condition

Skin depth:

$$k''\delta = 1$$

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$$\delta^{-} \qquad \delta = \frac{1}{k''} = \sqrt{\frac{2}{\omega\mu\sigma}} \quad \text{— (56)}$$

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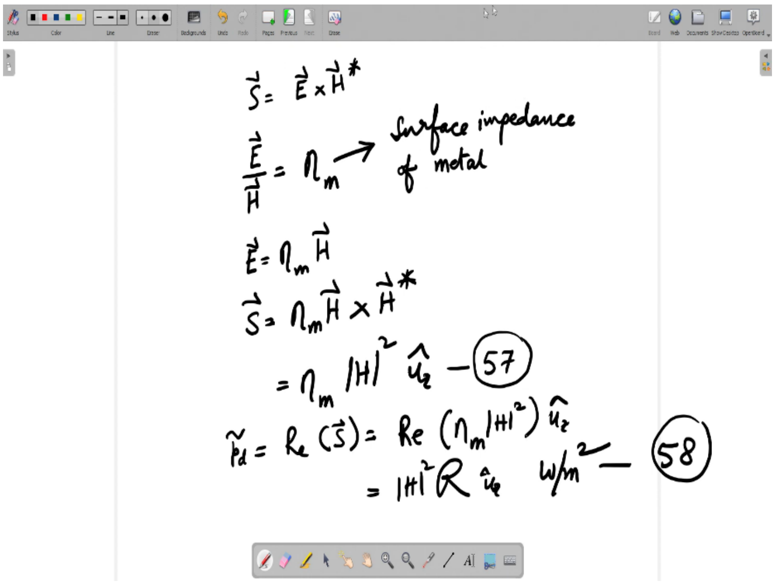

So, the density of power flow into the conductor which is ultimately converted to heat is given by

$$\vec{S} = \vec{E} \times \vec{H}^*$$

$$\frac{\vec{E}}{\vec{H}} = \eta_m \rightarrow \text{Surface impedance of metal}$$

$$\vec{E} = \eta_m \vec{H}$$

$$\vec{S} = \eta_m \vec{H} \times \vec{H}^*$$

$$= \eta_m |H|^2 \hat{u}_z \quad \text{— (57)}$$

So, let us call this equation 57.

So, let us call this equation 57 where H is the magnetic field at the surface. So, the time average power dissipation per unit area of cross section which is given by

$$\tilde{P}_d = R_e(\vec{S}) = Re\left(\eta_m |H|^2\right) \hat{u}_z$$
$$= |H|^2 R \, \hat{u}_z \quad W/m^2 \quad \text{— (58)}$$

Let us call this 58.

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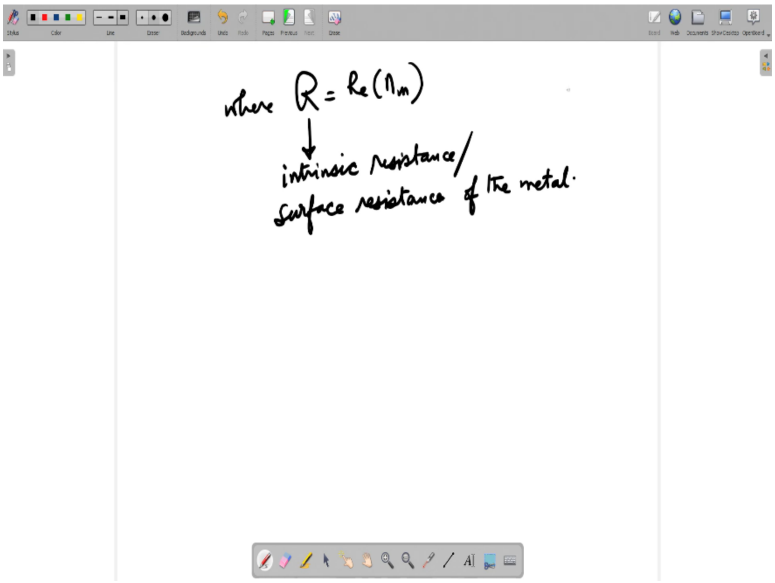
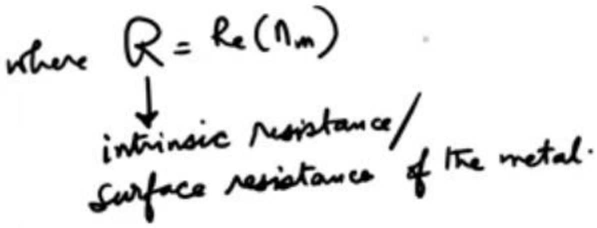


Where R is a



So, this being called the intrinsic resistance or the surface resistance of the metal.

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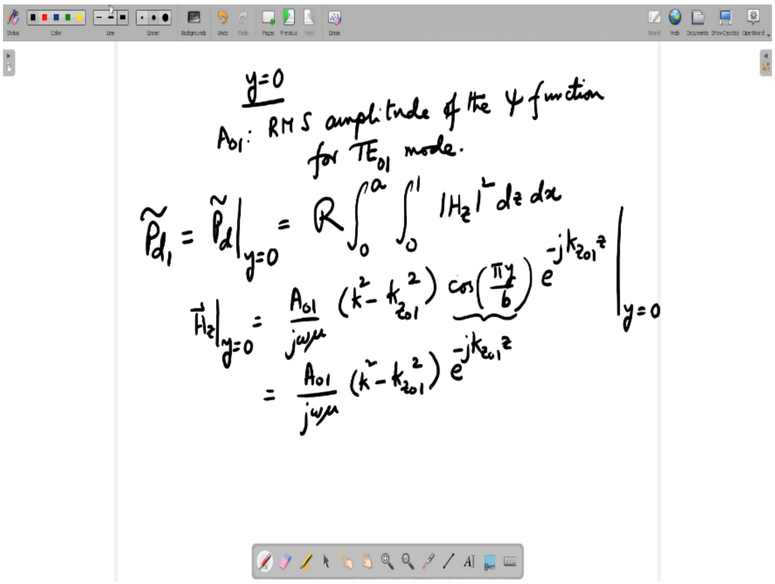

So, now, let us consider the TE_{01} mode of the waveguide and find out the power dissipated per unit length in the wall. So, let us consider the wall at y equal to 0 and let us consider A_{01}

to be the RMS amplitude of the psi function for the TE_{01} mode. So, we call the power dissipated per unit length as

$\underline{y=0}$

A_{01}: RMS amplitude of the ψ function for TE_{01} mode.

$$\tilde{P}_{d_1} = \tilde{P}_d\Big|_{y=0} = R\int_0^a\int_0^1 |H_z|^2\,dz\,dx$$

$$\vec{H}_z\Big|_{y=0} = \frac{A_{01}}{j\omega\mu}\left(k^2 - k_{z01}^2\right)\underbrace{\cos\left(\frac{\pi y}{b}\right)}\,e^{-jk_{z01}z}\Bigg|_{y=0}$$

$$= \frac{A_{01}}{j\omega\mu}\left(k^2 - k_{z01}^2\right)e^{-jk_{z01}}$$

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$$\tilde{P}_{d_1} = R\int_0^a\int_0^1 \frac{A_{01}^2}{\omega^2\mu^2}\left(k^2 - k_{z01}^2\right)^2 dz\,dx$$

$$= \frac{aRA_{01}^2}{\omega^2\mu^2}\left(k^2 - k_{z01}^2\right)^2$$

$$\because k^2 = k_{z01}^2 + \left(\frac{\pi}{b}\right)^2 = \omega^2\mu\epsilon$$

$$\tilde{P}_{d_1} = \frac{aRA_{01}^2}{\omega^2\mu^2}\left(\frac{\pi}{b}\right)^4 \quad \text{---} \quad (59)$$

An equal amount of power is dissipated at $y=b$.

And therefore,

$$\tilde{P}_{d_1} = R\int_0^a\int_0^1 \frac{A_{01}^2}{\omega^2\mu^2}\left(k^2 - k_{z01}^2\right)^2 dz\,dx$$

$$= \frac{aRA_{01}^2}{\omega^2\mu^2}\left(k^2 - k_{z01}^2\right)^2$$

$$\because k^2 = k_{z01}^2 + \left(\frac{\pi}{b}\right)^2 = \omega^2\mu\epsilon$$

$$\tilde{P}_{d_1} = \frac{aRA_{01}^2}{\omega^2\mu^2}\left(\frac{\pi}{b}\right)^4 \quad \text{---} \quad (59)$$

An equal amount of power is dissipated at $y=b$.

So, we can call this expression 59. And an equal amount of power will be dissipated at y equal to b.

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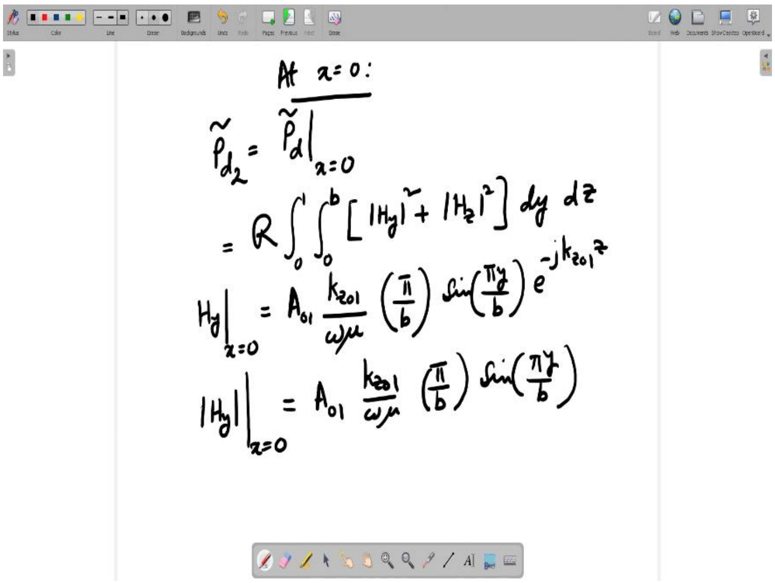
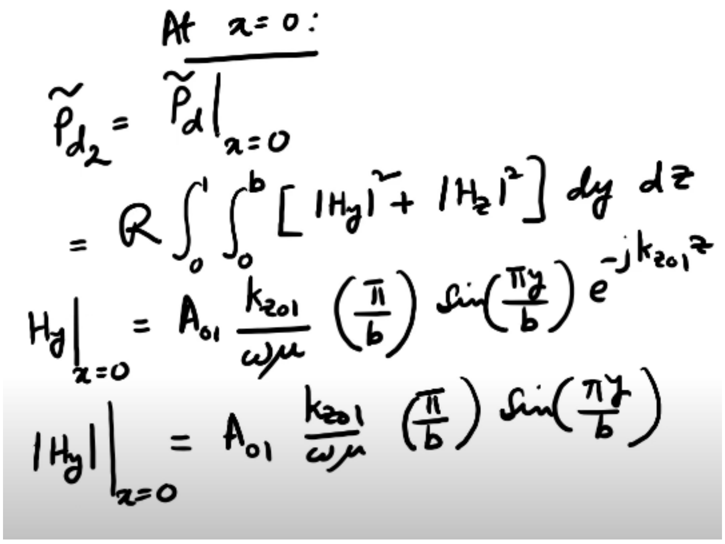


Then we next go to the power dissipated at the wall x equal to 0. So, at the wall x equal to 0, we have two tangential components of the magnetic field for the TE_{01} mode. One is H_y and another is H_z. So, therefore,



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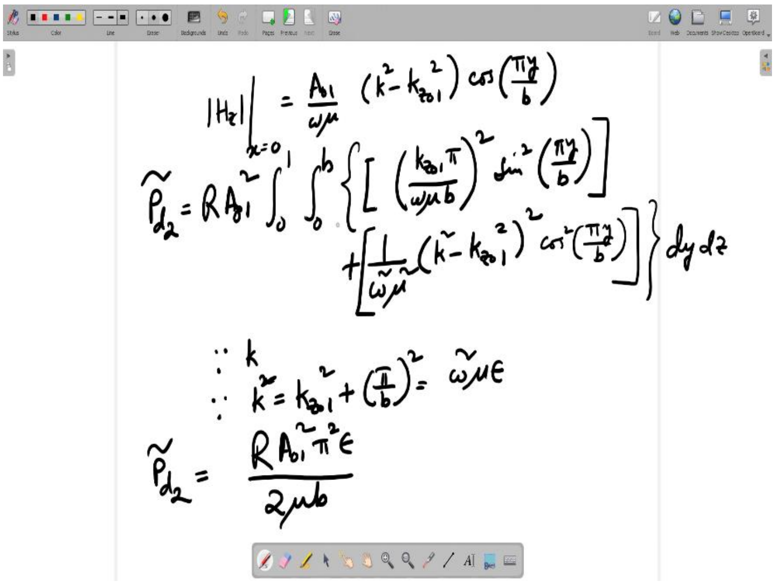
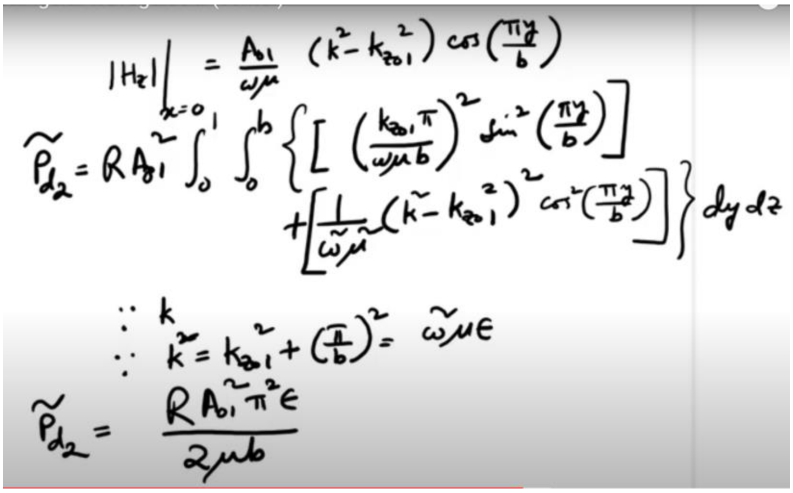


Similarly, mod Hz at x equal to 0 will be



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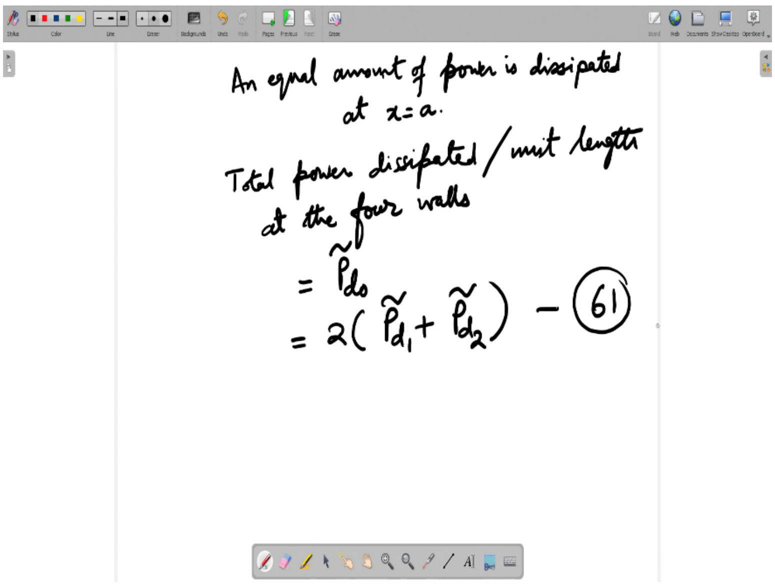

Unlike before an equal amount of power will be dissipated at x equal to A. So, the total power dissipated at the four walls of the waveguide, the total power dissipated per unit length at the four walls of the guide becomes equal to

An equal amount of power is dissipated at $x = a$.

Total power dissipated / unit length at the four walls

$$= \tilde{P}_{d0}$$

$$= 2(\tilde{P}_{d1} + \tilde{P}_{d2}) \quad \text{— (61)}$$

So, let us call this 61.

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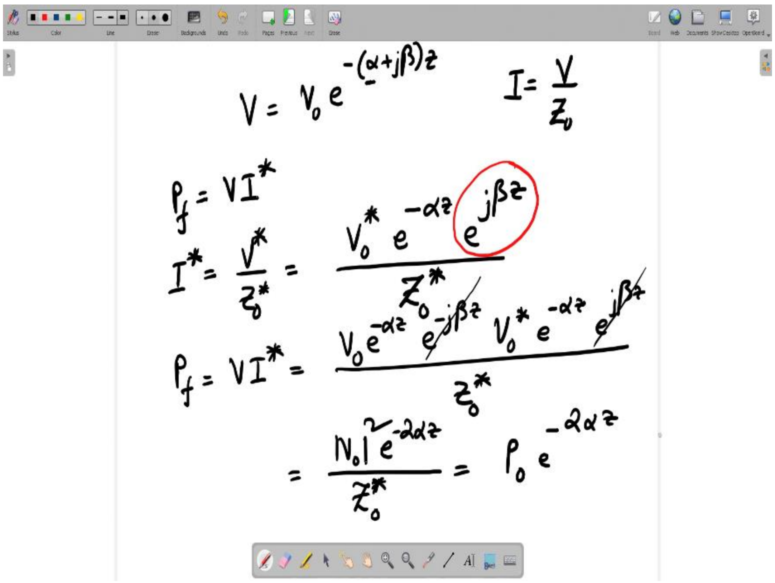
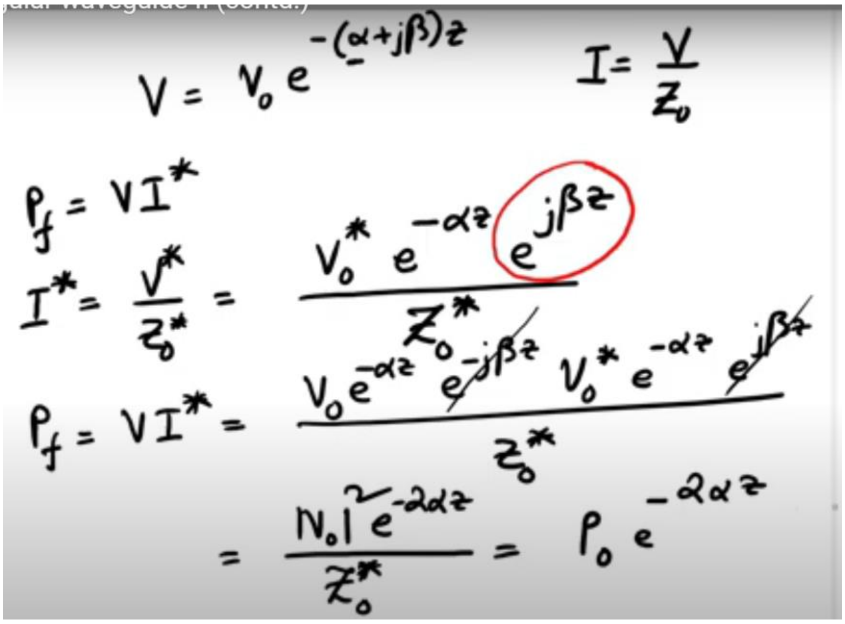


Now, considering the waveguide as a transmission line, we can consider the voltage variation along that line to be of the form



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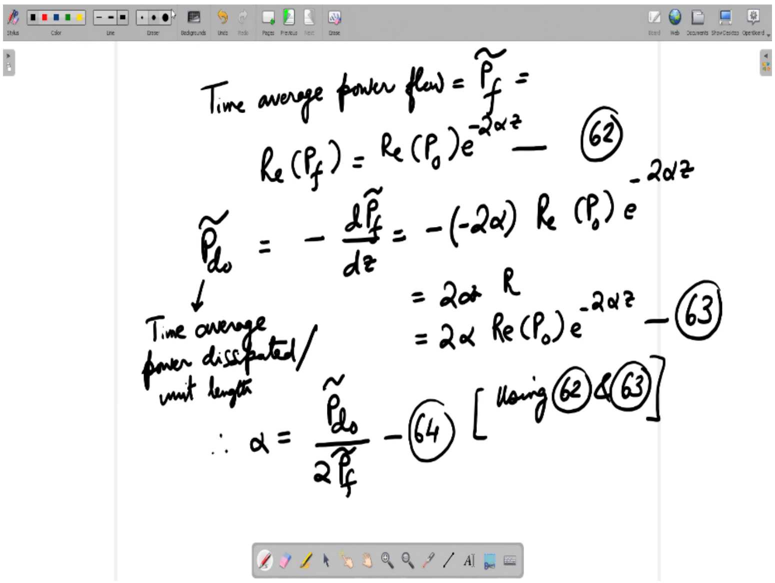
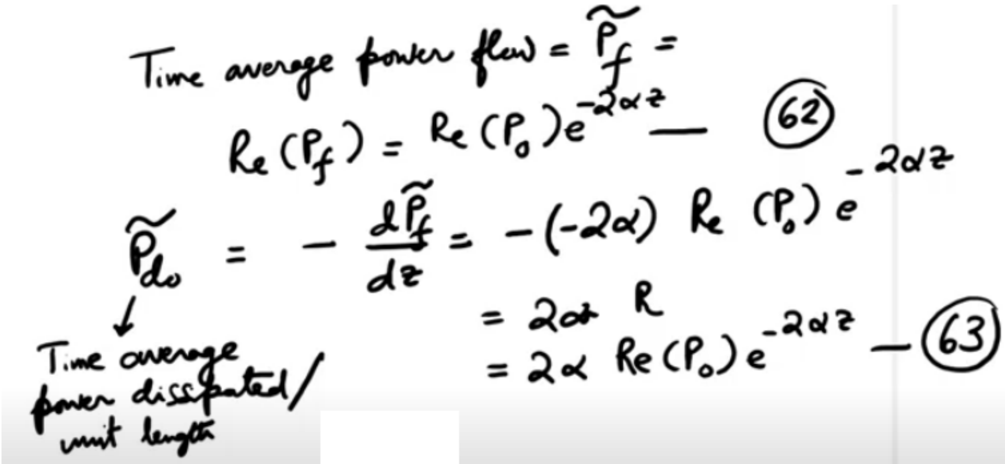


So, therefore, that time average power flow which is



So, let us call that 63.

So, therefore, that initial constant alpha can be written as

that is, using 62 and 63.

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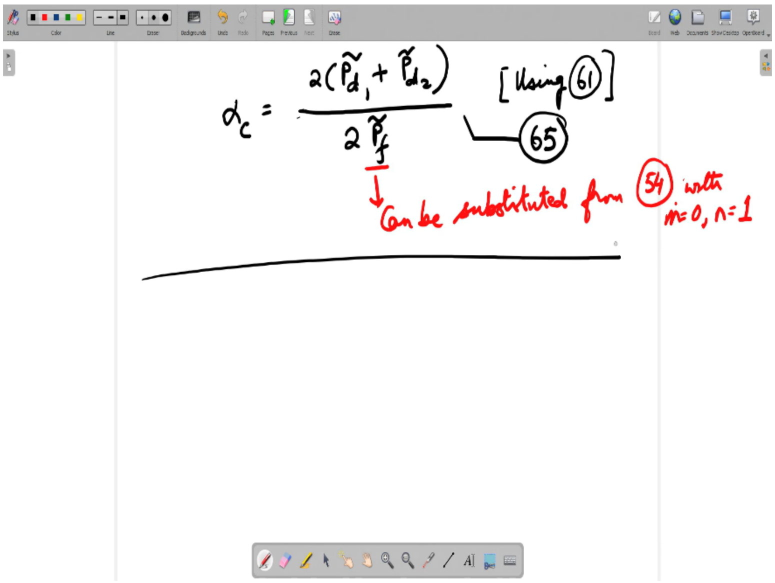

So, therefore, that the attenuation and therefore, the attenuation due to the conductor loss in the four walls written as

$$\alpha_c = \frac{2(\tilde{P}_{d_1} + \tilde{P}_{d_2})}{2\tilde{P}_f} \quad \text{[Using (61)]} \qquad (65)$$

Can be substituted from (54) with $m=0$, $n=1$

you can call this expression 65.

So, this expression evaluates the attenuation constant due to the power dissipation at the four walls of the rectangular waveguide. This concludes this section. We will stop here at this point. Thank you.