Advanced Microwave Guided-Structures and Analysis
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Lecture 36
Rectangular Waveguide II Tutorials

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Welcome to the next tutorial on the subject advanced microwave guided structures and analysis. So, this is the main set. Today we will do numerical problem on that calculation of wave impedance and power flow in rectangular waveguide.

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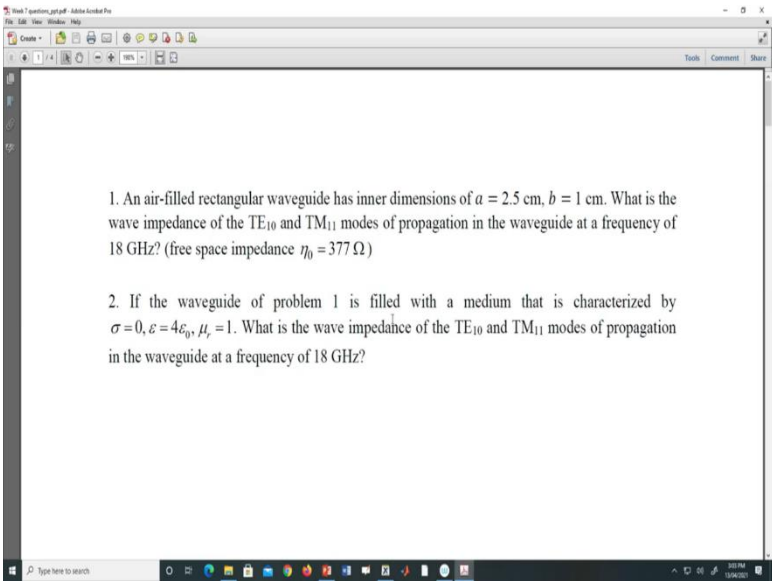

1. An air-filled rectangular waveguide has inner dimensions of $a = 2.5$ cm, $b = 1$ cm. What is the wave impedance of the TE_{10} and TM_{11} modes of propagation in the waveguide at a frequency of 18 GHz? (free space impedance $\eta_0 = 377\,\Omega$)

2. If the waveguide of problem 1 is filled with a medium that is characterized by $\sigma = 0, \varepsilon = 4\varepsilon_0, \mu_r = 1$. What is the wave impedance of the TE_{10} and TM_{11} modes of propagation in the waveguide at a frequency of 18 GHz?

So, this is the first problem.

1. An air-filled rectangular waveguide has inner dimensions of $a = 2.5$ cm, $b = 1$ cm. What is the wave impedance of the TE_{10} and TM_{11} modes of propagation in the waveguide at a frequency of 18 GHz? (free space impedance $\eta_0 = 377\,\Omega$)

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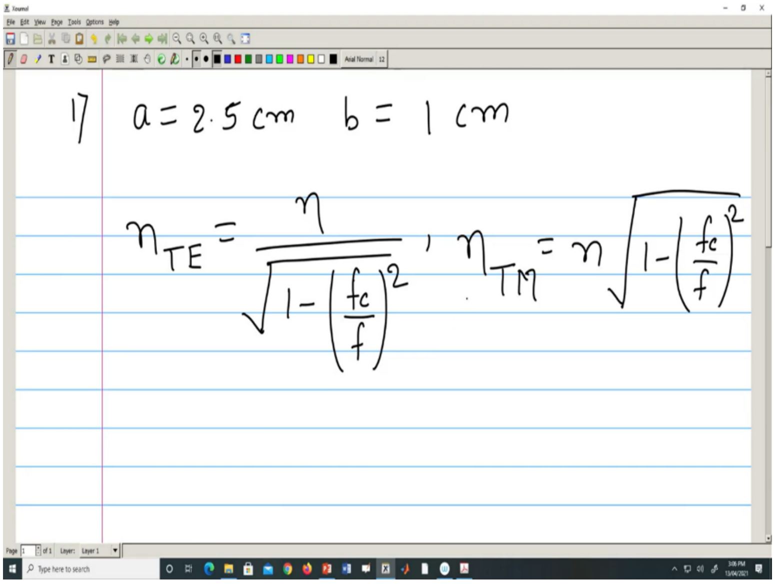

So, we have to calculate the wave impedance of TE_{10} and TM_{11} modes of propagation at 18 gigahertz. So, we know

$$a = 2.5 \text{ cm} \qquad b = 1 \text{ cm}$$

$$\eta_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}, \quad \eta_{TM} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

So, here we need cut-off frequency, we know cut-off frequency for each mode is different.

For TE_{10}, $m=1, n=0$

$$f_c = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

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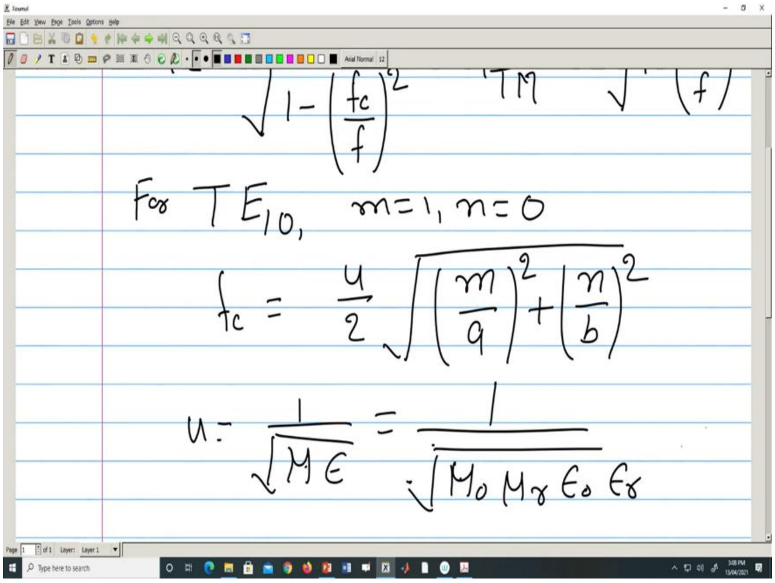

So, for here for TE_{10} that means m equal to 1 and n equal to 0. So, we can substitute the value and we will get the cut-off frequency for the TE_{10} mode.

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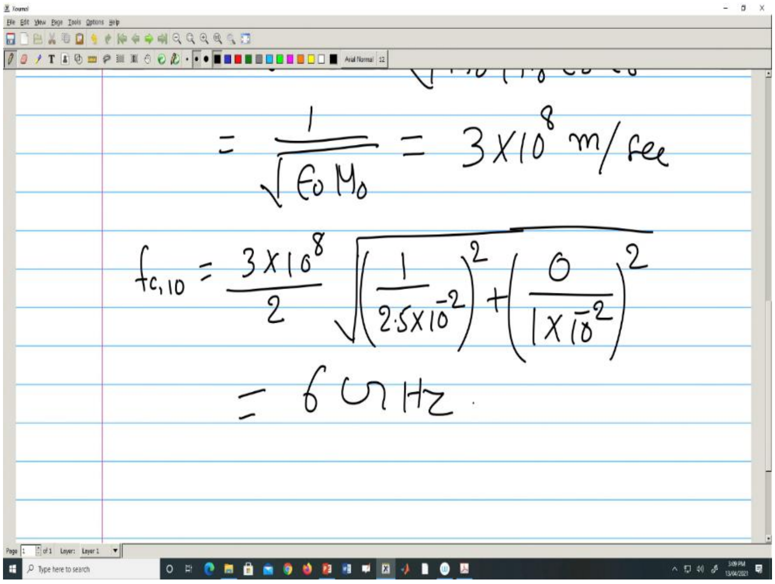

So, it will be

$$u = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}$$

$$= \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3\times10^8 \text{ m/sec}$$

So, from this we can calculate the cut-off frequency for TE_{10} mode.

$$f_{c,10} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{2.5\times10^{-2}}\right)^2+\left(\frac{0}{1\times10^{-2}}\right)^2}$$

$$= 6 \text{ GHz}$$

So, cut-off frequency for TE 10 mode will be 6 gigahertz.

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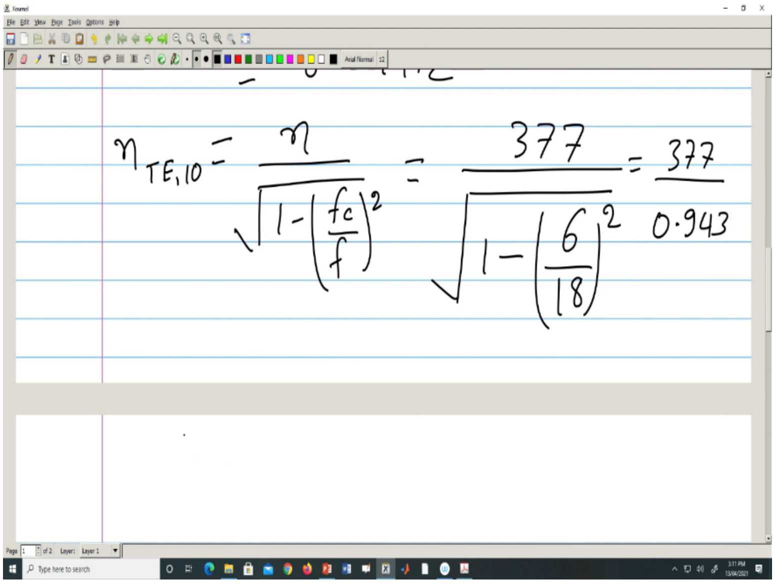

Similarly, cut-off frequency for this TM 11 mode,

$$\eta_{TE,10} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-\left(\frac{6}{18}\right)^2}} = \frac{377}{0.943}$$

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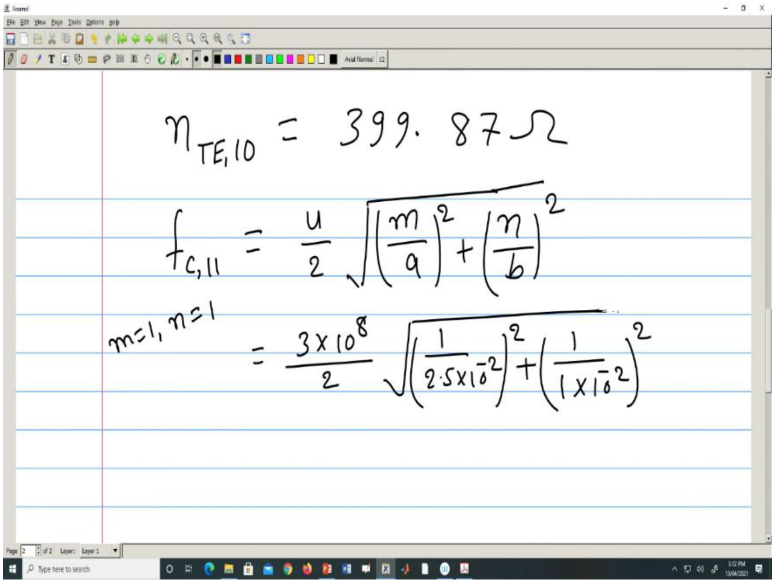

And answer for TE_{10} mode will be 399.87 ohm.

$$\eta_{TE,10} = 399.87\,\Omega$$

So, this is the answer for the first part that wave impedance for the TE_{10} mode.

Now, for the TM_{11}, again we have to calculate the cut-off frequency then we will get the wave impedance for the TM_{11} mode. For cut-off, to calculate the cut-off frequency, we can go by same method. So

$$f_{c,11} = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$m=1, n=1$

$$= \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{2.5\times10^{-2}}\right)^2 + \left(\frac{1}{1\times10^{-2}}\right)^2}$$

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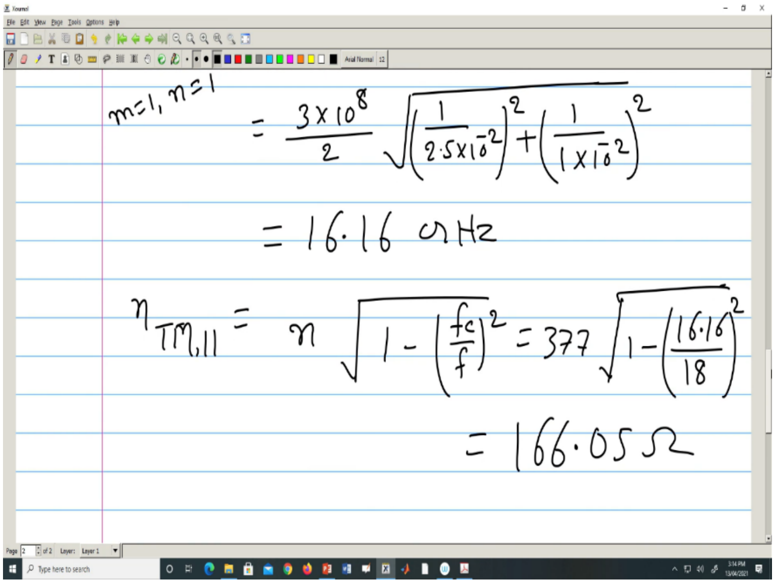
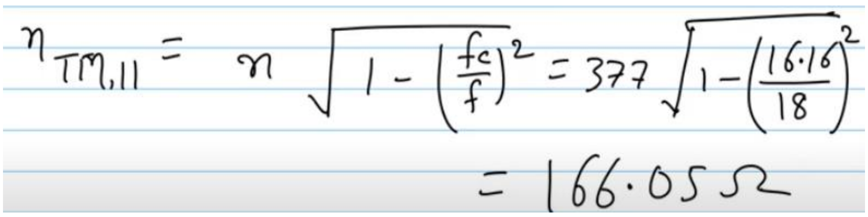


So, from this we will get the cut-off frequency for the TM_{11} mode, and this will come 16.16 gigahertz. From this, we will get the wave impedance actually. So,



So, we can see that the wave impedance for the TE_{10} mode is more so, and that is also greater than 377, that is wave impedance of the free space but for the TM mode it is less than free space wave impedance. So, this is the solution for the second part.

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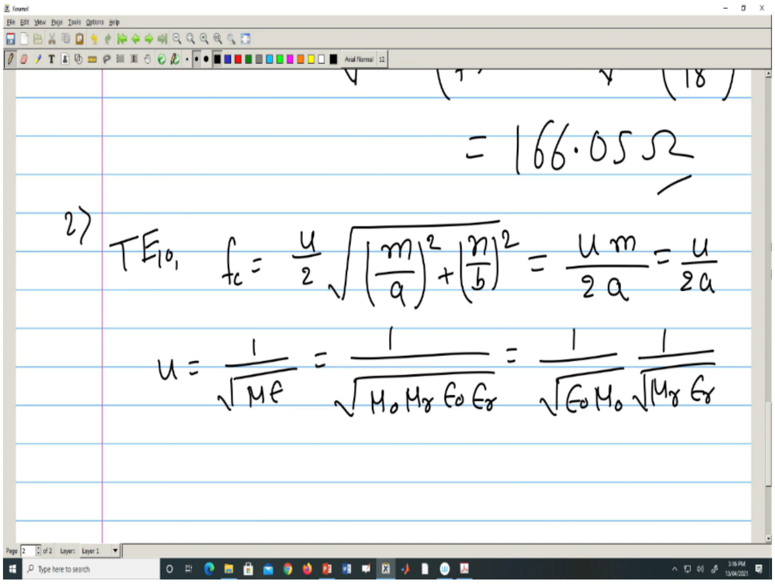
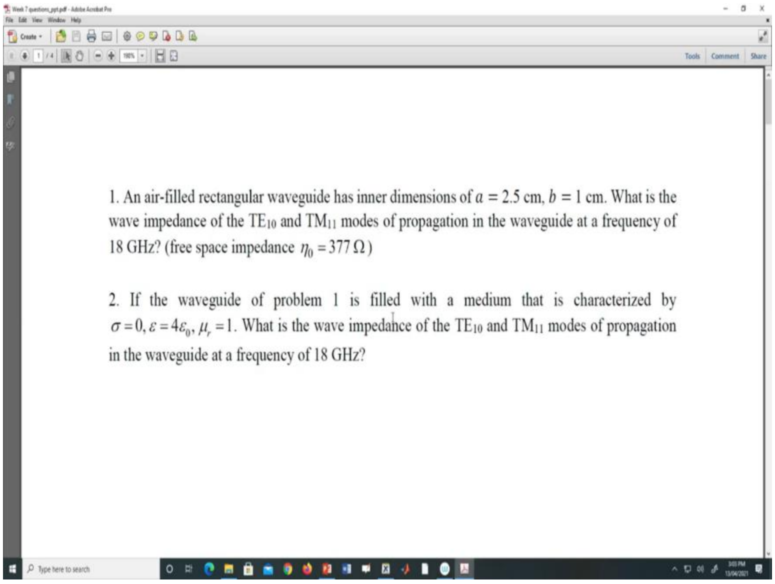
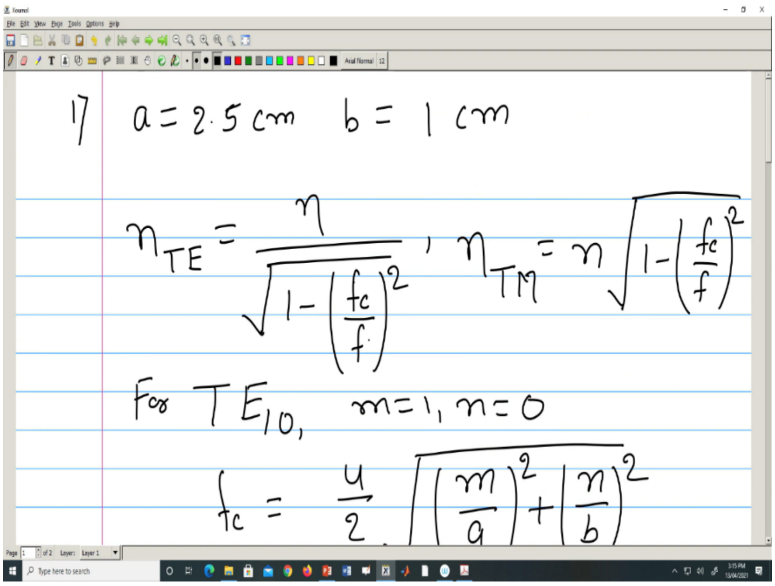




1. An air-filled rectangular waveguide has inner dimensions of $a = 2.5$ cm, $b = 1$ cm. What is the wave impedance of the TE_{10} and TM_{11} modes of propagation in the waveguide at a frequency of 18 GHz? (free space impedance $\eta_0 = 377\,\Omega$)

2. If the waveguide of problem 1 is filled with a medium that is characterized by $\sigma = 0, \varepsilon = 4\varepsilon_0, \mu_r = 1$. What is the wave impedance of the TE_{10} and TM_{11} modes of propagation in the waveguide at a frequency of 18 GHz?



$$\eta_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}, \quad \eta_{TM} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

For TE_{10}, $m=1, n=0$

$$f_c = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$$u = \frac{1}{\ldots} = \frac{1}{\ldots}$$

Now, we can go to the next second question.

1. An air-filled rectangular waveguide has inner dimensions of $a = 2.5$ cm, $b = 1$ cm. What is the wave impedance of the TE_{10} and TM_{11} modes of propagation in the waveguide at a frequency of 18 GHz? (free space impedance $\eta_0 = 377\,\Omega$)

2. If the waveguide of problem 1 is filled with a medium that is characterized by $\sigma = 0, \varepsilon = 4\varepsilon_0, \mu_r = 1$. What is the wave impedance of the TE_{10} and TM_{11} modes of propagation in the waveguide at a frequency of 18 GHz?

So, problem is same, just waveguide is filled with relative permittivity of 4. So, in the second part,

$$TE_{10}, \quad f_c = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{u\,m}{2a} = \frac{u}{2a}$$

$$u = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}} = \frac{1}{\sqrt{\epsilon_0\mu_0}}\,\frac{1}{\sqrt{\mu_r\epsilon_r}}$$

$$= 3\times10^8 \times \frac{1}{\sqrt{4}} = \frac{3}{2}\times10^8 = 1.5\times10^8 \text{ m/sec}$$

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$$u = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \frac{1}{\sqrt{\mu_r \epsilon_r}}$$

$$= 3\times10^8 \times \frac{1}{\sqrt{4}} = \frac{3}{2}\times10^8 = 1.5\times10^8 \ m/sec$$

$$f_{c,TE_{10}} = \frac{u}{2a} = \frac{1.5\times10^8}{2\times 2.5\times10^{-2}} = 3 \ GHz$$

So, Fc can be calculated, Fc will

$$f_{c,TE_{10}} = \frac{u}{2a} = \frac{1.5\times10^8}{2\times 2.5\times10^{-2}} = 3 \ GHz$$

$$\eta_{TE_{10}} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}, \quad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\eta = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \times \sqrt{\frac{\mu_r}{\epsilon_r}} = 120\pi \times \sqrt{\frac{1}{4}}$$

$$= 60\pi \ \Omega$$

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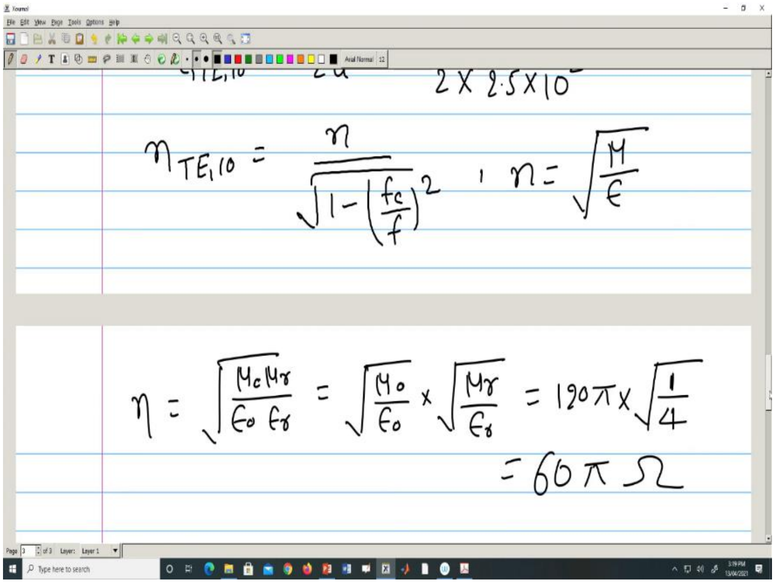

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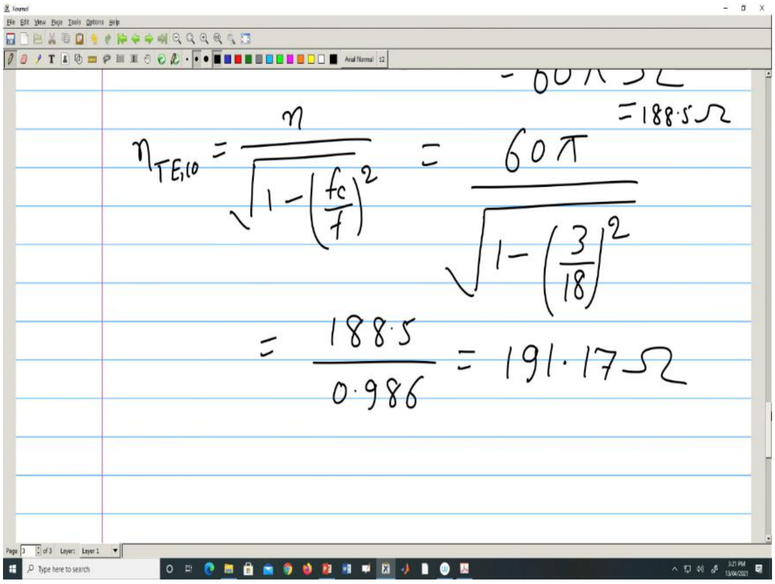

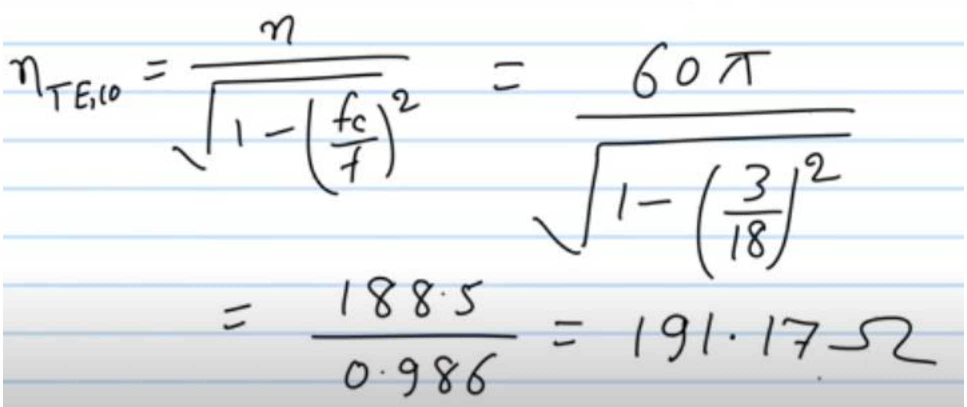

So, we can see that it is reduced, it is already reduced. So, 60 pi that means it is this 188.5. So, this 188.5 but it is for the TE it is more than eta. For TE 10 mode, it is more than eta, that 191.17.

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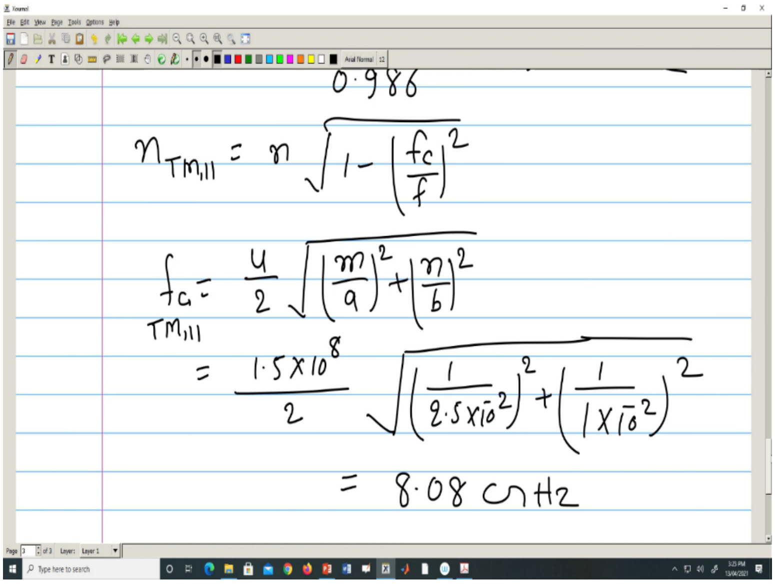
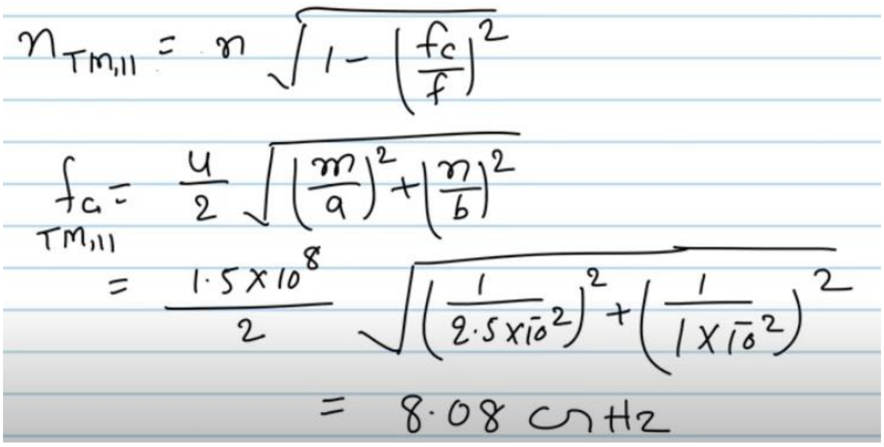


Similarly, we can calculate the wave impedance for the TM_{11} mode. For the TM_{11} mode, we know the formula



So after calculation it will come 8.08 Gigahertz.

So, this is the cut-off frequency for the TM_{11} mode.

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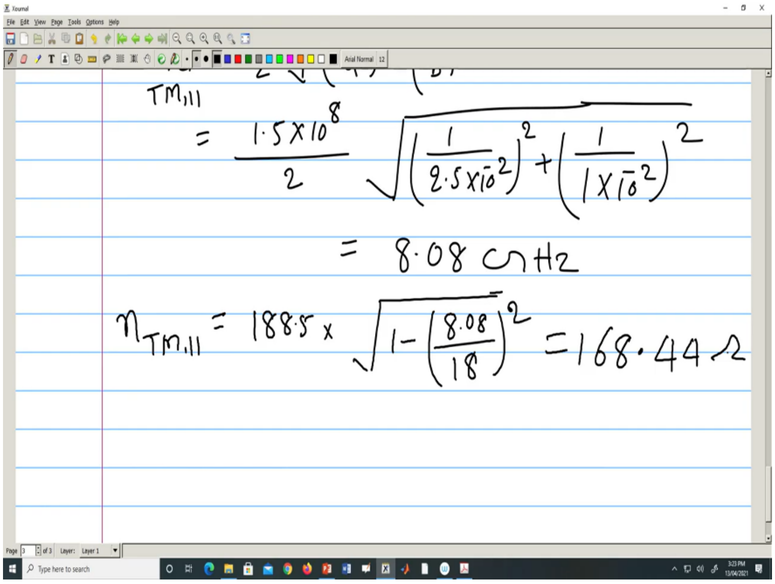
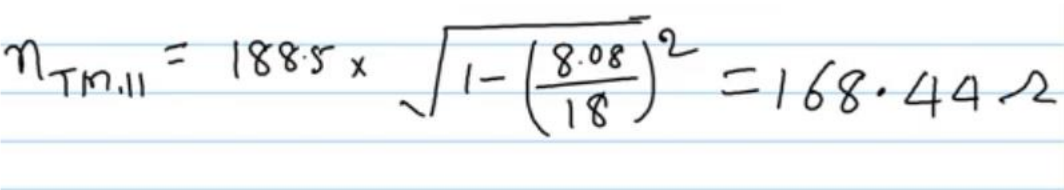


After this, so eta will also bw same like TE_{10} mode



So, this is the wave impedance for the TM_{11} mode.

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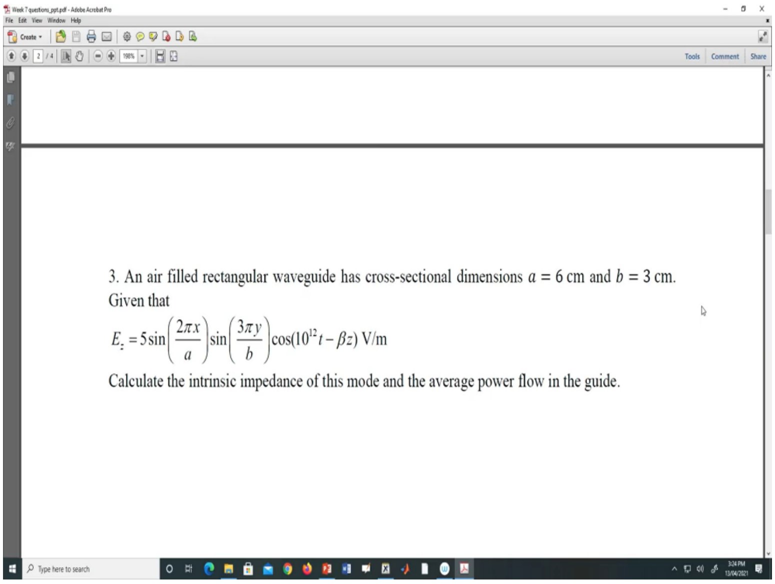
3. An air filled rectangular waveguide has cross-sectional dimensions a = 6 cm and b = 3 cm. Given that

$$E_z = 5\sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{3\pi y}{b}\right)\cos(10^{12}t - \beta z)\ \text{V/m}$$

Calculate the intrinsic impedance of this mode and the average power flow in the guide.

Now, we can go to the next problem.

3. An air filled rectangular waveguide has cross-sectional dimensions $a = 6$ cm and $b = 3$ cm. Given that

$$E_z = 5\sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{3\pi y}{b}\right)\cos(10^{12}t - \beta z)\ \text{V/m}$$

Calculate the intrinsic impedance of this mode and the average power flow in the guide.

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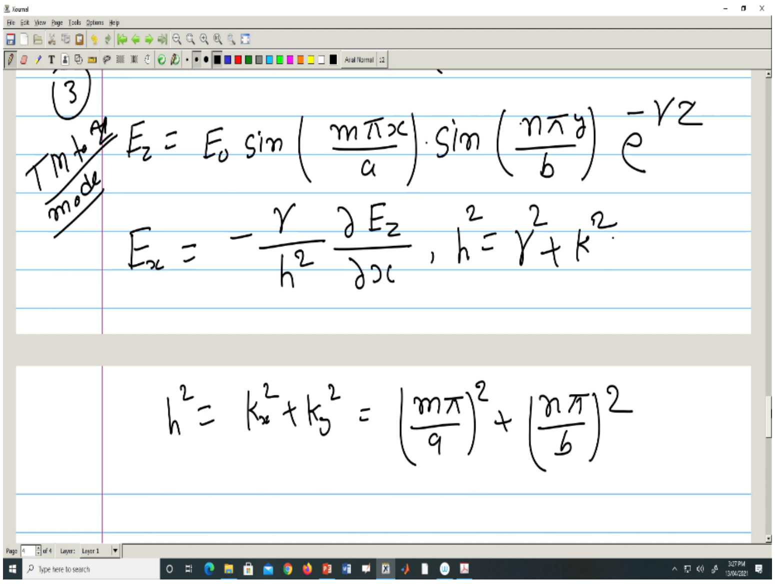

So, a standard equation it can be defined like this,

$$E_z = E_0 \sin\left(\frac{m\pi x}{a}\right) \cdot \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x}, \quad h^2 = \gamma^2 + k^2$$

This is for the TM to Z mode. So, for the given Ez,

$$E_x = \frac{-\gamma}{h^2}\left(\frac{m\pi}{a}\right) E_0 \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y}$$

$$= \frac{-\gamma}{h^2}\left(\frac{n\pi}{b}\right) E_0 \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

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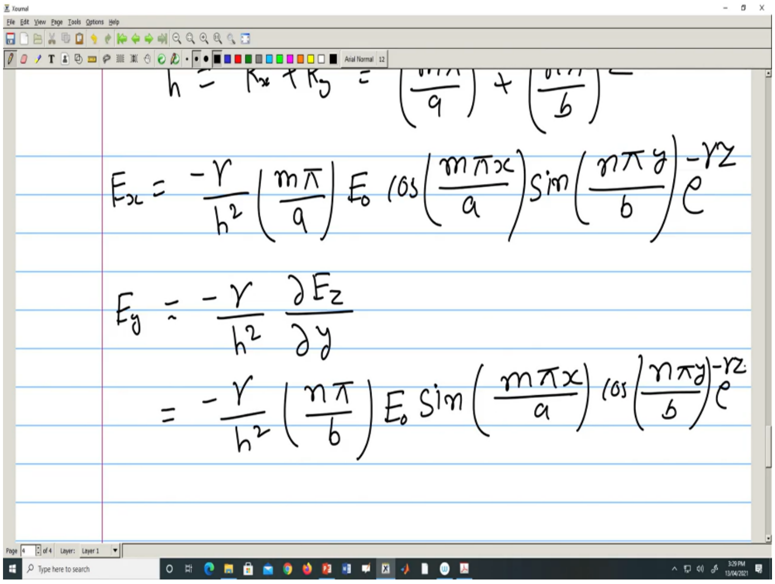
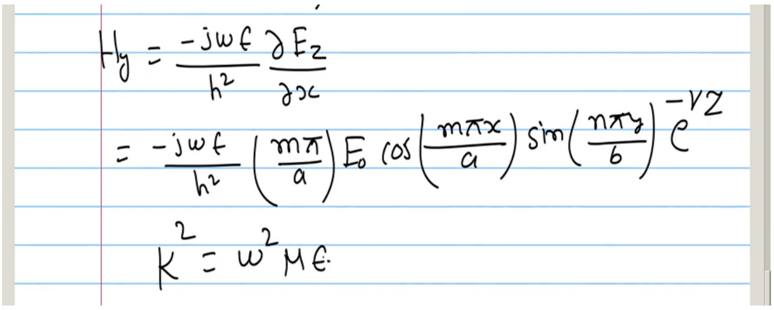


Again on calculation:

$$H_x = \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$= \frac{j\omega\epsilon}{h^2}\left(\frac{n\pi}{b}\right) E_0 \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$



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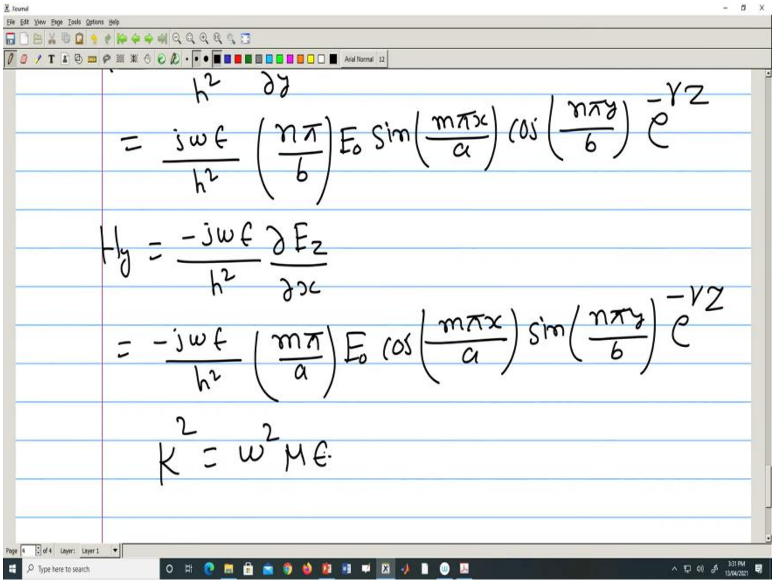

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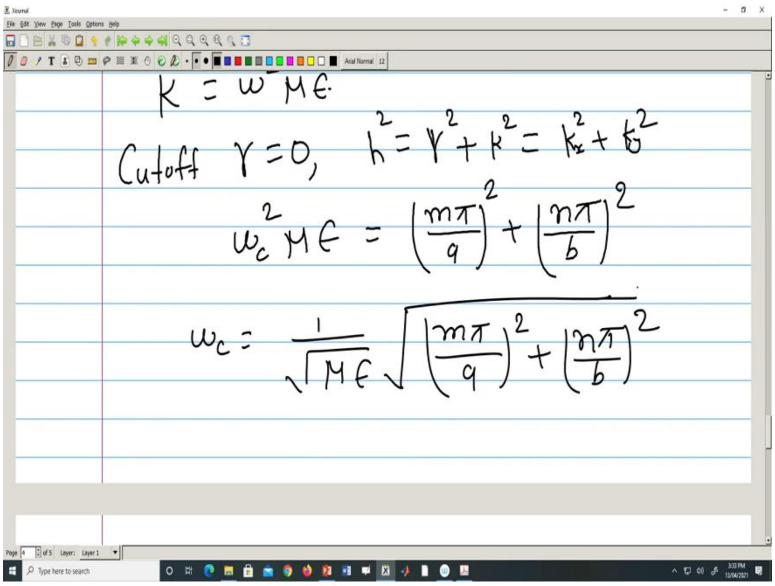
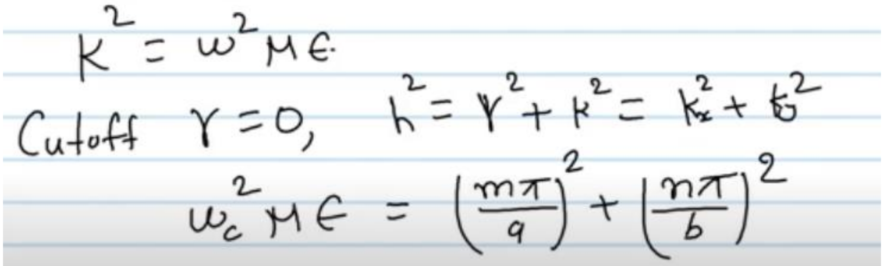


So, it can be means for the cut-off case means gamma will be 0 that is a gamma is a propagation constant in the z direction that is alpha plus j beta.



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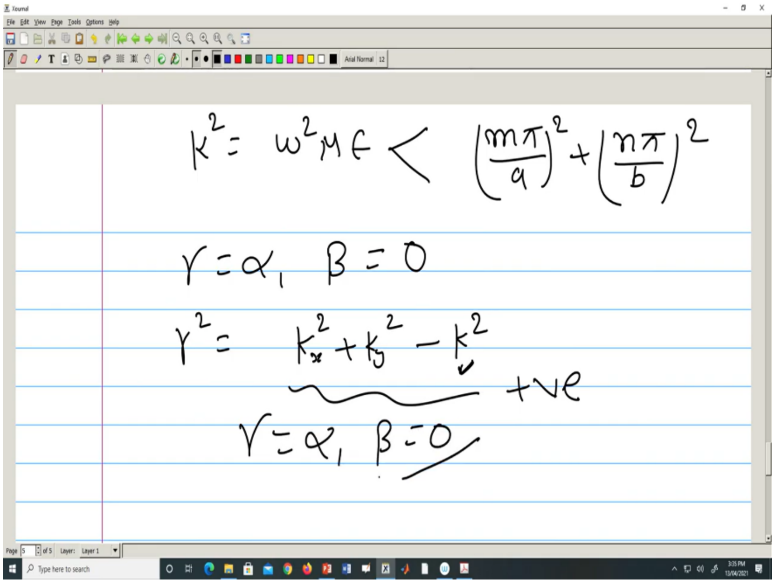
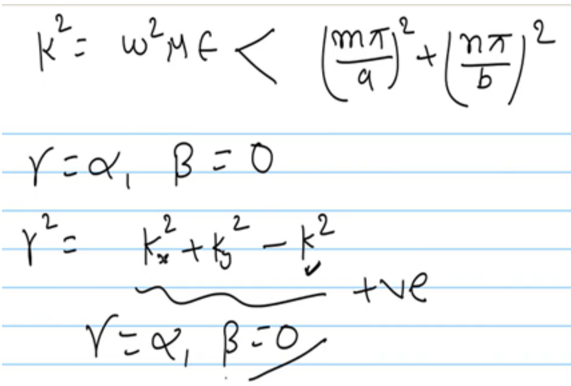


And in the second part,



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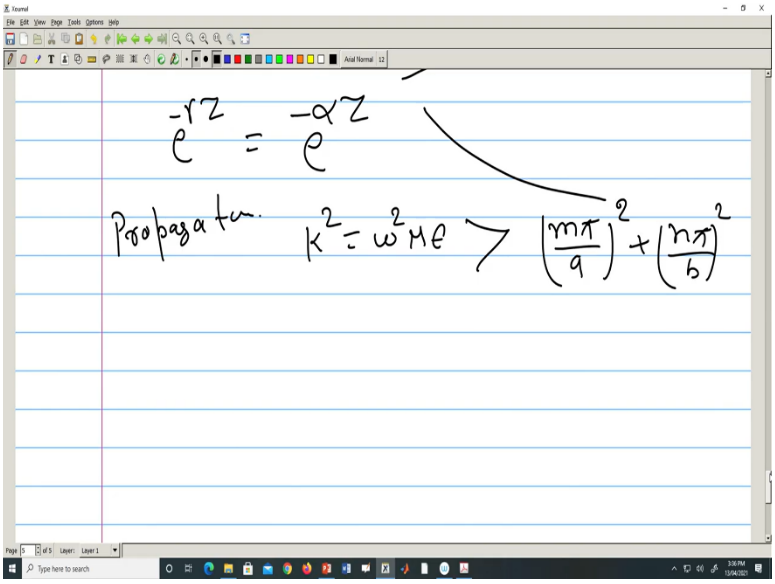
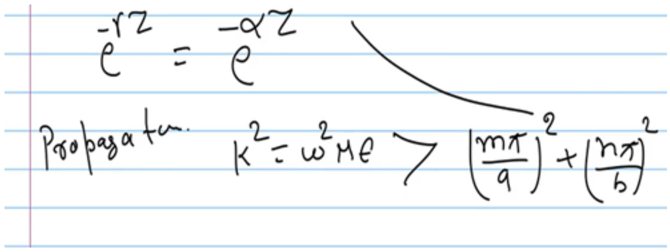


So, if alpha is some positive number so, it will get attenuated and it will not propagate inside the wave guide.

Next is the condition for the propagation



So, if k square is greater than this summation means kx square plus ky square that means, inside it will negative and this will be purely imaginary number.

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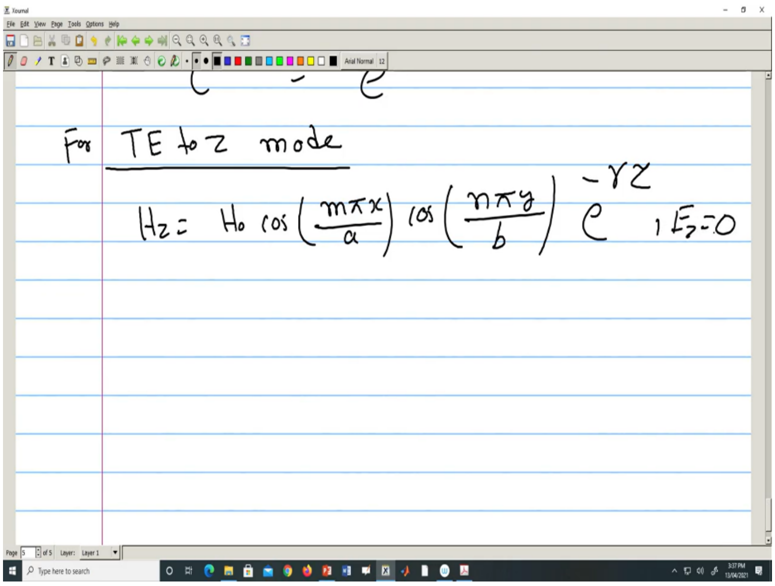

So, gamma will be purely imaginary and in that case gamma will be j beta and alpha will be 0. So, if gamma will be j beta and alpha will be 0, in that case we will get

$$\gamma = j\beta, \quad \alpha = 0$$

$$e^{-\gamma z} = e^{-j\beta z}$$

So, in this case it will not attenuate but it will propagate along the z direction. So, from this expression, we can compute the power flow in the waveguide.

Similarly, we can write this for the TM to Z mode.

for TE to z mode

$$H_z = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}, \quad E_z = 0$$

Okay we will continue in the next class. Thank you