

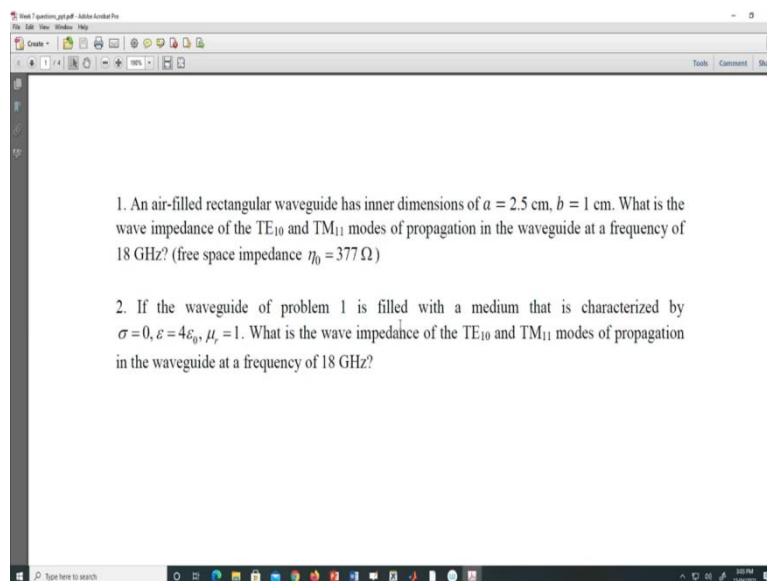
Advanced Microwave Guided-Structures and Analysis
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Lecture 36
Rectangular Waveguide II Tutorials

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Welcome to the next tutorial on the subject advanced microwave guided structures and analysis. So, this is the main set. Today we will do numerical problem on that calculation of wave impedance and power flow in rectangular waveguide.

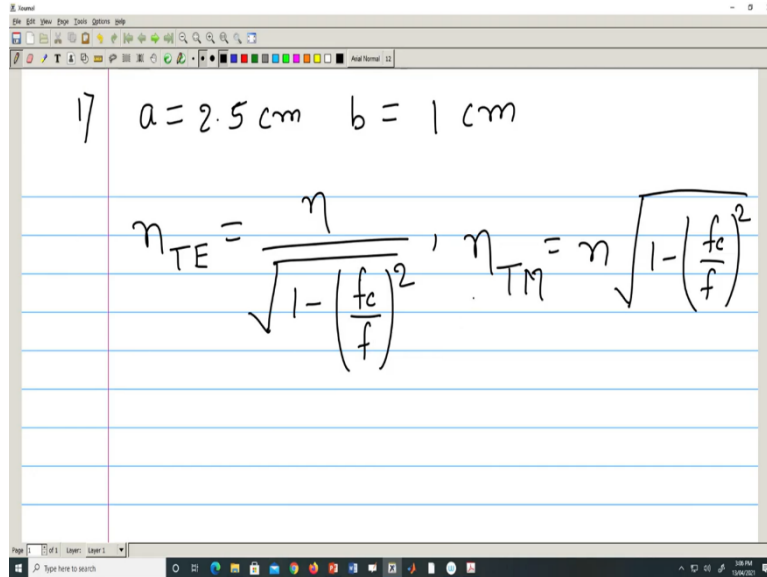
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So, this is the first problem.

1. An air-filled rectangular waveguide has inner dimensions of $a = 2.5 \text{ cm}$, $b = 1 \text{ cm}$. What is the wave impedance of the TE_{10} and TM_{11} modes of propagation in the waveguide at a frequency of 18 GHz? (free space impedance $\eta_0 = 377 \Omega$)

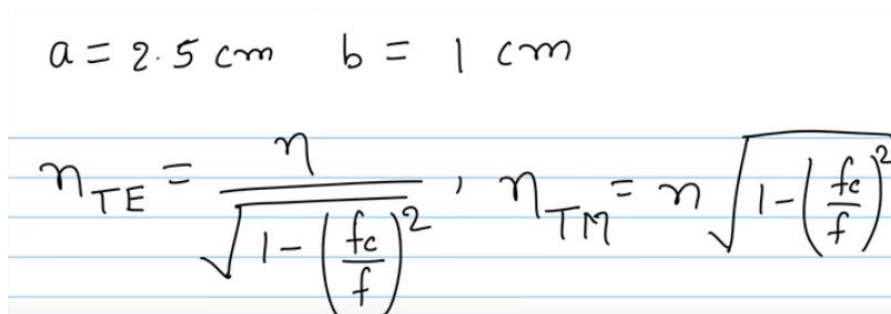
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17 $a = 2.5 \text{ cm}$ $b = 1 \text{ cm}$

$$\eta_{TE} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}, \quad \eta_{TM} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

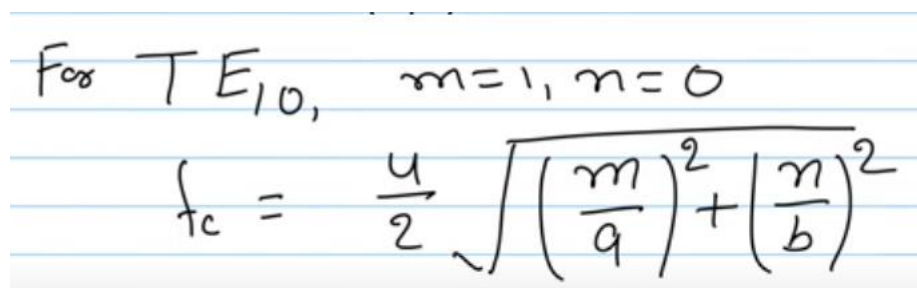
So, we have to calculate the wave impedance of TE_{10} and TM_{11} modes of propagation at 18 gigahertz. So, we know



$a = 2.5 \text{ cm}$ $b = 1 \text{ cm}$

$$\eta_{TE} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}, \quad \eta_{TM} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

So, here we need cut-off frequency, we know cut-off frequency for each mode is different.



For TE_{10} , $m=1, n=0$

$$f_c = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

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$$\sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad TM \quad \sqrt{f}$$

For TE_{10} , $m=1, n=0$

$$f_c = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$u = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}$$

So, for here for TE_{10} that means m equal to 1 and n equal to 0. So, we can substitute the value and we will get the cut-off frequency for the TE_{10} mode.

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The image shows a screenshot of a digital whiteboard with handwritten mathematical equations. The first equation is
$$= \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8 \text{ m/sec}$$
. The second equation is
$$f_{c,10} = \frac{3 \times 10^8}{2} \sqrt{\left(\frac{1}{2.5 \times 10^{-2}}\right)^2 + \left(\frac{0}{1 \times 10^{-2}}\right)^2}$$
. The final result is
$$= 6 \text{ GHz}$$
.

So, it will be

The image shows a handwritten derivation for phase velocity u . It starts with
$$u = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}$$
 and then simplifies to
$$= \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8 \text{ m/sec}$$
.

So, from this we can calculate the cut-off frequency for TE₁₀ mode.

The image shows a handwritten calculation for the cut-off frequency $f_{c,10}$. It is identical to the second equation in the first image:
$$f_{c,10} = \frac{3 \times 10^8}{2} \sqrt{\left(\frac{1}{2.5 \times 10^{-2}}\right)^2 + \left(\frac{0}{1 \times 10^{-2}}\right)^2}$$
 and
$$= 6 \text{ GHz}$$
.

So, cut-off frequency for TE 10 mode will be 6 gigahertz.

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$$\eta_{TE,10} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1 - \left(\frac{6}{18}\right)^2}} = \frac{377}{0.943}$$

Similarly, cut-off frequency for this TM 11 mode,

$$\eta_{TE,10} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1 - \left(\frac{6}{18}\right)^2}} = \frac{377}{0.943}$$

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$$\eta_{TE,10} = 399.87 \Omega$$

$$f_{c,11} = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$m=1, n=1 = \frac{3 \times 10^8}{2} \sqrt{\left(\frac{1}{2.5 \times 10^{-2}}\right)^2 + \left(\frac{1}{1 \times 10^{-2}}\right)^2}$$

And answer for TE₁₀ mode will be 399.87 ohm.

$$\eta_{TE,10} = 399.87 \Omega$$

So, this is the answer for the first part that wave impedance for the TE₁₀ mode.

Now, for the TM₁₁, again we have to calculate the cut-off frequency then we will get the wave impedance for the TM₁₁ mode. For cut-off, to calculate the cut-off frequency, we can go by same method. So

$$f_{c,11} = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$
$$m=1, n=1 \quad = \frac{3 \times 10^8}{2} \sqrt{\left(\frac{1}{2.5 \times 10^{-2}}\right)^2 + \left(\frac{1}{1 \times 10^{-2}}\right)^2}$$

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The image shows a screenshot of a digital whiteboard with handwritten mathematical equations. The first equation calculates the cut-off frequency for the TM_{11} mode, starting with $m=1, n=1$ and using the formula $f_c = \frac{3 \times 10^8}{2} \sqrt{\left(\frac{1}{2.5 \times 10^{-2}}\right)^2 + \left(\frac{1}{1 \times 10^{-2}}\right)^2}$, resulting in 16.16 GHz . The second equation calculates the wave impedance $\eta_{TM_{11}} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$ with $\eta = 377$ and $f = 18$, resulting in 166.05Ω .

$$m=1, n=1$$
$$= \frac{3 \times 10^8}{2} \sqrt{\left(\frac{1}{2.5 \times 10^{-2}}\right)^2 + \left(\frac{1}{1 \times 10^{-2}}\right)^2}$$
$$= 16.16 \text{ GHz}$$
$$\eta_{TM_{11}} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 377 \sqrt{1 - \left(\frac{16.16}{18}\right)^2}$$
$$= 166.05 \Omega$$

So, from this we will get the cut-off frequency for the TM_{11} mode, and this will come 16.16 gigahertz. From this, we will get the wave impedance actually. So,

The image shows a screenshot of a digital whiteboard with handwritten mathematical equations. The equation calculates the wave impedance $\eta_{TM_{11}} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$ with $\eta = 377$ and $f = 18$, resulting in 166.05Ω .

$$\eta_{TM_{11}} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 377 \sqrt{1 - \left(\frac{16.16}{18}\right)^2}$$
$$= 166.05 \Omega$$

So, we can see that the wave impedance for the TE_{10} mode is more so, and that is also greater than 377, that is wave impedance of the free space but for the TM mode it is less than free space wave impedance. So, this is the solution for the second part.

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$$= 166.05 \Omega$$

$$2) \text{ TE}_{10}, f_c = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{u m}{2a} = \frac{u}{2a}$$

$$u = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \frac{1}{\sqrt{\mu_r \epsilon_r}}$$

1. An air-filled rectangular waveguide has inner dimensions of $a = 2.5$ cm, $b = 1$ cm. What is the wave impedance of the TE_{10} and TM_{11} modes of propagation in the waveguide at a frequency of 18 GHz? (free space impedance $\eta_0 = 377 \Omega$)

2. If the waveguide of problem 1 is filled with a medium that is characterized by $\sigma = 0$, $\epsilon = 4\epsilon_0$, $\mu_r = 1$. What is the wave impedance of the TE_{10} and TM_{11} modes of propagation in the waveguide at a frequency of 18 GHz?

1) $a = 2.5$ cm $b = 1$ cm

$$\eta_{\text{TE}} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}, \quad \eta_{\text{TM}} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

For TE_{10} , $m=1, n=0$

$$f_c = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\eta_{TE} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}, \quad \eta_{TM} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

For TE_{10} , $m=1, n=0$

$$f_c = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$u = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}}$$

Now, we can go to the next second question.

1. An air-filled rectangular waveguide has inner dimensions of $a = 2.5$ cm, $b = 1$ cm. What is the wave impedance of the TE_{10} and TM_{11} modes of propagation in the waveguide at a frequency of 18 GHz? (free space impedance $\eta_0 = 377 \Omega$)

2. If the waveguide of problem 1 is filled with a medium that is characterized by $\sigma = 0$, $\epsilon = 4\epsilon_0$, $\mu_r = 1$. What is the wave impedance of the TE_{10} and TM_{11} modes of propagation in the waveguide at a frequency of 18 GHz?

So, problem is same, just waveguide is filled with relative permittivity of 4. So, in the second part,

$$TE_{10}, \quad f_c = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{u m}{2 a} = \frac{u}{2 a}$$

$$u = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}} = \frac{1}{\sqrt{\epsilon_0\mu_0}} \frac{1}{\sqrt{\mu_r\epsilon_r}}$$

$$= 3 \times 10^8 \times \frac{1}{\sqrt{4}} = \frac{3}{2} \times 10^8 = 1.5 \times 10^8 \text{ m/sec}$$

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$$u = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}} = \frac{1}{\sqrt{\epsilon_0\mu_0}\sqrt{\mu_r\epsilon_r}}$$

$$= 3 \times 10^8 \times \frac{1}{\sqrt{4}} = \frac{3}{2} \times 10^8 = 1.5 \times 10^8 \text{ m/sec}$$

$$f_{c,TE_{10}} = \frac{u}{2a} = \frac{1.5 \times 10^8}{2 \times 2.5 \times 10^{-2}} = 300 \text{ Hz}$$

So, f_c can be calculated, f_c will

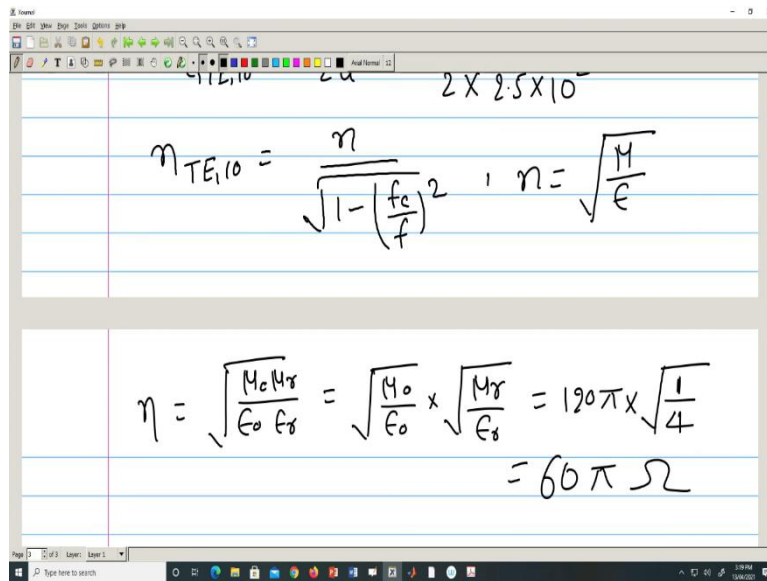
$$f_{c,TE_{10}} = \frac{u}{2a} = \frac{1.5 \times 10^8}{2 \times 2.5 \times 10^{-2}} = 300 \text{ Hz}$$

$$\eta_{TE_{10}} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}, \quad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\eta = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \times \sqrt{\frac{\mu_r}{\epsilon_r}} = 120\pi \times \sqrt{\frac{1}{4}}$$

$$= 60\pi \Omega$$

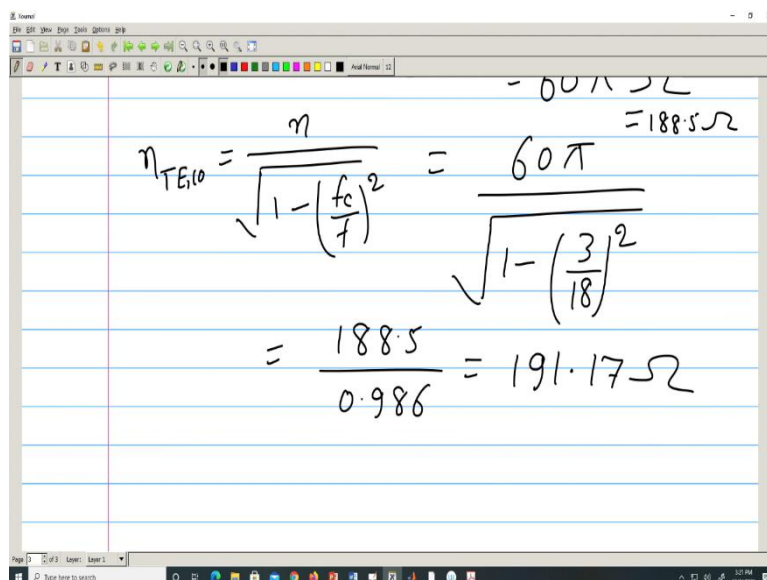
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The image shows a handwritten derivation of the TE₁₀ mode impedance. At the top, there are some faint notes: $\frac{1}{\sqrt{\epsilon_r}}$ and $2 \times 2.5 \times 10^{-2}$. The main derivation consists of two lines of equations:

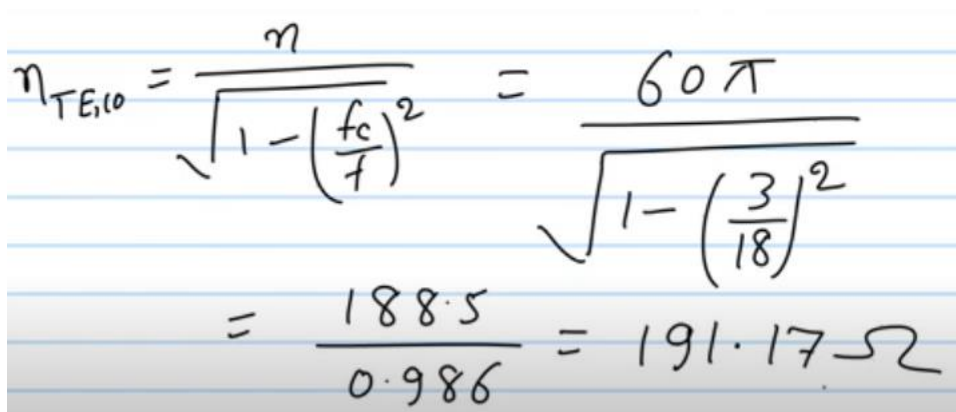
$$\eta_{TE_{10}} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \quad , \quad \eta = \sqrt{\frac{\mu}{\epsilon}}$$
$$\eta = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \times \sqrt{\frac{\mu_r}{\epsilon_r}} = 120\pi \times \sqrt{\frac{1}{4}} = 60\pi \Omega$$

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The image shows a handwritten calculation of the TE₁₀ mode impedance with numerical values. At the top, there are some faint notes: -0.01×2 and $= 188.5 \Omega$. The main calculation consists of three lines of equations:

$$\eta_{TE_{10}} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{60\pi}{\sqrt{1 - \left(\frac{3}{18}\right)^2}}$$
$$= \frac{188.5}{0.986} = 191.17 \Omega$$



The image shows a handwritten calculation of the TE₁₀ mode impedance with numerical values, identical to the previous one:

$$\eta_{TE_{10}} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{60\pi}{\sqrt{1 - \left(\frac{3}{18}\right)^2}}$$
$$= \frac{188.5}{0.986} = 191.17 \Omega$$

So, we can see that it is reduced, it is already reduced. So, 60 pi that means it is this 188.5. So, this 188.5 but it is for the TE it is more than eta. For TE 10 mode, it is more than eta, that 191.17.

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0.986

$$\eta_{TM_{11}} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$f_{c, TM_{11}} = \frac{4}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{1.5 \times 10^8}{2} \sqrt{\left(\frac{1}{2.5 \times 10^{-2}}\right)^2 + \left(\frac{1}{1 \times 10^{-2}}\right)^2}$$

$$= 8.08 \text{ GHz}$$

Similarly, we can calculate the wave impedance for the TM_{11} mode. For the TM_{11} mode, we know the formula

$$\eta_{TM_{11}} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$f_{c, TM_{11}} = \frac{4}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{1.5 \times 10^8}{2} \sqrt{\left(\frac{1}{2.5 \times 10^{-2}}\right)^2 + \left(\frac{1}{1 \times 10^{-2}}\right)^2}$$

$$= 8.08 \text{ GHz}$$

So after calculation it will come 8.08 Gigahertz.

So, this is the cut-off frequency for the TM_{11} mode.

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The image shows a screenshot of a digital whiteboard with handwritten mathematical derivations. The first part calculates the wave number $k_{TM_{11}}$ using the formula $k_{TM_{11}} = \frac{1.5 \times 10^8}{2} \sqrt{\left(\frac{1}{2.5 \times 10^{-2}}\right)^2 + \left(\frac{1}{1 \times 10^{-2}}\right)^2}$, resulting in 8.08 cm^{-1} . The second part calculates the intrinsic impedance $\eta_{TM_{11}} = 188.5 \times \sqrt{1 - \left(\frac{8.08}{18}\right)^2} = 168.44 \Omega$.

After this, so eta will also be same like TE_{10} mode

The image shows a handwritten equation on a whiteboard: $\eta_{TM_{11}} = 188.5 \times \sqrt{1 - \left(\frac{8.08}{18}\right)^2} = 168.44 \Omega$.

So, this is the wave impedance for the TM_{11} mode.

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The image shows a screenshot of a presentation slide with the following text: "3. An air filled rectangular waveguide has cross-sectional dimensions $a = 6 \text{ cm}$ and $b = 3 \text{ cm}$. Given that $E_z = 5 \sin\left(\frac{2\pi x}{a}\right) \sin\left(\frac{3\pi y}{b}\right) \cos(10^{12}t - \beta z) \text{ V/m}$. Calculate the intrinsic impedance of this mode and the average power flow in the guide."

Now, we can go to the next problem.

3. An air filled rectangular waveguide has cross-sectional dimensions $a = 6$ cm and $b = 3$ cm. Given that

$$E_z = 5 \sin\left(\frac{2\pi x}{a}\right) \sin\left(\frac{3\pi y}{b}\right) \cos(10^{12}t - \beta z) \text{ V/m}$$

Calculate the intrinsic impedance of this mode and the average power flow in the guide.

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(3)

$$E_z = E_0 \sin\left(\frac{m\pi x}{a}\right) \cdot \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$E_x = -\frac{\gamma}{h^2} \frac{\partial E_z}{\partial x}, \quad h^2 = \gamma^2 + k^2$$

$$h^2 = k_x^2 + k_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

So, a standard equation it can be defined like this,

$$E_z = E_0 \sin\left(\frac{m\pi x}{a}\right) \cdot \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$E_x = -\frac{\gamma}{h^2} \frac{\partial E_z}{\partial x}, \quad h^2 = \gamma^2 + k^2$$

This is for the TM to Z mode. So, for the given E_z ,

$$E_{zx} = -\frac{\gamma}{h^2} \left(\frac{m\pi}{a} \right) E_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$\begin{aligned} E_y &= -\frac{\gamma}{h^2} \frac{\partial E_z}{\partial y} \\ &= -\frac{\gamma}{h^2} \left(\frac{n\pi}{b} \right) E_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \end{aligned}$$

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$h = \sqrt{k_x^2 + k_y^2} = \left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2$
 $E_{zx} = -\frac{\gamma}{h^2} \left(\frac{m\pi}{a} \right) E_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$
 $E_y = -\frac{\gamma}{h^2} \frac{\partial E_z}{\partial y}$
 $= -\frac{\gamma}{h^2} \left(\frac{n\pi}{b} \right) E_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$

Again on calculation:

$$\begin{aligned} H_x &= \frac{j\omega\epsilon}{h^2} \frac{\partial E_z}{\partial y} \\ &= \frac{j\omega\epsilon}{h^2} \left(\frac{n\pi}{b} \right) E_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \end{aligned}$$

$$H_y = \frac{-j\omega\epsilon}{h^2} \frac{\partial E_z}{\partial x}$$

$$= \frac{-j\omega\epsilon}{h^2} \left(\frac{m\pi}{a}\right) E_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$K^2 = \omega^2 \mu\epsilon$$

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$$h^2 \frac{\partial^2}{\partial y^2}$$

$$= \frac{j\omega\epsilon}{h^2} \left(\frac{n\pi}{b}\right) E_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_y = \frac{-j\omega\epsilon}{h^2} \frac{\partial E_z}{\partial x}$$

$$= \frac{-j\omega\epsilon}{h^2} \left(\frac{m\pi}{a}\right) E_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$K^2 = \omega^2 \mu\epsilon$$

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$$K = \omega \sqrt{\mu\epsilon}$$

$$\text{Cutoff } \gamma = 0, \quad h^2 = \gamma^2 + K^2 = k_x^2 + k_y^2$$

$$\omega_c^2 \mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\omega_c = \frac{1}{\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

So, it can be means for the cut-off case means gamma will be 0 that is a gamma is a propagation constant in the z direction that is alpha plus j beta.

$$k^2 = \omega^2 \mu \epsilon$$

Cutoff $\gamma = 0$, $h^2 = \gamma^2 + k^2 = k_x^2 + k_y^2$

$$\omega_c^2 \mu \epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

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A screenshot of a digital whiteboard interface. The whiteboard contains the following handwritten text:

$$k^2 = \omega^2 \mu \epsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma = \alpha, \beta = 0$$

$$\gamma^2 = \underbrace{k_x^2 + k_y^2}_{\text{+ve}} - k^2$$

$$\underline{\gamma = \alpha, \beta = 0}$$

And in the second part,

$$k^2 = \omega^2 \mu \epsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma = \alpha, \beta = 0$$

$$\gamma^2 = \underbrace{k_x^2 + k_y^2}_{\text{+ve}} - k^2$$

$$\underline{\gamma = \alpha, \beta = 0}$$

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$$e^{-\gamma z} = e^{-\alpha z}$$

Propagation $k^2 = \omega^2 \mu \epsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

So, if alpha is some positive number so, it will get attenuated and it will not propagate inside the wave guide.

Next is the condition for the propagation

$$e^{-\gamma z} = e^{-\alpha z}$$

Propagation $k^2 = \omega^2 \mu \epsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

So, if k square is greater than this summation means k_x square plus k_y square that means, inside it will negative and this will be purely imaginary number.

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$$H_z = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \quad E_z = 0$$

So, gamma will be purely imaginary and in that case gamma will be j beta and alpha will be 0. So, if gamma will be j beta and alpha will be 0, in that case we will get

$$\gamma = j\beta, \quad \alpha = 0$$

$$e^{-\gamma z} = e^{-j\beta z}$$

So, in this case it will not attenuate but it will propagate along the z direction. So, from this expression, we can compute the power flow in the waveguide.

Similarly, we can write this for the TM to Z mode.

For TE to Z mode

$$H_z = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}, \quad E_z = 0$$

Okay we will continue in the next class. Thank you