

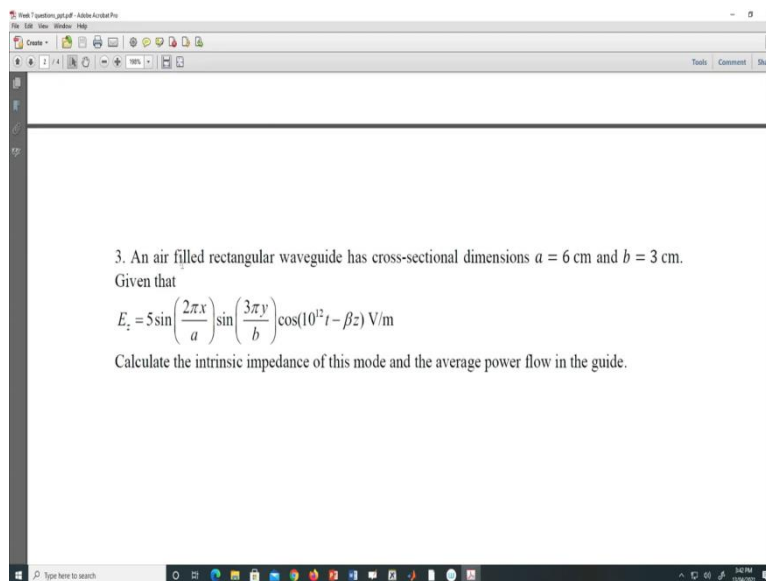
**Advanced Microwave Guided-Structures and Analysis**  
**Professor Bratin Ghosh**  
**Department of Electronics & Electrical Communication Engineering**  
**Indian Institute of Technology, Kharagpur**  
**Lecture 37**  
**Rectangular Waveguide II Tutorials (contd.)**

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Welcome to the next tutorial class. So, that is on the power flow in a rectangular waveguide. We will continue from the previous class.

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So, we have for the TE to Z mode. So, our problem is an air-filled rectangular waveguide has cross-sectional dimensions, a is equal to 6 and b is equal to 3. So, here  $E_z$  is given, so that

means it is for TM to Z mode because here electric field is given,  $E_z$  is given that means no magnetic field is given So,  $H_z$  will be 0 and this is for the TM to Z mode. So, we have computed that field expression for the TE to Z mode, TM to Z mode.

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The image shows a screenshot of a digital whiteboard with the following handwritten equations:

$$H_z = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}, \quad E_z = 0$$

$$E_x = -\frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y}$$

$$= \frac{j\omega\mu}{h^2} \frac{n\pi}{b} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

And here, we can calculate also for the TE to Z mode. For the TE to Z mode,  $H_z$  will be this. So, this is a standard format for the  $H_z$ .

The image shows a screenshot of a digital whiteboard with the following handwritten equations:

$$H_z = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}, \quad E_z = 0$$

$$E_x = -\frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y}$$

$$= \frac{j\omega\mu}{h^2} \frac{n\pi}{b} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

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$$= \frac{j\omega\mu}{h^2} \frac{n\pi}{b} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$E_y = \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

$$= -\frac{j\omega\mu}{h^2} \frac{m\pi}{a} H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

So, this is for the  $E_z$  and  $E_y$  will be

$$E_y = \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

$$= -\frac{j\omega\mu}{h^2} \frac{m\pi}{a} H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

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$$= -\frac{j\omega\mu}{h^2} \frac{m\pi}{a} H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_x = -\frac{\gamma}{h^2} \frac{\partial H_z}{\partial x}$$

$$= \frac{\gamma}{h^2} \left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

Similarly, we can calculate the  $H_x$  and  $H_y$ .  $H_x$  will be minus of gamma h square. This is for the TE to Z mode. So, in terms of  $H_z$ , all the field expression will come.

$$H_x = -\gamma \frac{\partial H_z}{\partial x}$$

$$= \frac{\gamma}{h^2} \left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_y = -\gamma \frac{\partial H_z}{\partial y}$$

$$= \frac{\gamma}{h^2} \left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

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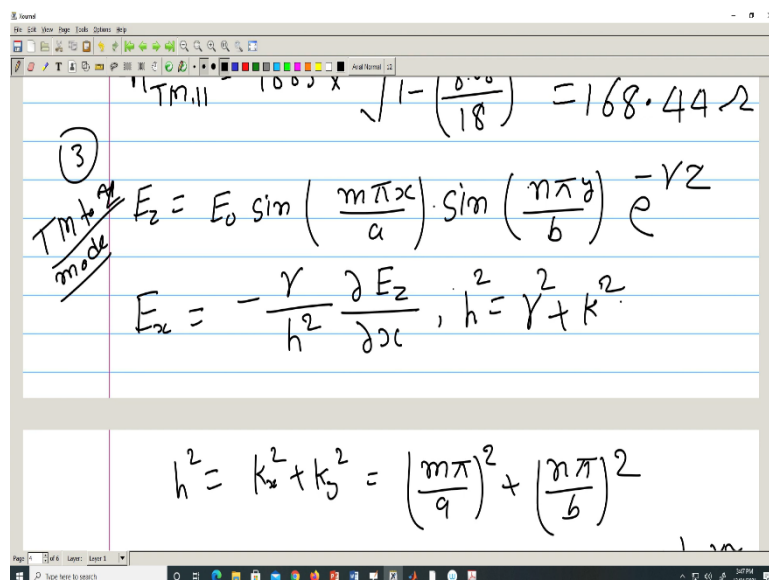
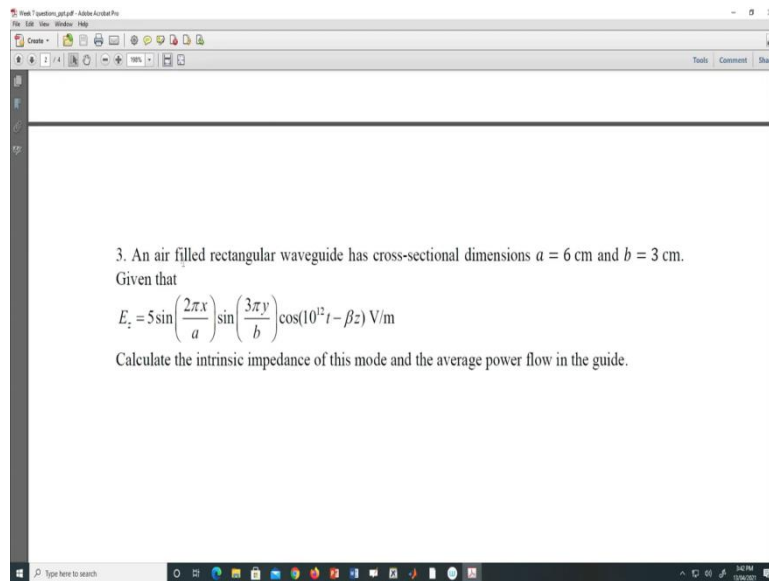
The screenshot shows a presentation slide with the following mathematical derivations:

$$H_x = -\gamma \frac{\partial H_z}{\partial x}$$

$$= \frac{\gamma}{h^2} \left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_y = -\gamma \frac{\partial H_z}{\partial y}$$

$$= \frac{\gamma}{h^2} \left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$



Now, we can come to the problem. In the problem it is given,  $E_z$  is equal

$$E_z = 5 \sin\left(\frac{2\pi x}{a}\right) \sin\left(\frac{3\pi y}{b}\right) \cos(10^{12} t - \beta z) \text{ V/m}$$

We can compare this with our standard equation

$$E_z = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

So, after comparing from this, we will get  $m$  equal to 2 and  $n$  is equal to 3 and because  $E_z$  is given, so this will be so  $TM_{23}$  mode.

So, this will be TM<sub>23</sub> and calculate the, first we will calculate the intrinsic impedance of this mode. So, we identified the mode that is TM<sub>23</sub> and for TM<sub>23</sub> mode, we will calculate the wave impedance, intrinsic wave impedance for TM<sub>23</sub>.

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Handwritten notes on a digital whiteboard:

TM<sub>23</sub> mode

$$\eta_{TM} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$f_{c,23} = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3 \times 10^8}{2} \sqrt{\left(\frac{2}{6 \times 10^{-2}}\right)^2 + \left(\frac{3}{3 \times 10^{-2}}\right)^2}$$

$m=2$   
 $n=3$

$$f_{c,23} = 15.81 \text{ GHz}$$


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3. An air filled rectangular waveguide has cross-sectional dimensions  $a = 6 \text{ cm}$  and  $b = 3 \text{ cm}$ . Given that

$$E_z = 5 \sin\left(\frac{2\pi x}{a}\right) \sin\left(\frac{3\pi y}{b}\right) \cos(10^{12} t - \beta z) \text{ V/m}$$

Calculate the intrinsic impedance of this mode and the average power flow in the guide.

For TM<sub>23</sub> mode, we will calculate

TM<sub>23</sub> mode

$$\eta_{TM} = n \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$f_{c,23} = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3 \times 10^8}{2} \sqrt{\left(\frac{2}{6 \times 10^{-2}}\right)^2 + \left(\frac{3}{3 \times 10^{-2}}\right)^2}$$

$m=2$   
 $n=3$

$$f_{c,23} = 15.81 \text{ GHz}$$

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$f_{c,23} = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3 \times 10^8}{2} \sqrt{\left(\frac{2}{6 \times 10^{-2}}\right)^2 + \left(\frac{3}{3 \times 10^{-2}}\right)^2}$

$m=2$   
 $n=3$

$$f_{c,23} = 15.81 \text{ GHz}$$

$$\omega = 10^{12}, \quad 2\pi f = 10^{12}$$

$$f = \frac{10^{12}}{2\pi} \text{ Hz} = 159.2 \text{ GHz}$$

So, after calculation we will get  $f_c$  for TM<sub>23</sub>, that is, 15.81 gigahertz. So, now

$$\omega = 10^{12}, \quad 2\pi f = 10^{12}$$

$$f = \frac{10^{12}}{2\pi} \text{ Hz} = 159.2 \text{ GHz}$$

So, this will come 159.2 gigahertz. So, this is in the millimetre wavelength.

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$$\eta_{TM_{23}} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 377 \sqrt{1 - \left(\frac{15.81}{159.2}\right)^2}$$

$$= 375.1 \Omega$$

$$\eta_{TM_{23}} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 375.1 \Omega$$

$$P_{ave} = \frac{1}{2} \iint \operatorname{Re}(\vec{E} \times \vec{H}^*) \cdot d\vec{S}$$

$$\vec{E} \times \vec{H}^* = \begin{matrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ E_x & E_y & E_z \end{matrix}$$

3. An air filled rectangular waveguide has cross-sectional dimensions  $a = 6$  cm and  $b = 3$  cm. Given that

$$E_z = 5 \sin\left(\frac{2\pi x}{a}\right) \sin\left(\frac{3\pi y}{b}\right) \cos(10^{12}t - \beta z) \text{ V/m}$$

Calculate the intrinsic impedance of this mode and the average power flow in the guide.

So, for this we can calculate the eta.



$$\eta_{TM_{23}} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 377 \sqrt{1 - \left(\frac{15.81}{159.2}\right)^2}$$

$$= 375.1 \Omega$$

This will give 375.1 Ohm. So, this is the solution for the first part.

Now, we have to calculate the average power flow in the guide, for this mode. That is also more dependant. So to determine the power average flow in the guide, we have to use

$$P_{ave} = \frac{1}{2} \iint \operatorname{Re}(\vec{E} \times \vec{H}^*) \cdot d\vec{S}$$

So, first we can calculate

$$\vec{E} \times \vec{H}^* = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ E_x & E_y & E_z \\ H_x^* & H_y^* & 0 \end{vmatrix}$$

$$= \hat{a}_x (0 - E_z H_y^*) + \hat{a}_y (E_z H_x^* - 0) + \hat{a}_z (E_x H_y^* - E_y H_x^*)$$

$$dS = dx dy \hat{a}_z$$

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The image shows a presentation slide with a white background and blue horizontal lines. The content is handwritten in black ink. At the top, it shows the curl of a vector field  $\vec{E} \times \vec{H}^*$  as a determinant of a 3x3 matrix. The first row contains the unit vectors  $\hat{a}_x$ ,  $\hat{a}_y$ , and  $\hat{a}_z$ . The second row contains the components  $E_x$ ,  $E_y$ , and  $E_z$ . The third row contains the components  $H_x^*$ ,  $H_y^*$ , and  $0$ . Below the determinant, the expansion is shown as the sum of three terms:  $\hat{a}_x(0 - E_z H_y^*) + \hat{a}_y(E_z H_x^* - 0) + \hat{a}_z(E_x H_y^* - E_y H_x^*)$ . At the bottom, the differential surface element is given as  $dS = dx dy \hat{a}_z$ . The slide is framed by a software interface with a menu bar at the top and a taskbar at the bottom.

$$\vec{E} \times \vec{H}^* = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ E_x & E_y & E_z \\ H_x^* & H_y^* & 0 \end{vmatrix}$$
$$= \hat{a}_x(0 - E_z H_y^*) + \hat{a}_y(E_z H_x^* - 0) + \hat{a}_z(E_x H_y^* - E_y H_x^*)$$
$$dS = dx dy \hat{a}_z$$

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$$P_{ave} = \frac{1}{2} \int_a^b \int_0^L \operatorname{Re} (E_x H_y^* - E_y H_x^*) \hat{a}_z \cdot dx \cdot dy \cdot \hat{a}_z$$

$$\eta = \frac{E_x}{H_y}, \quad \eta = -\frac{E_y}{H_x}$$

$$= \frac{1}{2} \int_a^b \int_0^L \frac{|E_x|^2 + |E_y|^2}{\eta} dy \cdot dx.$$

Power average will be

$$P_{ave} = \frac{1}{2} \int_a^b \int_0^L \operatorname{Re} (E_x H_y^* - E_y H_x^*) \hat{a}_z \cdot dx \cdot dy \cdot \hat{a}_z$$

$$\eta = \frac{E_x}{H_y}, \quad \eta = -\frac{E_y}{H_x}$$

$$= \frac{1}{2} \int_a^b \int_0^L \frac{|E_x|^2 + |E_y|^2}{\eta} dy \cdot dx.$$

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$$E_x = -\frac{j\beta}{h^2} \left( \frac{m\pi}{a} \right) E_0 \cos \left( \frac{2\pi x}{a} \right) \sin \left( \frac{3\pi y}{b} \right) e^{-j\beta z}$$

$$E_y = -\frac{j\beta}{h^2} \left( \frac{n\pi}{b} \right) E_0 \sin \left( \frac{2\pi x}{a} \right) \cos \left( \frac{3\pi y}{b} \right) e^{-j\beta z}$$

So,  $E_x$  and  $E_y$

$$E_x = -\frac{j\beta}{h^2} \left( \frac{m\pi}{a} \right) E_0 \cos\left(\frac{2\pi x}{a}\right) \sin\left(\frac{3\pi y}{b}\right) e^{-j\beta z}$$

$$E_y = -\frac{j\beta}{h^2} \left( \frac{n\pi}{b} \right) E_0 \sin\left(\frac{2\pi x}{a}\right) \cos\left(\frac{3\pi y}{b}\right) e^{-j\beta z}$$

$$P_{ave} = \frac{1}{2\eta_{TM}} \frac{\beta^2 E_0^2}{h^4} \int_{x=0}^a \int_{y=0}^b \left[ \left( \frac{2\pi}{a} \right)^2 \cos^2\left(\frac{2\pi x}{a}\right) \sin^2\left(\frac{3\pi y}{b}\right) + \left( \frac{3\pi}{b} \right)^2 \sin^2\left(\frac{2\pi x}{a}\right) \cos^2\left(\frac{3\pi y}{b}\right) \right] dy \cdot dx$$

$$\int_{x=0}^a \int_{y=0}^b \cos^2\left(\frac{m\pi x}{a}\right) \sin^2\left(\frac{n\pi y}{b}\right) \cdot dy \cdot dx = \frac{ab}{4}$$

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The screenshot shows a presentation slide with the following content:

$$P_{ave} = \frac{1}{2\eta_{TM}} \frac{\beta^2 E_0^2}{h^4} \int_{x=0}^a \int_{y=0}^b \left[ \left( \frac{2\pi}{a} \right)^2 \cos^2\left(\frac{2\pi x}{a}\right) \sin^2\left(\frac{3\pi y}{b}\right) + \left( \frac{3\pi}{b} \right)^2 \sin^2\left(\frac{2\pi x}{a}\right) \cos^2\left(\frac{3\pi y}{b}\right) \right] dy \cdot dx$$

$$\int_{x=0}^a \int_{y=0}^b \cos^2\left(\frac{m\pi x}{a}\right) \sin^2\left(\frac{n\pi y}{b}\right) \cdot dy \cdot dx = \frac{ab}{4}$$

Total integration value will be  $ab$  by 4. So, we can substitute all this value there in the upper equation.

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$$= \frac{1}{2\gamma_{TM}} \frac{\beta^2 E_0^2}{h^4} \frac{ab}{4} \left[ \frac{4\pi^2}{a^2} + \frac{9\pi^2}{b^2} \right]$$

$$\beta = \frac{\omega}{v} = \frac{\omega}{c} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \frac{10}{3 \times 10^8} \sqrt{1 - \left(\frac{15.81}{159.2}\right)^2}$$

$$= 3.317 \times 10^{-3}$$

$$h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = (2\pi)^2 + \left(\frac{3\pi}{b}\right)^2$$

So, it will be

$$= \frac{1}{2\gamma_{TM}} \frac{\beta^2 E_0^2}{h^4} \frac{ab}{4} \left[ \frac{4\pi^2}{a^2} + \frac{9\pi^2}{b^2} \right]$$

$$\beta = \frac{\omega}{v} = \frac{\omega}{c} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \frac{10}{3 \times 10^8} \sqrt{1 - \left(\frac{15.81}{159.2}\right)^2}$$

$$= 3.317 \times 10^{-3}$$

So, this is the value of beta.

and h square will be

$$h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \left(\frac{2\pi}{a}\right)^2 + \left(\frac{3\pi}{b}\right)^2$$

$$= \frac{4\pi^2}{a^2} + \frac{9\pi^2}{b^2}$$

So, h square value is this one.

Total expression then will be

$$P_{\text{ave}} = \frac{\beta E_0^2 ab}{8 \eta_{TM_{23}} h^2}$$

$$= \frac{(3.317 \times 10^3)^2 \times 25 \times 18 \times 10^{-4}}{8 \times 375.1 \times 1.097 \times 10^5}$$

$$= 1.502 \text{ mW}$$

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$$P_{\text{ave}} = \frac{\beta E_0^2 ab}{8 \eta_{TM_{23}} h^2}$$

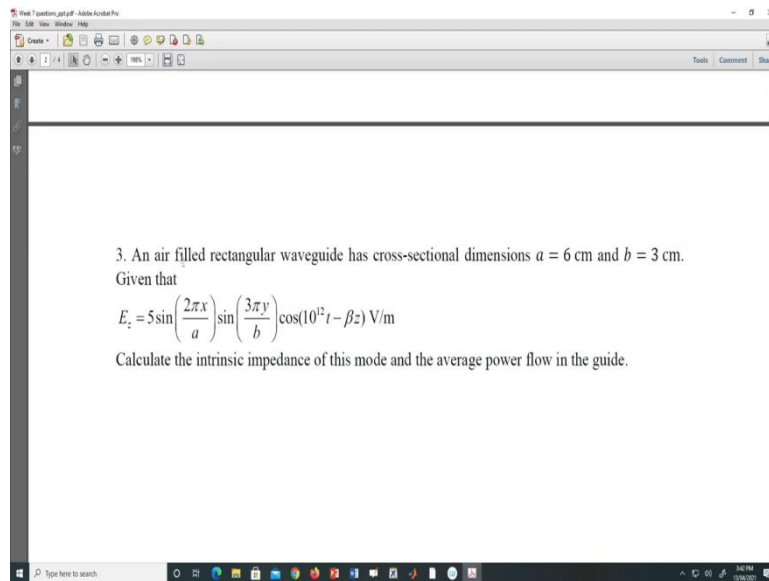
$$= \frac{(3.317 \times 10^3)^2 \times 25 \times 18 \times 10^{-4}}{8 \times 375.1 \times 1.097 \times 10^5}$$

$$\iint_{x=0, y=0}^{x=a, y=b} \cos^2\left(\frac{m\pi x}{a}\right) \sin^2\left(\frac{n\pi y}{b}\right) dx dy = \frac{4}{4}$$

$$= \frac{1}{2\eta_{TM}} \frac{\beta^2 E_0^2}{h^4} \frac{ab}{4} \left[ \frac{4\pi^2}{a^2} + \frac{9\pi^2}{b^2} \right]$$

$$\beta = \frac{\omega}{v} = \frac{\omega}{c} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \frac{10^{12}}{3 \times 10^8} \sqrt{1 - \left(\frac{1581}{1592}\right)^2}$$

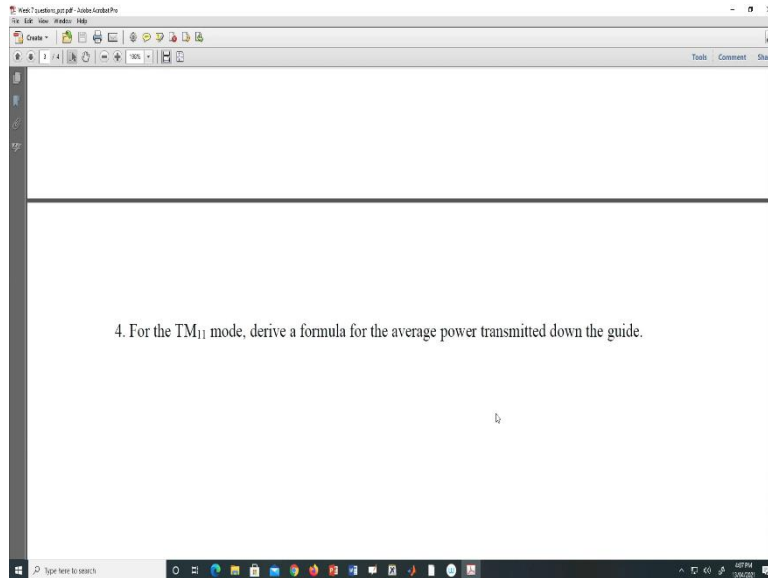
$$= 3.317 \times 10^3$$



This will come 1.502 milliwatt.

So, the average power flow is 1.502 milliwatt.

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Now, next problem is

4. For the  $TM_{11}$  mode, derive a formula for the average power transmitted down the guide.

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4)

$$P_{av} = \frac{1}{2\eta} \int_{y=0}^b \int_{x=0}^a (|E_x|^2 + |E_y|^2) dx dy$$

$$E_x = -\frac{j\beta}{h^2} \left(\frac{\pi}{a}\right) E_0 \cos\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right) e^{-j\beta z}$$

$$E_y = -\frac{j\beta}{h^2} \left(\frac{\pi}{b}\right) E_0 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right) e^{-j\beta z}$$

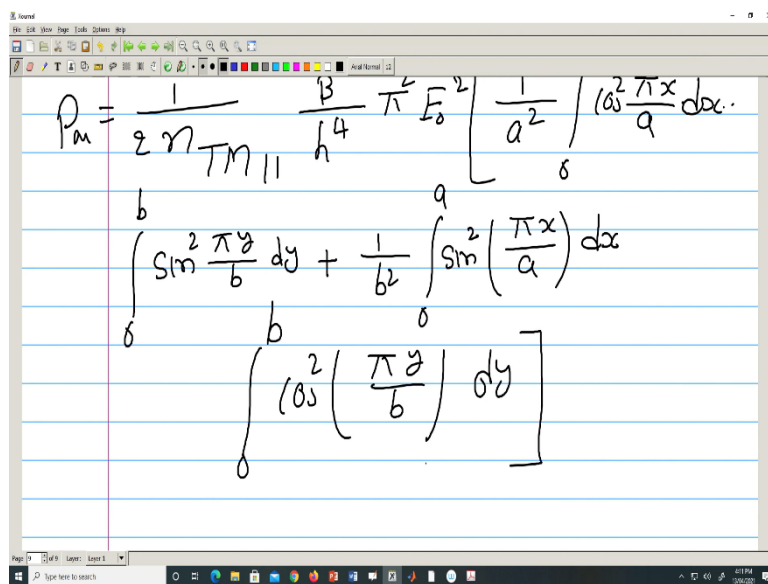
For the  $TM_{11}$  mode,



$$\begin{aligned}
 4) \quad P_m &= \frac{1}{2\eta} \int_{y=0}^b \int_{x=0}^a (|E_x|^2 + |E_y|^2) dx dy \\
 E_x &= \frac{-j\beta}{h^2} \left(\frac{\pi}{a}\right) E_0 \cos\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right) e^{-j\beta z} \\
 E_y &= \frac{-j\beta}{h^2} \left(\frac{\pi}{b}\right) E_0 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right) e^{-j\beta z}
 \end{aligned}$$

So, this is the  $E_x$  and  $E_y$ .

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$$\begin{aligned}
 P_m &= \frac{1}{2\eta_{TM11}} \frac{\beta}{h^4} \pi E_0^2 \left[ \frac{1}{a^2} \int_0^a \cos^2\left(\frac{\pi x}{a}\right) dx \cdot \int_0^b \sin^2\left(\frac{\pi y}{b}\right) dy \right. \\
 &\quad \left. + \frac{1}{b^2} \int_0^a \sin^2\left(\frac{\pi x}{a}\right) dx \cdot \int_0^b \cos^2\left(\frac{\pi y}{b}\right) dy \right]
 \end{aligned}$$

We can substitute the  $E_x$  and  $E_y$  in the power average formula and we will get power average equal to

$$\begin{aligned}
 P_m &= \frac{1}{2\eta_{TM11}} \frac{\beta}{h^4} \pi E_0^2 \left[ \frac{1}{a^2} \int_0^a \cos^2\left(\frac{\pi x}{a}\right) dx \cdot \int_0^b \sin^2\left(\frac{\pi y}{b}\right) dy \right. \\
 &\quad \left. + \frac{1}{b^2} \int_0^a \sin^2\left(\frac{\pi x}{a}\right) dx \cdot \int_0^b \cos^2\left(\frac{\pi y}{b}\right) dy \right]
 \end{aligned}$$

$$= \frac{1}{2 \eta_{TM,11}} \frac{\beta^2 \pi^2}{h^4} E_0^2 \left[ \frac{1}{a^2} + \frac{1}{b^2} \right] \frac{ab}{4}$$

$$h^2 = \frac{\pi^2}{a^2} + \frac{\pi^2}{b^2} = \left( \frac{a^2 + b^2}{a^2 b^2} \right) \pi^2$$

$$P_{ave} = \frac{1}{2 \eta_{TM,11}} \frac{\beta^2 E_0^2}{h^2} \frac{ab}{4}$$

$$= \frac{1}{2 \eta_{TM,11}} \frac{\beta^2 E_0^2 ab}{4} \cdot \frac{a^2 b^2}{(a^2 + b^2) \pi^2}$$

$$= \frac{\beta^2 E_0^2 a^3 b^3}{8 \pi^2 \eta_{TM,11} (a^2 + b^2)}$$

So, this is the average power flow in the waveguide.

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$$= \frac{1}{2 \eta_{TM,11}} \frac{\beta^2 \pi^2}{h^4} E_0^2 \left[ \frac{1}{a^2} + \frac{1}{b^2} \right] \frac{ab}{4}$$

$$h^2 = \frac{\pi^2}{a^2} + \frac{\pi^2}{b^2} = \left( \frac{a^2 + b^2}{a^2 b^2} \right) \pi^2$$

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$$\begin{aligned}
 P_{ave} &= \frac{1}{2\eta_{TM_{11}}} \frac{\beta^2 E_0^2}{h^2} \frac{ab}{4} \\
 &= \frac{1}{2\eta_{TM_{11}}} \frac{\beta^2 E_0^2 ab}{4} \cdot \frac{a^2 b^2}{(a^2 + b^2) \pi^2} \\
 &= \frac{\beta^2 E_0^2 a^3 b^3}{8\pi^2 \eta_{TM_{11}} (a^2 + b^2)}
 \end{aligned}$$

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$$P_m = \frac{1}{2\eta} \int_{y=0}^b \int_{x=0}^a (|E_x|^2 + |E_y|^2) dx dy$$

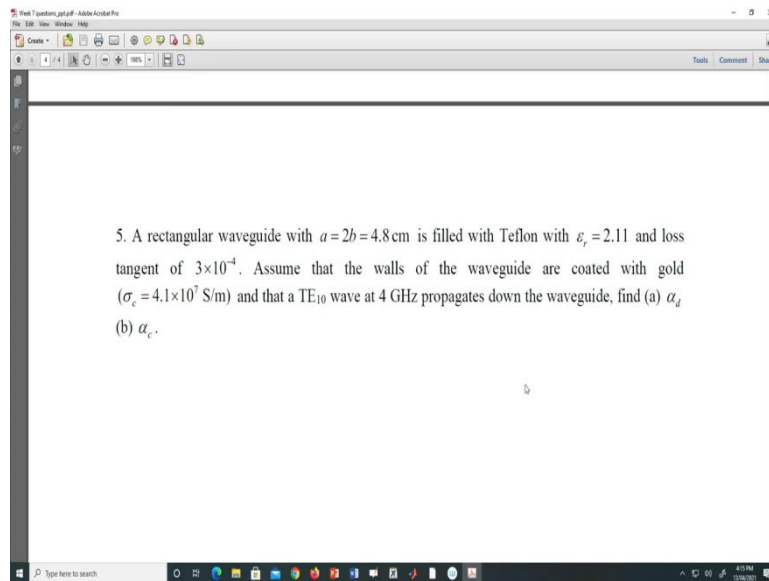
$$E_x = -\frac{j\beta}{h^2} \left(\frac{\pi}{a}\right) E_0 \cos\left(\frac{\pi x}{a}\right) \cdot \sin\left(\frac{\pi y}{b}\right) e^{-j\beta z}$$

$$E_y = -\frac{j\beta}{h^2} \left(\frac{\pi}{b}\right) E_0 \sin\left(\frac{\pi x}{a}\right) \cdot \cos\left(\frac{\pi y}{b}\right) e^{-j\beta z}$$

$$P_m = \frac{1}{2\eta_{TM_{11}}} \frac{\beta^2}{h^4} \pi^2 E_0^2 \left[ \frac{1}{a^2} \int_0^a \cos^2 \frac{\pi x}{a} dx \cdot \dots \right]$$

So, like this we can calculate for the TE mode.

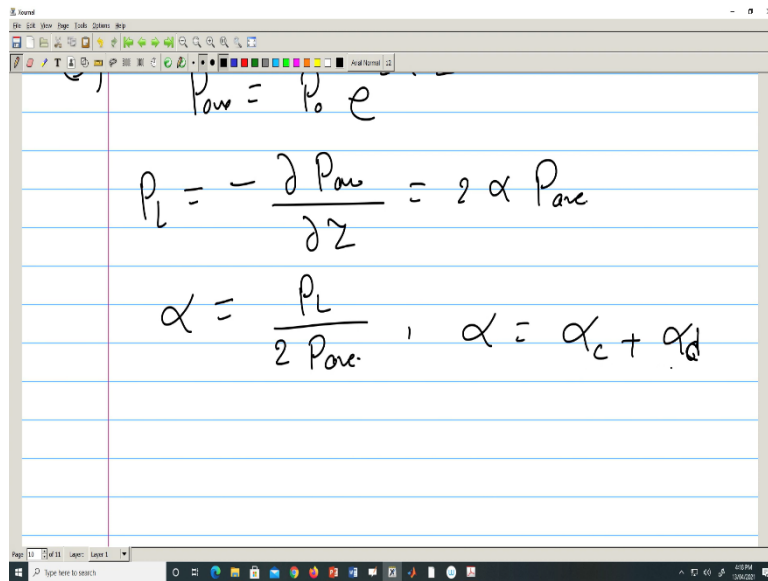
(Refer Slide Time: 33:08)



Now we can see the next problem.

5. A rectangular waveguide with  $a = 2b = 4.8$  cm is filled with Teflon with  $\epsilon_r = 2.11$  and loss tangent of  $3 \times 10^{-4}$ . Assume that the walls of the waveguide are coated with gold ( $\sigma_c = 4.1 \times 10^7$  S/m) and that a  $TE_{10}$  wave at 4 GHz propagates down the waveguide, find (a)  $\alpha_d$  (b)  $\alpha_c$ .

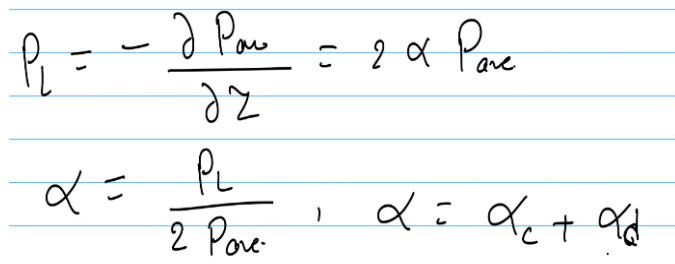
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The screenshot shows a digital whiteboard with the following handwritten equations:

$$P_{\text{ave}} = P_0 e^{-\alpha z}$$
$$P_L = -\frac{\partial P_{\text{ave}}}{\partial z} = 2\alpha P_{\text{ave}}$$
$$\alpha = \frac{P_L}{2 P_{\text{ave}}}, \quad \alpha = \alpha_c + \alpha_d$$

So, this is a power flow in the waveguide with attenuation:

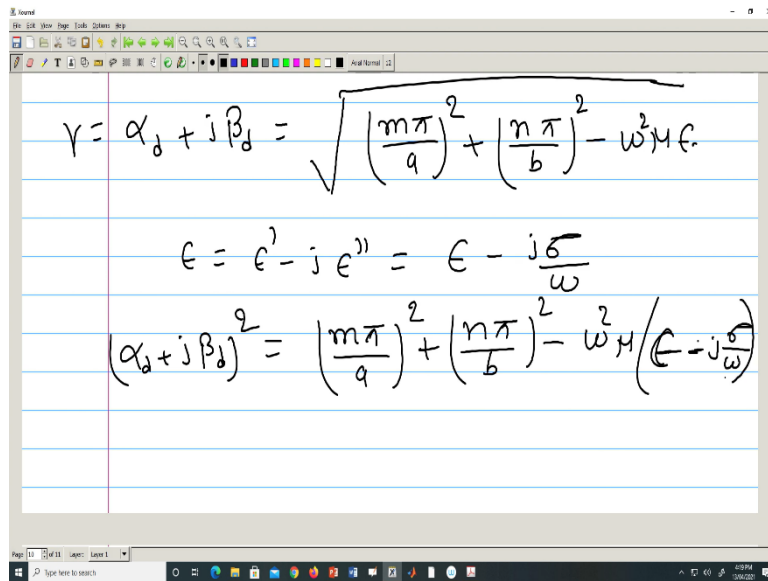


The image shows the same handwritten equations as the screenshot above:

$$P_L = -\frac{\partial P_{\text{ave}}}{\partial z} = 2\alpha P_{\text{ave}}$$
$$\alpha = \frac{P_L}{2 P_{\text{ave}}}, \quad \alpha = \alpha_c + \alpha_d$$

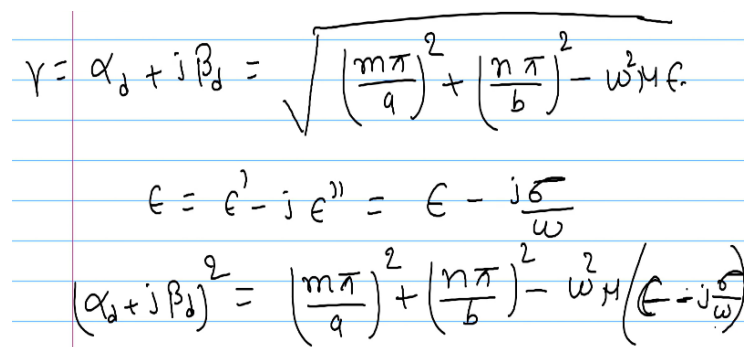
here alpha c is the attenuation constant due to ohmic or conduction loss and alpha d is the attenuation constant due to the dielectric loss.

(Refer Slide Time: 36:16)



The image shows a digital whiteboard with three lines of handwritten mathematical equations. The first line is  $\gamma = \alpha_d + j\beta_d = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$ . The second line is  $\epsilon = \epsilon' - j\epsilon'' = \epsilon - \frac{j\sigma}{\omega}$ . The third line is  $(\alpha_d + j\beta_d)^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\left(\epsilon - \frac{j\sigma}{\omega}\right)$ . The whiteboard interface includes a toolbar at the top and a Windows taskbar at the bottom.

So, we know



The image shows a whiteboard with three lines of handwritten mathematical equations, identical to the ones in the screenshot above. The first line is  $\gamma = \alpha_d + j\beta_d = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$ . The second line is  $\epsilon = \epsilon' - j\epsilon'' = \epsilon - \frac{j\sigma}{\omega}$ . The third line is  $(\alpha_d + j\beta_d)^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\left(\epsilon - \frac{j\sigma}{\omega}\right)$ .

(Refer Slide Time: 37:52)

$$\alpha_d^2 - \beta_d^2 + 2j\alpha_d\beta_d = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega\mu\epsilon + j\omega\mu\sigma$$
$$\alpha_d^2 - \beta_d^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon$$
$$2\alpha_d\beta_d = \omega\mu\sigma \rightarrow \alpha_d = \frac{\omega\mu\sigma}{2\beta_d}$$
$$\alpha_d^2 \ll \beta_d^2$$

So, calculating further and equating the real and imaginary part both side:

$$\alpha_d^2 - \beta_d^2 + 2j\alpha_d\beta_d = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega\mu\epsilon + j\omega\mu\sigma$$
$$\alpha_d^2 - \beta_d^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon$$
$$2\alpha_d\beta_d = \omega\mu\sigma \rightarrow \alpha_d = \frac{\omega\mu\sigma}{2\beta_d}$$
$$\alpha_d^2 \ll \beta_d^2$$

we can assume that alpha d is very very less than means alpha d whole square is very very less than beta d whole square, because beta d is a propagation constant and we want minimum attenuation means minimum loss be with the waveguide as a power flow to send the power from one place to another place.

So, we can assume that attenuation is constant inside that dielectric that is very small and beta d's frequency dependent so, it will come in the very large compared to the alpha d. So, from this if alpha d is very very small so, we can write this expression.

$$-\beta_d^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon$$

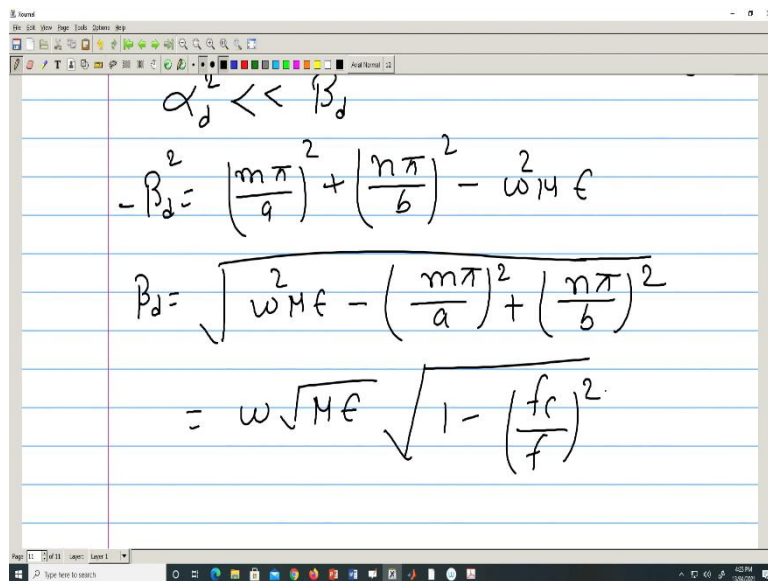
$$\beta_d = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$$= \omega \sqrt{\mu \epsilon} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\alpha_d = \frac{\sigma - \omega}{2 \sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$\alpha_c = \frac{2 R_s}{b \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \left[ \frac{1}{2} + \frac{b}{a} \left(\frac{f_c}{f}\right)^2 \right]$$

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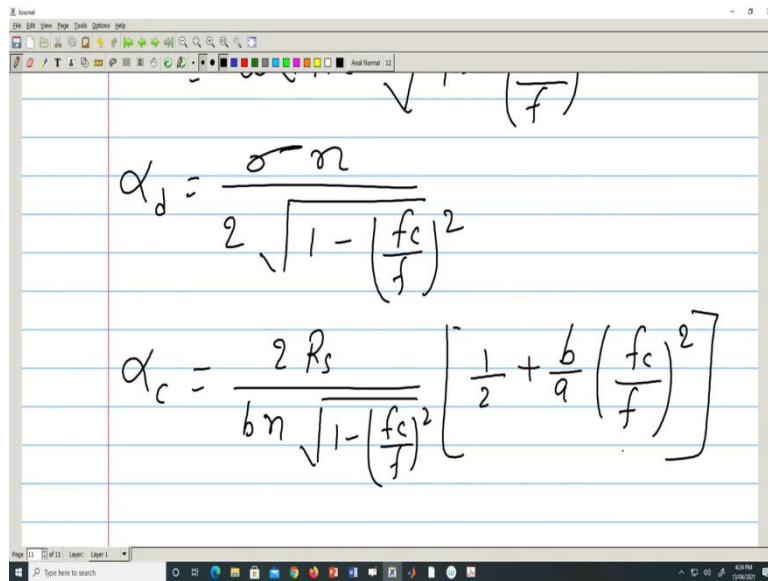


$\alpha_d \ll \beta_d$

$$-\beta_d^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon$$
$$\beta_d = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$
$$= \omega \sqrt{\mu \epsilon} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

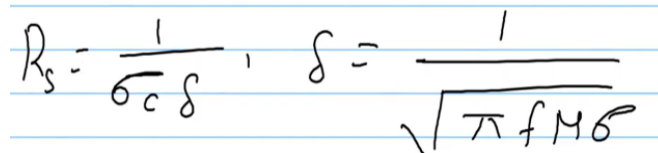


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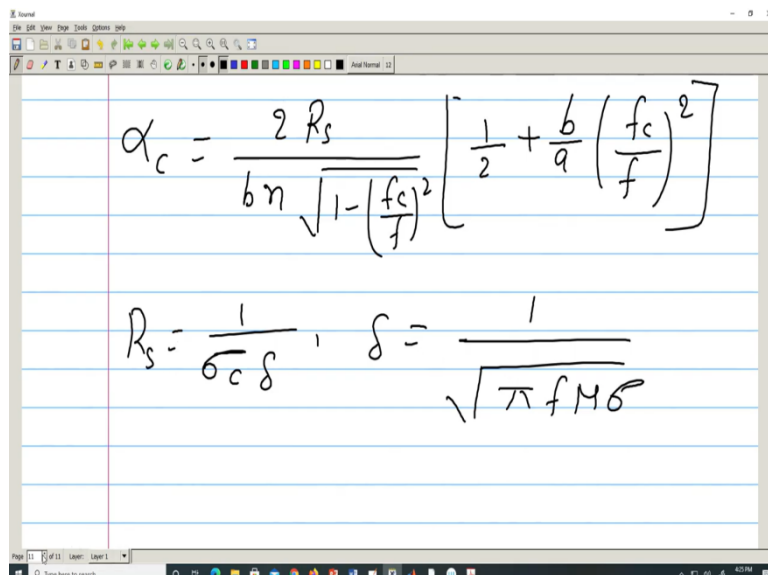
The screenshot shows a presentation slide with two handwritten equations. The first equation is  $\alpha_d = \frac{\sigma \eta}{2 \sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$ . The second equation is  $\alpha_c = \frac{2 R_s}{b \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \left[ \frac{1}{2} + \frac{b}{a} \left(\frac{f_c}{f}\right)^2 \right]$ . The equations are written in blue ink on a white background with horizontal lines.

So, from this we will get the alpha d and alpha c I am not deriving, and here  $R_s$  is the skin resistance of the wall.

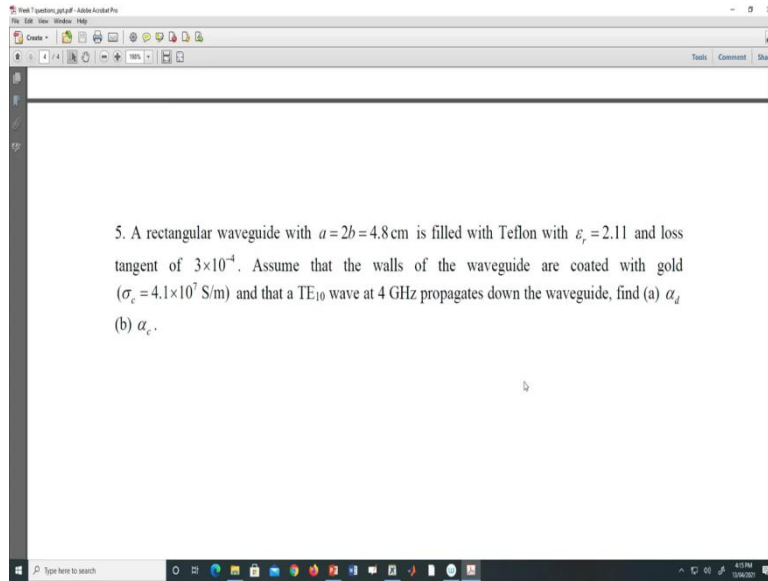


The image shows two handwritten equations:  $R_s = \frac{1}{\sigma_c \delta}$  and  $\delta = \frac{1}{\sqrt{\pi f M \sigma}}$ . The equations are written in blue ink on a white background with horizontal lines.

(Refer Slide Time: 42:34)



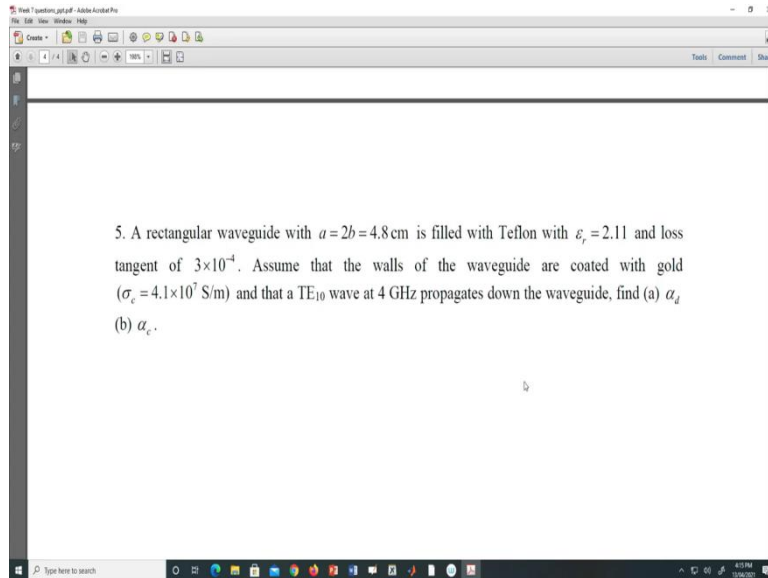
The screenshot shows a presentation slide with three handwritten equations. The first equation is  $\alpha_c = \frac{2 R_s}{b \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \left[ \frac{1}{2} + \frac{b}{a} \left(\frac{f_c}{f}\right)^2 \right]$ . The second equation is  $R_s = \frac{1}{\sigma_c \delta}$ . The third equation is  $\delta = \frac{1}{\sqrt{\pi f M \sigma}}$ . The equations are written in blue ink on a white background with horizontal lines.



So,  $R_s$  is the skin resistance

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$$f_c = \frac{c}{2a\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{2 \times 4.8 \times 10^{-2} \times \sqrt{2.11}}$$
$$= 2.151 \text{ GHz}$$



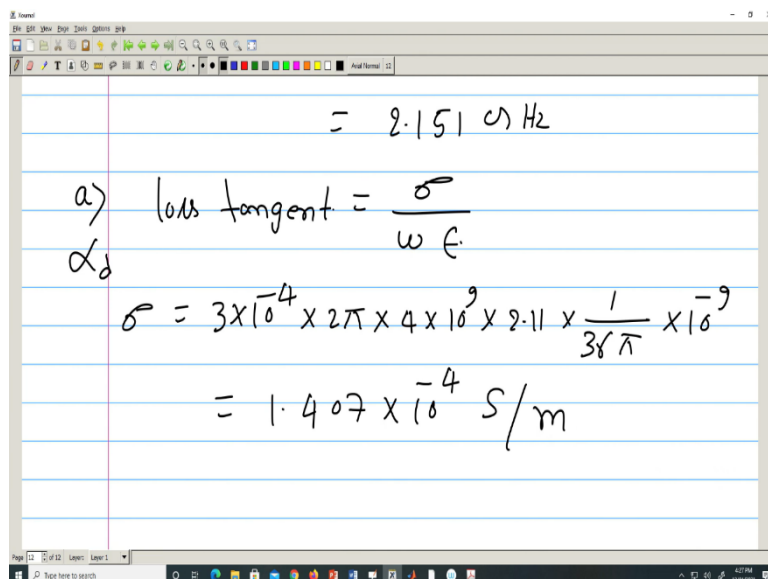
So, for the TE 10 mode,  $f_c$  will be

$$f_c = \frac{c}{2a\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{2 \times 4.8 \times 10^{-2} \times \sqrt{2.11}}$$

$$= 2.151 \text{ GHz}$$

So, this will come 2.151 gigahertz. So, this is the value of  $f_c$ .

(Refer Slide Time: 43:58)



$$\begin{aligned} a) \quad \text{loss tangent} &= \frac{\sigma}{\omega \epsilon} \\ \alpha_d \\ \sigma &= 3 \times 10^{-4} \times 2\pi \times 4 \times 10^9 \times 2.11 \times \frac{1}{36\pi} \times 10^{-9} \\ &= 1.407 \times 10^{-4} \text{ S/m} \end{aligned}$$

So, we will continue this in the next class. Thank you.