Advanced Microwave Guided-Structures and Analysis
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Lecture 37
Rectangular Waveguide II Tutorials (contd.)

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Welcome to the next tutorial class. So, that is on the power flow in a rectangular waveguide. We will continue from the previous class.

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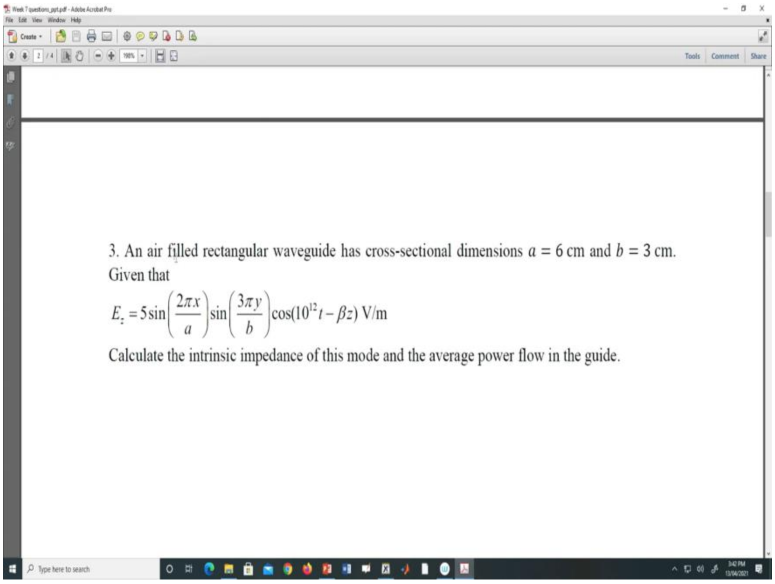

So, we have for the TE to Z mode. So, our problem is an air-filled rectangular waveguide has cross-sectional dimensions, a is equal to 6 and b is equal to 3. So, here Ez is given, so that means it is for TM to Z mode because here electric field is given, Ez is given that means no magnetic field is given So, Hz will be 0 and this is for the TM to Z mode. So, we have computed that field expression for the TE to Z mode, TM to Z mode.

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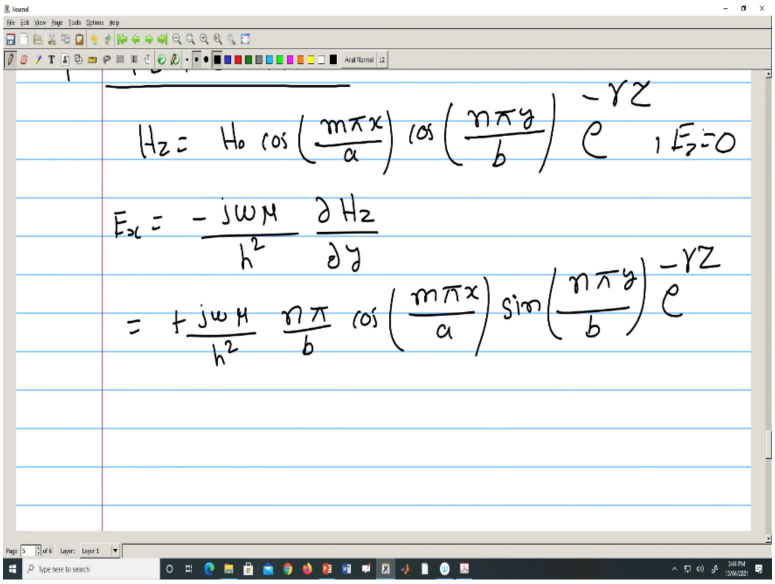
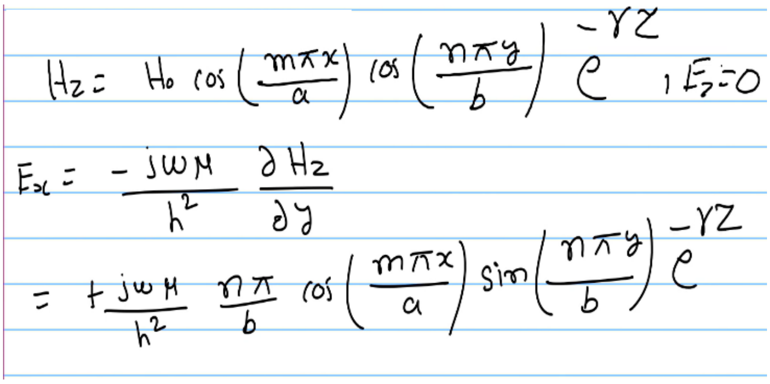


And here, we can calculate also for the TE to Z mode. For the TE to Z mode, Hz will be this. So, this is a standard format for the Hz.



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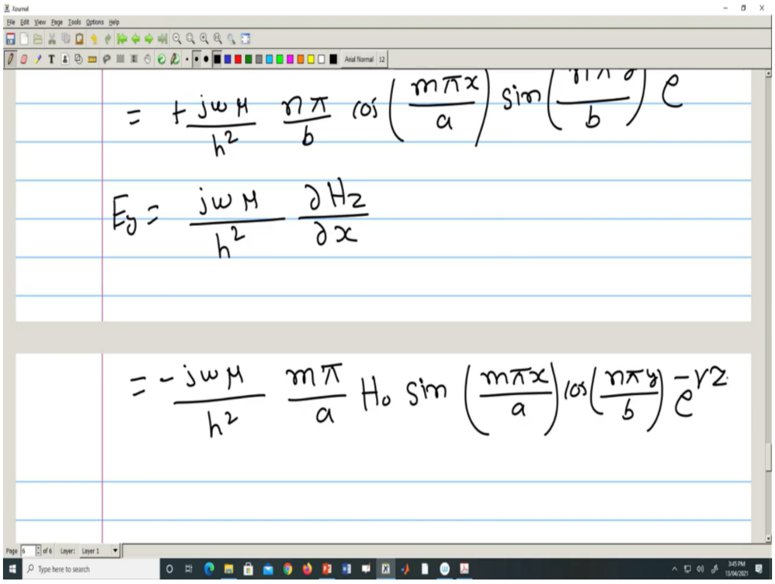
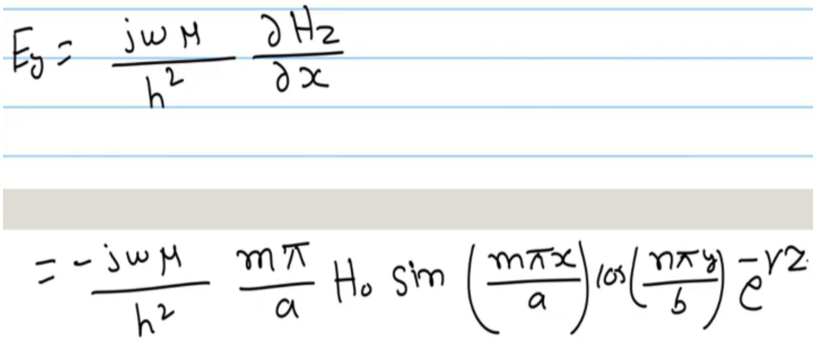


So, this is for the Ez and Ey will be



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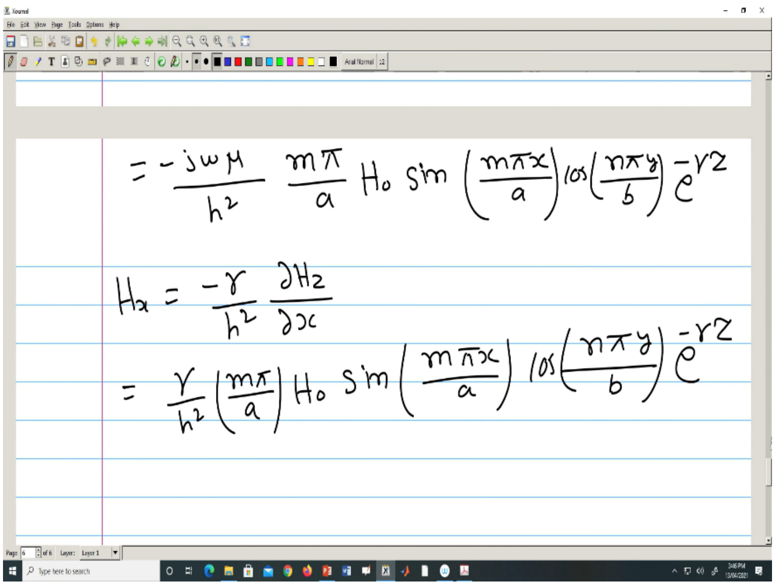

Similarly, we can calculate the Hz and Hy. Hx will we minus of gamma h square. This is for the TE to Z mode. So, in terms of Hz, all the field expression will come.

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x}$$

$$= \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y}$$

$$= \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

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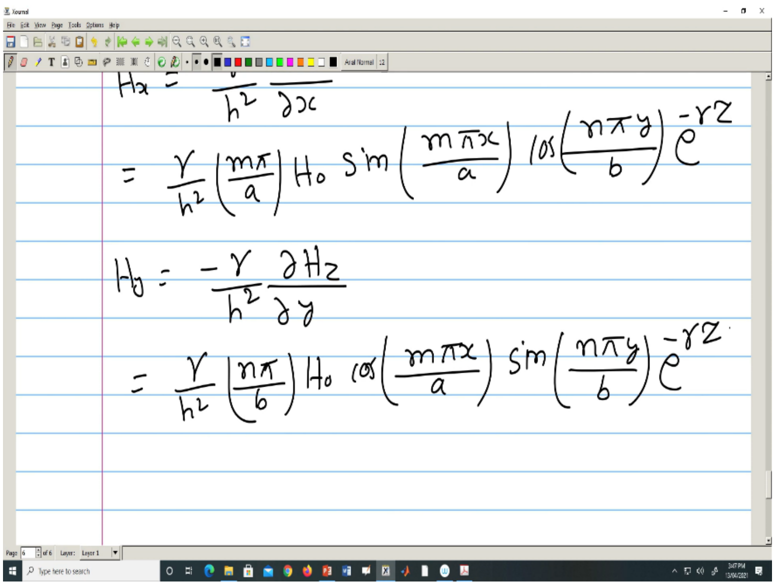

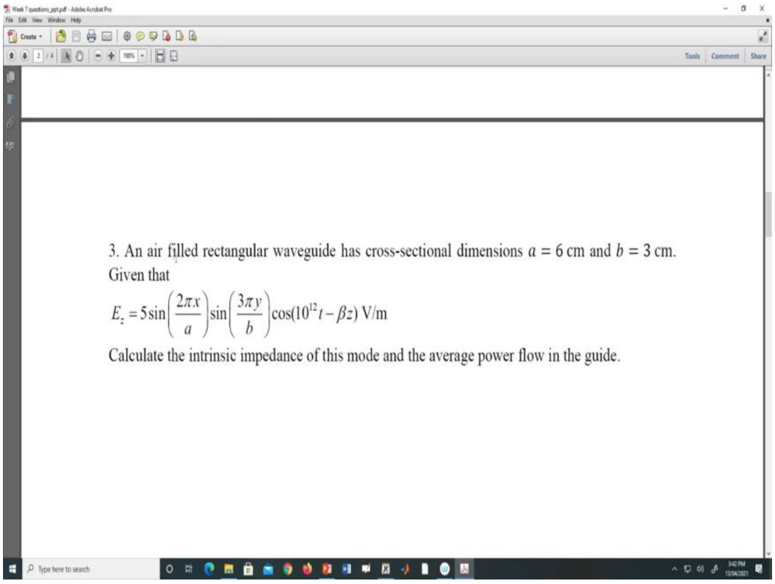

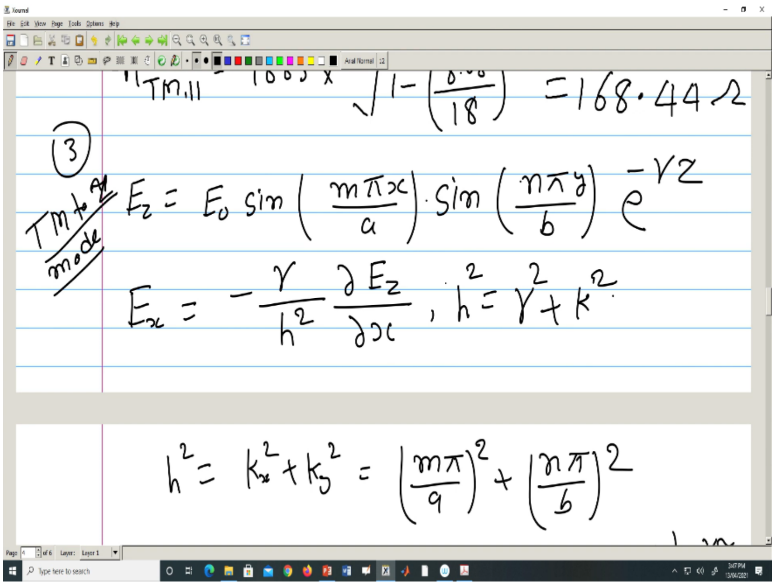

Now, we can come to the problem. In the problem it is given, Ez is equal

$$E_z = 5\sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{3\pi y}{b}\right)\cos(10^{12}t - \beta z)\ \text{V/m}$$

We can compare this with our standard equation

$$E_z = E_0 \sin\left(\frac{m\pi x}{a}\right) \cdot \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

So, after comparing from this, we will get m equal to 2 and n is equal to 3 and because Ez is given, so this will be so TM_{23} mode.

So, this will be TM_{23} and calculate the, first we will calculate the intrinsic impedance of this mode. So, we identified the mode that is TM_{23} and for TM_{23} mode, we will calculate the wave impedance, intrinsic wave impedance for TM_{23}.

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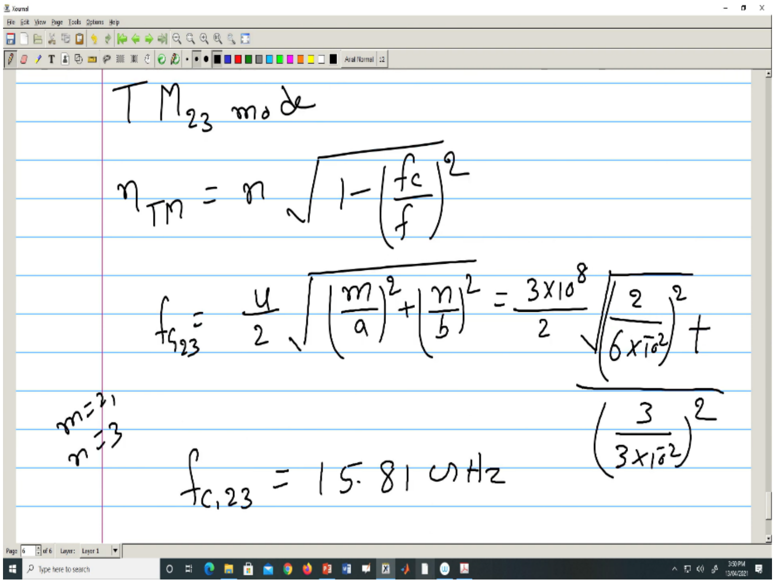
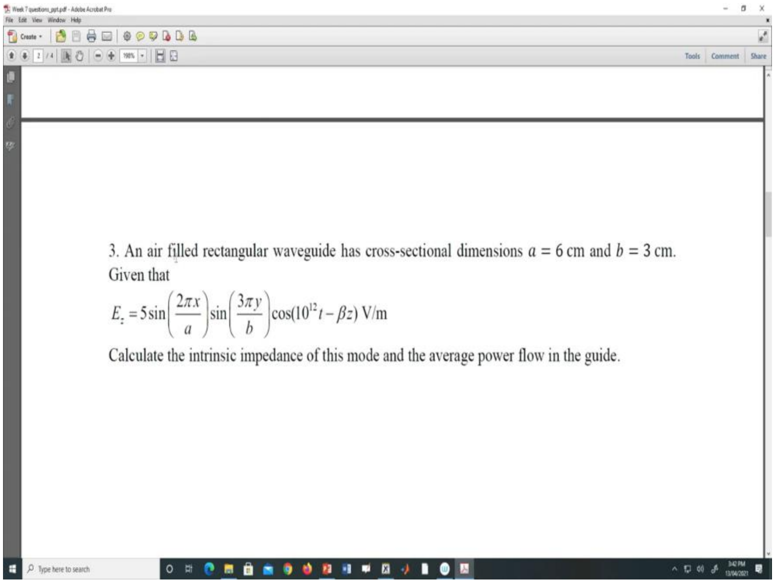
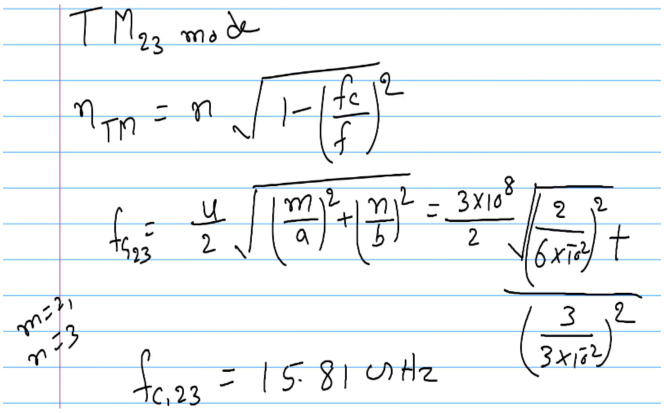




For TM23 mode, we will calculate



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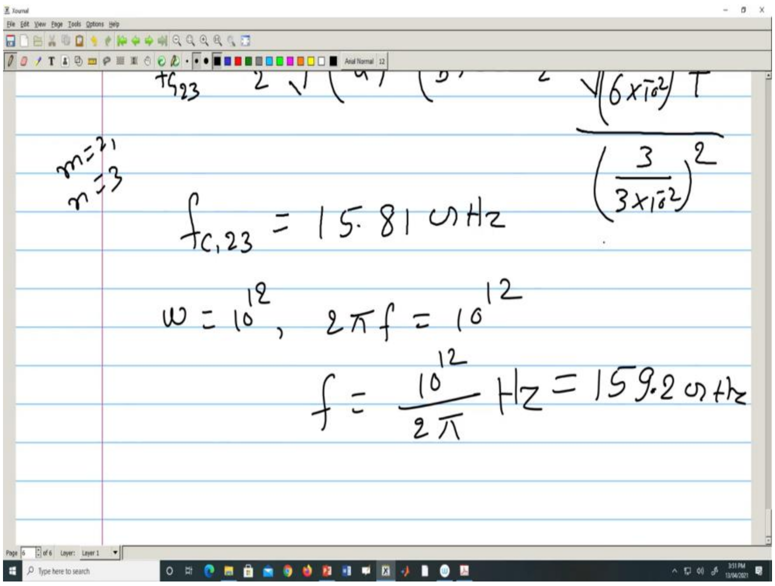
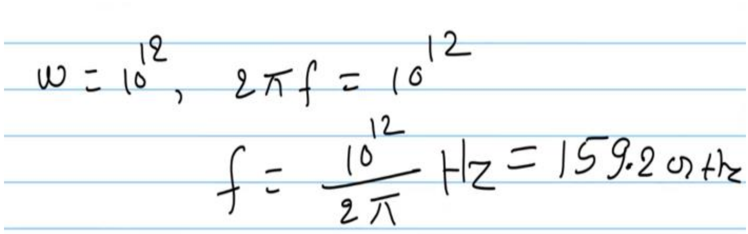


So, after calculation we will get fc for TM_{23}, that is, 15.81 gigahertz. So, now



So, this will come 159.2 gigahertz. So, this is in the millimetre wavelength.

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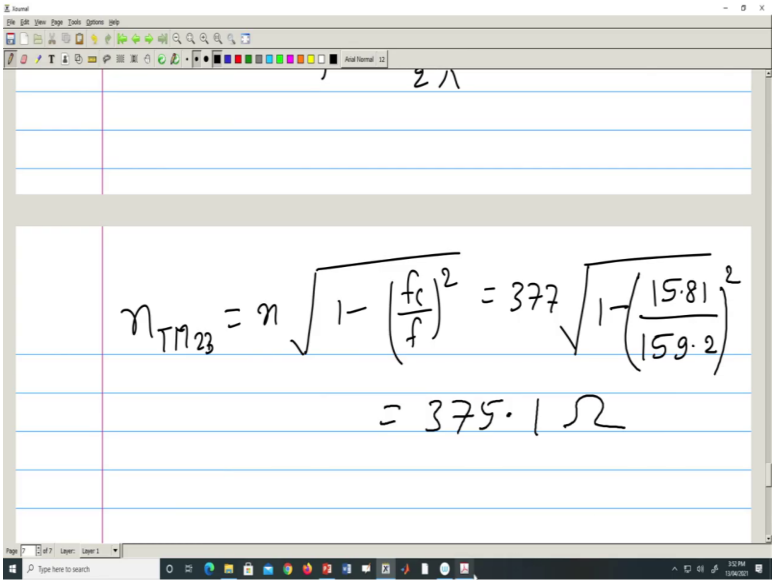

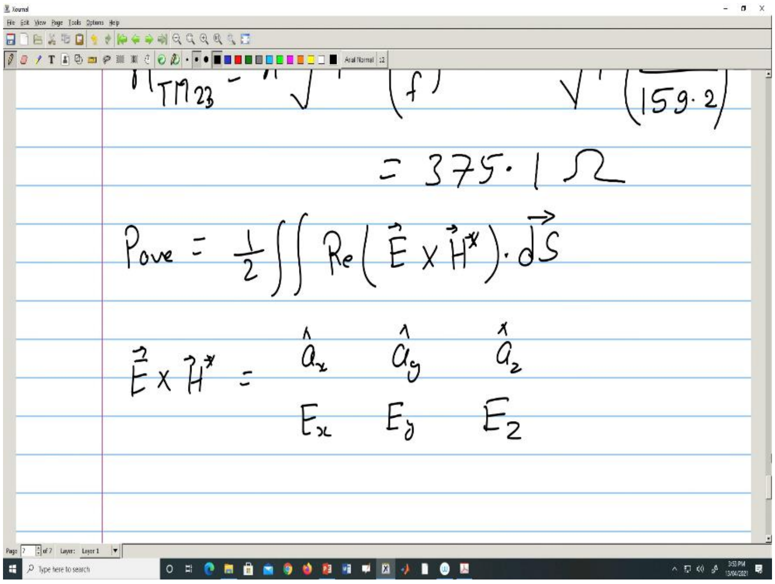

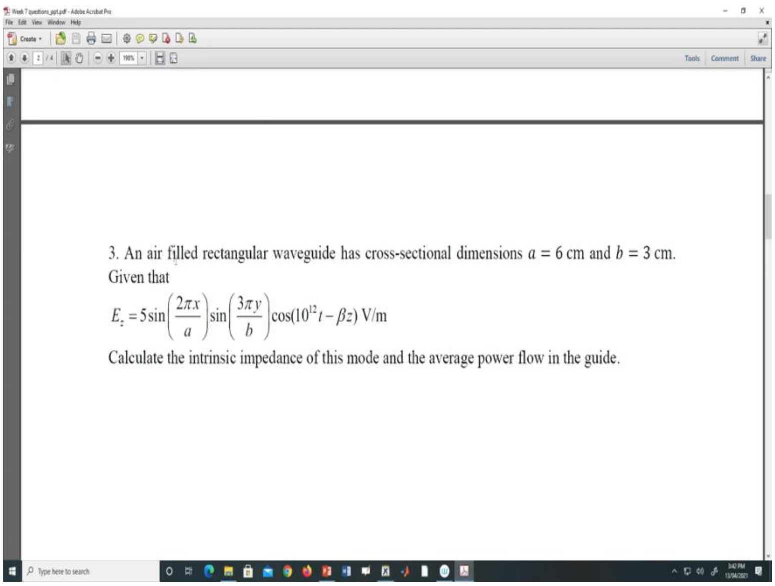

3. An air filled rectangular waveguide has cross-sectional dimensions $a = 6$ cm and $b = 3$ cm. Given that

$$E_z = 5\sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{3\pi y}{b}\right)\cos(10^{12}t - \beta z) \text{ V/m}$$

Calculate the intrinsic impedance of this mode and the average power flow in the guide.

So, for this we can calculate the eta.

$$\eta_{TM_{23}} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 377\sqrt{1-\left(\frac{15.81}{159.2}\right)^2}$$

$$= 375.1\ \Omega$$

This will give 375.1 Ohm. So, this is the solution for the first part.

Now, we have to calculate the average power flow in the guide, for this mode. That is also more dependant. So to determine the power average flow in the guide, we have to use

$$P_{ave} = \frac{1}{2}\iint Re\left(\vec{E} \times \vec{H}^*\right) \cdot \vec{dS}$$

So, first we can calculate

$$\vec{E} \times \vec{H}^* = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ E_x & E_y & E_z \\ H_x^* & H_y^* & 0 \end{vmatrix}$$

$$= \hat{a}_x\left(0 - E_z H_y^*\right) + \hat{a}_y\left(E_z H_x^* - 0\right) + \hat{a}_z\left(E_x H_y^* - E_y H_x^*\right)$$

$$dS = dx\, dy\, \hat{a}_z$$

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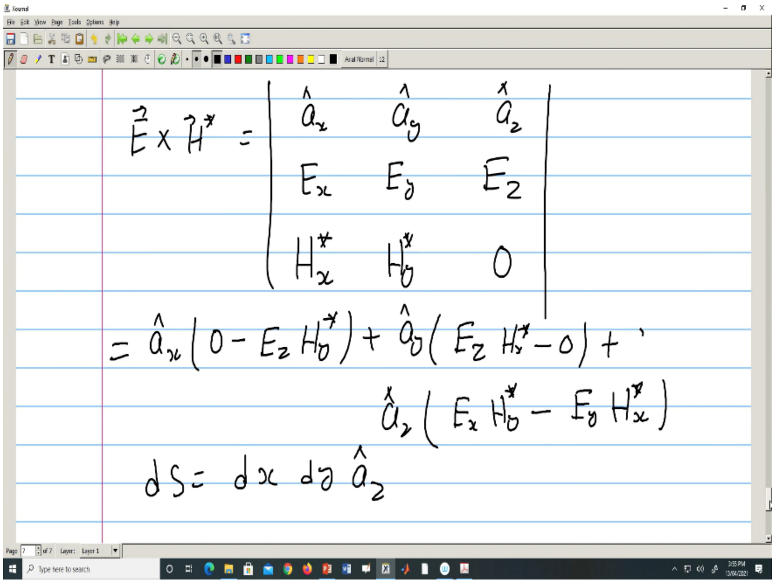

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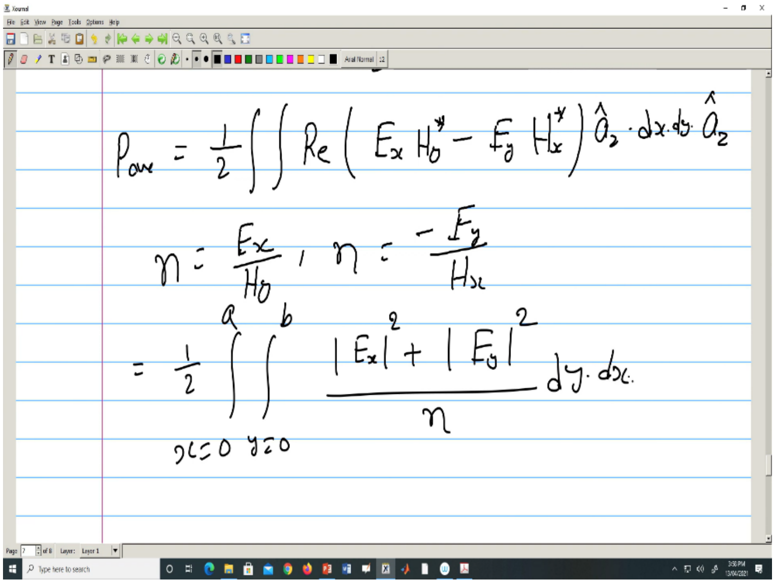

Power average will be

$$P_{avg} = \frac{1}{2}\int\int Re\left(E_x H_y^* - E_y H_x^*\right)\hat{a}_z \cdot dx \cdot dy \cdot \hat{a}_z$$

$$\eta = \frac{E_x}{H_y}, \quad \eta = \frac{-E_y}{H_x}$$

$$= \frac{1}{2}\int_{x=0}^{a}\int_{y=0}^{b} \frac{|E_x|^2 + |E_y|^2}{\eta}\, dy \cdot dx$$

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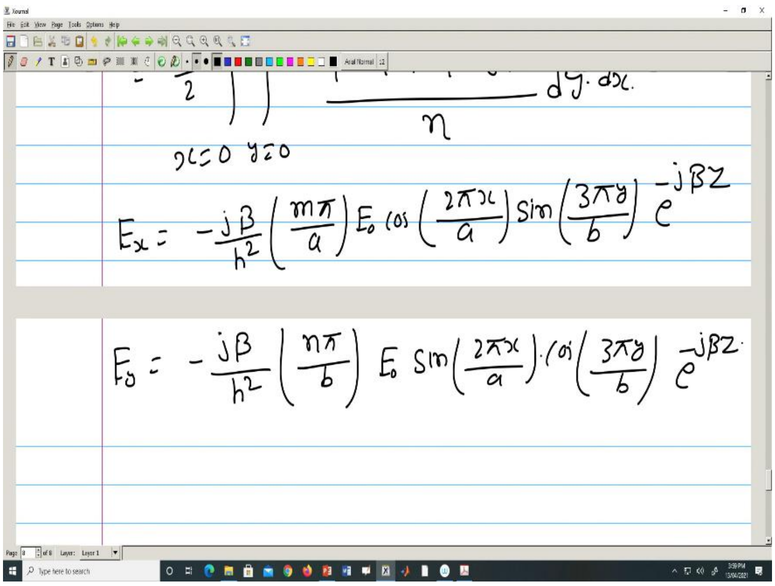

So, Ex and Ey

$$E_x = -\frac{j\beta}{h^2}\left(\frac{m\pi}{a}\right) E_0 \cos\left(\frac{2\pi x}{a}\right) \sin\left(\frac{3\pi y}{b}\right) e^{-j\beta z}$$

$$E_y = -\frac{j\beta}{h^2}\left(\frac{n\pi}{b}\right) E_0 \sin\left(\frac{2\pi x}{a}\right) \cdot \cos\left(\frac{3\pi y}{b}\right) e^{-j\beta z}$$

$$P_{ave} = \frac{1}{2\eta_{TM}} \frac{\beta^2 E_0^2}{h^4} \int_{x=0}^{a}\int_{y=0}^{b} \left[\left(\frac{2\pi}{a}\right)^2 \cdot \cos^2\left(\frac{2\pi x}{a}\right) \cdot \sin^2\left(\frac{3\pi y}{b}\right) + \left(\frac{3\pi}{b}\right)^2 \sin^2\left(\frac{2\pi x}{a}\right) \cos^2\left(\frac{3\pi y}{b}\right)\right] dy \cdot dx$$

$$\int_{x=0}^{a}\int_{y=0}^{b} \cos^2\left(\frac{m\pi x}{a}\right) \sin^2\left(\frac{n\pi y}{b}\right) \cdot dy \cdot dx = \frac{ab}{4}$$

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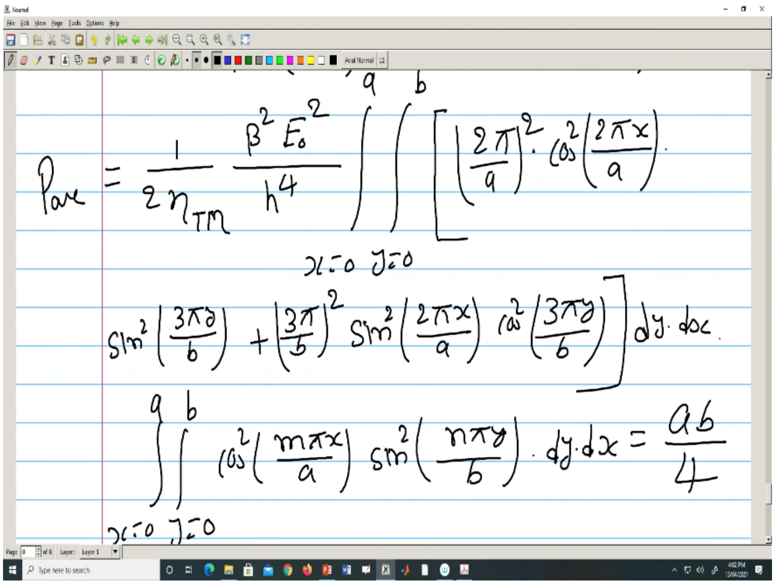

Total integration value will be ab by 4. So, we can substitute all this value there in the upper equation.

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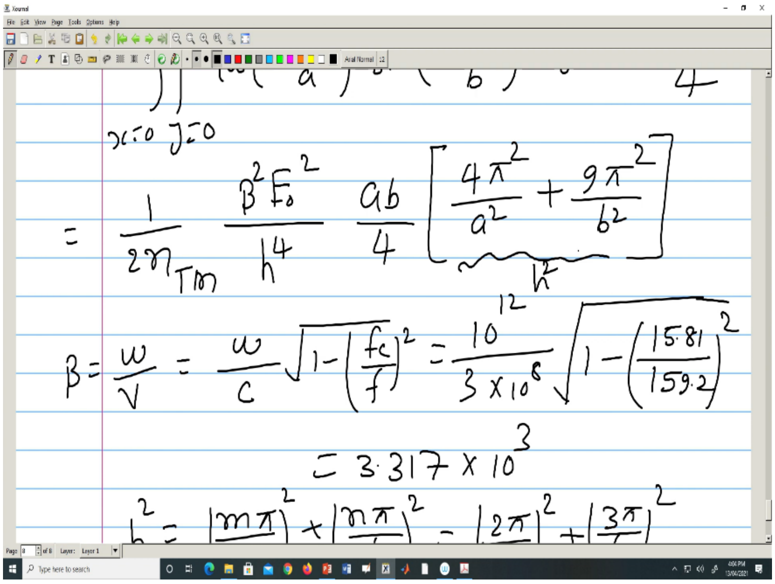
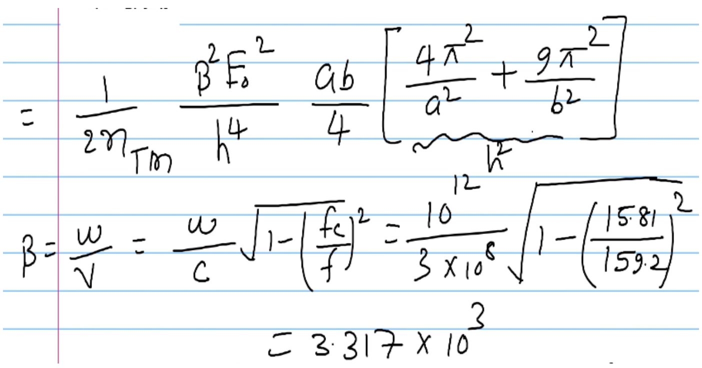
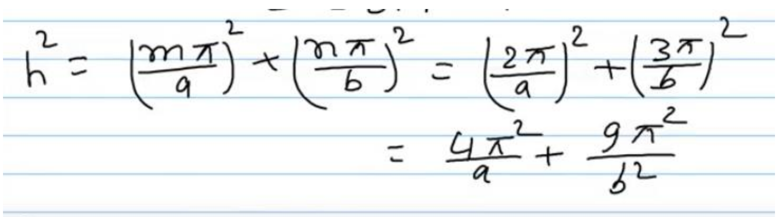


So, it will be



So, this is the value of beta.

and h square will be



So, h square value is this one.

Total expression then will be

$$P_{or} = \frac{\beta E_0^2 ab}{8 \eta_{TM_{23}} h^2}$$

$$= \frac{(3.317 \times 10^3)^2 \times 25 \times 18 \times 10^{-4}}{8 \times 375.1 \times 1.097 \times 10^5}$$

$$= 1.502 \text{ mW}$$

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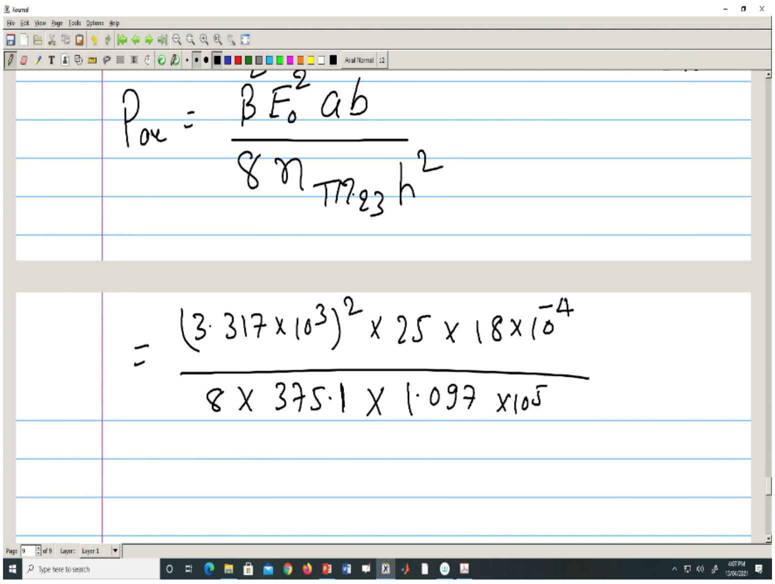

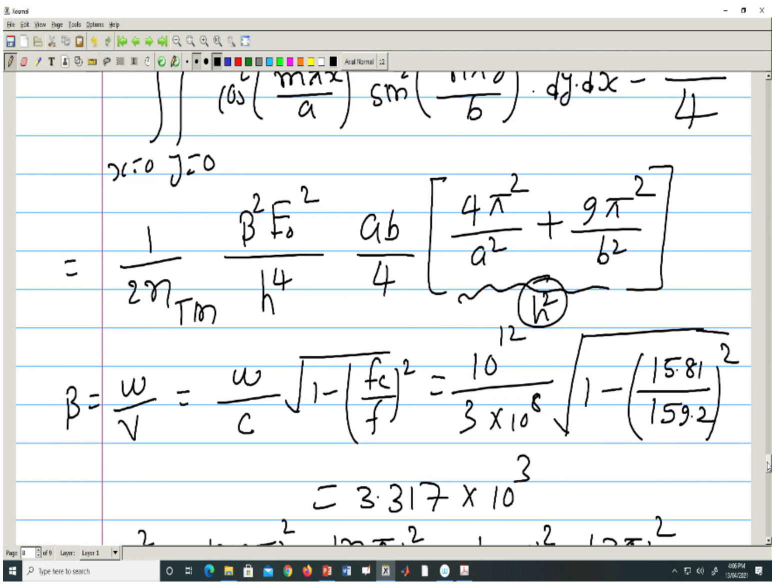

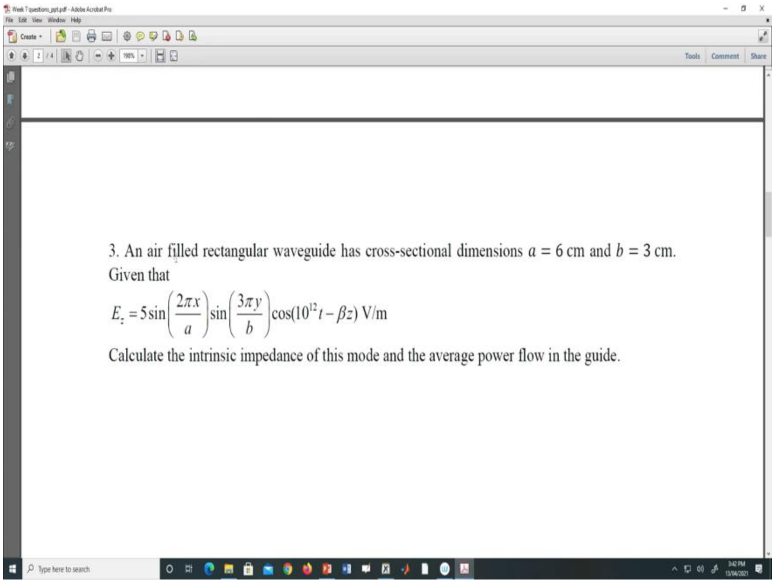

3. An air filled rectangular waveguide has cross-sectional dimensions a = 6 cm and b = 3 cm. Given that

$$E_z = 5\sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{3\pi y}{b}\right)\cos(10^{12}t - \beta z) \text{ V/m}$$

Calculate the intrinsic impedance of this mode and the average power flow in the guide.

This will come 1.502 milliwatt.

So, the average power flow is 1.502 milliwatt.

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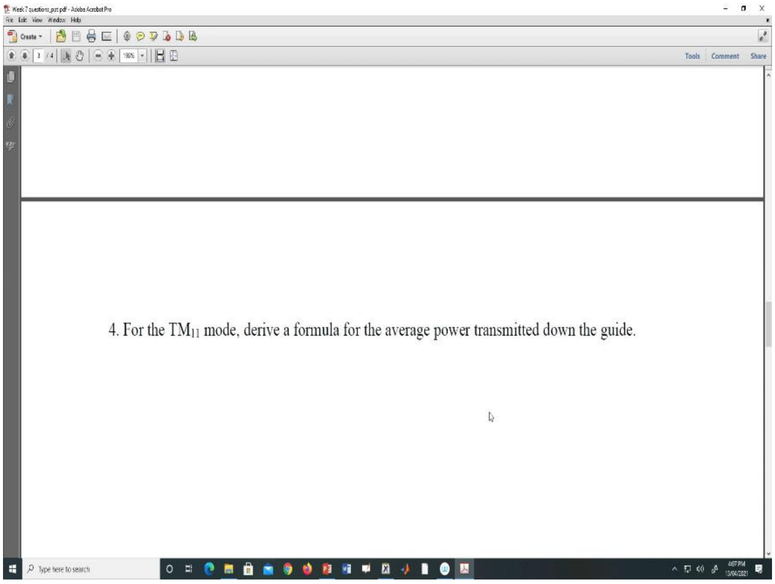

Now, next problem is

4. For the TM_{11} mode, derive a formula for the average power transmitted down the guide.

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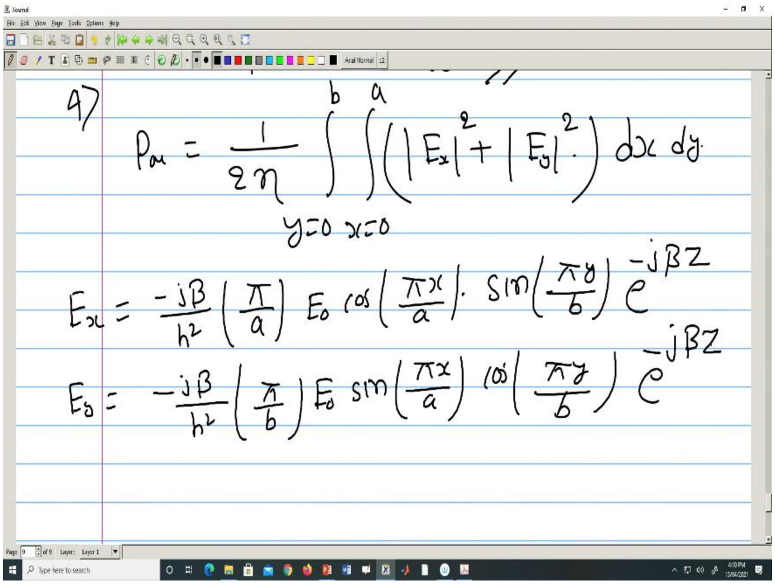

For the TM 11 mode,

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$$P_{av} = \frac{1}{2\eta}\int_{y=0}^{b}\int_{x=0}^{a}\left(|E_x|^2+|E_y|^2\right)dx\,dy$$

$$E_x = \frac{-j\beta}{h^2}\left(\frac{\pi}{a}\right)E_0\cos\left(\frac{\pi x}{a}\right)\cdot\sin\left(\frac{\pi y}{b}\right)e^{-j\beta z}$$

$$E_y = \frac{-j\beta}{h^2}\left(\frac{\pi}{b}\right)E_0\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)e^{-j\beta z}$$

So, this is the Ex and Ey.

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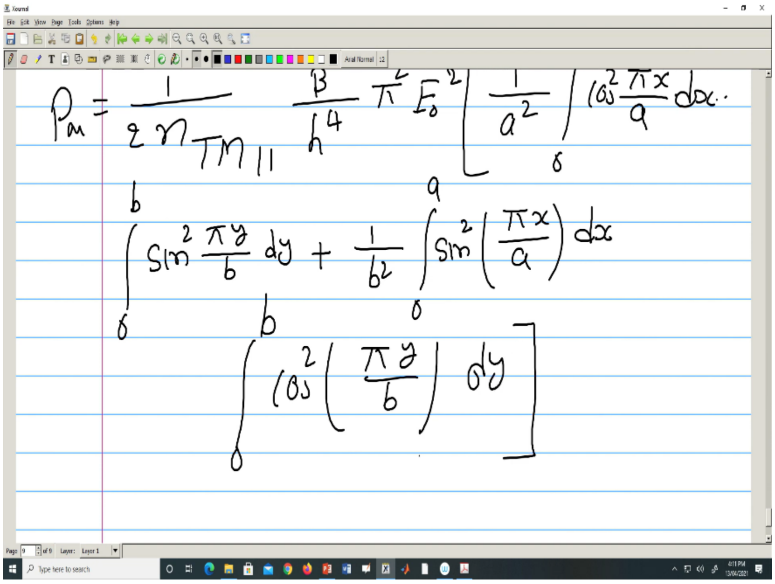

We can substitute the Ex and Ey in the power average formula and we will get power average equal to

$$P_{av} = \frac{1}{2\eta_{TM_{11}}}\frac{\beta^2}{h^4}\pi^2E_0^2\left[\frac{1}{a^2}\int_0^{a}\cos^2\frac{\pi x}{a}dx\int_0^b\sin^2\frac{\pi y}{b}dy+\frac{1}{b^2}\int_0^a\sin^2\left(\frac{\pi x}{a}\right)dx\int_0^b\cos^2\left(\frac{\pi y}{b}\right)dy\right]$$

$$= \frac{1}{2\eta_{TM,11}} \frac{\beta^2 \pi^2}{h^4} E_0^2 \left[\frac{1}{a^2} + \frac{1}{b^2}\right] \frac{ab}{4}$$

$$h^2 = \frac{\pi^2}{a^2} + \frac{\pi^2}{b^2} = \left(\frac{a^2+b^2}{a^2 b^2}\right) \pi^2$$

$$P_{ave} = \frac{1}{2\eta_{TM_{11}}} \frac{\beta^2 E_0^2}{h^2} \frac{ab}{4}$$

$$= \frac{1}{2\eta_{TM_{11}}} \frac{\beta^2 E_0^2 ab}{4} \cdot \frac{a^2 b^2}{(a^2+b^2)\pi^2}$$

$$= \frac{\beta^2 E_0^2 a^3 b^3}{8\pi^2 \eta_{TM_{11}}} \cdot \frac{1}{(a^2+b^2)}$$

So, this is the average power flow in the waveguide.

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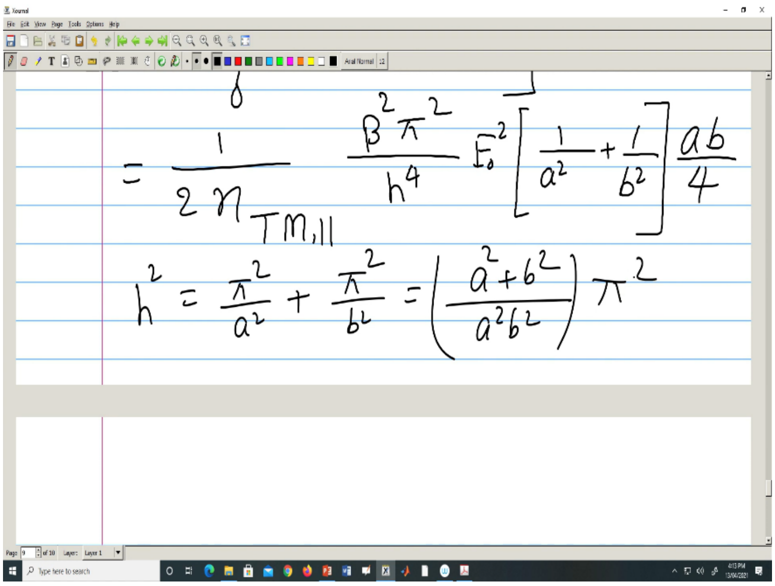

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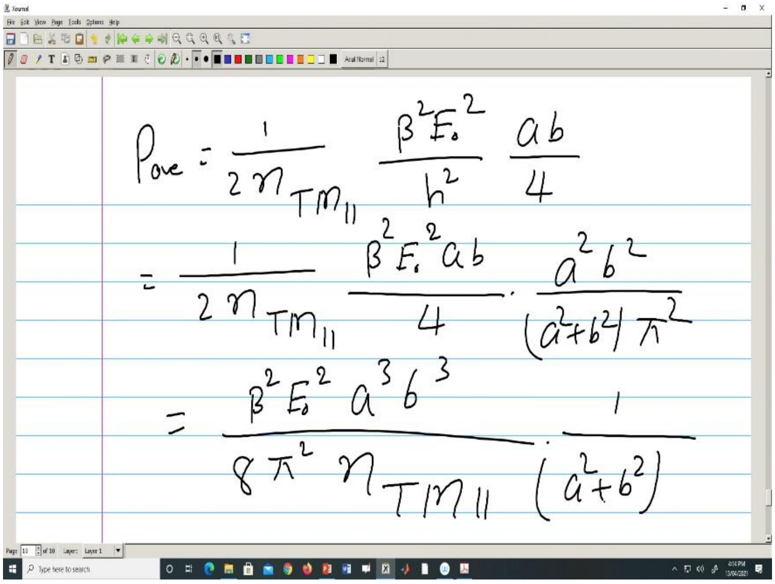

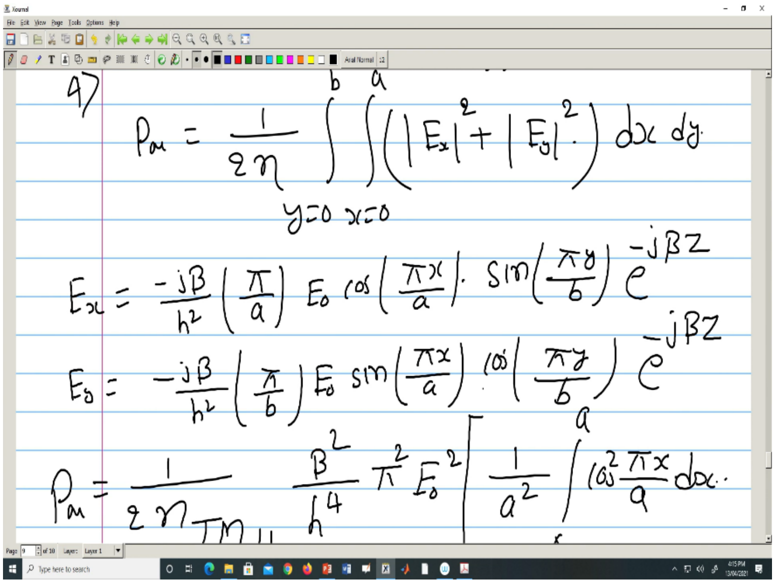

So, like this we can calculate for the TE mode.

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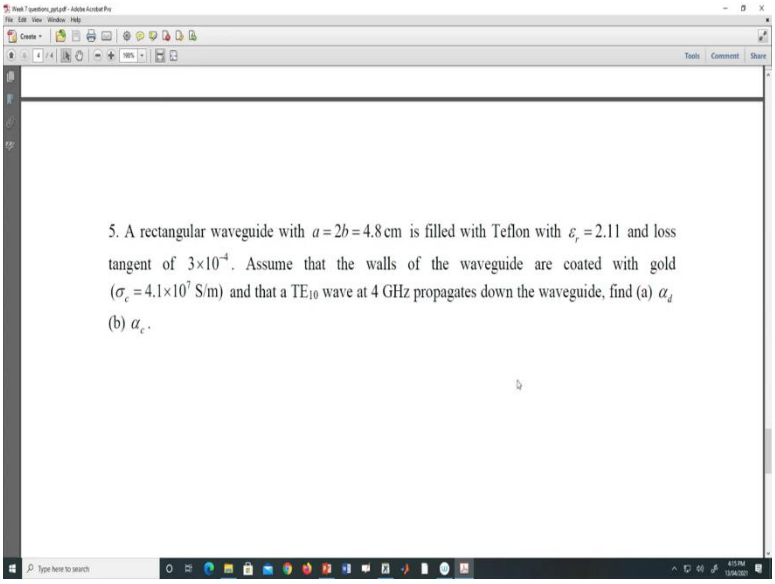

5. A rectangular waveguide with $a = 2b = 4.8\,\text{cm}$ is filled with Teflon with $\varepsilon_r = 2.11$ and loss tangent of 3×10^{-4}. Assume that the walls of the waveguide are coated with gold ($\sigma_c = 4.1\times10^{7}$ S/m) and that a TE_{10} wave at 4 GHz propagates down the waveguide, find (a) α_d (b) α_c.

Now we can see the next problem.

5. A rectangular waveguide with $a = 2b = 4.8\,\text{cm}$ is filled with Teflon with $\varepsilon_r = 2.11$ and loss tangent of 3×10^{-4}. Assume that the walls of the waveguide are coated with gold ($\sigma_c = 4.1\times10^{7}$ S/m) and that a TE_{10} wave at 4 GHz propagates down the waveguide, find (a) α_d (b) α_c.

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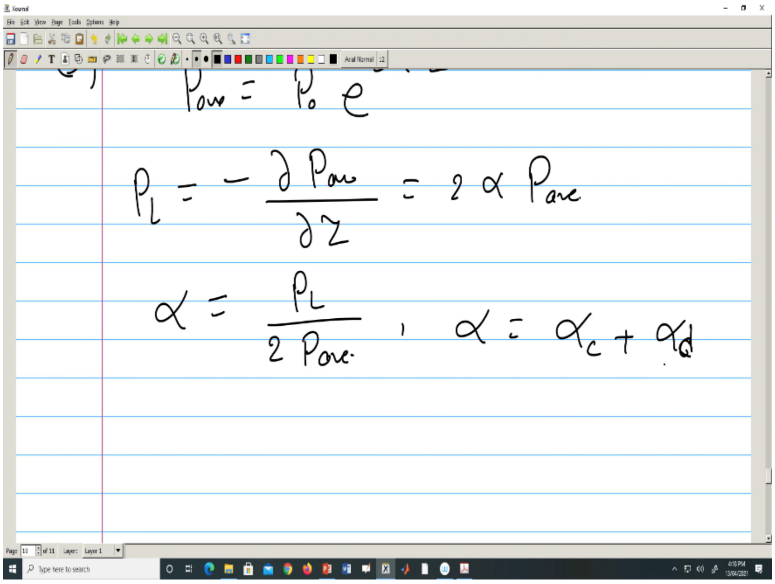

So, this is a power flow in the waveguide with attenuation:

$$P_L = -\frac{\partial P_{av}}{\partial z} = 2\alpha P_{ave}$$

$$\alpha = \frac{P_L}{2 P_{ave.}}, \quad \alpha = \alpha_c + \alpha_d$$

here alpha c is the attenuation constant due to ohmic or conduction loss and alpha d is the attenuation constant due to the dielectric loss.

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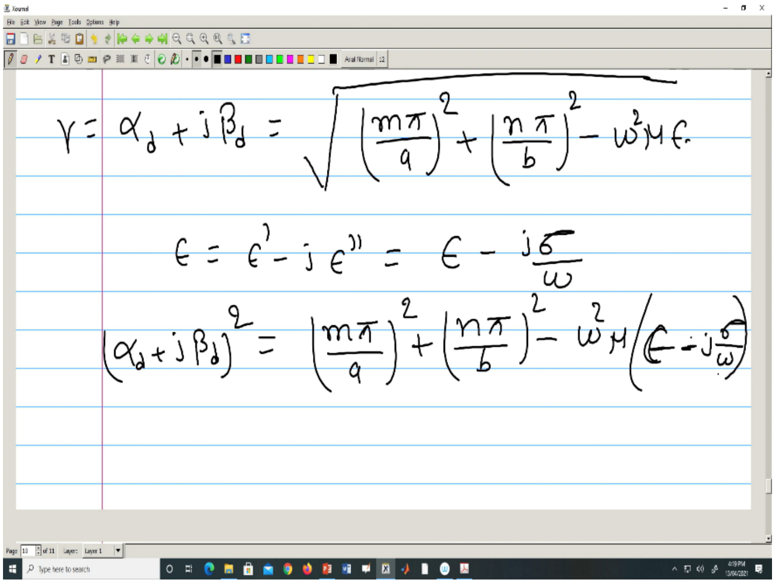

So, we know

$$\gamma = \alpha_d + j\beta_d = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$$

$$\epsilon = \epsilon' - j\epsilon'' = \epsilon - j\frac{\sigma}{\omega}$$

$$(\alpha_d + j\beta_d)^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\left(\epsilon - j\frac{\sigma}{\omega}\right)$$

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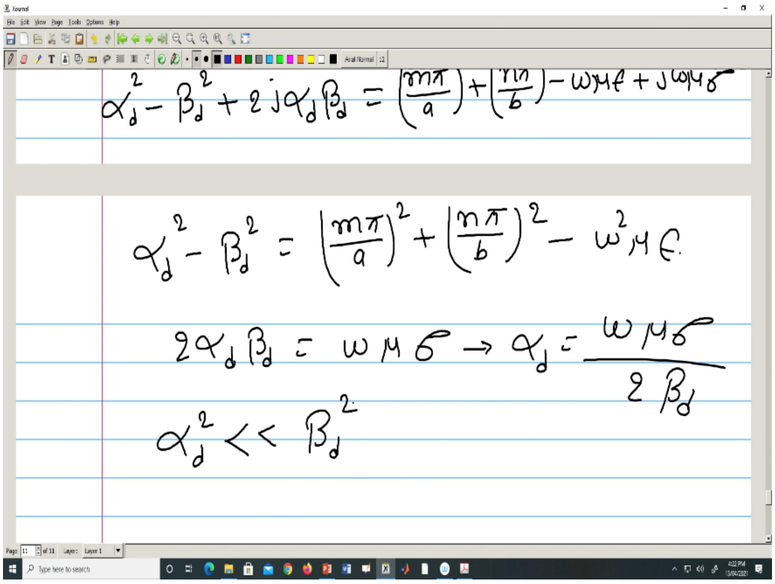
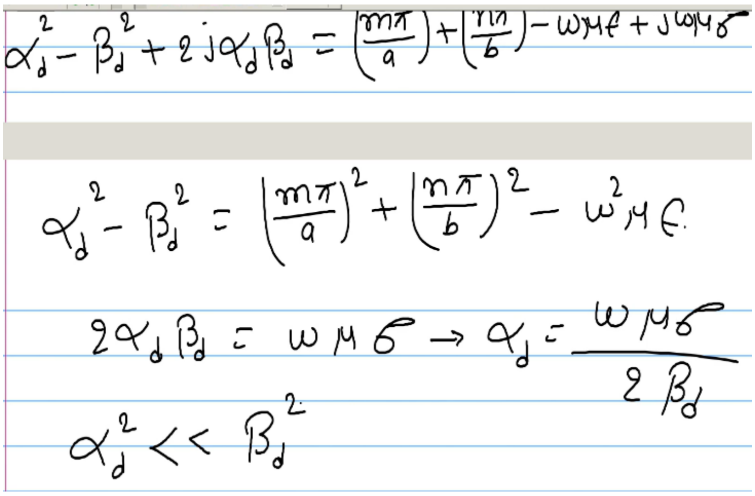


So, calculating further and equating the real and imaginary part both side:



we can assume that alpha d is very very less than means alpha d whole square is very very less than beta d whole square, because beta d is a propagation constant and we want minimum attenuation means minimum loss be with the waveguide as a power flow to send the power from one place to another place.

So, we can assume that attenuation is constant inside that dielectric that is very small and beta d's frequency dependent so, it will come in the very large compared to the alpha d. So, from this if alpha d is very very small so, we can write this expression.

$$-\beta_d^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon$$

$$\beta_d = \sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$= \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\alpha_d = \frac{\sigma\eta}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\alpha_c = \frac{2R_s}{b\eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[\frac{1}{2} + \frac{b}{a}\left(\frac{f_c}{f}\right)^2\right]$$

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$$\alpha_d^2 << \beta_d$$

$$-\beta_d^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon$$

$$\beta_d = \sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$= \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

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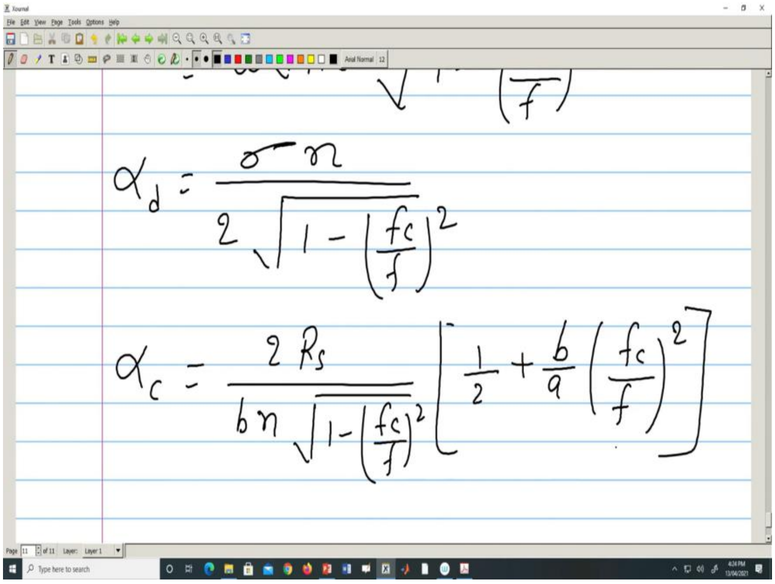

So, from this we will get the alpha d and alpha c I am not deriving, and here Rs is the skin resistance of the wall.

$$R_s = \frac{1}{\sigma_c \delta}, \quad \delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

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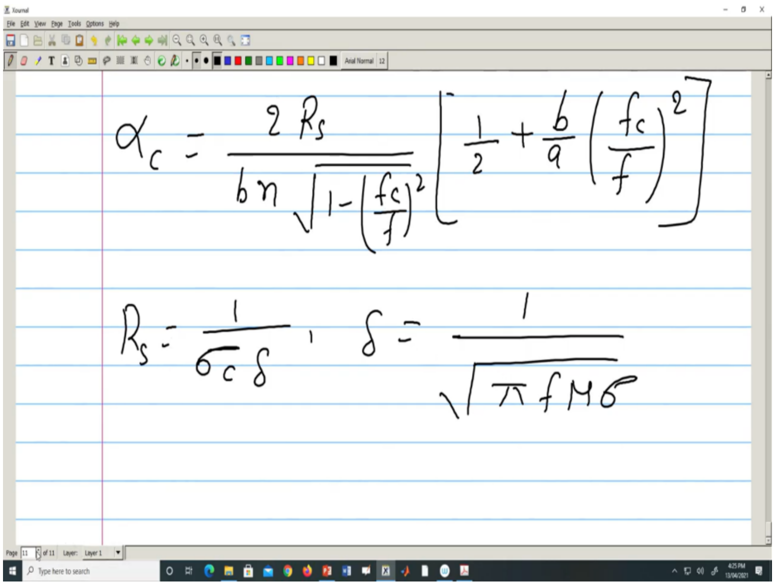

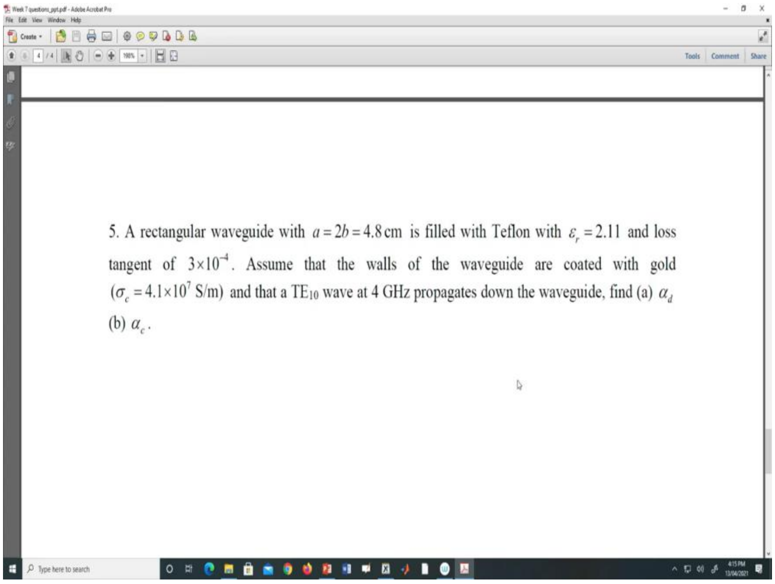

So, Rs is the skin resistance

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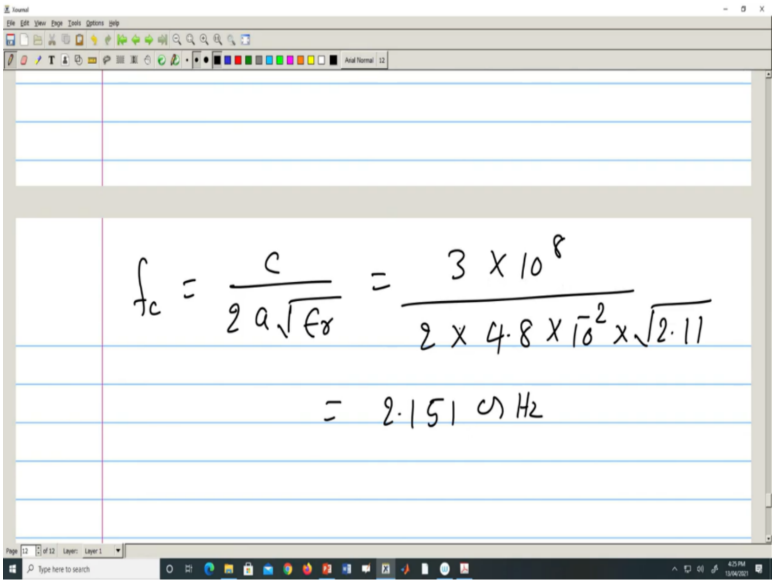

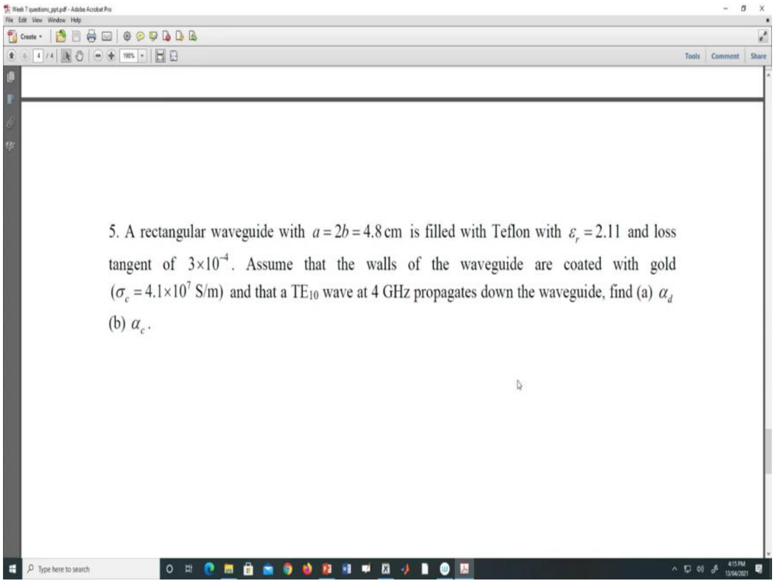

So, for the TE 10 mode, fc will be

$$f_c = \frac{c}{2a\sqrt{\epsilon_r}} = \frac{3\times10^{8}}{2\times4.8\times10^{-2}\times\sqrt{2.11}}$$

$$= 2.151 \text{ GHz}$$

So, this will come 2.151 gigahertz. So, this is the value of fc.

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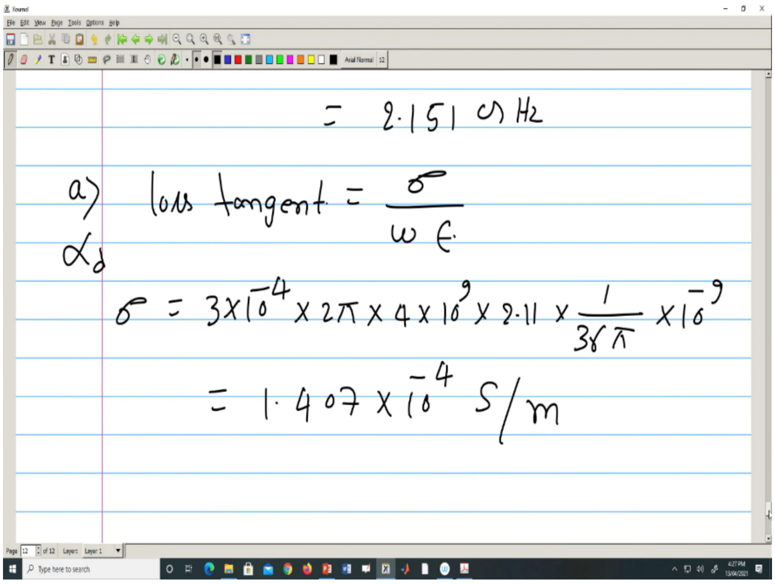

a) $\text{loss tangent} = \dfrac{\sigma}{\omega \epsilon}$

α_d

$$\sigma = 3 \times 10^{-4} \times 2\pi \times 4 \times 10^{9} \times 2.11 \times \frac{1}{36\pi} \times 10^{-9}$$

$$= 1.407 \times 10^{-4} \ S/m$$

So, we will continue this in the next class. Thank you.