Advanced Microwave Guided-Structures and Analysis
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Lecture 38
Rectangular Cavity Resonator

Welcome to the session on the rectangular cavity resonator. We are going to cover the potential functions in the rectangular cavity in this session as well as the field computation of the modes in the cavity and we will also cover the quality factor, the stored energy in the cavity so, let us go to the lecture.

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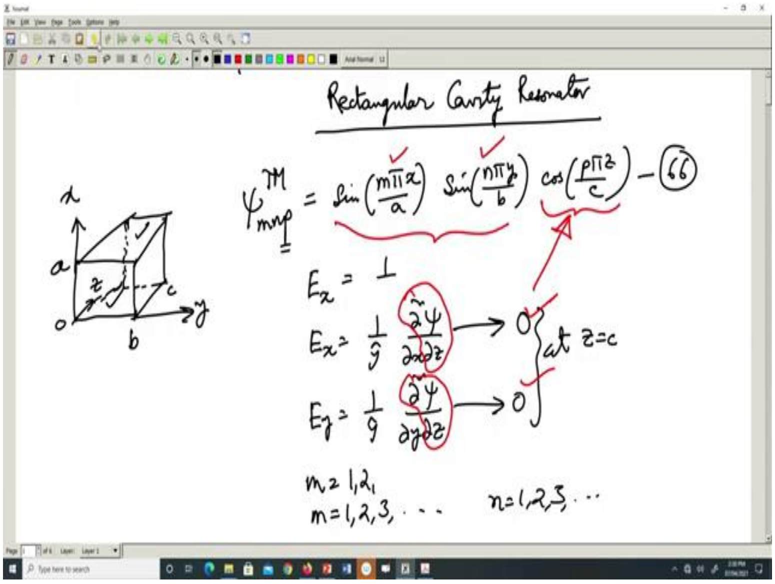

So, the first thing we want to understand about the rectangular cavity resonator is, derivation of the modes in the rectangular cavity resonator. We did the same thing for the rectangular waveguide, we first of all found out the potential functions for the rectangular waveguide. And from the potential functions we found out the, the field distributions of the TE to z and the TM to z mode sets in the rectangular waveguide.

So, we are now going to do the same thing for the cavity resonator only the fact that is different in this case is that the cavity comprises of two shorted metallic plates at z equal to 0 and let us suppose at 0 equal to c. So, the open waveguide structure if we terminate that open waveguide structure with two metallic plates at z equal to 0 and z equal to c, then the propagation along the z direction stops and we are standing waves along the z direction and then we found, and thereby we can find the potential functions of this new structure, and from the potential functions of the new structure.

We can derive the modes of the cavity so the first thing to understand is that, the what are the TE to z and that TM to z mode potential functions for the cavity. So, let us now concentrate on the TM to z modes of the cavity. So, the potential function

$$\psi^{TM}_{mnp} = \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{c}\right) \quad -(66)$$

So, we called this equation 66. Looking at the electric field distribution of the TM to z mode we will find that

$$E_x = \frac{1}{\hat{y}}\frac{\partial^2 \psi}{\partial x \partial z} \rightarrow 0$$
$$E_y = \frac{1}{\hat{y}}\frac{\partial^2 \psi}{\partial y \partial z} \rightarrow 0 \quad \text{at } z=c$$

$m = 1, 2,$
$m = 1, 2, 3, \ldots \qquad n = 1, 2, 3, \ldots$

So, both of these have to be 0 at z equal to c so,

So, because we have the derivative along the z direction, this derivative operation along the z direction, in order for E_x and E_y to vanish at z equal to c, we must have this potential function. So, here m is 1 so, here m is 1, 2, 3 etcetera and n is 1, 2, 3 etcetera.

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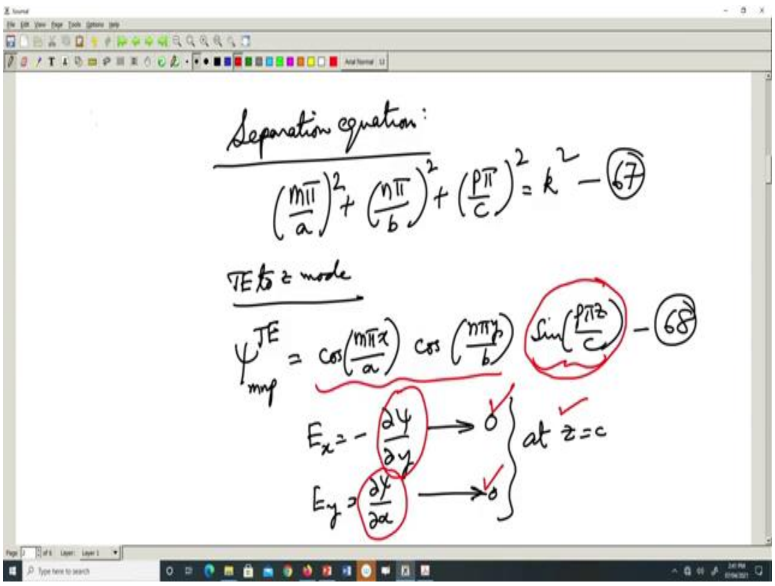

We have the separation equation,

Separation equation:

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2 = k^2 \quad (67)$$

let us call this equation 67. Mode so, similarly for the TE to z mode.

TE to z mode

$$\psi^{TE}_{mnp} = \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{c}\right) \quad (68)$$

$$\left.\begin{aligned} E_x &= -\frac{\partial \psi}{\partial y} \rightarrow 0 \\ E_y &= \frac{\partial \psi}{\partial x} \rightarrow 0 \end{aligned}\right\} \text{at } z = c$$

which is equation 68.

So, you see now that there is no derivative with respect to z, there is no derivative with respect to z so, therefore the z variation of the potential function is chosen like this, in order for E_x and E_y to be 0 at z equal to c.

Similarly, the other two potential functions are unaffected because at x equal to 0, x equal to a, and at y equal to 0, y equal to b, the boundary conditions for the rectangular cavity resonator is the same as that of the rectangular waveguide.

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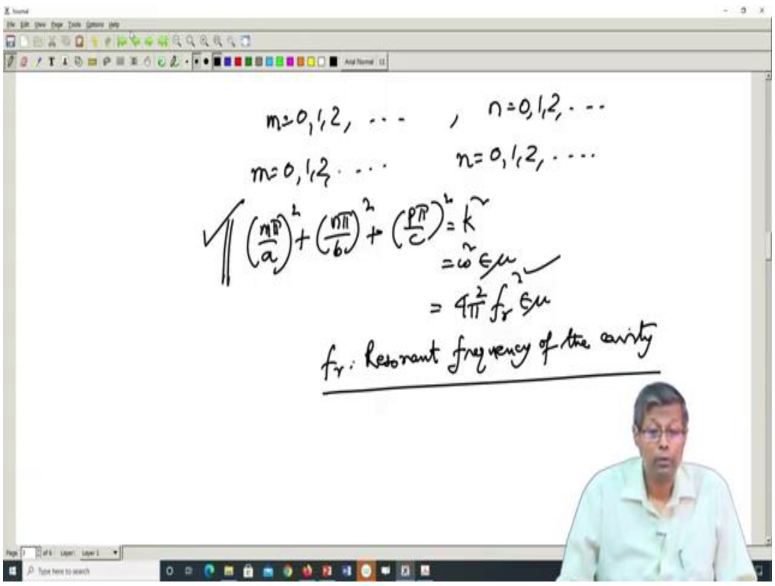

So, here m is equal to 0, 1, 2 and n equal to 0, 1, 2 and the same separation equation holds

$$m=0,1,2,\ \ldots\ ,\quad n=0,1,2,\ \ldots$$

$$m=0,1,2\ \ldots\qquad n=0,1,2,\ \ldots$$

$$\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2+\left(\frac{p\pi}{c}\right)^2=\tilde{k}^2=\tilde{\omega}^2\epsilon\mu=4\pi^2 f_r^2\epsilon\mu$$

f_r: Resonant frequency of the cavity

Where, f r is the resonant frequency of the cavity.

So, therefore the resonant frequency is going to vary with m, n and p so, different modes of the cavity are going to be characterized by different resonant frequencies. So, for distinct values of m, n and p, f_r is also going to be distinct. The resonant frequencies are again independent, of whether the mode is TE to z or TM to z. So, both TE to z and TM to z modes of the cavity will satisfy the same separation equation, and thereby will have the same resonant frequency so, the TE_{mnp} mode and the TM_{mnp} mode are going to have the same resonant frequency.

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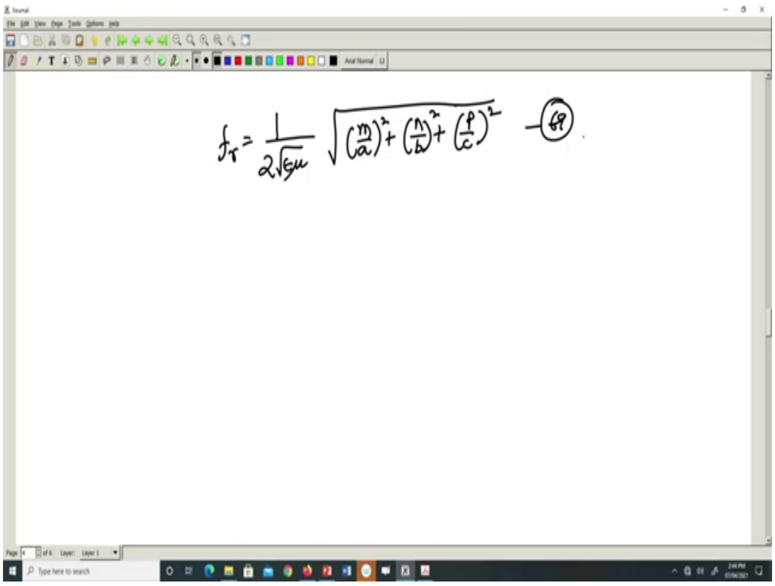

So, we can from the previous equation, write down the expression for the resonant frequency as

$$f_r=\frac{1}{2\sqrt{\epsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{p}{c}\right)^2}\qquad (68)$$

So, now therefore once the potential functions are known, we can find out the expressions for the electric and magnetic fields for any given TE_{mnp} or TM_{mnp} modes.

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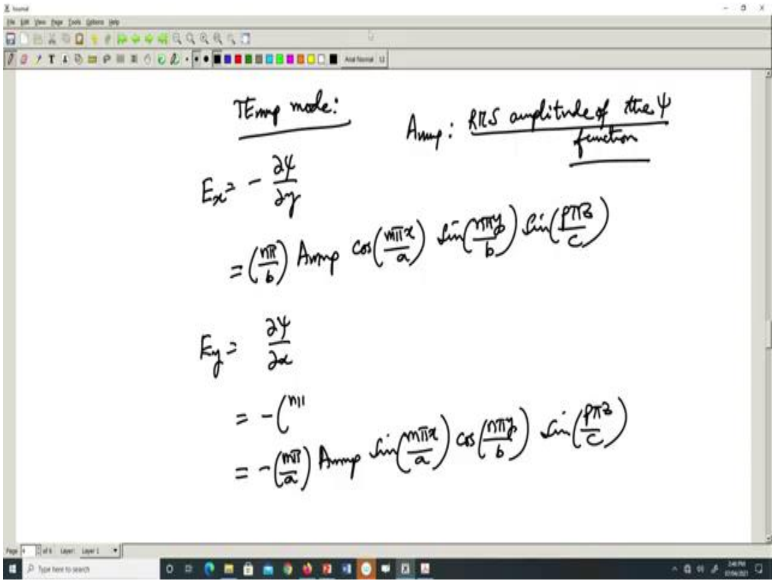
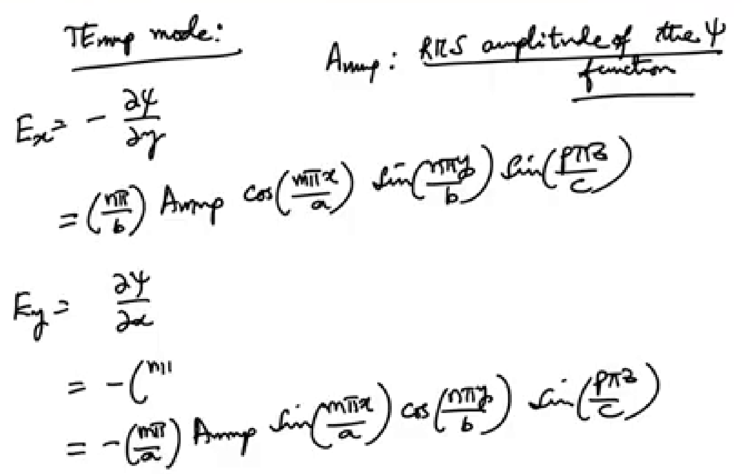


So, let us consider for example, the TE_{mnp} mode is given



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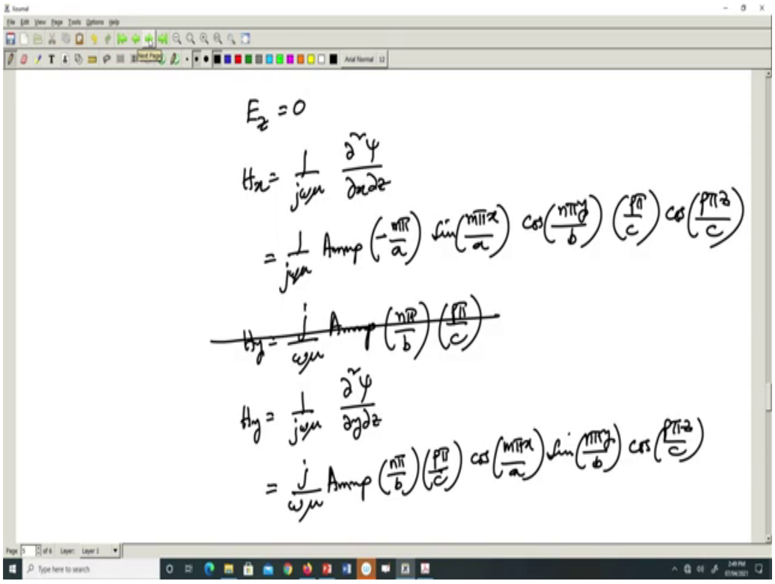
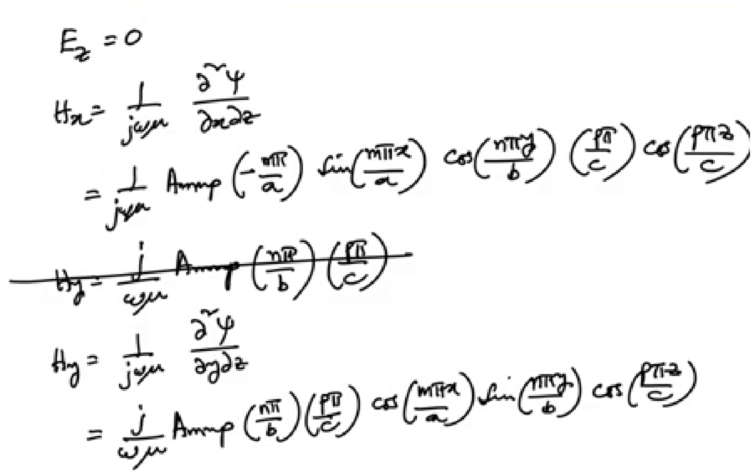


Similarly, E_z will be 0 because, it is the TE mode H x will be



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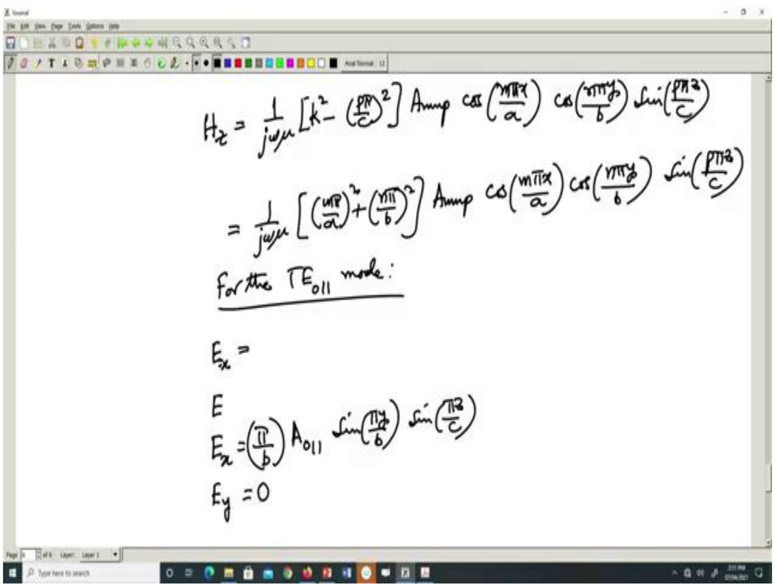

And H z will be

$$H_z = \frac{1}{j\omega\mu}\left[k^2 - \left(\frac{p\pi}{c}\right)^2\right] A_{mnp} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{c}\right)$$
$$= \frac{1}{j\omega\mu}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] A_{mnp} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{c}\right)$$

Now, if we specialize for the TE_{011} mode if we specialize for the TE_{011} mode, we will find by substituting m as 0, n as 1 p as 1 we will get

$$E_x = \left(\frac{\pi}{b}\right) A_{011} \sin\left(\frac{\pi y}{b}\right) \sin\left(\frac{\pi z}{c}\right)$$
$$E_y = 0$$

$$E_z = 0$$
$$H_x = 0$$
$$H_y = \frac{j}{\omega\mu} A_{011} \left(\frac{\pi}{b}\right)\left(\frac{\pi}{c}\right) \sin\left(\frac{\pi y}{b}\right) \cos\left(\frac{\pi z}{c}\right)$$
$$H_z = \frac{\pi^2}{j\omega\mu b^2} A_{011} \cos\left(\frac{\pi y}{b}\right) \sin\left(\frac{\pi z}{c}\right)$$

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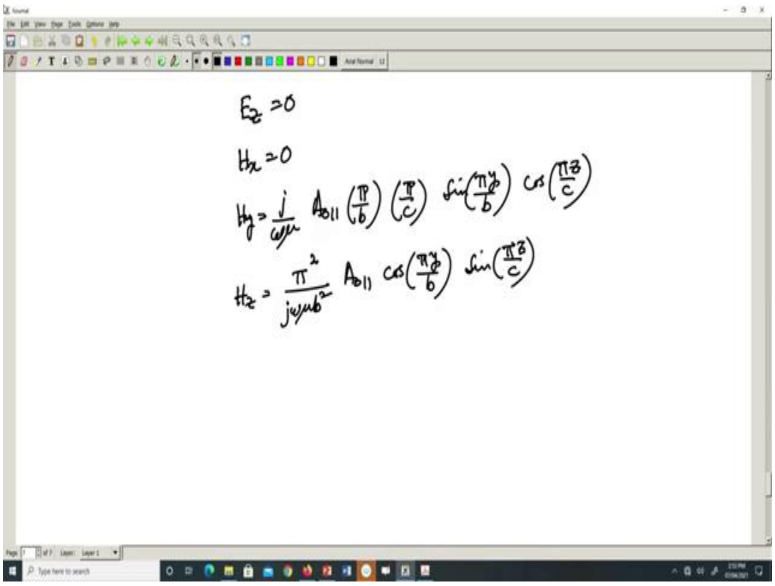

so, from this the average stored electric energy can be evaluated the average stored electric energy can be evaluated inside the cavity.

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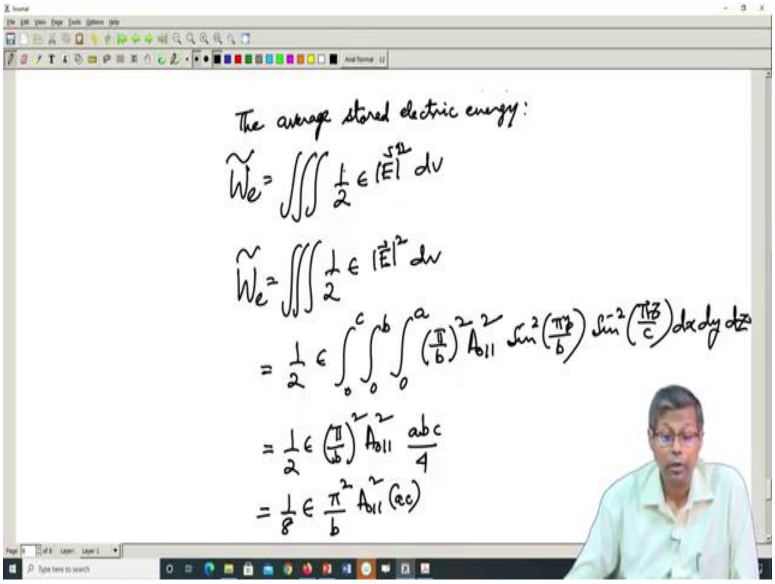
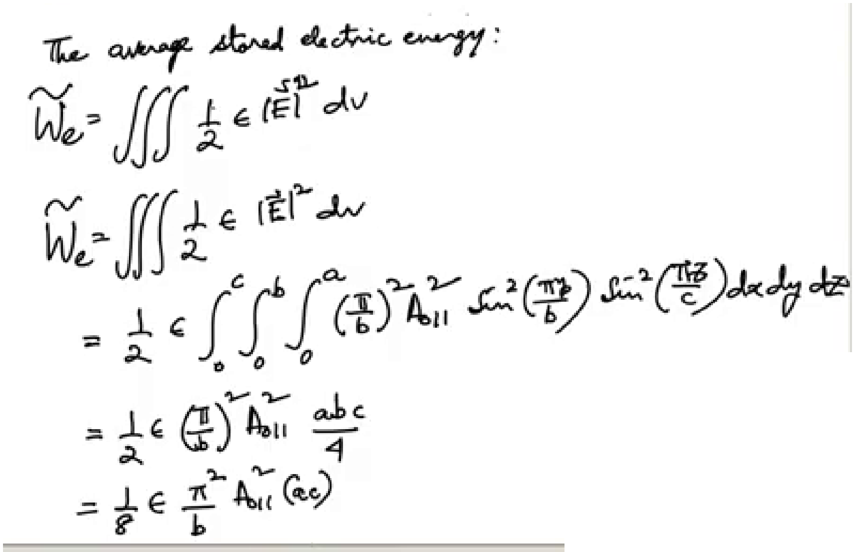


So, the average stored electric energy will be



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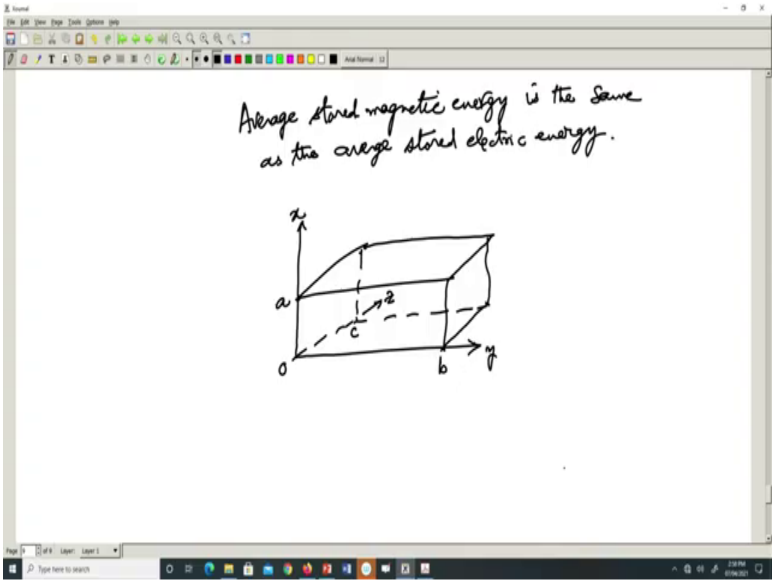
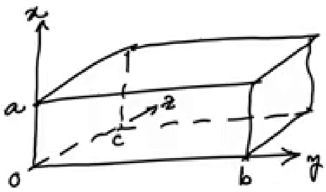


So, you can verify that so, please verify that the Average stored magnetic energy, is the same as the average stored electric energy.

So, now in order to find the q factor of the cavity, we need to find out the conductor loss that is the power dissipated on the six walls of the cavity, so for that, let us draw the cavity once again.



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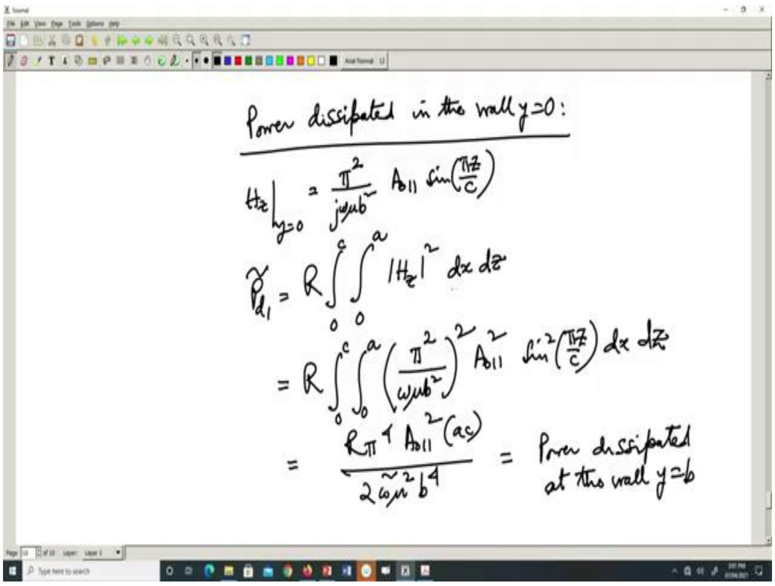

So, now for this structure power dissipated in the wall y equal to 0, we find that the only tangential component of the magnetic field, which is, which is present at y equal to 0.

Power dissipated in the wall $y=0$:

$$H_z\Big|_{y=0} = \frac{\pi^2}{j\omega\mu b^2} A_{11} \sin\left(\frac{\pi z}{c}\right)$$

$$\tilde{P}_{d_1} = R\int_0^c\int_0^a |H_z|^2 \, dx\, dz$$

$$= R\int_0^c\int_0^a \left(\frac{\pi^2}{\omega\mu b^2}\right)^2 A_{11}^2 \sin^2\left(\frac{\pi z}{c}\right) dx\, dz$$

$$= \frac{R\pi^4 A_{11}^2 (ac)}{2\omega^2\mu^2 b^4} = \text{Power dissipated at the wall } y=b$$

And an equal amount of power will be dissipated at y equal to b. So, similarly we can find the power dissipated at the surface x equal to 0.

Power dissipated at $x=0$:

$$\tilde{P}_{d_2} = R\int_0^c\int_0^b \left\{|H_y|^2 + |H_z|^2\right\} dy\, dz$$

$$= R\int_0^c\int_0^b \left\{\frac{1}{\omega^2\mu^2} A_{11}^2 \frac{\pi^4}{b^2 c^2} \sin^2\left(\frac{\pi y}{b}\right)\cos^2\left(\frac{\pi z}{c}\right) + \frac{\pi^4}{\omega^2\mu^2 b^4} A_{11}^2 \cos^2\left(\frac{\pi y}{b}\right)\sin^2\left(\frac{\pi z}{c}\right)\right\} dy\, dz$$

$$= R\frac{A_{11}^2\pi^4}{4\omega^2\mu^2 b}\left[\frac{1}{c} + \frac{c}{b^2}\right] = \text{Power dissipated at } x=a$$

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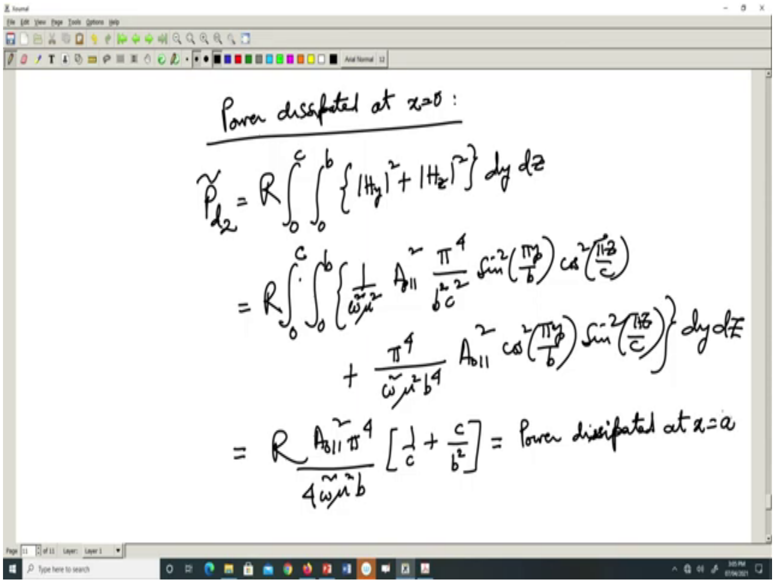

And that is the same power dissipated at x equal to a.

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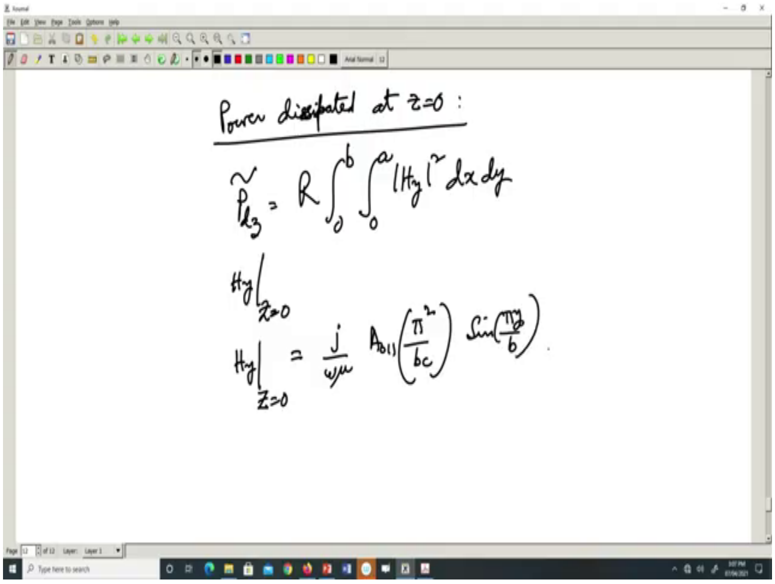

So, now for the third surface power dissipated at the surface z equal to 0.

Power dissipated at $z=0$:

$$\tilde{P}_{d_3} = R\int_0^b\int_0^a |H_y|^2 \, dx\, dy$$

$$H_y\Big|_{z=0}$$

$$H_y\Big|_{z=0} = \frac{j}{\omega\mu} A_{011}\left(\frac{\pi^2}{bc}\right)\sin\left(\frac{\pi y}{b}\right)$$

$$\tilde{P}_{d_3} = R\int_0^b\int_0^a \frac{1}{\omega^2\mu^2} A_{11}^2 \frac{\pi^4}{b^2c^2}\sin^2\left(\frac{\pi y}{b}\right) dx\, dy$$

$$= R\frac{1}{\omega^2\mu^2} A_{011}^2 \frac{a\pi^4}{2bc^2}$$

$$= \text{Power dissipated at } z=c$$

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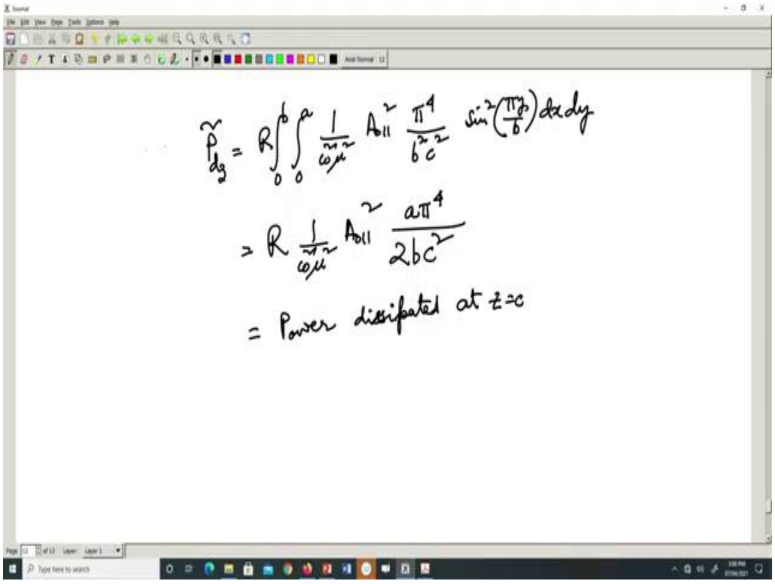

And the same power will be dissipated at z equal to c. So, from this we are in a position to find the q factor of the cavity.

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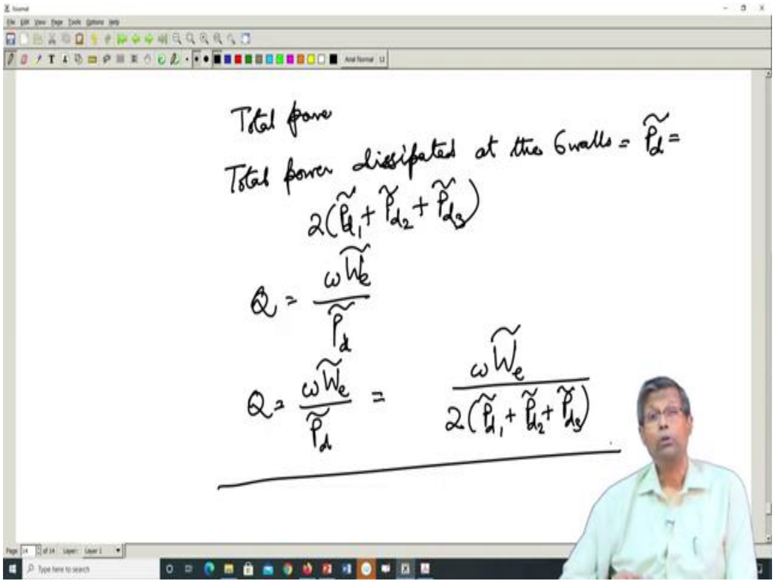

Before that the total, the total before that, the total power dissipated at the 6 walls of the cavity, becomes

Total powe

Total power dissipated at the 6 walls = $\tilde{P}_d = 2(\tilde{P}_{d_1}+\tilde{P}_{d_2}+\tilde{P}_{d_3})$

$$Q = \frac{\omega \tilde{W}_e}{\tilde{P}_d}$$

$$Q = \frac{\omega \tilde{W}_e}{\tilde{P}_d} = \frac{\omega \tilde{W}_e}{2(\tilde{P}_{d_1}+\tilde{P}_{d_2}+\tilde{P}_{d_3})}$$

So, this completes the derivation of the q factor of the cavity taking into account the conductor loss on the cavity walls, thank you.