Advanced Microwave Guided-Structures and Analysis
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Lecture 39
Rectangular Cavity Resonator Tutorials

Welcome to the next tutorial class now on waveguide resonator.

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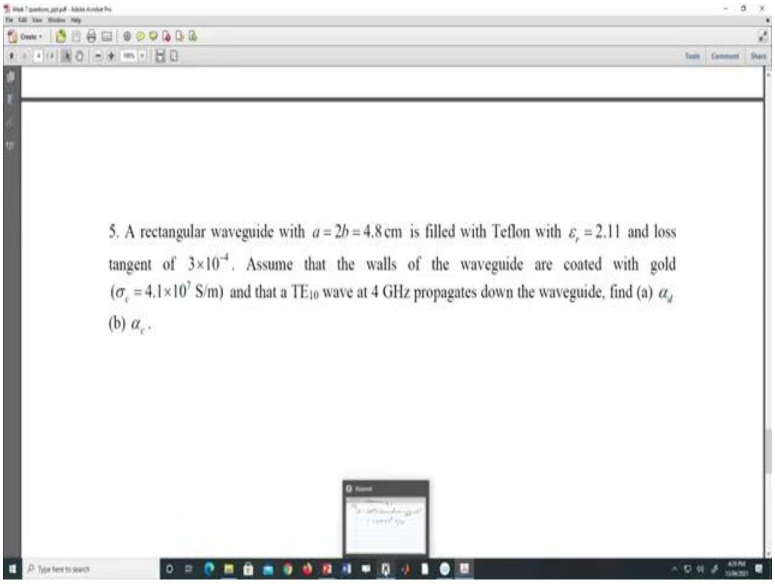

So, before go to the waveguide resonator so, we have to know the answer for this so alpha d and alpha c.

5. A rectangular waveguide with $a = 2b = 4.8\,\text{cm}$ is filled with Teflon with $\varepsilon_r = 2.11$ and loss tangent of 3×10^{-4}. Assume that the walls of the waveguide are coated with gold ($\sigma_c = 4.1\times10^7$ S/m) and that a TE_{10} wave at 4 GHz propagates down the waveguide, find (a) α_d (b) α_c.

$$\sigma = 3\times10^{-4}\times 2\pi \times 4\times10^{9}\times 2.11 \times \frac{1}{36\pi}\times10^{-9}$$

$$= 1.407\times10^{-4}\ S/m$$

$$\eta = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{120\pi}{\sqrt{2.11}} = 259.53\ \Omega$$

$$\alpha_d = \frac{\sigma\eta}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{1.407\times10^{-4}\times 259.53}{2\sqrt{1-\left(\frac{2.151}{4}\right)^2}}$$

$$= 2.165\times10^{-2}\ Np/m$$

(b) $$R_s = \sqrt{\frac{\pi f \mu}{\sigma_c}} = \sqrt{\frac{\pi\times4\times10^{9}\times4\pi\times10^{-7}}{4.1\times10^{7}}}$$

$$= 1.9625\times10^{-2}\ S/m$$

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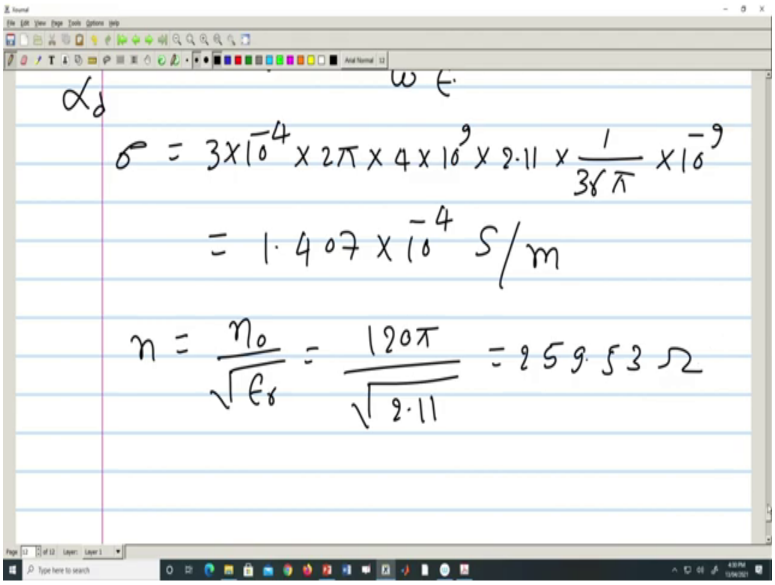

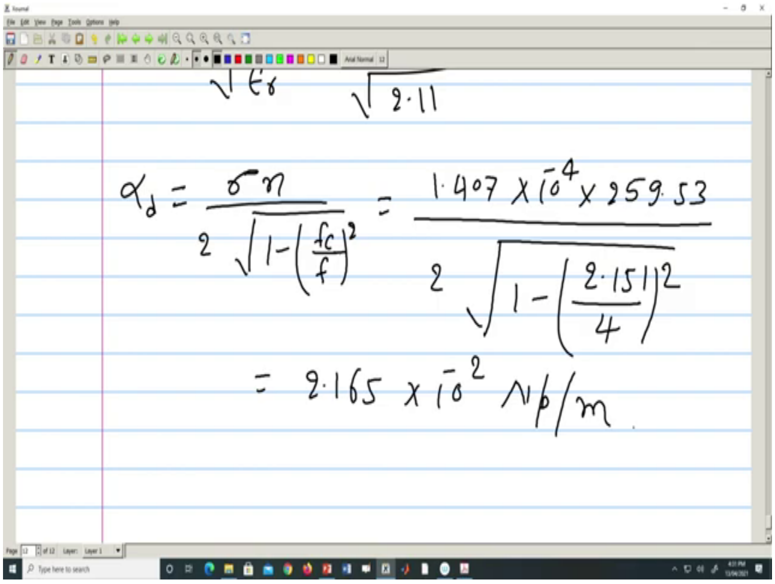

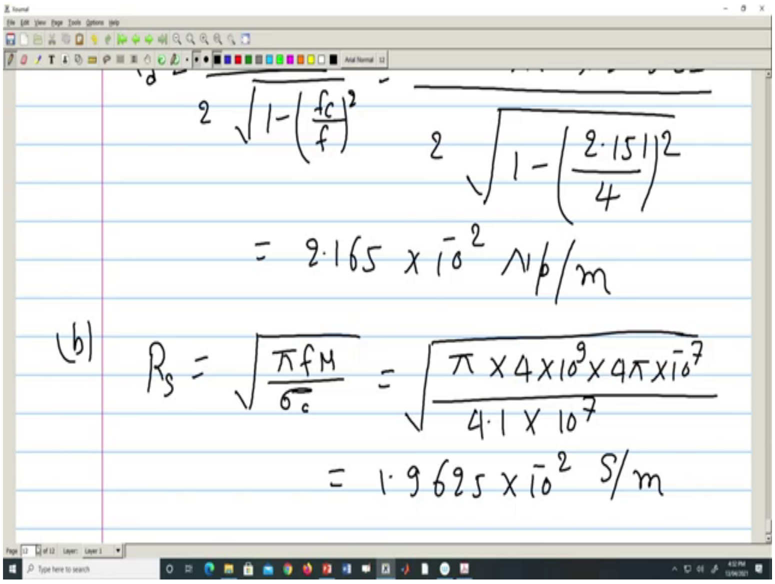

In the next part that is a calculation of Alpha c for that we need R_s.

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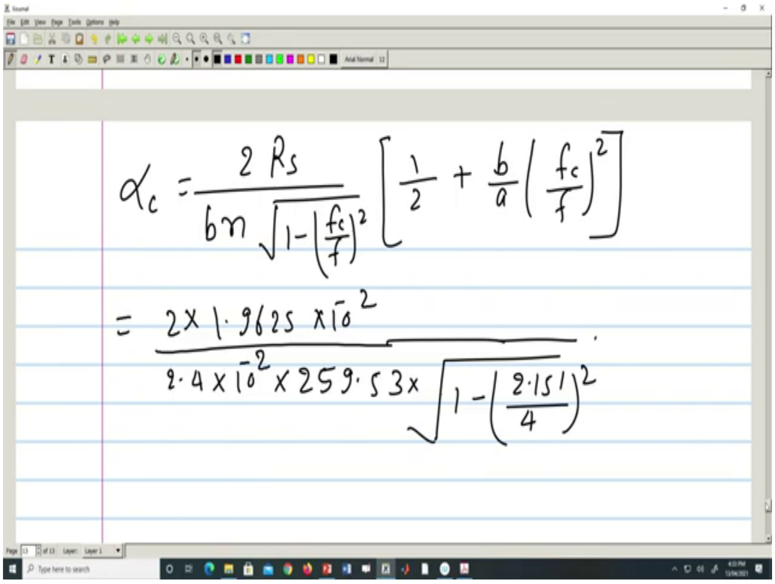

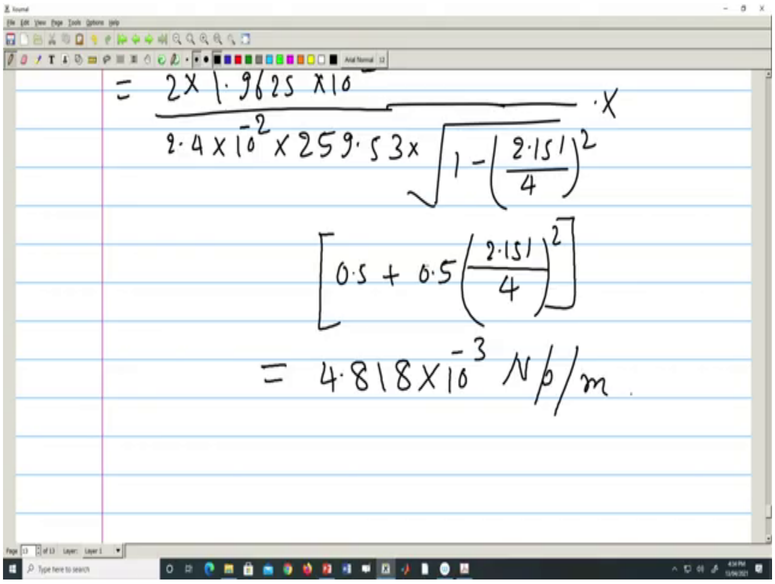

So, like this we can calculate the Alpha c and Alpha d.

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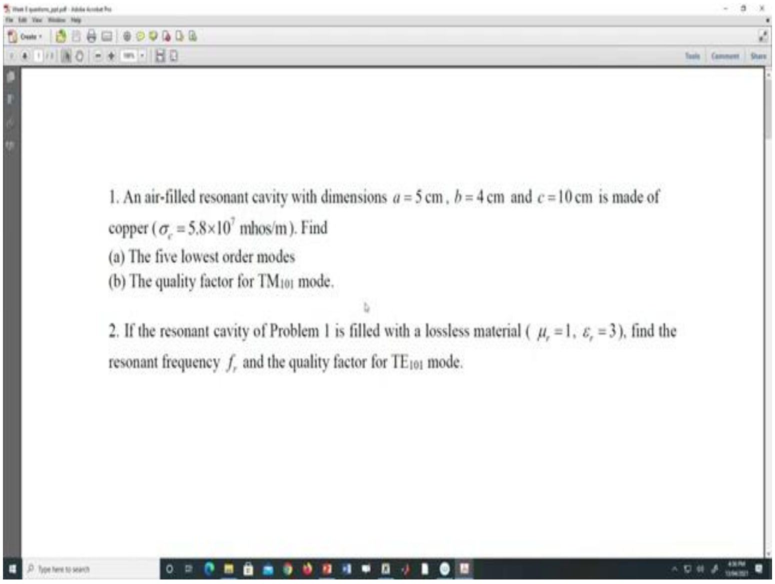
1. An air-filled resonant cavity with dimensions $a = 5$ cm , $b = 4$ cm and $c = 10$ cm is made of copper ($\sigma_c = 5.8\times10^7$ mhos/m). Find
(a) The five lowest order modes
(b) The quality factor for TM_{101} mode.

2. If the resonant cavity of Problem 1 is filled with a lossless material ($\mu_r = 1$, $\varepsilon_r = 3$), find the resonant frequency f_r and the quality factor for TE_{101} mode.

So, now we are going to do, the problem on waveguide resonator. So, first problem is,

1. An air-filled resonant cavity with dimensions $a = 5$ cm , $b = 4$ cm and $c = 10$ cm is made of copper ($\sigma_c = 5.8\times10^7$ mhos/m). Find
(a) The five lowest order modes
(b) The quality factor for TM_{101} mode.

For TM to z mode:

1) TM to z mode

$$E_z = E_0 \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{c}\right)$$

$$\beta^2 = k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2$$

$m = 1, 2, \ldots$, $n = 1, 2, \ldots$, $p = 0, 1, 2, 3$

$$\beta^2 = \omega^2 \mu \varepsilon$$

$$f_r = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

TE to z mode

$$H_z = H_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{c}\right)$$

$m = 0, 1, 2, \ldots$, $n = 0, 1, 2, \ldots$ $p = 1, 2, 3 \ldots$

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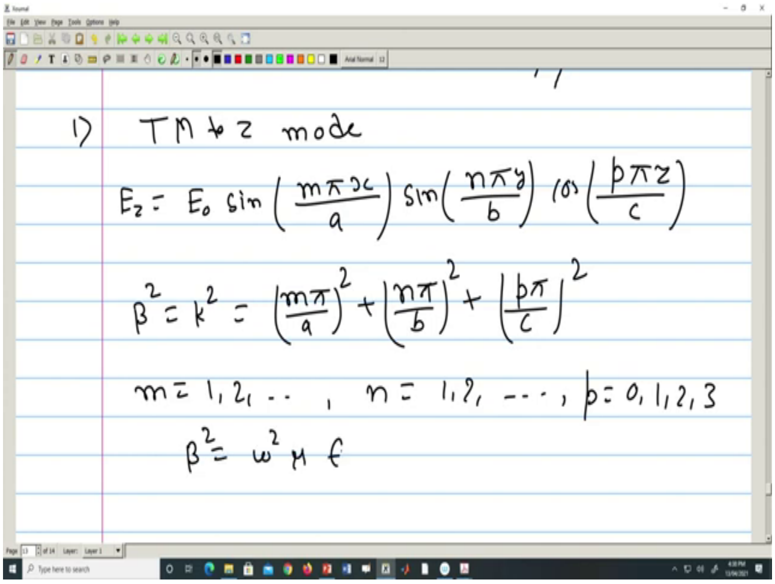
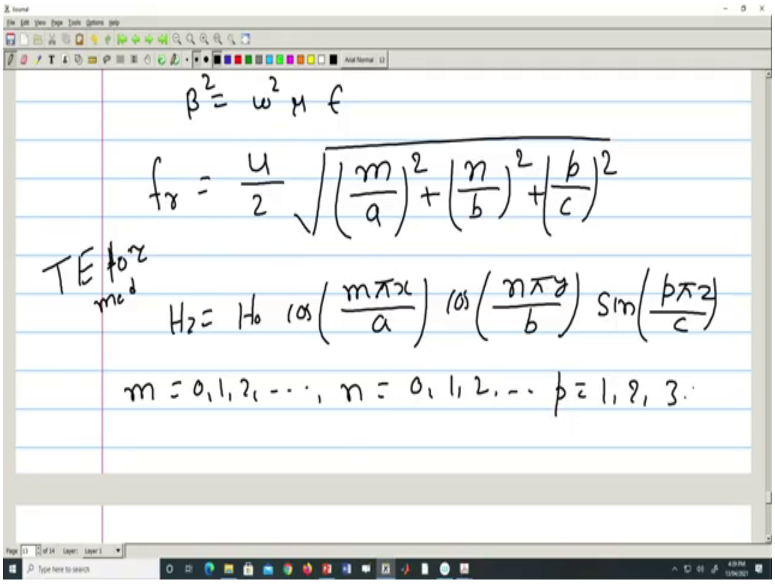
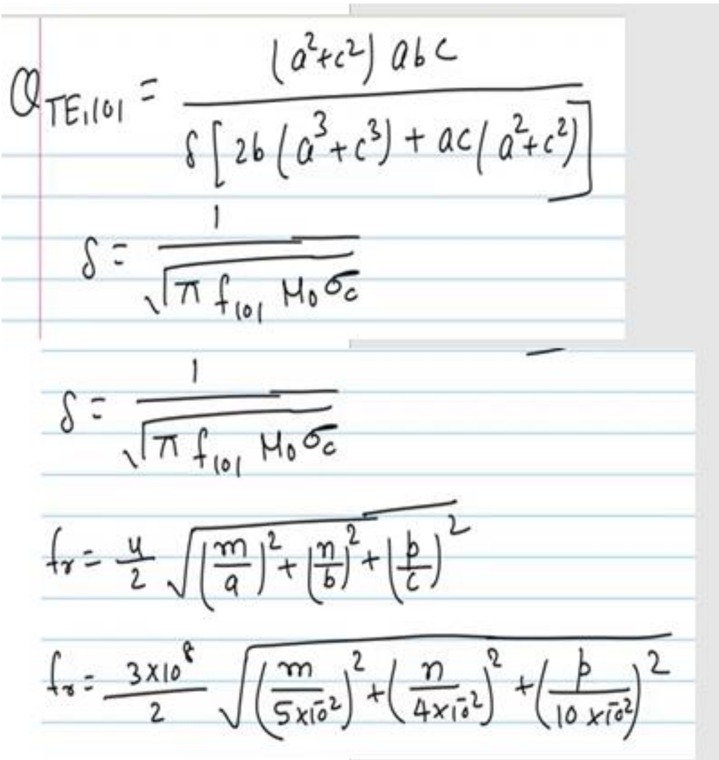




Quality factor is defined as:



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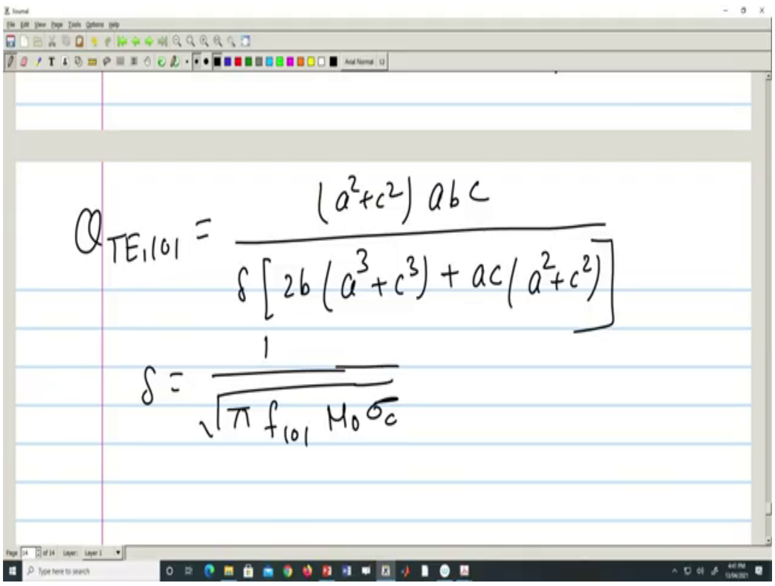
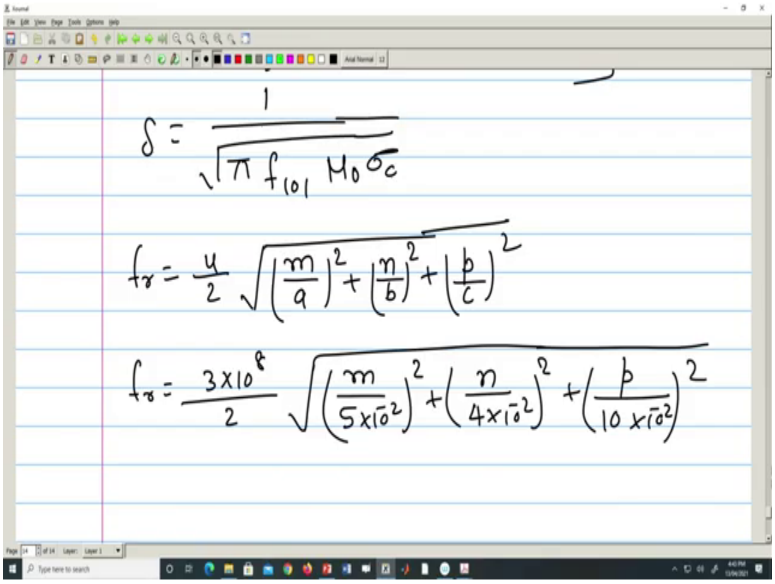
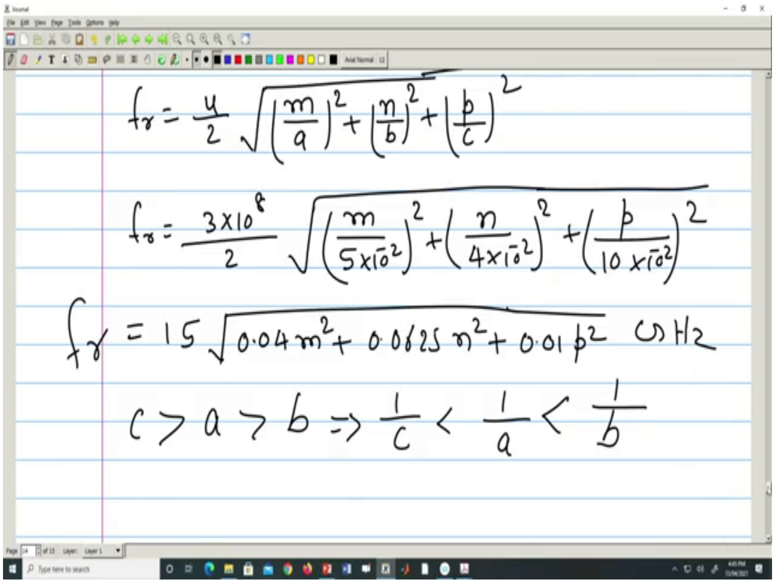
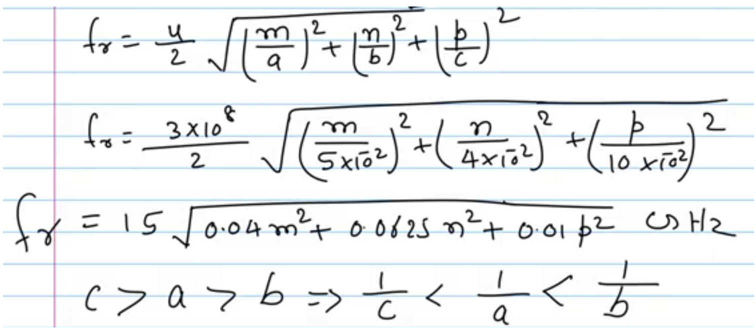






F_r is defined as



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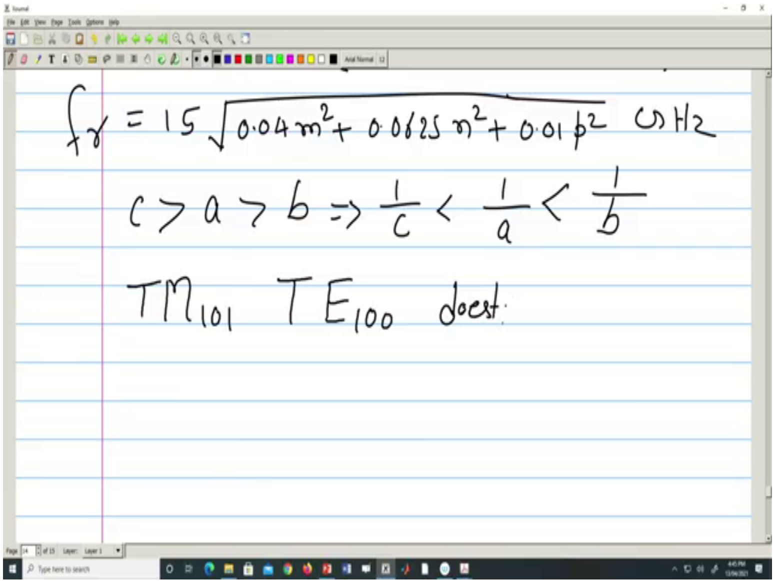
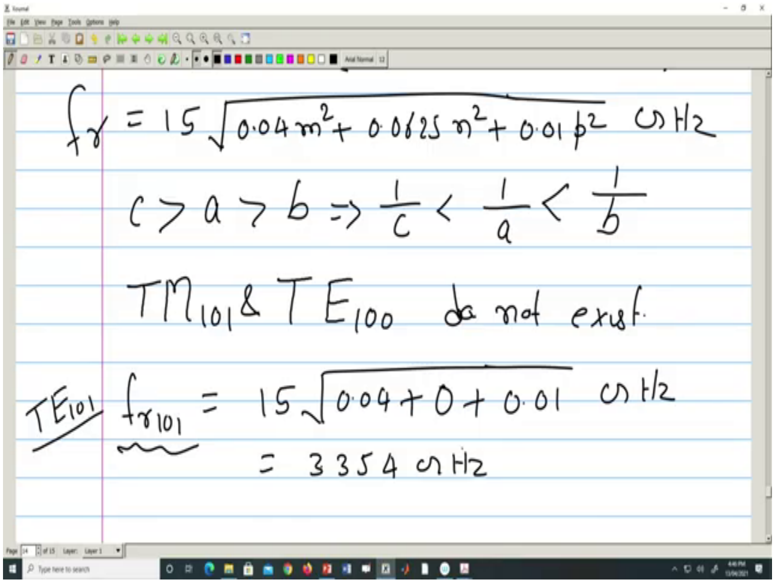
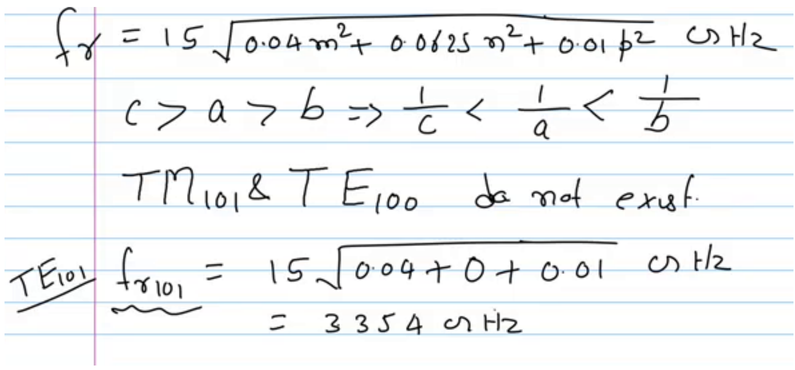




And TM 101 mode that will not exist TM 101 mode, that will not exist TE 100 also will not exist because, for TE case this last should be, last one should be start from 1, but here it is 0. So, does not exist so, this will be does not exist so, this mode and this mode. So, this will be not exist, so this will be not exist, and fr, fr 101 now we can calculate the fr 101 so, this will be 15 so, this for the means for the TE 101 mode, a TE 101 mode, either it can be for the TM 101 mode also. same formula we can apply but, this will not exist so, this is for the TE 101 mode, so 15 so, m will be 1, m will be 1 so, this will be 0.04 m will be 1, plus 0 because middle part n is 0, plus 0.01 so, and p will be 1. So, this Giga Hertz so, after calculation it will 3.354 Giga Hertz. So, this is the fr 101 net higher order mode will be a TE 01 mode, 011 mode.



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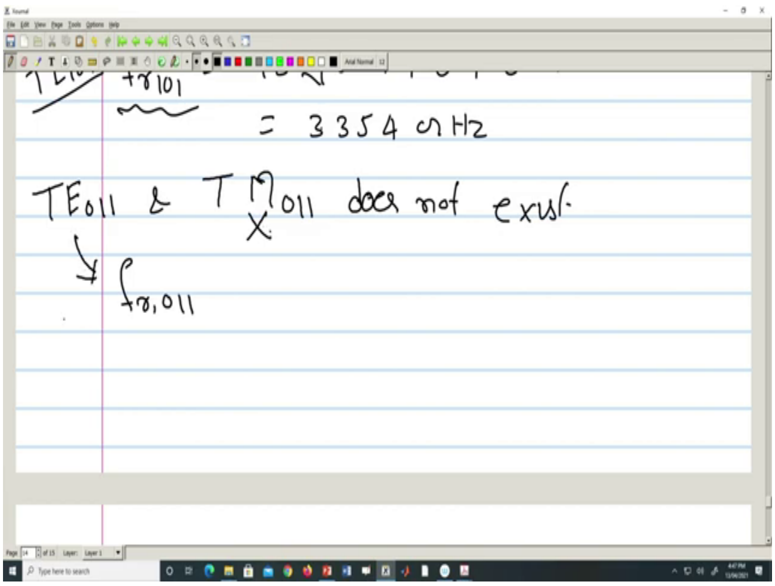
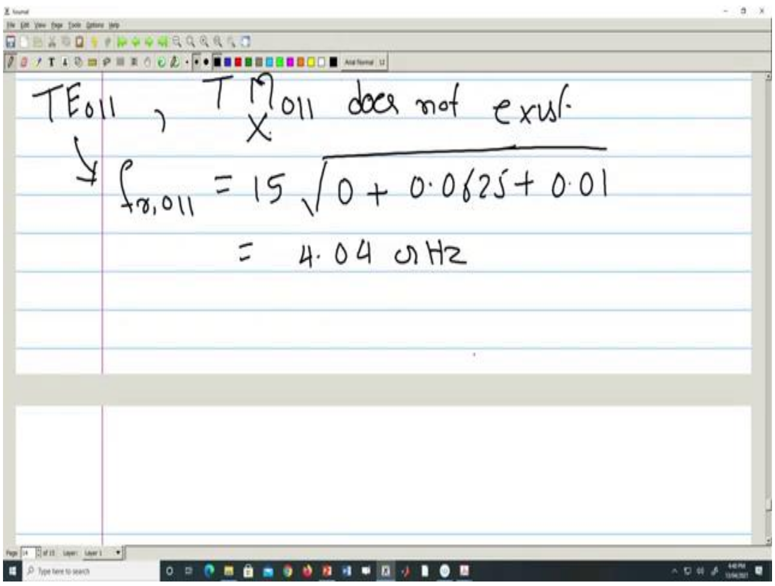
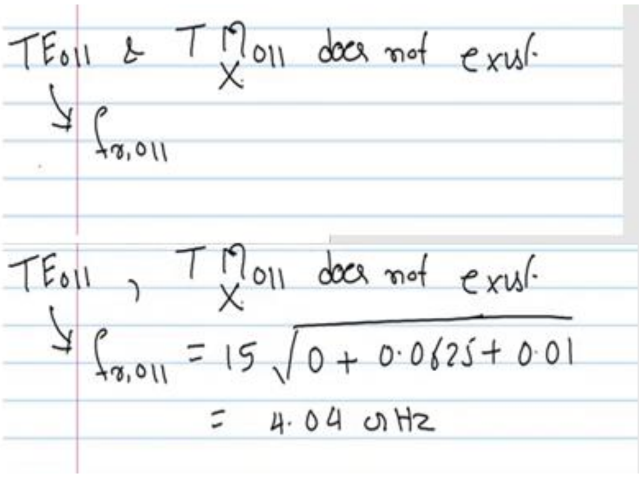




So, next higher order mode will be, TE 011 mode and TM if we can see the TM 011 so, this will not exist because, for the TM case, first this m should start from 1 so, this will be does not exist. So, this will be does not exist so, fr for 011 that means this does not exist but, this will exist so, this is not and, so this will exist. Fr 011 will be 15 a square root of so, m will be

0, plus 0.0625 plus n will be, n will be 1 so, 0.01. This will be 4.04 Giga Hertz. So, fr 011 will be 4.04 Giga Hertz.



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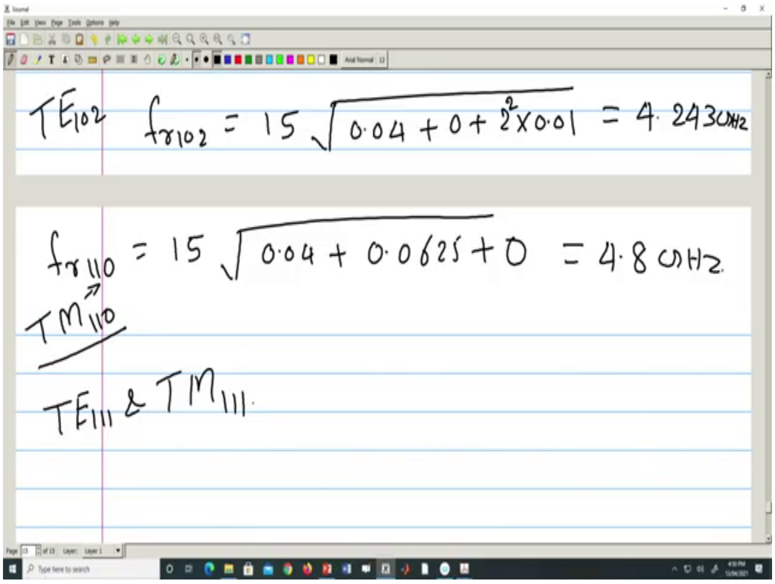
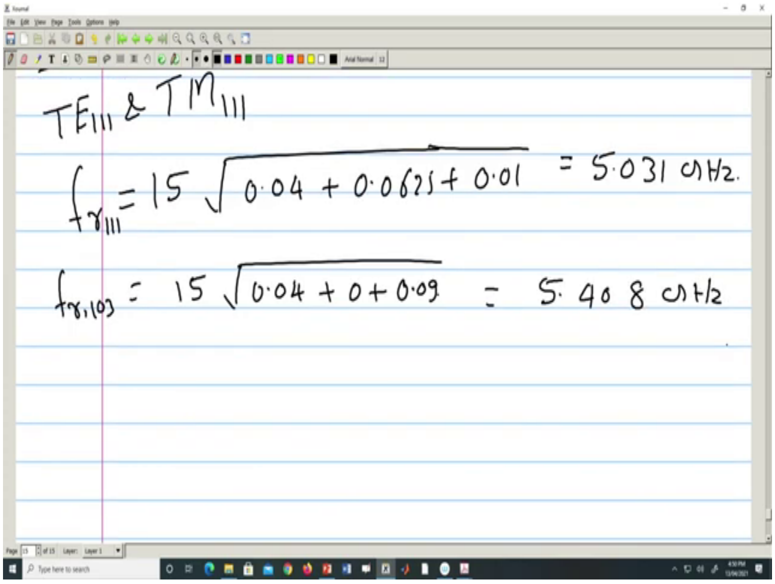
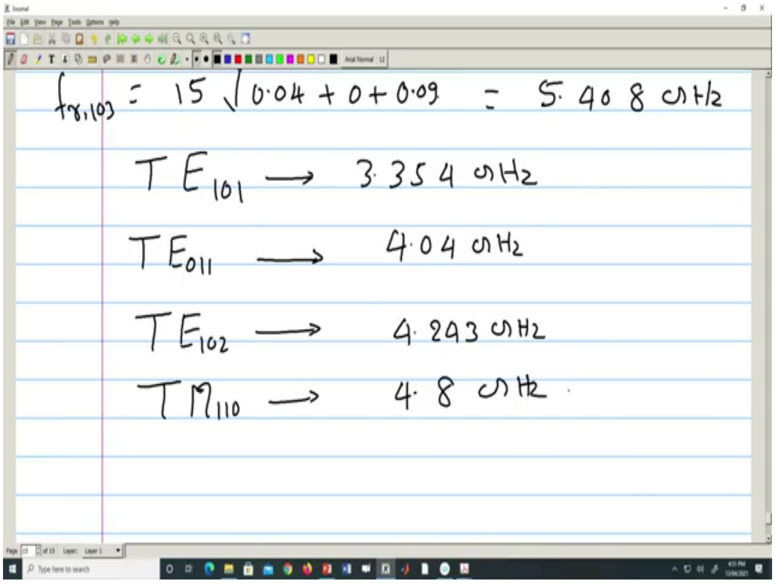
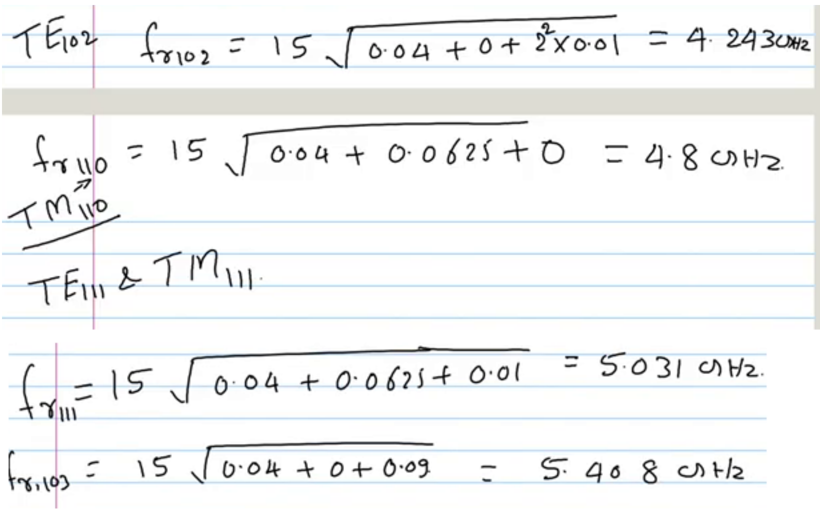


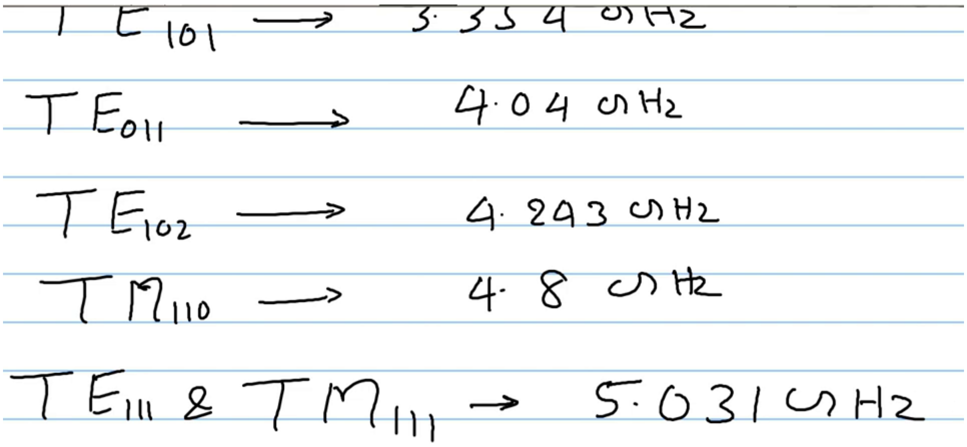




Similarly



So, that is



for the TM 103, so we can write for all the resonant frequencies, 101 resonant frequency 3.354 Giga Hertz next is TE 011 for that it is 4.04 Giga Hertz. Next is TE 102 that is 4.243 Giga Hertz, next is TM 110 that is 4.8 Giga Hertz.

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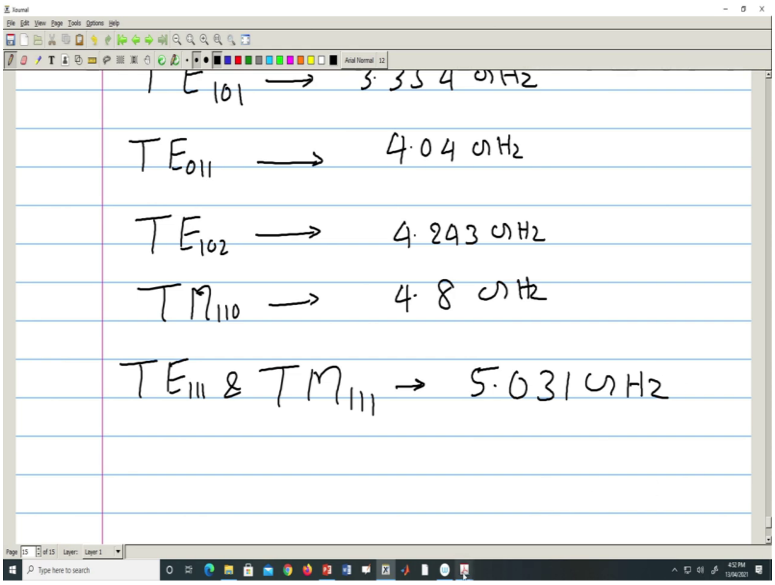

Next one is TE 111 that is degenerate mode, or TM 111, that both have same resonant frequencies it will be 5.031 Giga Hertz. So, these are the five lowest resonant frequencies.

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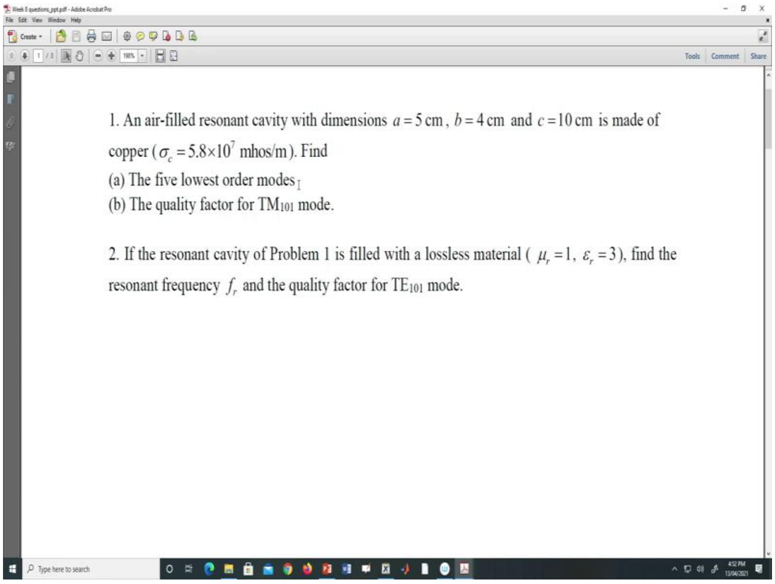
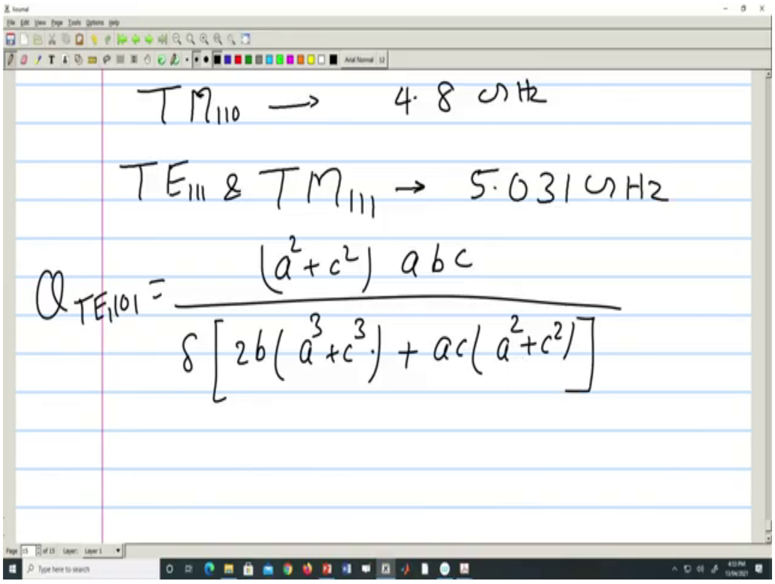


1. An air-filled resonant cavity with dimensions $a = 5$ cm, $b = 4$ cm and $c = 10$ cm is made of copper ($\sigma_c = 5.8\times10^7$ mhos/m). Find

(a) The five lowest order modes

(b) The quality factor for TM_{101} mode.

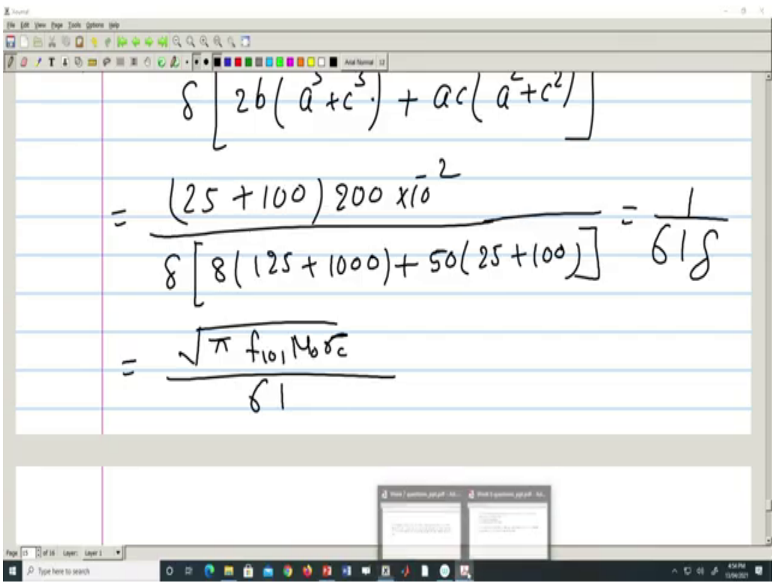
2. If the resonant cavity of Problem 1 is filled with a lossless material ($\mu_r = 1$, $\varepsilon_r = 3$), find the resonant frequency f_r and the quality factor for TE_{101} mode.



$TM_{110} \rightarrow 4.8$ GHz

TE_{111} & $TM_{111} \rightarrow 5.031$ GHz

$$Q_{TE_{101}} = \frac{(a^2+c^2)\,abc}{\delta\left[2b(a^3+c^3) + ac(a^2+c^2)\right]}$$



$$= \frac{(25+100)\,200\times10^{-2}}{\delta\left[8(125+1000)+50(25+100)\right]} = \frac{1}{61\delta}$$

$$= \frac{\sqrt{\pi f_{101}\mu_0\sigma_c}}{61}$$

We know the formula that

$$Q_{TE_{101}} = \frac{(a^2+c^2)\,abc}{\delta\left[2b(a^3+c^3)+ac(a^2+c^2)\right]}$$

$$= \frac{(25+100)\,200\times10^{-2}}{\delta\left[8(125+1000)+50(25+100)\right]} = \frac{1}{61\delta}$$

$$= \frac{\sqrt{\pi f_{101}\mu_0\sigma_c}}{61}$$

This will come after simplification

$$= \frac{\sqrt{\pi\times3.354\times10^{9}\times4\pi\times10^{-7}\times5.8\times10^{7}}}{61}$$

$$= 14366$$

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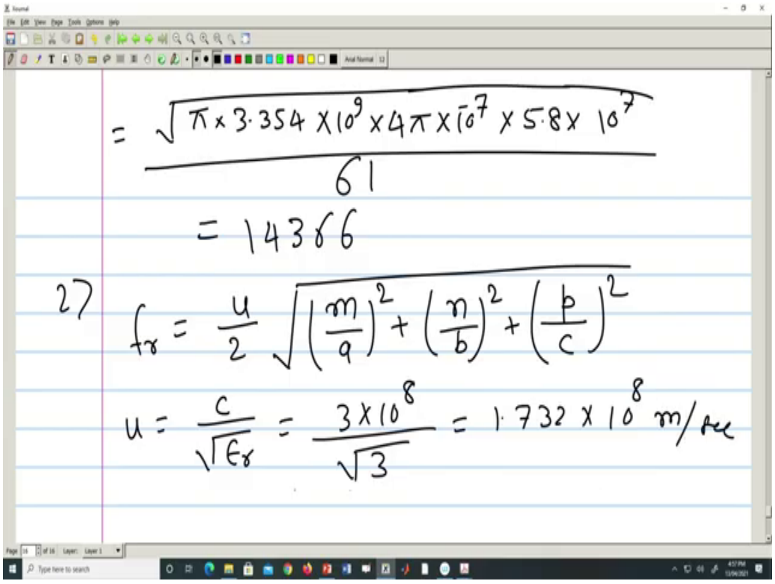
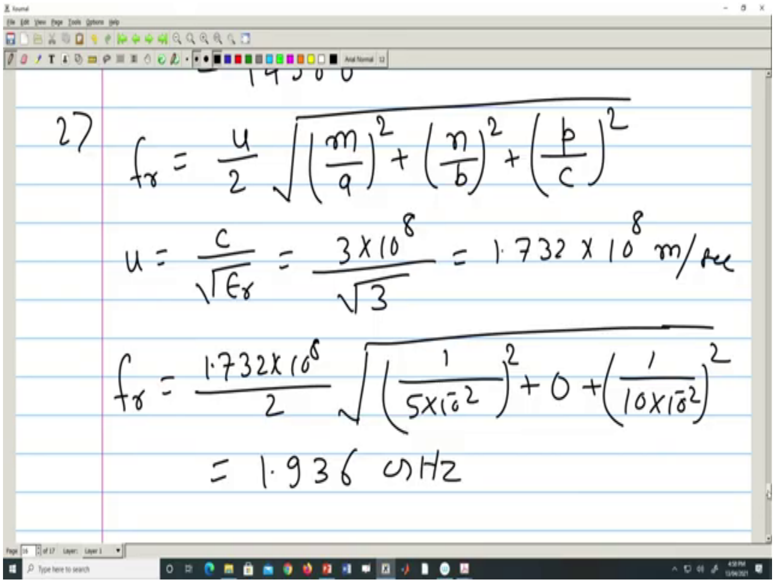
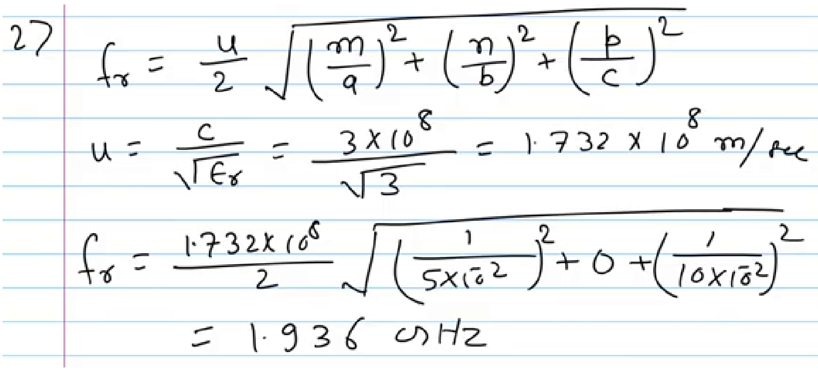




And now, in the second question,

2. If the resonant cavity of Problem 1 is filled with a lossless material ($\mu_r = 1$, $\varepsilon_r = 3$), find the resonant frequency f_r and the quality factor for TE_{101} mode.

So, for this case we have to first find out the resonant frequency fr, for TE 101 mode and quality factor for TE 101 mode. Now, in this case all the dimensions will be same and that material will also same, conductivity will also same. For this case,



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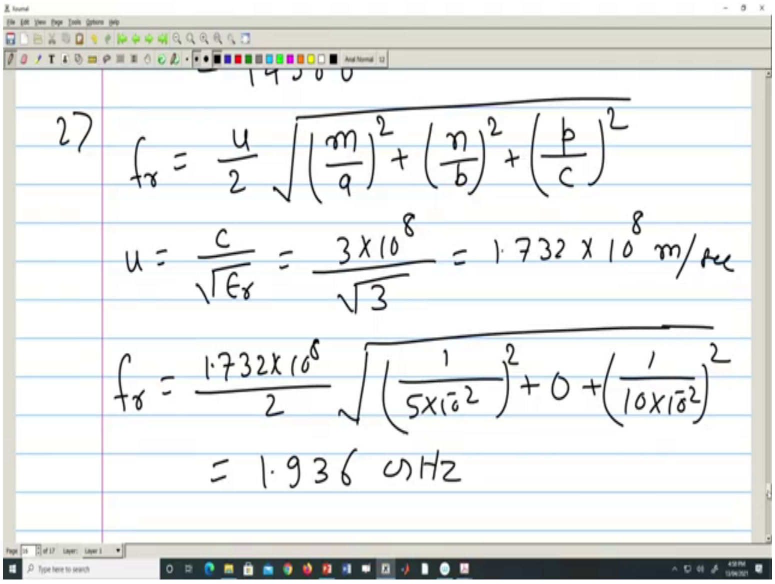

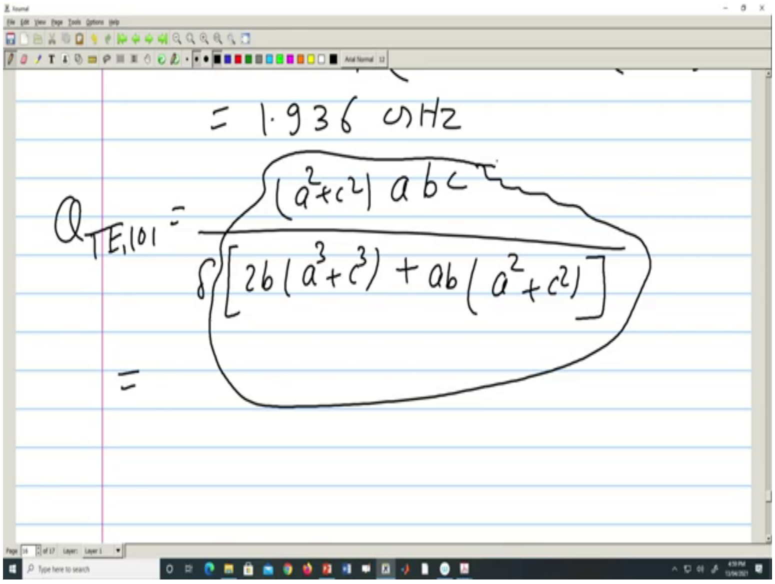

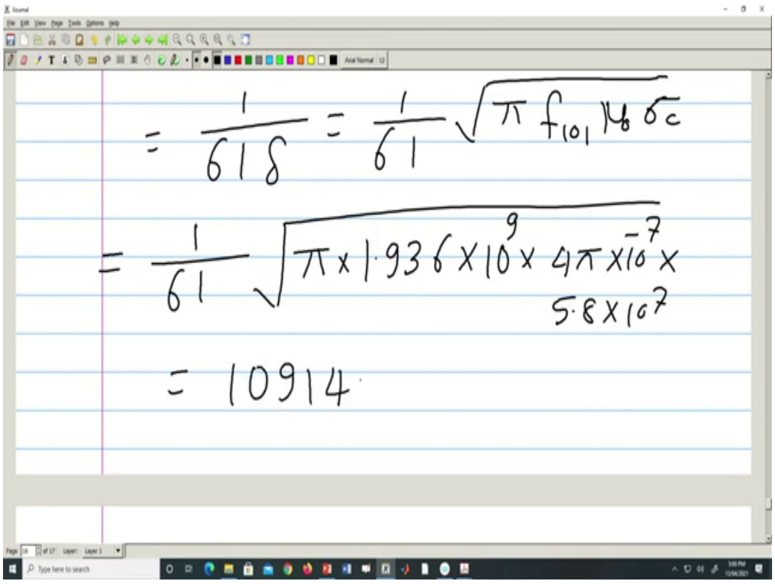

This will give, 1.936 Giga Hertz.

Now, quality factor can be calculated as:

$$Q_{TE,101} = \frac{(a^2+c^2)\,abc}{\delta\left[2b(a^3+c^3) + ab(a^2+c^2)\right]}$$

$$= \frac{1}{61\delta} = \frac{1}{61}\sqrt{\pi f_{101}\mu_0 \sigma_c}$$

$$= \frac{1}{61}\sqrt{\pi \times 1.936\times 10^{9} \times 4\pi \times 10^{-7} \times 5.8\times 10^{7}}$$

$$= 10914$$

So, answer will be 10914. So, for this we got this quality factor 10914. And in the previous case, we have got 14000 so, we can say that quality factor is reduced due to the loading of the this permittivity. So, this has been reduced, we will do some problem in the next class, thank you.