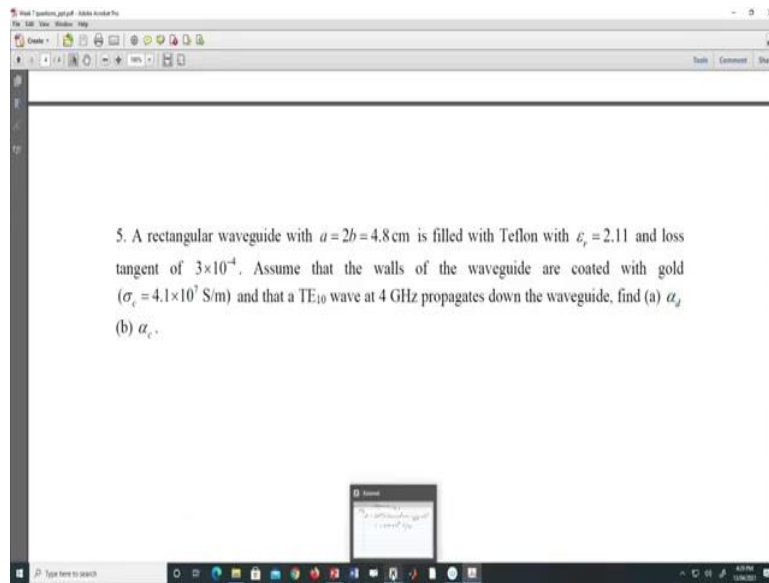


Advanced Microwave Guided-Structures and Analysis
Professor Bratin Ghosh
Department of Electronics & Electrical Communication Engineering
Indian Institute of Technology Kharagpur
Lecture 39
Rectangular Cavity Resonator Tutorials

Welcome to the next tutorial class now on waveguide resonator.

(Refer Slide Time: 00:21)



So, before go to the waveguide resonator so, we have to know the answer for this so alpha d and alpha c.

5. A rectangular waveguide with $a = 2b = 4.8$ cm is filled with Teflon with $\epsilon_r = 2.11$ and loss tangent of 3×10^{-4} . Assume that the walls of the waveguide are coated with gold ($\sigma_c = 4.1 \times 10^7$ S/m) and that a TE_{10} wave at 4 GHz propagates down the waveguide, find (a) α_d (b) α_c .

$$\sigma = 3 \times 10^{-4} \times 2\pi \times 4 \times 10^9 \times 2.11 \times \frac{1}{38\pi} \times 10^{-9}$$

$$= 1.407 \times 10^{-4} \text{ S/m}$$

$$\eta = \frac{\eta_0}{\sqrt{\epsilon_s}} = \frac{120\pi}{\sqrt{2.11}} = 259.53 \Omega$$

$$\alpha_d = \frac{\sigma \eta}{2 \sqrt{1 - \left(\frac{fc}{f}\right)^2}} = \frac{1.407 \times 10^{-4} \times 259.53}{2 \sqrt{1 - \left(\frac{2.151}{4}\right)^2}}$$

$$= 2.165 \times 10^{-2} \text{ Np/m}$$

(b)

$$R_s = \sqrt{\frac{\pi f M}{\sigma_c}} = \sqrt{\frac{\pi \times 4 \times 10^9 \times 4\pi \times 10^{-7}}{4.1 \times 10^7}}$$

$$= 1.9625 \times 10^{-2} \text{ S/m}$$

(Refer Slide Time: 00:31)

A screenshot of a digital whiteboard application. The whiteboard has a light blue background with horizontal lines. The calculations from the first image are written in black ink. At the top left, there is a small label α_d . The equations are:

$$\sigma = 3 \times 10^{-4} \times 2\pi \times 4 \times 10^9 \times 2.11 \times \frac{1}{38\pi} \times 10^{-9}$$

$$= 1.407 \times 10^{-4} \text{ S/m}$$

$$\eta = \frac{\eta_0}{\sqrt{\epsilon_s}} = \frac{120\pi}{\sqrt{2.11}} = 259.53 \Omega$$

The whiteboard interface includes a toolbar at the top with various drawing tools and a Windows taskbar at the bottom.

$$\alpha_d = \frac{\sigma_m}{2 \sqrt{1 - \left(\frac{fc}{f}\right)^2}} = \frac{1.407 \times 10^4 \times 259.53}{2 \sqrt{1 - \left(\frac{2.1512}{4}\right)^2}}$$

$$= 2.165 \times 10^2 \text{ Np/m}$$

$$R_s = \sqrt{\frac{\pi f M}{6 \sigma_c}} = \sqrt{\frac{\pi \times 4 \times 10^9 \times 4\pi \times 10^7}{4.1 \times 10^7}}$$

$$= 1.9625 \times 10^2 \text{ S/m}$$

In the next part that is a calculation of Alpha c for that we need R_s.

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$$\alpha_c = \frac{2 R_s}{b m \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \left[\frac{1}{2} + \frac{b}{a} \left(\frac{f_c}{f}\right)^2 \right]$$

$$= \frac{2 \times 1.9625 \times 10^2}{2.4 \times 10^{-2} \times 259.53 \times \sqrt{1 - \left(\frac{2.151}{4}\right)^2}}$$

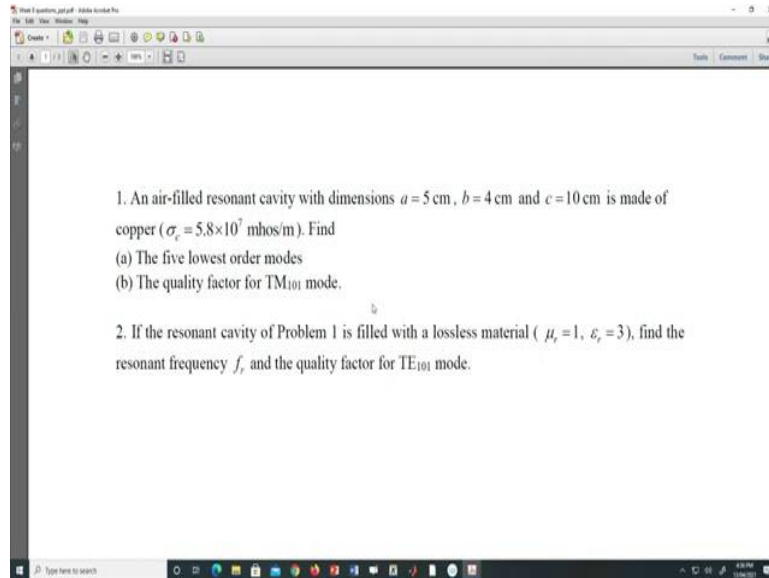
$$= \frac{2 \times 1.9625 \times 10^2}{2.4 \times 10^{-2} \times 259.53 \times \sqrt{1 - \left(\frac{2.151}{4}\right)^2}} \cdot X$$

$$\left[0.5 + 0.5 \left(\frac{2.151}{4}\right)^2 \right]$$

$$= 4.818 \times 10^{-3} \text{ N/m}.$$

So, like this we can calculate the Alpha c and Alpha d.

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So, now we are going to do, the problem on waveguide resonator. So, first problem is,

- An air-filled resonant cavity with dimensions $a = 5 \text{ cm}$, $b = 4 \text{ cm}$ and $c = 10 \text{ cm}$ is made of copper ($\sigma_c = 5.8 \times 10^7 \text{ mhos/m}$). Find
 - The five lowest order modes
 - The quality factor for TM_{101} mode.

For TM to z mode:

1) TM to z mode

$$E_z = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \cos\left(\frac{p\pi z}{c}\right)$$

$$\beta^2 = k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2$$

$m = 1, 2, \dots$, $n = 1, 2, \dots$, $p = 0, 1, 2, 3$

$$\beta^2 = \omega^2 \mu \epsilon$$

$$f_r = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

TE to z mode

$$H_z = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{c}\right)$$

$m = 0, 1, 2, \dots$, $n = 0, 1, 2, \dots$, $p = 1, 2, 3$

(Refer Slide Time: 06:49)

The image consists of two screenshots of a digital whiteboard. The top screenshot shows the following handwritten text:

1) TM to z mode

$$E_z = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \cos\left(\frac{p\pi z}{c}\right)$$
$$\beta^2 = k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2$$

$m = 1, 2, \dots$, $n = 1, 2, \dots$, $p = 0, 1, 2, 3$

$$\beta^2 = \omega^2 \mu \epsilon$$

The bottom screenshot shows the following handwritten text:

$$\beta^2 = \omega^2 \mu \epsilon$$
$$f_c = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

TE_{to z} mode

$$H_z = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{c}\right)$$

$m = 0, 1, 2, \dots$, $n = 0, 1, 2, \dots$, $p = 1, 2, 3$

Quality factor is defined as:

$$Q_{TE_{101}} = \frac{(a^2+c^2)abc}{\delta [2b(a^3+c^3) + ac(a^2+c^2)]}$$

$$\delta = \frac{1}{\sqrt{\pi f_{101} \mu_0 \sigma_c}}$$

$$f_r = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$f_r = \frac{3 \times 10^8}{2} \sqrt{\left(\frac{m}{5 \times 10^{-2}}\right)^2 + \left(\frac{n}{4 \times 10^{-2}}\right)^2 + \left(\frac{p}{10 \times 10^{-2}}\right)^2}$$

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The screenshot shows a presentation slide with the following content:

$$Q_{TE_{101}} = \frac{(a^2+c^2)abc}{\delta [2b(a^3+c^3) + ac(a^2+c^2)]}$$

$$\delta = \frac{1}{\sqrt{\pi f_{101} \mu_0 \sigma_c}}$$

$$\delta = \frac{1}{\sqrt{\pi f_{101} M_0 \sigma_c}}$$

$$f_r = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$f_r = \frac{3 \times 10^8}{2} \sqrt{\left(\frac{m}{5 \times 10^2}\right)^2 + \left(\frac{n}{4 \times 10^2}\right)^2 + \left(\frac{p}{10 \times 10^2}\right)^2}$$

$$f_r = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$f_r = \frac{3 \times 10^8}{2} \sqrt{\left(\frac{m}{5 \times 10^2}\right)^2 + \left(\frac{n}{4 \times 10^2}\right)^2 + \left(\frac{p}{10 \times 10^2}\right)^2}$$

$$f_r = 15 \sqrt{0.04 m^2 + 0.0625 n^2 + 0.01 p^2} \text{ Hz}$$

$$c > a > b \Rightarrow \frac{1}{c} < \frac{1}{a} < \frac{1}{b}$$

F_r is defined as

$$f_r = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$f_r = \frac{3 \times 10^8}{2} \sqrt{\left(\frac{m}{5 \times 10^2}\right)^2 + \left(\frac{n}{4 \times 10^2}\right)^2 + \left(\frac{p}{10 \times 10^2}\right)^2}$$

$$f_r = 15 \sqrt{0.04 m^2 + 0.0625 n^2 + 0.01 p^2} \text{ Hz}$$

$$c > a > b \Rightarrow \frac{1}{c} < \frac{1}{a} < \frac{1}{b}$$

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$$f_c = 15 \sqrt{0.04 \text{ m}^2 + 0.0625 \text{ m}^2 + 0.01 \text{ p}^2} \text{ GHz}$$

$$c > a > b \Rightarrow \frac{1}{c} < \frac{1}{a} < \frac{1}{b}$$

TM₁₀₁ TE₁₀₀ doest.

$$f_c = 15 \sqrt{0.04 \text{ m}^2 + 0.0625 \text{ m}^2 + 0.01 \text{ p}^2} \text{ GHz}$$

$$c > a > b \Rightarrow \frac{1}{c} < \frac{1}{a} < \frac{1}{b}$$

TM₁₀₁ & TE₁₀₀ do not exist.

TE₁₀₁ $f_{c101} = 15 \sqrt{0.04 + 0 + 0.01} \text{ GHz}$
 $= 3.354 \text{ GHz}$

And TM 101 mode that will not exist TM 101 mode, that will not exist TE 100 also will not exist because, for TE case this last should be, last one should be start from 1, but here it is 0. So, does not exist so, this will be does not exist so, this mode and this mode. So, this will be not exist, so this will be not exist, and fr, fr 101 now we can calculate the fr 101 so, this will be 15 so, this for the means for the TE 101 mode, a TE 101 mode, either it can be for the TM 101 mode also. same formula we can apply but, this will not exist so, this is for the TE 101 mode, so 15 so, m will be 1, m will be 1 so, this will be 0.04 m will be 1, plus 0 because middle part n is 0, plus 0.01 so, and p will be 1. So, this Giga Hertz so, after calculation it will 3.354 Giga Hertz. So, this is the fr 101 net higher order mode will be a TE 01 mode, 011 mode.

$$f_c = 15 \sqrt{0.04 \text{ m}^2 + 0.0625 \text{ m}^2 + 0.01 \text{ p}^2} \text{ GHz}$$

$$c > a > b \Rightarrow \frac{1}{c} < \frac{1}{a} < \frac{1}{b}$$

TM₁₀₁ & TE₁₀₀ does not exist.

TE₀₁₁ $f_{c,011} = 15 \sqrt{0.04 + 0 + 0.01} \text{ GHz}$
 $= 3.354 \text{ GHz}$

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TE₀₁₁ $f_{c,011} = 3.354 \text{ GHz}$

TE₀₁₁ & TM₀₁₁ does not exist.

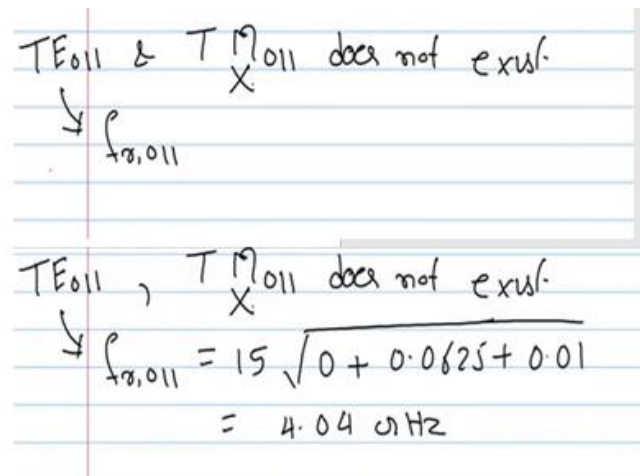
↓ $f_{c,011}$

TE₀₁₁ , TM₀₁₁ does not exist.

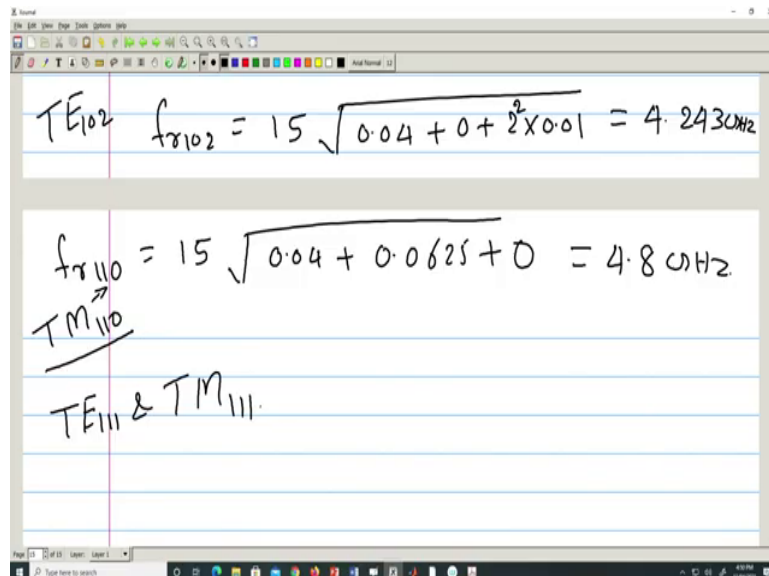
↓ $f_{c,011} = 15 \sqrt{0 + 0.0625 + 0.01}$
 $= 4.04 \text{ GHz}$

So, next higher order mode will be, TE 011 mode and TM if we can see the TM 011 so, this will not exist because, for the TM case, first this m should start from 1 so, this will be does not exist. So, this will be does not exist so, fr for 011 that means this does not exist but, this will exist so, this is not and, so this will exist. Fr 011 will be 15 a square root of so, m will be

0, plus 0.0625 plus n will be, n will be 1 so, 0.01. This will be 4.04 Giga Hertz. So, fr 011 will be 4.04 Giga Hertz.



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TE₁₁₁ & TM₁₁₁

$$f_{r_{111}} = 15 \sqrt{0.04 + 0.0625 + 0.01} = 5.031 \text{ MHz}$$

$$f_{r_{103}} = 15 \sqrt{0.04 + 0 + 0.09} = 5.408 \text{ MHz}$$

$$f_{r_{103}} = 15 \sqrt{0.04 + 0 + 0.09} = 5.408 \text{ MHz}$$

TE₁₀₁ → 3.354 MHz

TE₀₁₁ → 4.04 MHz

TE₁₀₂ → 4.243 MHz

TM₁₁₀ → 4.8 MHz

Similarly

TE₁₀₂ $f_{r_{102}} = 15 \sqrt{0.04 + 0 + 2^2 \times 0.01} = 4.243 \text{ MHz}$

$f_{r_{110}} = 15 \sqrt{0.04 + 0.0625 + 0} = 4.8 \text{ MHz}$

TM₁₁₀

TE₁₁₁ & TM₁₁₁

$$f_{r_{111}} = 15 \sqrt{0.04 + 0.0625 + 0.01} = 5.031 \text{ MHz}$$

$$f_{r_{103}} = 15 \sqrt{0.04 + 0 + 0.09} = 5.408 \text{ MHz}$$

So, that is

$$TE_{101} \rightarrow 3.354 \text{ GHz}$$

$$TE_{011} \rightarrow 4.04 \text{ GHz}$$

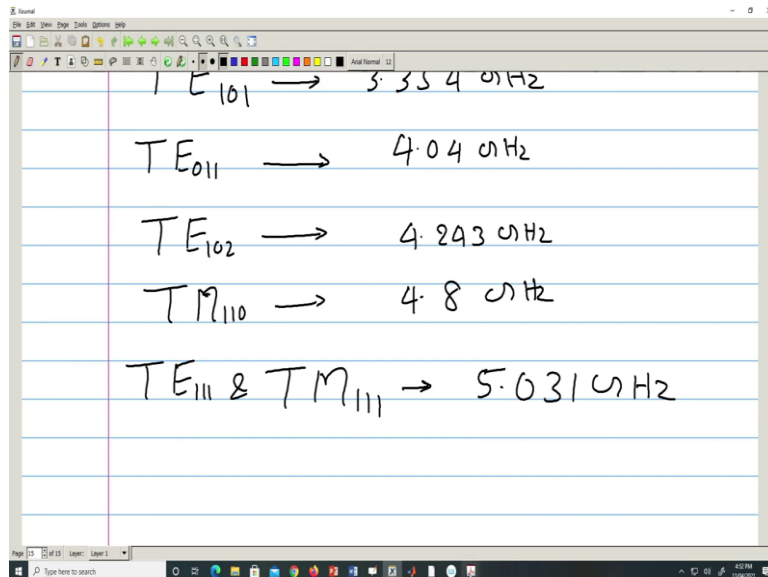
$$TE_{102} \rightarrow 4.243 \text{ GHz}$$

$$TM_{110} \rightarrow 4.8 \text{ GHz}$$

$$TE_{111} \& TM_{111} \rightarrow 5.031 \text{ GHz}$$

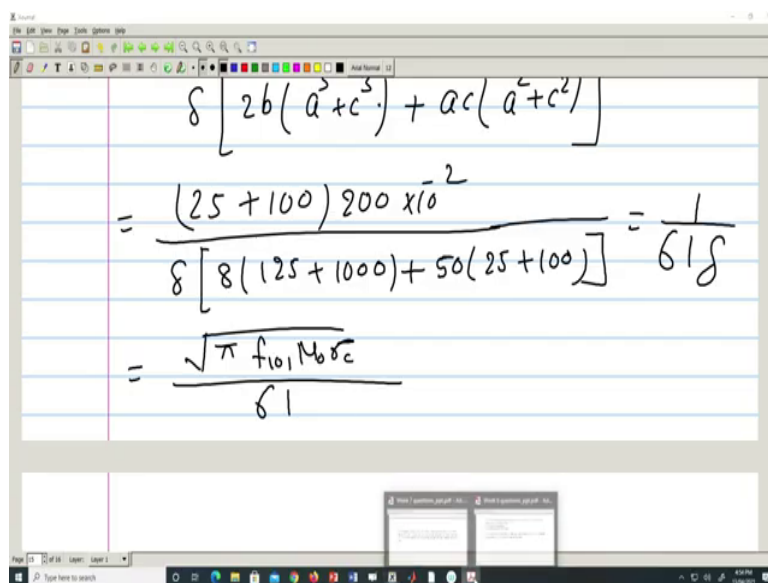
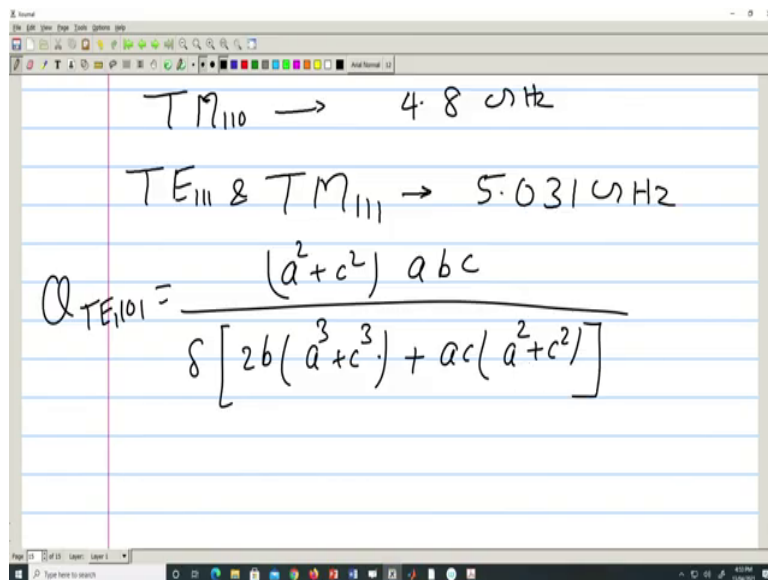
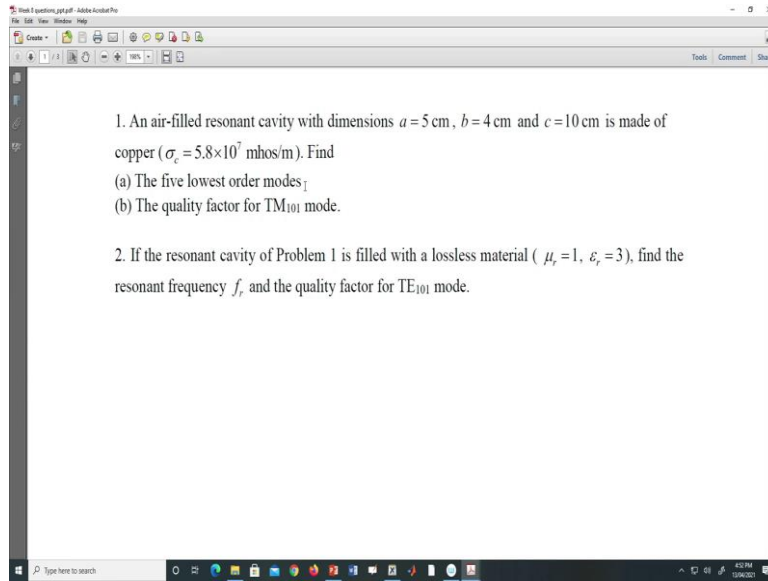
for the TM 103, so we can write for all the resonant frequencies, 101 resonant frequency 3.354 Giga Hertz next is TE 011 for that it is 4.04 Giga Hertz. Next is TE 102 that is 4.243 Giga Hertz, next is TM 110 that is 4.8 Giga Hertz.

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Next one is TE 111 that is degenerate mode, or TM 111, that both have same resonant frequencies it will be 5.031 Giga Hertz. So, these are the five lowest resonant frequencies.

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We know the formula that

$$Q_{TE_{101}} = \frac{(a^2 + c^2) abc}{\delta [2b(a^3 + c^3) + ac(a^2 + c^2)]}$$

$$= \frac{(25 + 100) 200 \times 10^{-2}}{\delta [8(125 + 1000) + 50(25 + 100)]} = \frac{1}{61 \delta}$$

$$= \frac{\sqrt{\pi f_{101} \mu_0 \delta c}}{61}$$

This will come after simplification

$$= \frac{\sqrt{\pi \times 3.354 \times 10^9 \times 4\pi \times 10^{-7} \times 5.8 \times 10^{-7}}}{61}$$

$$= 14386$$

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The screenshot shows a presentation slide with the following content:

$$= \frac{\sqrt{\pi \times 3.354 \times 10^9 \times 4\pi \times 10^{-7} \times 5.8 \times 10^{-7}}}{61}$$

$$= 14386$$

27)
$$\gamma_r = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$u = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{\sqrt{3}} = 1.732 \times 10^8 \text{ m/sec}$$

$$27 \quad f_r = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$u = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{\sqrt{3}} = 1.732 \times 10^8 \text{ m/sec}$$

$$f_r = \frac{1.732 \times 10^8}{2} \sqrt{\left(\frac{1}{5 \times 10^{-2}}\right)^2 + 0 + \left(\frac{1}{10 \times 10^{-2}}\right)^2}$$

$$= 1.936 \text{ MHz}$$

And now, in the second question,

2. If the resonant cavity of Problem 1 is filled with a lossless material ($\mu_r = 1$, $\epsilon_r = 3$), find the resonant frequency f_r and the quality factor for TE₁₀₁ mode.

So, for this case we have to first find out the resonant frequency f_r , for TE 101 mode and quality factor for TE 101 mode. Now, in this case all the dimensions will be same and that material will also same, conductivity will also same. For this case,

$$27 \quad f_r = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$u = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{\sqrt{3}} = 1.732 \times 10^8 \text{ m/sec}$$

$$f_r = \frac{1.732 \times 10^8}{2} \sqrt{\left(\frac{1}{5 \times 10^{-2}}\right)^2 + 0 + \left(\frac{1}{10 \times 10^{-2}}\right)^2}$$

$$= 1.936 \text{ MHz}$$

(Refer Slide Time: 28:13)

$$27 \quad f_g = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$u = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{\sqrt{3}} = 1.732 \times 10^8 \text{ m/sec}$$

$$f_g = \frac{1.732 \times 10^8}{2} \sqrt{\left(\frac{1}{5 \times 10^{-2}}\right)^2 + 0 + \left(\frac{1}{10 \times 10^{-2}}\right)^2}$$

$$= 1.936 \text{ GHz}$$

$$= 1.936 \text{ GHz}$$

$$Q_{TE_{101}} = \frac{(a^2 + c^2) abc}{\delta [2b(a^3 + c^3) + ab(a^2 + c^2)]}$$

$$=$$

$$= \frac{1}{618} = \frac{1}{61} \sqrt{\pi f_{101} \mu_0 \delta c}$$

$$= \frac{1}{61} \sqrt{\pi \times 1.936 \times 10^9 \times 4\pi \times 10^{-7} \times 5.8 \times 10^{-7}}$$

$$= 10914$$

This will give, 1.936 Giga Hertz.

Now, quality factor can be calculated as:

$$\begin{aligned}
 Q_{TE_{101}} &= \frac{(a^2+c^2)abc}{\delta [2b(a^3+c^3) + ab(a^2+c^2)]} \\
 &= \\
 &= \frac{1}{\delta 18} = \frac{1}{61} \sqrt{\pi f_{101} \mu_0 \sigma_c} \\
 &= \frac{1}{61} \sqrt{\pi \times 1.936 \times 10^9 \times 4\pi \times 10^{-7} \times 5.8 \times 10^7} \\
 &= 10914
 \end{aligned}$$

So, answer will be 10914. So, for this we got this quality factor 10914. And in the previous case, we have got 14000 so, we can say that quality factor is reduced due to the loading of the this permittivity. So, this has been reduced, we will do some problem in the next class, thank you.