Advanced Microwave Guided – Structures and Analysis
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Lecture 4
Scattering Matrix Concepts (Contd.)

So, welcome back to the Scattering Matrix Concepts. In this session we will investigate the scattering matrix parameters for elements like the shunt element and the series elements.

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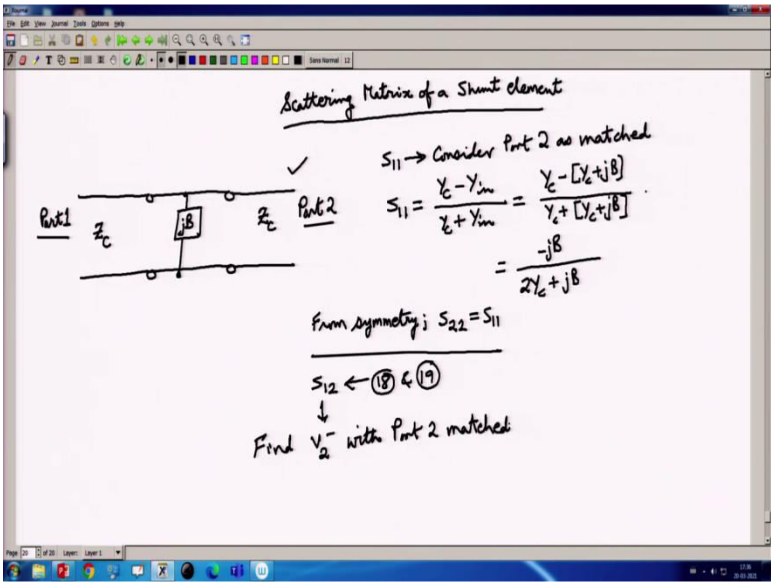


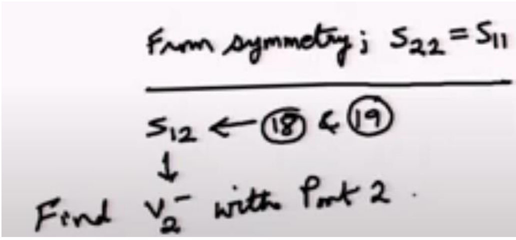
So, let us begin for the shunt element, we will write the scattering matrix of a shunt element. So, in this case I have a transmission line. So, let us have a shunt element connected on either sides to transmission line. So, the input transmission line has a characteristic impedance Zc, the output transmission line has a characteristic impedance of Zc

So, to find S_{11} of this network we consider port 2 to be matched. So, S_{11} because this is a shunt element, we will discuss this in terms of the admittance. Looking at towards the right side from port 1.

$S_{11} \rightarrow$ Consider Port 2 as matched

$$S_{11} = \frac{Y_c - Y_{in}}{Y_c + Y_{in}} = \frac{Y_c - [Y_c + jB]}{Y_c + [Y_c + jB]}$$
$$= \frac{-jB}{2Y_c + jB}$$

Now, from symmetry, we can see S_{22} will be equal to S_{11}. You can easily see that this network is symmetrical from the port 1 and port 2. So, from symmetry we can write S_{22} equal to S_{11}. So, S_{12} can be evaluated by using 18 and 19. So, that can be obtained from 18 and 19 or S_{12} can be found also from the network by finding V_2^- with port 2 matched.



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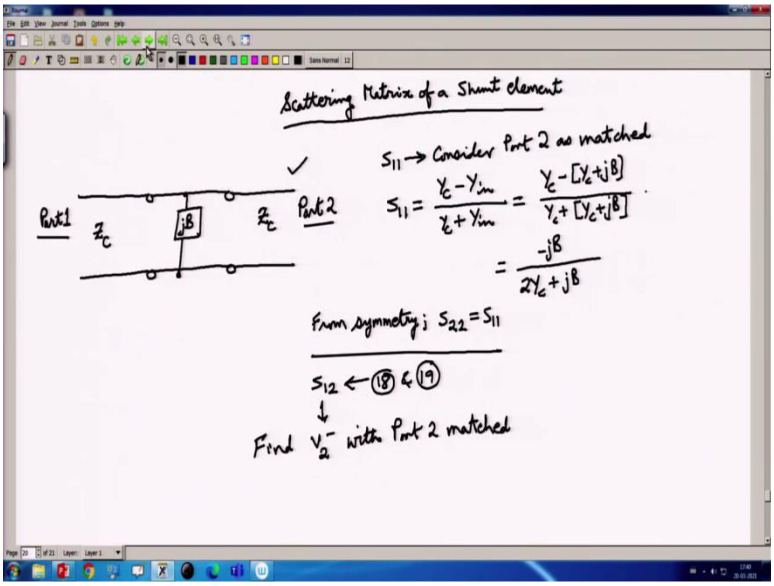
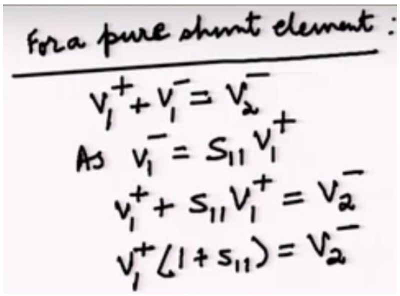
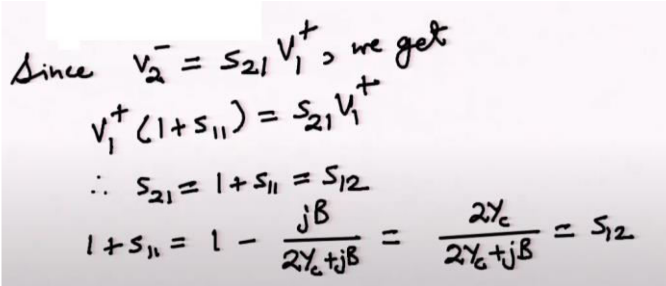


Now, for a pure shunt element, we must have the total input voltage will be equal to V_2^- because port 2 is matched. So, there is no reflected wave from port 2. Now, as $V_1^- = S_{11}V_1^+$ we can write



Now, since,



So, this is for the pure shunt element.

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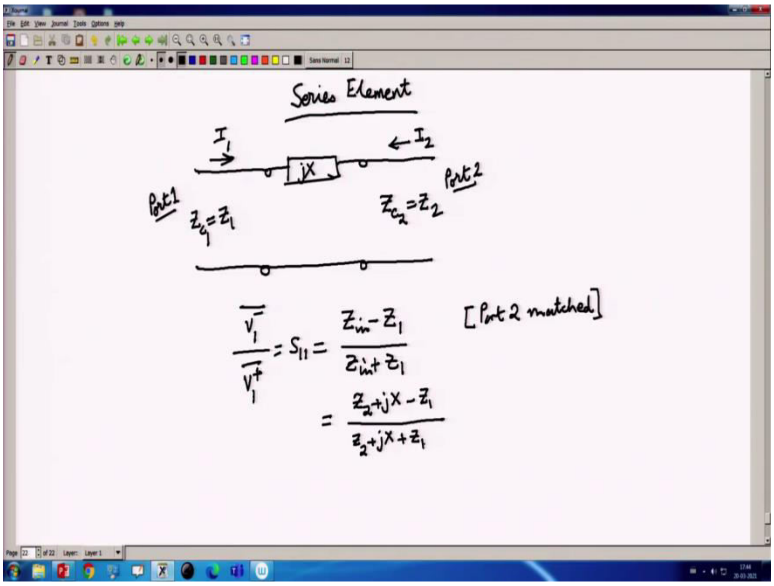
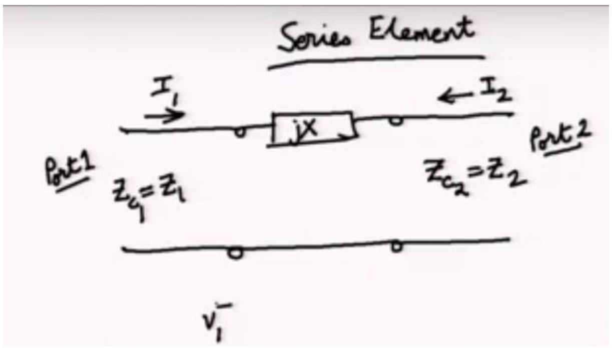
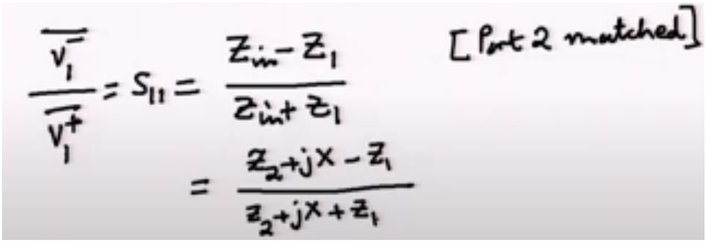


Next we take the case of a series element, and in this case we have this kind of a network. So, the two characteristic impedances of the transmission line on the two sides, they are different. Zc1 is the characteristic impedance of the transmission line on the left hand side it is Z1 and Zc2 is the characteristic impedance of the transmission on the right hand side and that is Z2. The left is port 1 and the right is port 2.



So, we can write that



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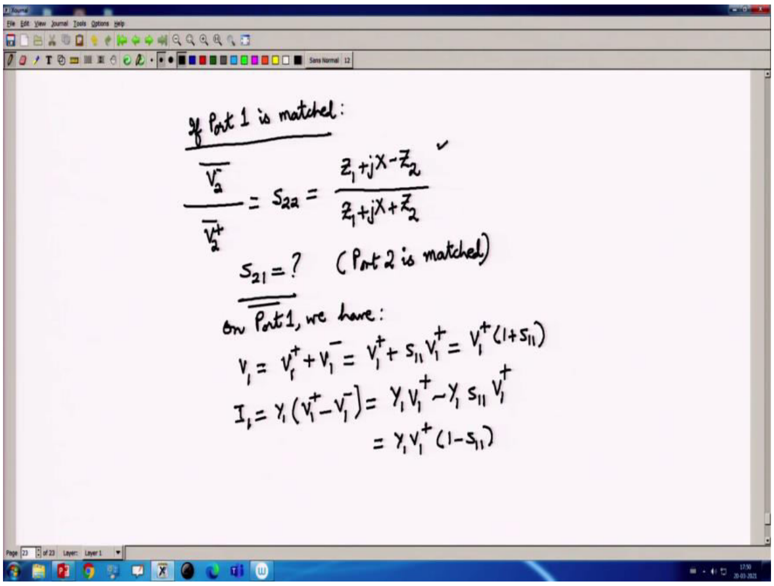
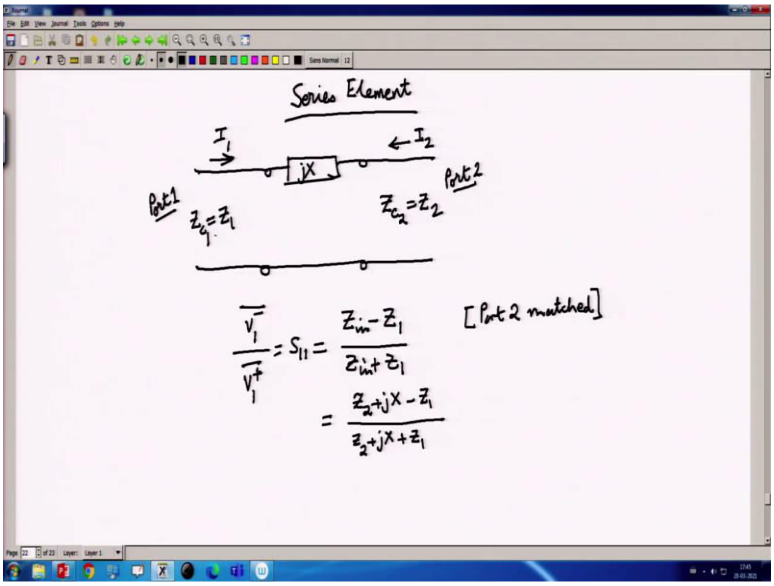
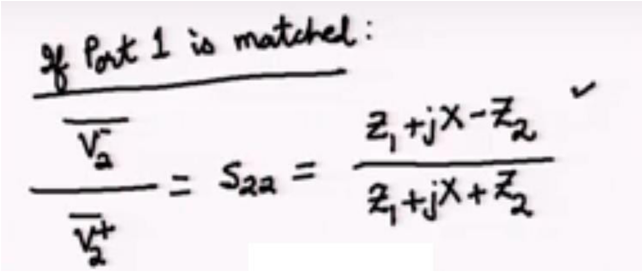
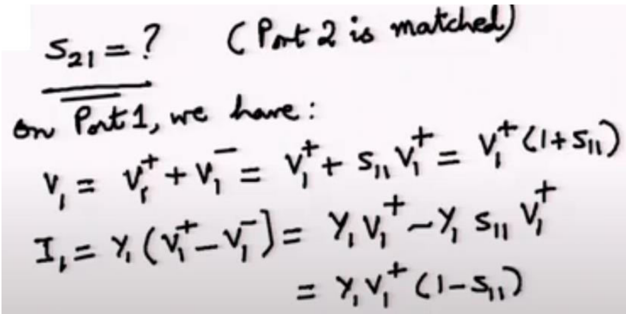




Similarly, if port 1 is matched, we can write



In order to find S_{21}, we again consider port 2 is matched. So, on port 1, what do we have? It is



.So, this is the Y1 is the admittance of the transmission line to the left.

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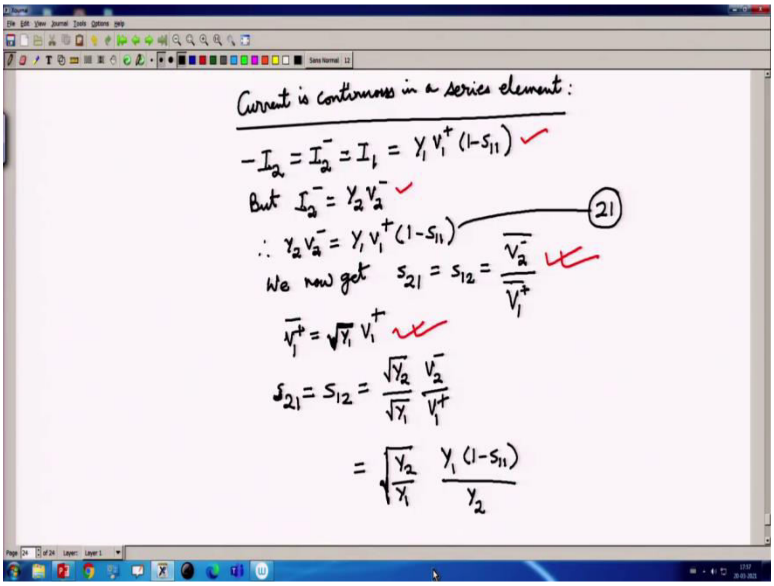

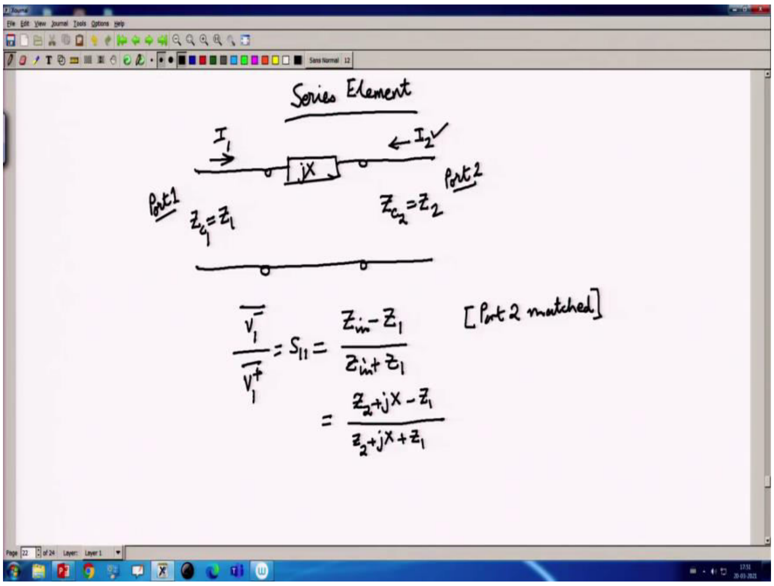

Now, we know the current is continuous in a series element. So, let us write that down.

Current is continuous in a series element:

$$-I_2 = I_2^- = I_1 = Y_1 V_1^+ (1-S_{11})$$

But $I_2^- = Y_2 V_2^-$

$$\therefore\ Y_2 V_2^- = Y_1 V_1^+ (1-S_{11})$$

We just equate the two, we just equate these two. So, we now get

We now get $S_{21} = S_{12} = \frac{\bar{V}_2^-}{\bar{V}_1^+}$

Now, if I choose a normalized voltage, the normalized voltage is related to the unnormalized voltage

$$\bar{V}_1^+ = \sqrt{Y_1}\, V_1^+$$

So, we can write:

$$s_{21} = s_{12} = \frac{\sqrt{Y_2}}{\sqrt{Y_1}} \frac{V_2^-}{V_1^+}$$

$$= \sqrt{\frac{Y_2}{Y_1}} \frac{Y_1 (1 - S_{11})}{Y_2}$$

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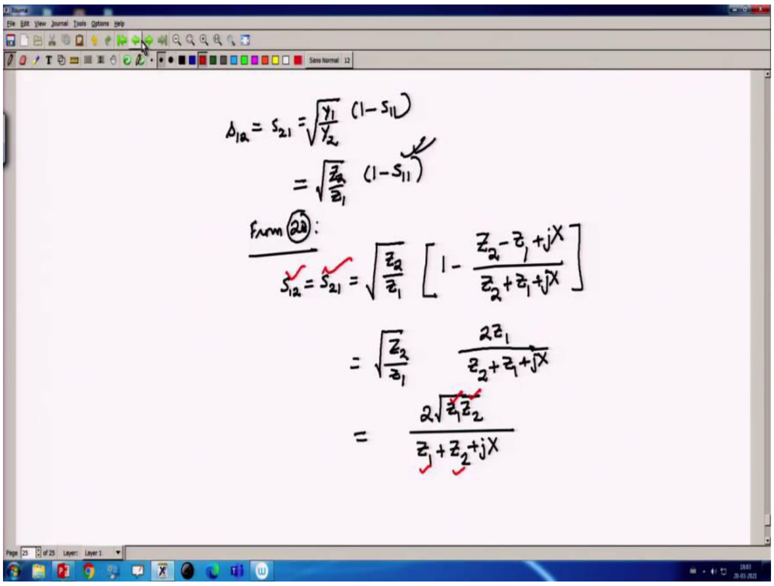

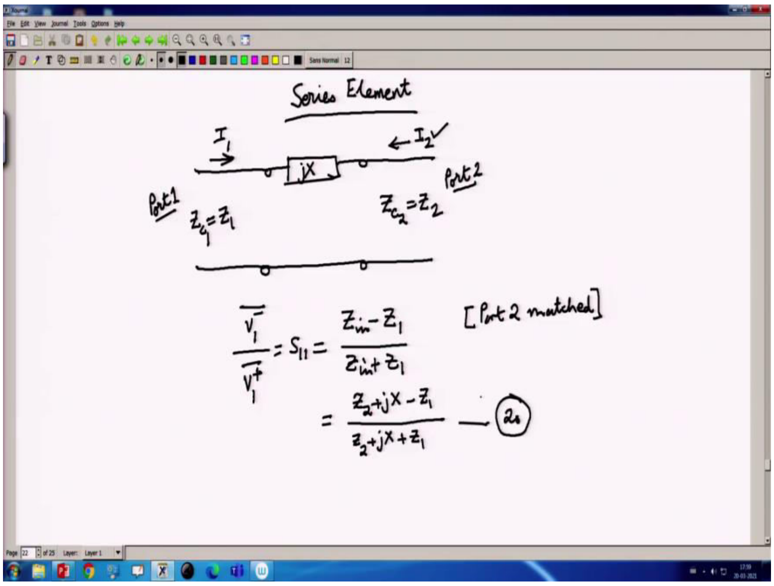

And next we can write

$$S_{12} = S_{21} = \sqrt{\frac{Y_1}{Y_2}}\,(1 - S_{11})$$

$$= \sqrt{\frac{Z_2}{Z_1}}\,(1 - S_{11})$$

From (22):

$$S_{12} = S_{21} = \sqrt{\frac{Z_2}{Z_1}}\left[1 - \frac{Z_2 - Z_1 + jX}{Z_2 + Z_1 + jX}\right]$$

$$= \sqrt{\frac{Z_2}{Z_1}}\;\frac{2Z_1}{Z_2 + Z_1 + jX}$$

$$= \frac{2\sqrt{Z_1 Z_2}}{Z_1 + Z_2 + jX}$$

So, the symmetricity of S_{11}, S_{12} and S_{21} appears because of the way the expressions for Z_1 and Z_2 appear in this equation. So, they are symmetric. So, you see that the S_{12} and S_{21} they are symmetric. So, we had used here normalized voltages. So, if unnormalized values are used or if unnormalized voltages are used the same expressions will be obtained for S_{11} and S_{22} which we had obtained now, but the expressions for S_{12} and S_{21} would get changed.

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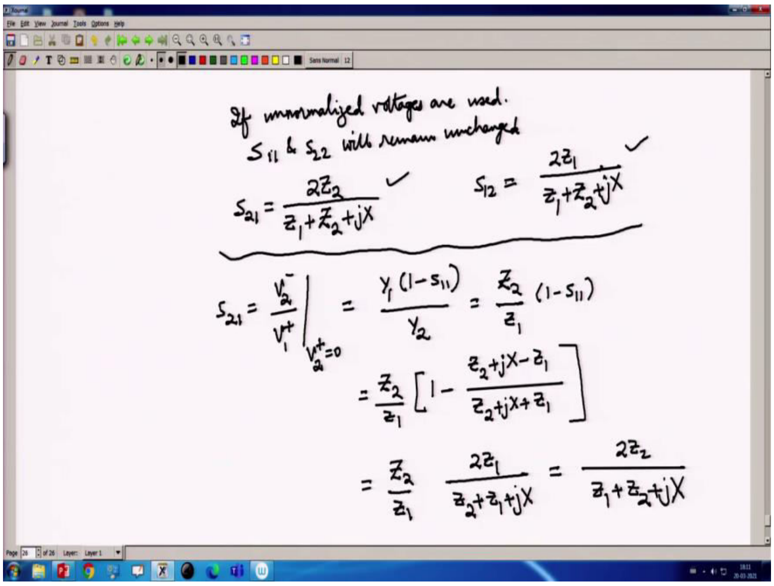
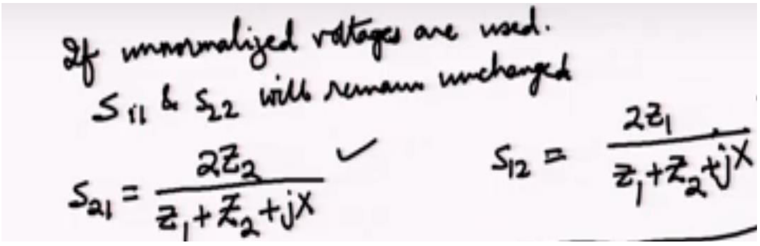
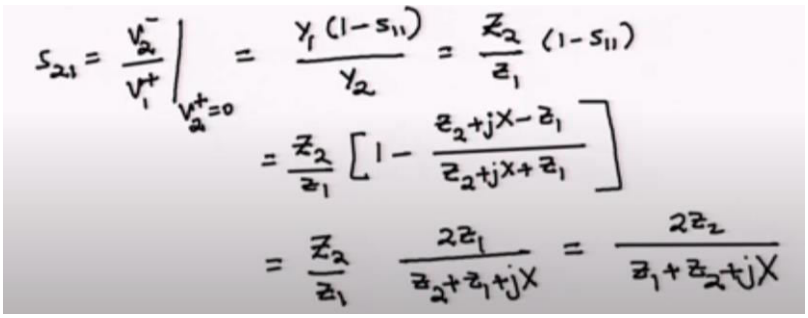


In fact, in that case if unnormalized voltages are used S_{11} and S_{22} will remain unchanged. But, S_{21} will be equal



So, in that case if unnormalized voltages are used, we can find S_{21}



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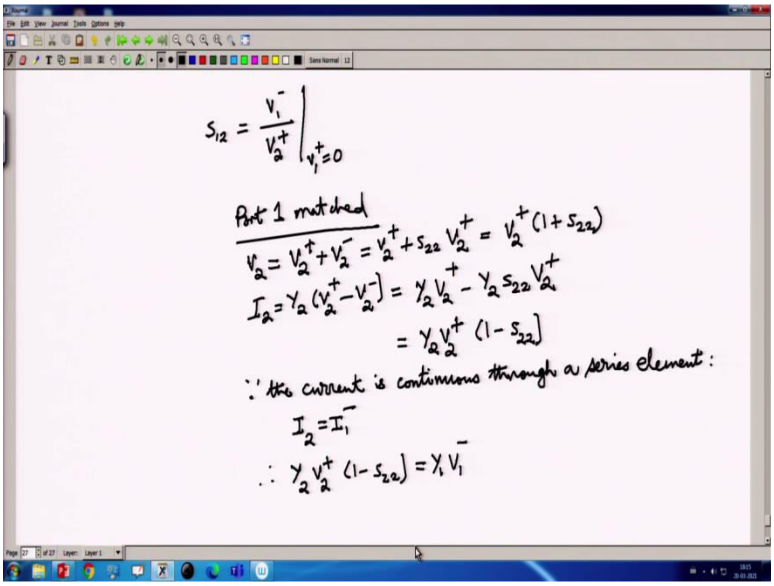
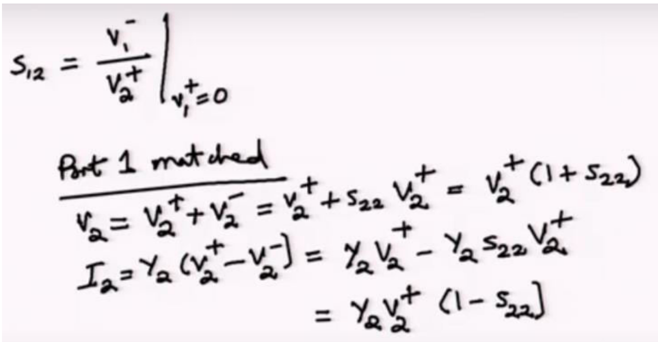
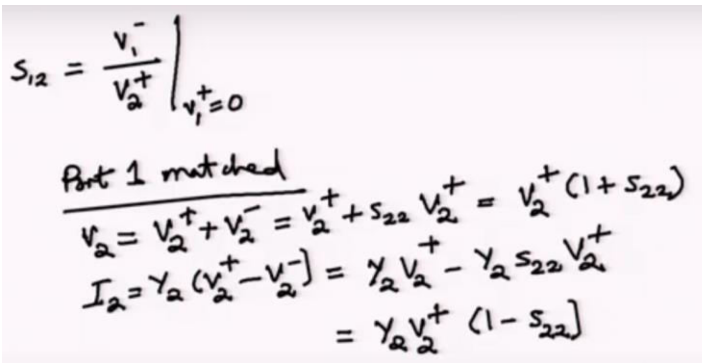


In order to find S_{12}, it is nothing but



Now since, the current is continuous through a series element,



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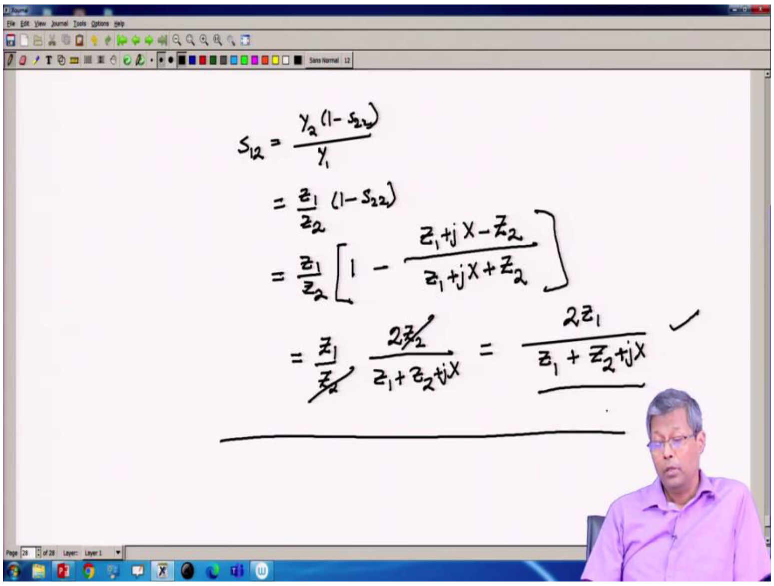

And from here, I can write down, S_{12} is

$$
\begin{aligned}
S_{12} &= \frac{Y_2(1-S_{22})}{Y_1} \\
&= \frac{Z_1}{Z_2}(1-S_{22}) \\
&= \frac{Z_1}{Z_2}\left[1 - \frac{Z_1+jX-Z_2}{Z_1+jX+Z_2}\right] \\
&= \frac{Z_1}{\cancel{Z_2}} \frac{2\cancel{Z_2}}{Z_1+Z_2+jX} = \frac{2Z_1}{Z_1+Z_2+jX}
\end{aligned}
$$

So, this completes the derivation of S_{12} for the unnormalized voltage case. So, this ends the section on the concept of scattering matrix. Thank you.