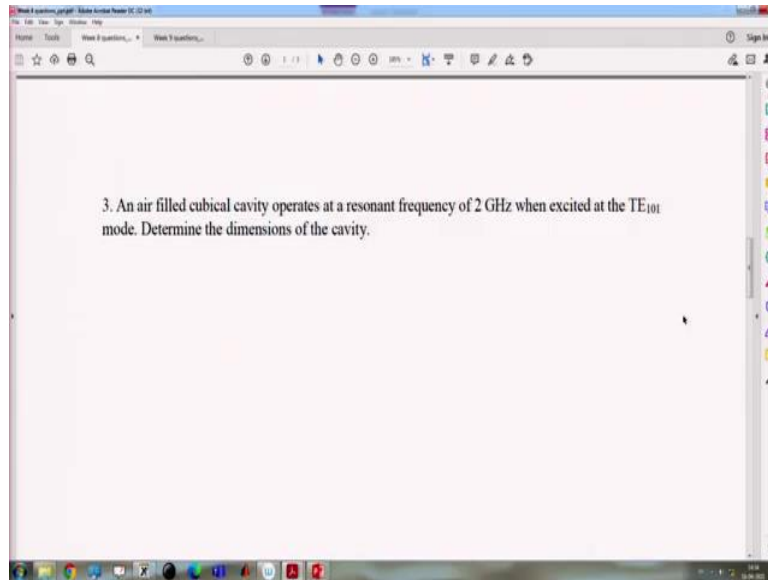


**Advanced Microwave Guided-Structures and Analysis**  
**Professor Bratin Ghosh**  
**Department of Electronics and Electrical Communication Engineering**  
**Indian Institute of Technology Kharagpur**  
**Lecture 40**  
**Rectangular Cavity Resonator Tutorials (Contd.)**

(Refer Slide Time: 00:12)



Welcome to the next tutorial class, so we will do today some numerical problems on waveguide resonator. We have already completed two problems and now, here is the third problem so, third problem is that,

**3. An air filled cubical cavity operates at a resonant frequency of 2 GHz when excited at the TE<sub>101</sub> mode. Determine the dimensions of the cavity.**

(Refer Slide Time: 00:54)

$$f_r = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

TE<sub>101</sub>    m=1, n=0, p=1

$$f_r = \frac{u}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2}$$

$$f_r = \frac{u}{2} \frac{\sqrt{2}}{a} = \frac{u}{\sqrt{2} a}$$

$$a = \frac{u}{\sqrt{2} f_r} = \frac{3 \times 10^8}{\sqrt{2} \times 2 \times 10^9}$$

$$= 10.61 \text{ cm} = b = c$$

So, to calculate the resonant frequency we know that formula is,

$$f_c = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

TE<sub>101</sub>    m=1, n=0, p=1

$$f_c = \frac{c}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2}$$

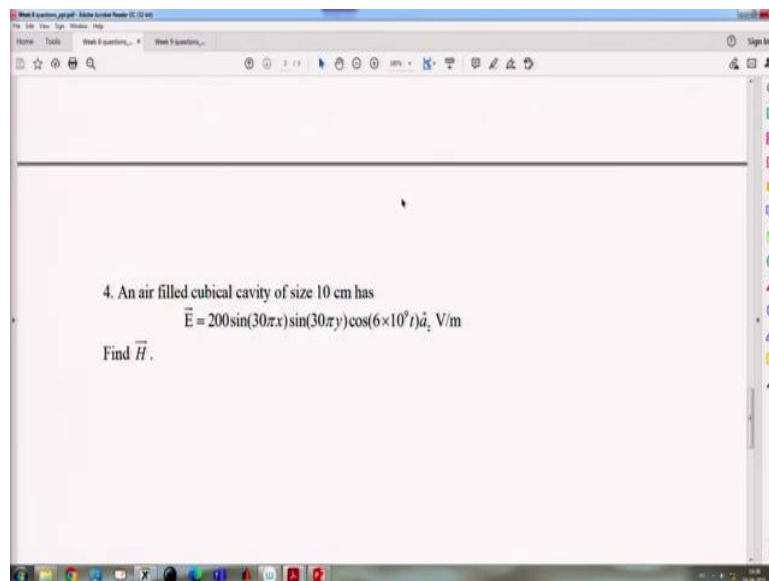
$$f_c = \frac{c}{2} \frac{\sqrt{2}}{a} = \frac{c}{\sqrt{2} a}$$

$$a = \frac{c}{\sqrt{2} f_c} = \frac{3 \times 10^8}{\sqrt{2} \times 2 \times 10^9}$$

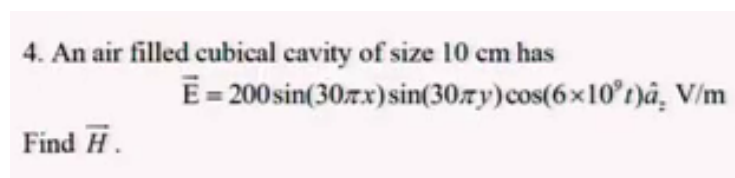
$$= 10.61 \text{ cm} = b = c$$

So, this will come 10.61 centimeter, so this is a that means equal to b equal to c so, this is the dimensions of the rectangular this cubical waveguide, cavity, cubical cavity.

(Refer Slide Time: 04:07)



Next problem is:



So, here  $\mathbf{a}_z$  is given that means here, only electric field is present in the z direction. That means, no magnetic field in the z direction that means this is for the TM to z mode. So, this is for the TM to z mode, and for this case we have to find out the vector magnetic field. So, vector magnetic field will be.

$$4) \quad \vec{\nabla} \times \vec{E} = -j\omega\mu \vec{H}$$

$$\vec{H} = \frac{-1}{j\omega\mu} (\vec{\nabla} \times \vec{E})$$

$$= \frac{j}{\omega\mu} \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_z(x, y) \end{vmatrix}$$

$$= \frac{j}{\omega\mu} \left[ a_x \left( \frac{\partial}{\partial y} E_z - 0 \right) + a_y \left( 0 - \frac{\partial E_z}{\partial x} \right) + a_z (0 - 0) \right]$$

$$= \frac{j}{\omega \mu} \left[ \hat{a}_x \frac{\partial E_z}{\partial y} - \hat{a}_y \frac{\partial E_z}{\partial x} \right]$$

$$\frac{j}{\omega \mu} = \frac{j}{6 \times 10^9 \times 4\pi \times 10^{-7}} = j \frac{10^{-2}}{24\pi}$$

$$\vec{H} = \frac{j 10^{-2}}{24\pi} 200 \left[ 30\pi \sin(30\pi x) \cos(30\pi y) \hat{a}_x - \right.$$

$$\left. 30\pi \cos(30\pi x) \sin(30\pi y) \hat{a}_y \right]$$

$$= j 25 \left[ \sin(30\pi x) \cos(30\pi y) \hat{a}_x - \cos(30\pi x) \sin(30\pi y) \hat{a}_y \right]$$

(Refer Slide Time: 05:06)

$$= 10.61 \text{ cm} = b = c$$

$$4) \quad \vec{\nabla} \times \vec{E} = -j\omega \mu \vec{H}$$

$$\vec{H} = \frac{-1}{j\omega \mu} (\vec{\nabla} \times \vec{E})$$

$$\begin{aligned}
 & j\omega\mu \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_z(x,y) \end{vmatrix} \\
 & = \frac{j}{\omega\mu} \left[ a_x \left( \frac{\partial}{\partial y} E_z - 0 \right) + a_y \left( 0 - \frac{\partial E_z}{\partial x} \right) + a_z (0-0) \right]
 \end{aligned}$$

(Refer Slide Time: 08:29)

$$\begin{aligned}
 & = \frac{j}{\omega\mu} \left[ \hat{a}_x \frac{\partial E_z}{\partial y} - \hat{a}_y \frac{\partial E_z}{\partial x} \right] \\
 \frac{j}{\omega\mu} & = \frac{j}{6 \times 10^9 \times 4\pi \times 10^{-7}} = j \frac{10^{-2}}{24\pi} \\
 \vec{H} & = j \frac{10^{-2}}{24\pi} 200 \left[ 30\pi \sin(30\pi x) \cos(30\pi y) \hat{a}_x - \right.
 \end{aligned}$$

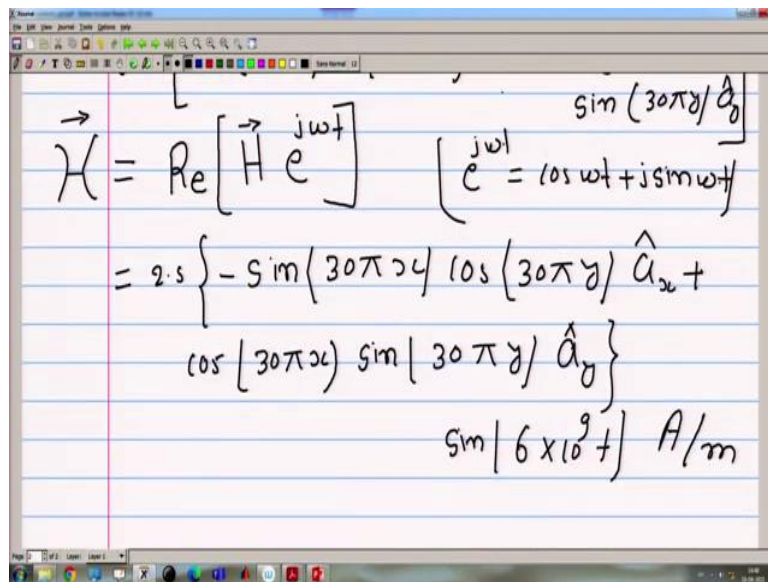
$$\begin{aligned}
 & \left. 30\pi \cos(30\pi x) \sin(30\pi y) \hat{a}_y \right] \\
 & = j 25 \left[ \sin(30\pi x) \cos(30\pi y) \hat{a}_x - \cos(30\pi x) \sin(30\pi y) \hat{a}_y \right]
 \end{aligned}$$

So, this is the air-filled component that is the complex part, and for instantaneous part we can calculate.

$$\vec{H} = \text{Re}[\vec{H} e^{j\omega t}] \quad \left[ e^{j\omega t} = \cos \omega t + j \sin \omega t \right]$$

$$= 2.5 \left\{ -\sin(30\pi z) \cos(30\pi y) \hat{a}_x + \cos(30\pi z) \sin(30\pi y) \hat{a}_y \right\} \sin(6 \times 10^9 t) \text{ A/m}$$

(Refer Slide Time: 12:43)



$$\vec{H} = \text{Re}[\vec{H} e^{j\omega t}] \quad \left[ e^{j\omega t} = \cos \omega t + j \sin \omega t \right]$$

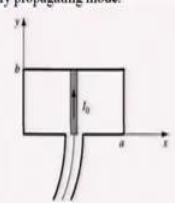
$$= 2.5 \left\{ -\sin(30\pi z) \cos(30\pi y) \hat{a}_x + \cos(30\pi z) \sin(30\pi y) \hat{a}_y \right\} \sin(6 \times 10^9 t) \text{ A/m}$$

So, this is the instantaneous magnetic field component.

From this we can go to the week 9 numerical problems.

(Refer Slide Time: 14:47)

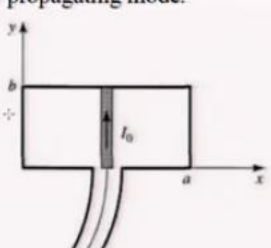
1. For the probe-fed rectangular waveguide shown in below figure, determine the amplitudes of the forward and backward traveling  $TE_{10}$  modes, and the input resistance seen by the probe. Assume that the  $TE_{10}$  mode is the only propagating mode.



A uniform current probe in a rectangular waveguide.

Here is the, first problem

1. For the probe-fed rectangular waveguide shown in below figure, determine the amplitudes of the forward and backward traveling  $TE_{10}$  modes, and the input resistance seen by the probe. Assume that the  $TE_{10}$  mode is the only propagating mode.

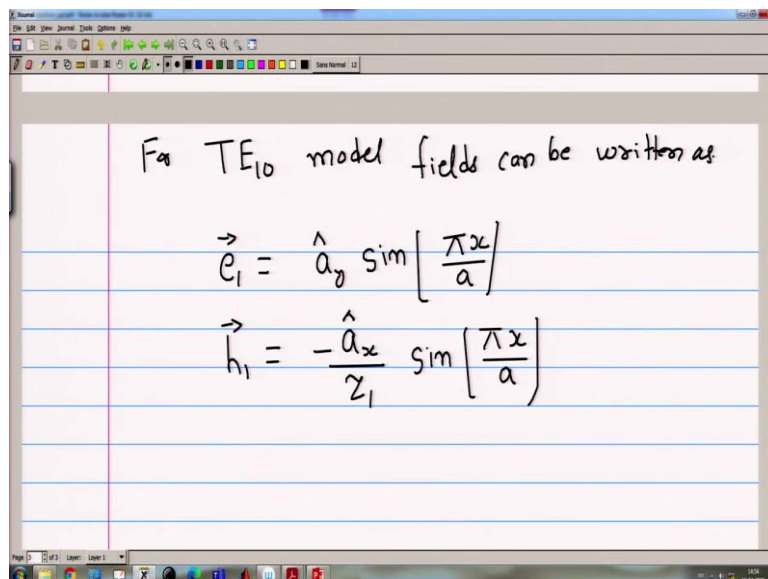
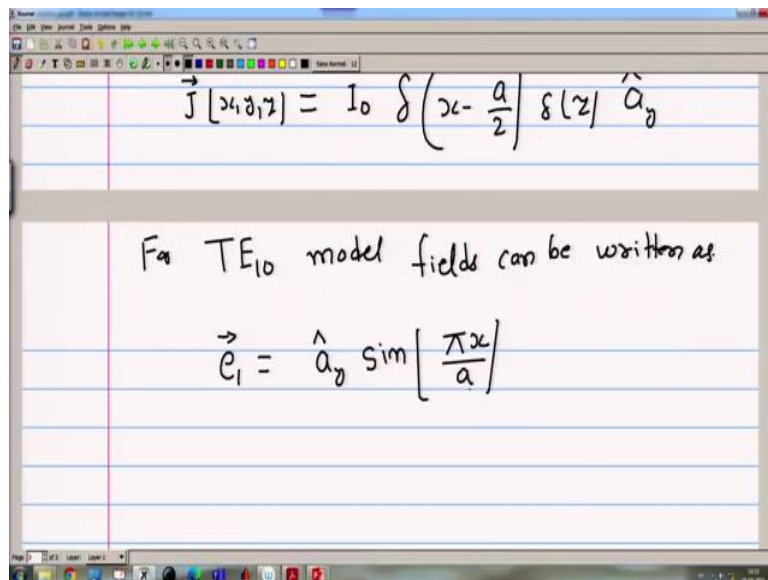
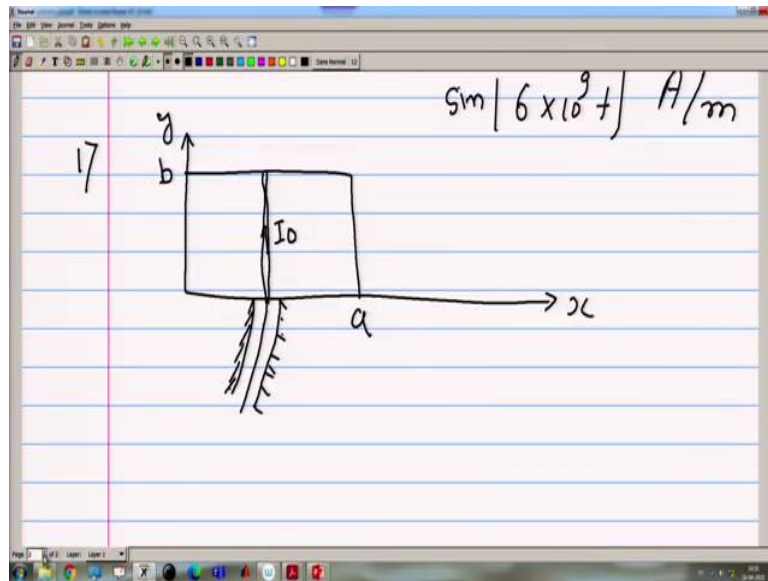


A uniform current probe in a rectangular waveguide.

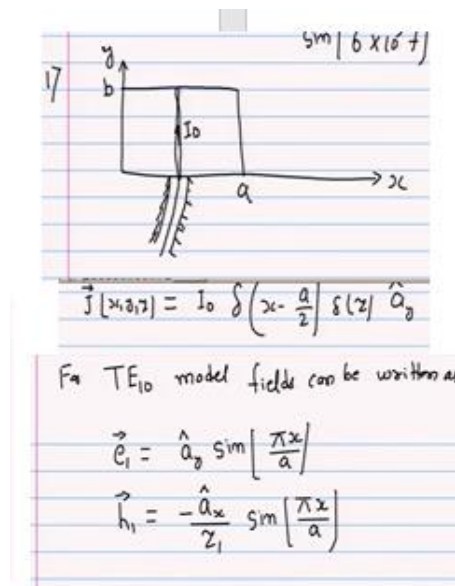
So, first we have to find out the amplitude of the forward and backward traveling,  $TE_{10}$  mode.



(Refer Slide Time: 16:25)

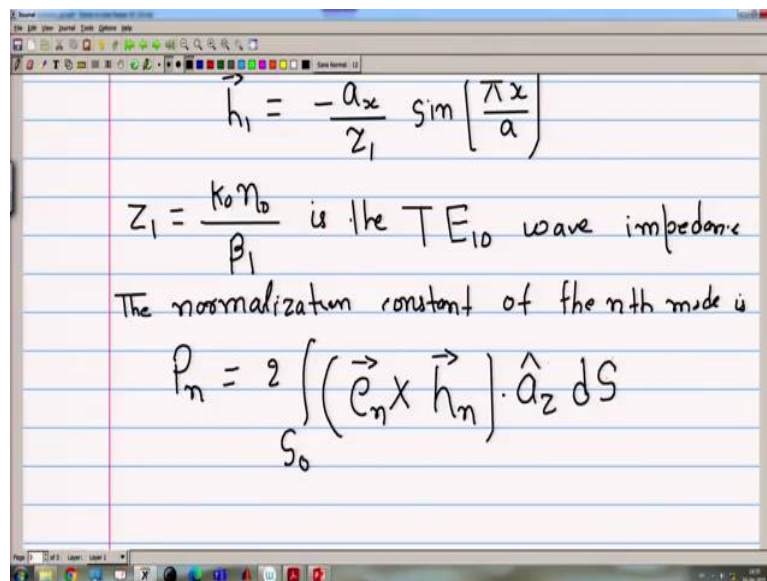


So, here is the waveguide,



For only, z directed current we can write, surface current on the probe surface as  $\mathbf{J}$ .

(Refer Slide Time: 19:28)



For TE<sub>10</sub> mode

$$P_1 = 2 \int_{S_0} (\vec{e}_1 \times \vec{h}_1) \cdot \hat{a}_z \, dS$$

$$= 2 \int_{S_0} \left[ a_y \sin\left(\frac{\pi x}{a}\right) \times -\frac{a_x}{Z_1} \sin\left(\frac{\pi x}{a}\right) \right] \cdot \hat{a}_z \, dS$$

If  $Z_1$  is the TE<sub>10</sub> wave impedance then

$$Z_1 = \frac{k_0 \eta_0}{\beta_1} \text{ is the TE}_{10} \text{ wave impedance}$$

The normalization constant of the  $n$ th mode is

$$P_n = 2 \int_{S_0} (\vec{e}_n \times \vec{h}_n) \cdot \hat{a}_z \, dS$$

And for the TE<sub>10</sub> mode,

For TE<sub>10</sub> mode

$$P_1 = 2 \int_{S_0} (\vec{e}_1 \times \vec{h}_1) \cdot \hat{a}_z \, dS$$

$$= 2 \int_{S_0} \left[ a_y \sin\left(\frac{\pi x}{a}\right) \times -\frac{a_x}{Z_1} \sin\left(\frac{\pi x}{a}\right) \right] \cdot \hat{a}_z \, dS$$

(Refer Slide Time: 23:32)

$$\begin{aligned} &= 2 \int_{S_0} \left[ a_y \sin\left(\frac{\pi x}{a}\right) \right] \times \frac{-a_x}{Z_1} \sin\left(\frac{\pi x}{a}\right) \cdot u_z \, dS \\ &= \frac{2}{Z_1} \int_0^a \int_0^b \sin^2 \frac{\pi x}{a} \cdot dx \, dy \\ &= \frac{2}{Z_1} \int_0^a \int_0^b \left( \frac{1 - \cos \frac{2\pi x}{a}}{2} \right) dx \, dy \end{aligned}$$

$$= \frac{2}{Z_1} \times \frac{a}{2} \times b = \frac{ab}{Z_1}$$

Amplitude of the positive traveling wave

$$A_n^+ = \frac{-1}{P_n} \int_V \left( \vec{e}_n - \hat{a}_z e_{zn} \right) \cdot \vec{J} e^{j\beta_n z} \, dV$$

$$\begin{aligned}
&= 2 \int_{S_0} \left[ a_y \sin\left(\frac{\pi x}{a}\right) \times \frac{-ax}{Z_1} \sin\left(\frac{\pi x}{a}\right) \cdot u_2 \, dS \right. \\
&= \frac{2}{Z_1} \int_{x=0}^a \int_{y=0}^b \sin^2 \frac{\pi x}{a} \cdot dx \, dy \\
&= \frac{2}{Z_1} \int_{x=0}^a \int_{y=0}^b \left( \frac{1 - \cos \frac{2\pi x}{a}}{2} \right) dx \, dy \\
&= \frac{2}{Z_1} \times \frac{a}{2} \times b = \frac{ab}{Z_1}
\end{aligned}$$

Amplitude of the positive traveling wave

$$A_n^+ = \frac{-1}{P_n} \int_V \left( \vec{e}_n - \hat{a}_z e_{zn} \right) \cdot \vec{J} e^{j\beta_n z} \, dV$$

(Refer Slide Time: 27:22)

For TE<sub>10</sub>

$$\begin{aligned}
A_1^+ &= \frac{-1}{P_1} \int_V \left( \vec{e}_1 - \hat{a}_z e_{z1} \right) \cdot \vec{J} e^{j\beta_1 z} \, dV \\
&= \frac{-1}{P_1} \int_V \left( \hat{a}_y \sin \frac{\pi x}{a} - \hat{a}_z \cdot 0 \right)
\end{aligned}$$

$$\begin{aligned}
 A_1^+ &= -\frac{1}{\rho_1} \int_V \left( \vec{e}_1 - \hat{a}_2 e_{z1} \right) \vec{J} e^{j\beta_1 z} dV \\
 &= -\frac{1}{\rho_1} \int_V \left( \hat{a}_y \sin \frac{\pi x}{a} - \hat{a}_z \cdot 0 \right) \\
 &\quad I_0 \delta \left( x - \frac{a}{2} \right) \delta(z) \hat{a}_y e^{j\beta_1 z} dV
 \end{aligned}$$

$$\begin{aligned}
 &= -\frac{1}{\rho_1} \int_V \sin \frac{\pi x}{a} e^{j\beta_1 z} I_0 \delta \left( x - \frac{a}{2} \right) \delta(z) dx dy dz \\
 &= -\frac{I_0}{\rho_1} \times 1 \times b \times 1 = -\frac{b I_0}{\rho_1} \\
 &\sin \left[ \frac{\pi}{a} \frac{a}{2} \right]
 \end{aligned}$$

So, for the TE<sub>10</sub> mode, it can be written as,

$$\begin{aligned}
 \text{For TE}_{10} \\
 A_1^+ &= -\frac{1}{\rho_1} \int_V \left( \vec{e}_1 - \hat{a}_2 e_{z1} \right) \vec{J} e^{j\beta_1 z} dV \\
 &= -\frac{1}{\rho_1} \int_V \left( \hat{a}_y \sin \frac{\pi x}{a} - \hat{a}_z \cdot 0 \right)
 \end{aligned}$$

$$\begin{aligned}
 A_1^+ &= -\frac{1}{\rho_1} \int_V \left( \hat{e}_1 - \hat{a}_2 e_{z1} \right) \vec{J} e^{j\beta_1 z} dV \\
 &= -\frac{1}{\rho_1} \int_V \left( \hat{a}_y \sin \frac{\pi x}{a} - \hat{a}_z \cdot 0 \right) \\
 &\quad I_0 \delta \left( x - \frac{a}{2} \right) \delta(z) \hat{a}_y \\
 &\quad e^{j\beta_1 z} dV \\
 &= -\frac{1}{\rho_1} \int_V \sin \frac{\pi x}{a} e^{j\beta_1 z} I_0 \delta \left( x - \frac{a}{2} \right) \delta(z) \\
 &\quad dx dy dz \\
 &= -\frac{I_0}{\rho_1} \times 1 \times b \times 1 = -\frac{b I_0}{\rho_1} \\
 &\quad \sin \left| \frac{\pi x}{a} \right|
 \end{aligned}$$

(Refer Slide Time: 32:05)

The screenshot shows a digital whiteboard with the following handwritten equations:

$$\begin{aligned}
 &= -\frac{I_0}{\rho_1} \times 1 \times b \times 1 = -\frac{b I_0}{\rho_1} \\
 &\sin \left| \frac{\pi x}{a} \right| = -\frac{b I_0}{a \beta} z_1 = -\frac{z_1 I_0}{a} \\
 \\
 &A_1^+ = -\frac{z_1 I_0}{a}
 \end{aligned}$$

$$\sin \left[ \frac{\pi \phi}{2} \right] = -\frac{Z_1 I_0}{a \beta} \gamma_1 = -\frac{Z_1 I_0}{a}$$

$$A_1^+ = \frac{-Z_1 I_0}{a}$$

so this is for the positive traveling wave, similar to this we can calculate the amplitude of the negative traveling wave so, that we will calculate in the next class, thank you.