Advanced Microwave Guided-Structures and Analysis
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Indian Institute of Technology Kharagpur
Lecture 40
Rectangular Cavity Resonator Tutorials (Contd.)

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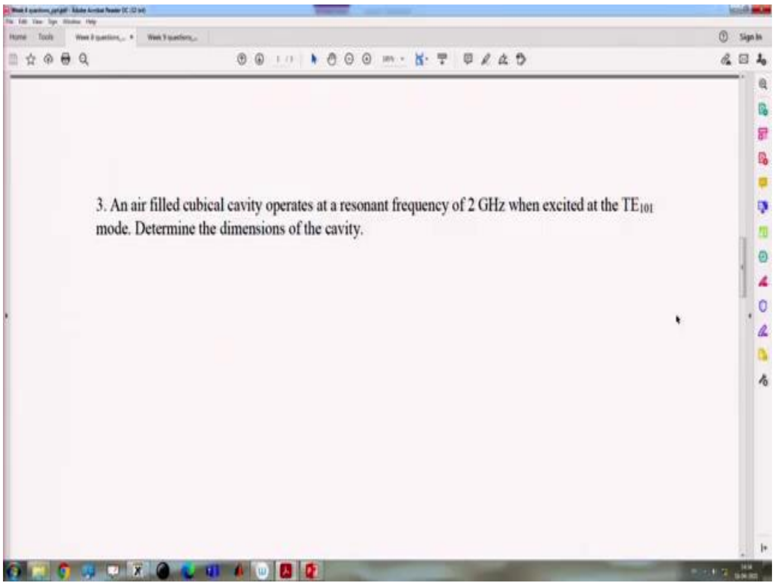

Welcome to the next tutorial class, so we will do today some numerical problems on waveguide resonator. We have already completed two problems and now, here is the third problem so, third problem is that,

3. An air filled cubical cavity operates at a resonant frequency of 2 GHz when excited at the TE_{101} mode. Determine the dimensions of the cavity.

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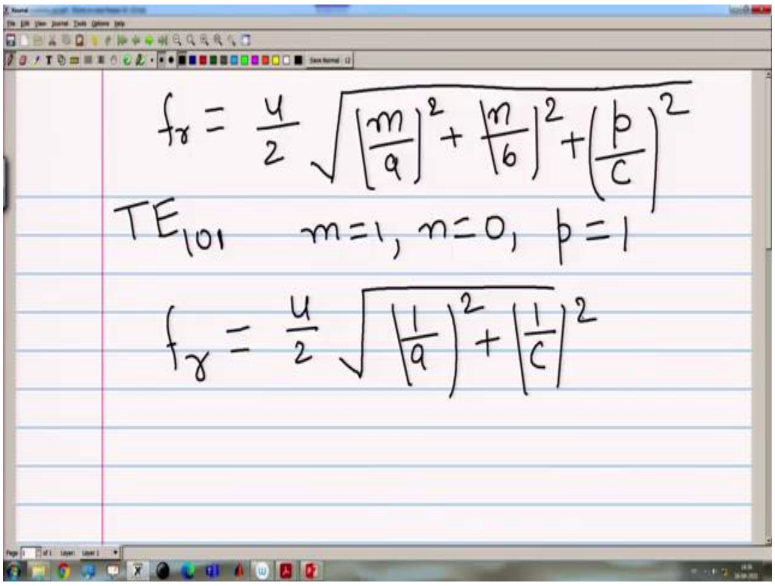

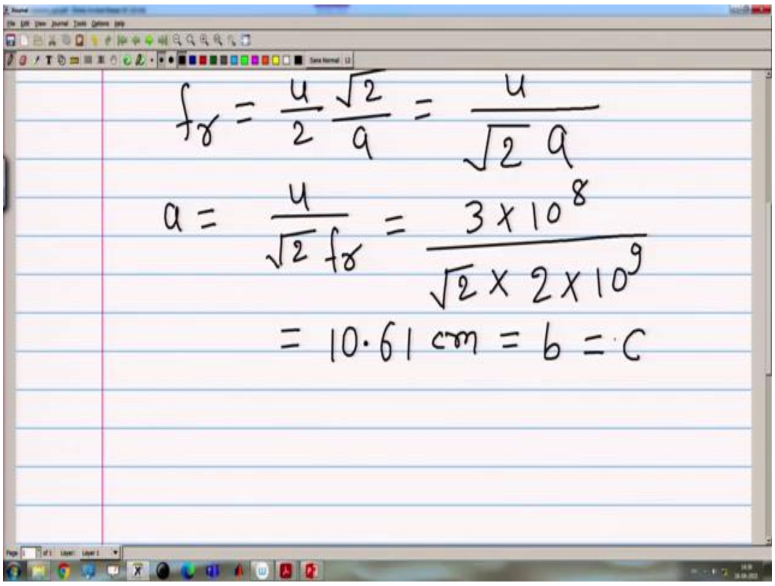

So, to calculate the resonant frequency we know that formula is,

$$f_r = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

TE_{101} $\quad m=1,\ n=0,\ p=1$

$$f_r = \frac{u}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2}$$

$$f_r = \frac{u}{2}\frac{\sqrt{2}}{a} = \frac{u}{\sqrt{2}\,a}$$

$$a = \frac{u}{\sqrt{2}\,f_r} = \frac{3\times 10^8}{\sqrt{2}\times 2\times 10^9}$$

$$= 10.61\ \text{cm} = b = c$$

So, this will come 10.61 centimeter, so this is a that means equal to b equal to c so, this is the dimensions of the rectangular this cubical waveguide, cavity, cubical cavity.

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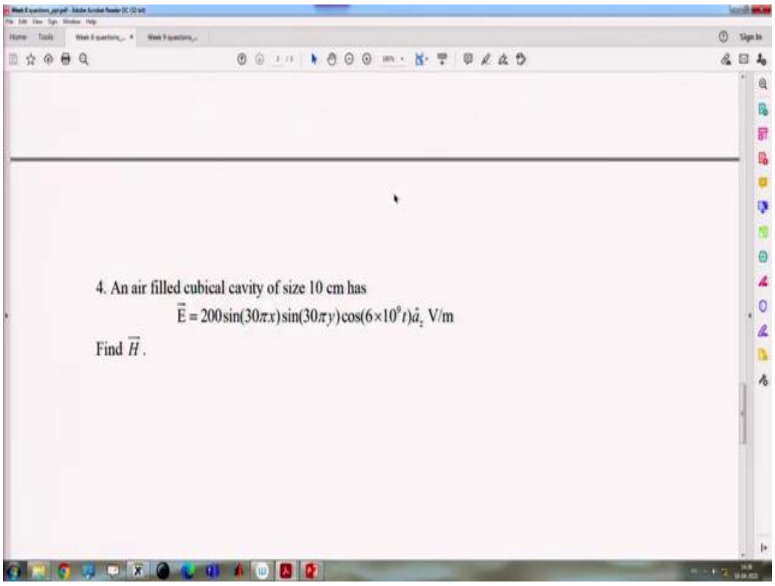


Next problem is:

4. An air filled cubical cavity of size 10 cm has

$$\vec{E} = 200\sin(30\pi x)\sin(30\pi y)\cos(6\times10^{9}t)\hat{a}_z \text{ V/m}$$

Find $\vec{H}$.

So, here $\mathbf{a_z}$ is given that means here, only electric field is present in the z direction. That means, no magnetic field in the z direction that means this is for the TM to z mode. So, this is for the TM to z mode, and for this case we have to find out the vector magnetic field. So, vector magnetic field will be.

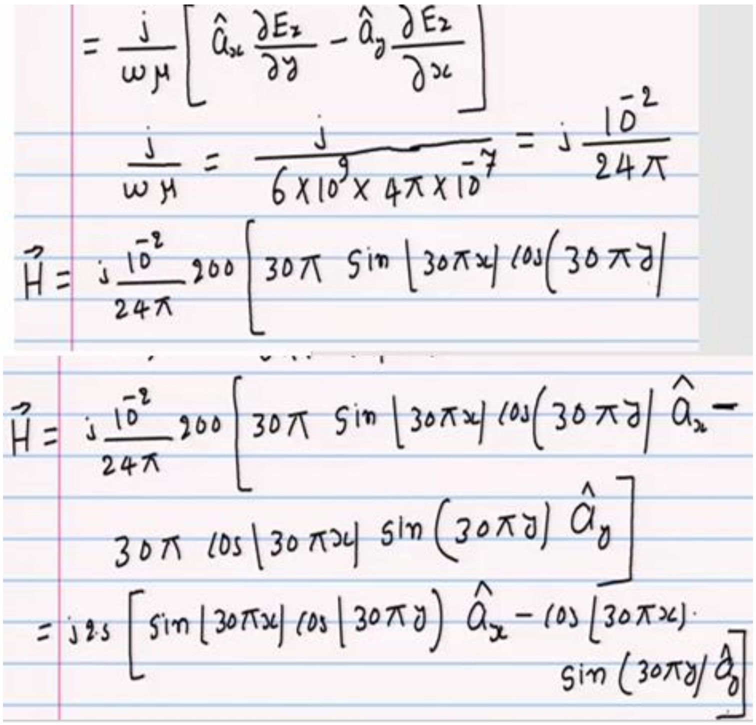
4)

$$\vec{\nabla} \times \vec{E} = -j\omega\mu \vec{H}$$

$$\vec{H} = -\frac{1}{j\omega\mu}\left(\vec{\nabla} \times \vec{E}\right)$$

$$= \frac{j}{\omega\mu}\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_z(x,y) \end{vmatrix}$$

$$= \frac{j}{\omega\mu}\left[a_x\left(\frac{\partial}{\partial y}E_z - 0\right) + a_y\left(0 - \frac{\partial E_z}{\partial x}\right) + a_z(0-0)\right]$$



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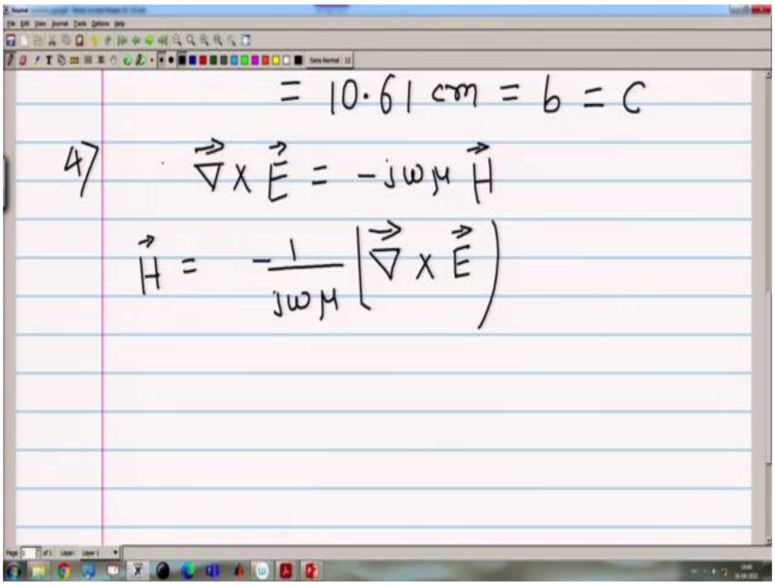

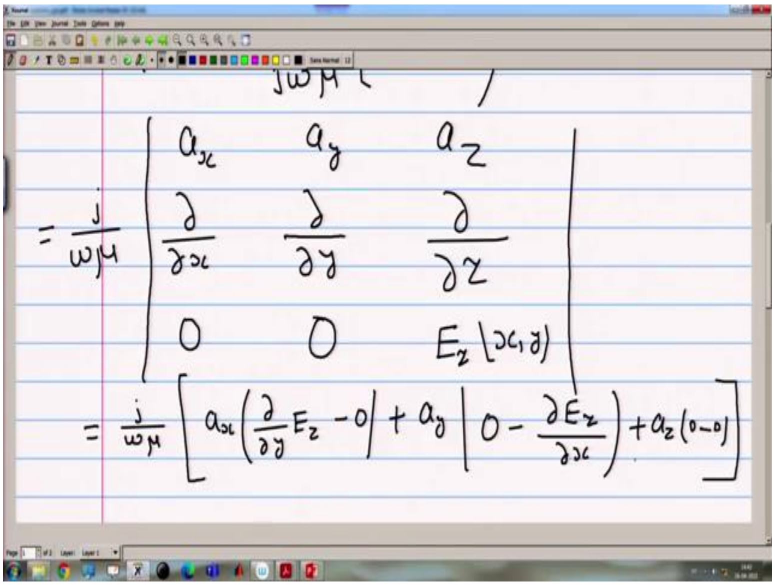

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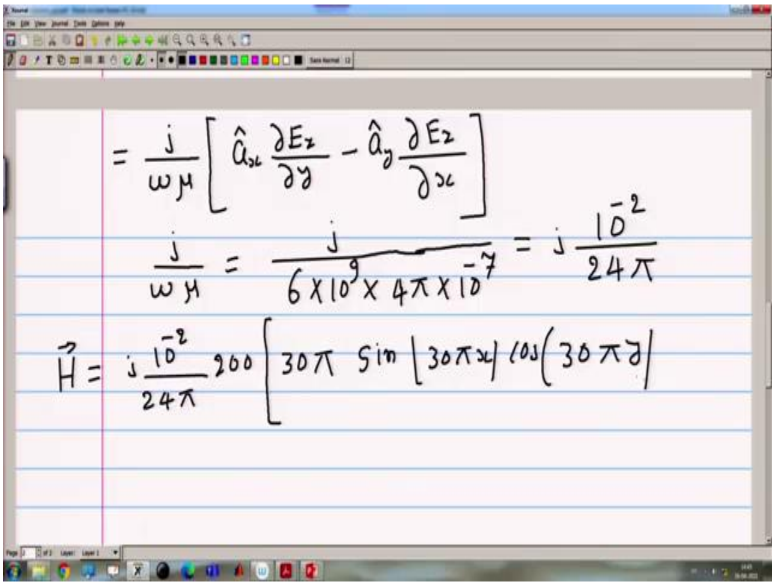
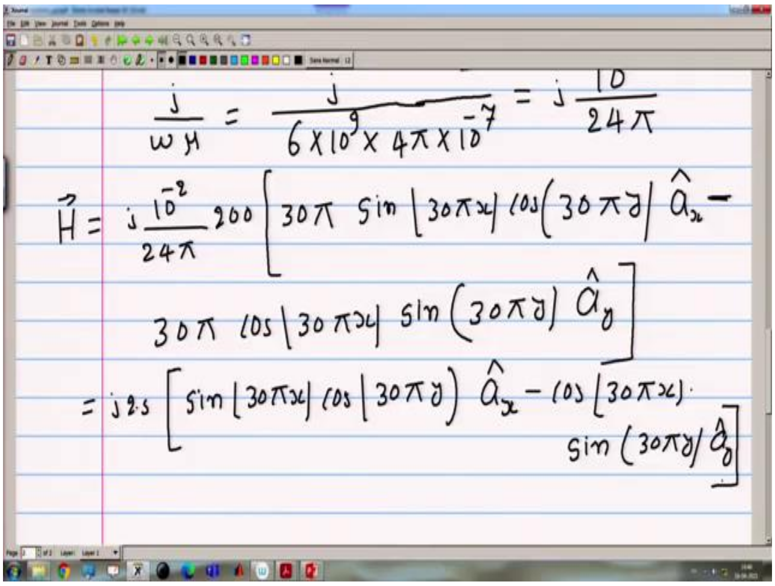
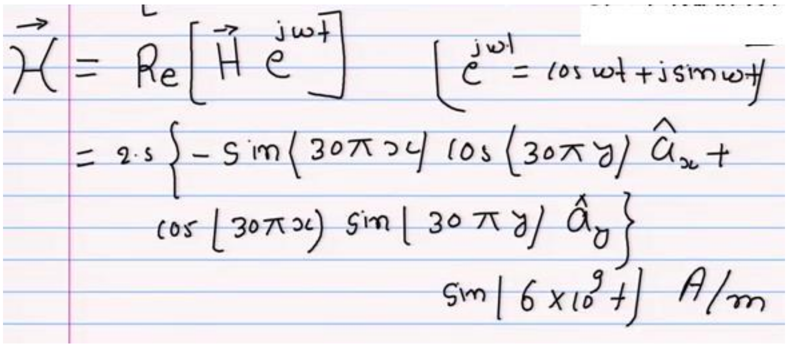




So, this is the air-filled component that is the complex part, and for instantaneous part we can calculate.



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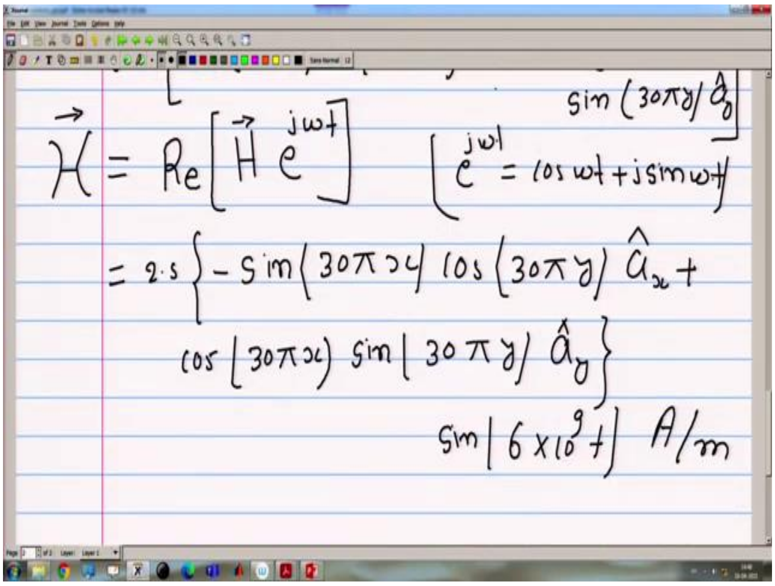

So, this is the instantaneous magnetic field component.

From this we can go to the week 9 numerical problems.

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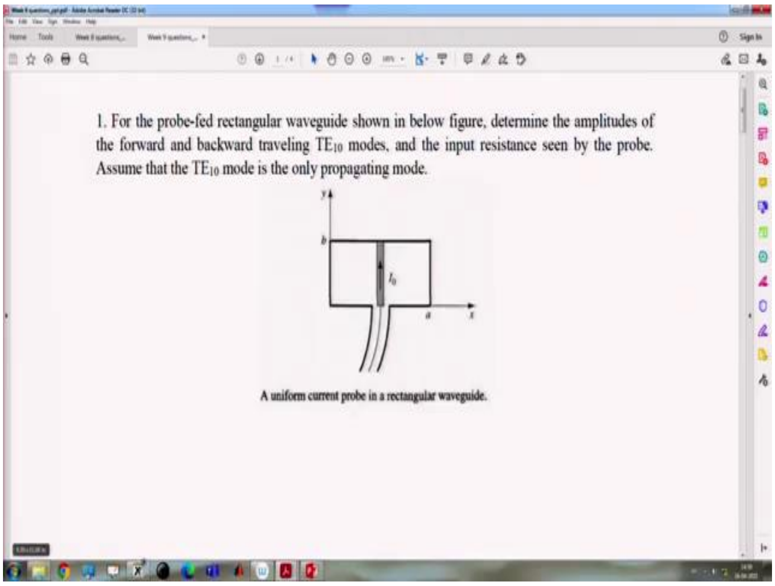
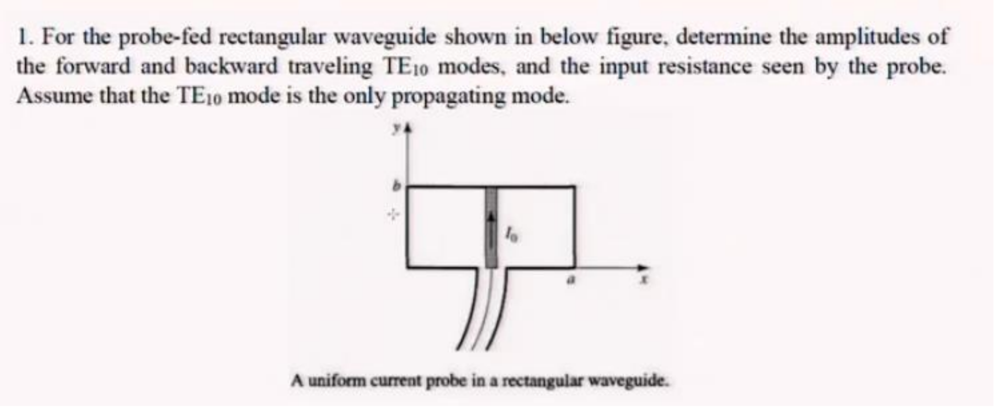


1. For the probe-fed rectangular waveguide shown in below figure, determine the amplitudes of the forward and backward traveling TE_{10} modes, and the input resistance seen by the probe. Assume that the TE_{10} mode is the only propagating mode.

A uniform current probe in a rectangular waveguide.

Here is the, first problem



1. For the probe-fed rectangular waveguide shown in below figure, determine the amplitudes of the forward and backward traveling TE_{10} modes, and the input resistance seen by the probe. Assume that the TE_{10} mode is the only propagating mode.

A uniform current probe in a rectangular waveguide.

So, first we have to find out the amplitude of the forward and backward traveling, TE_{10} mode.

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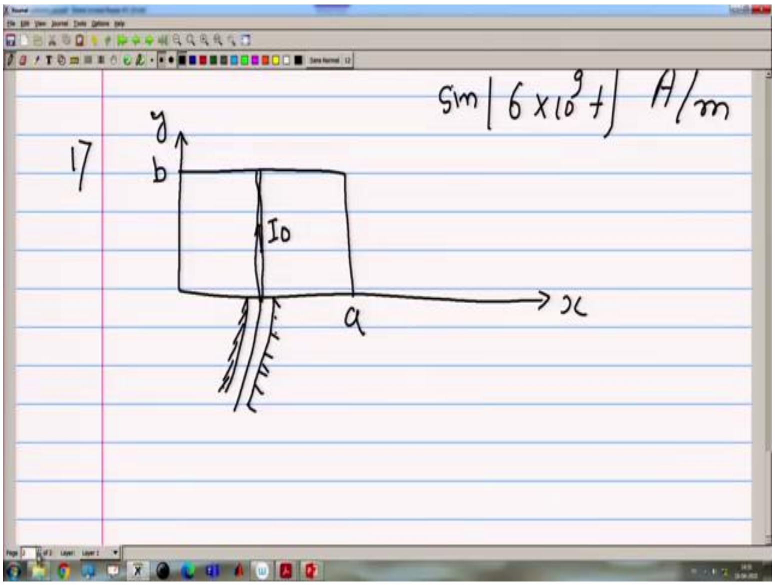
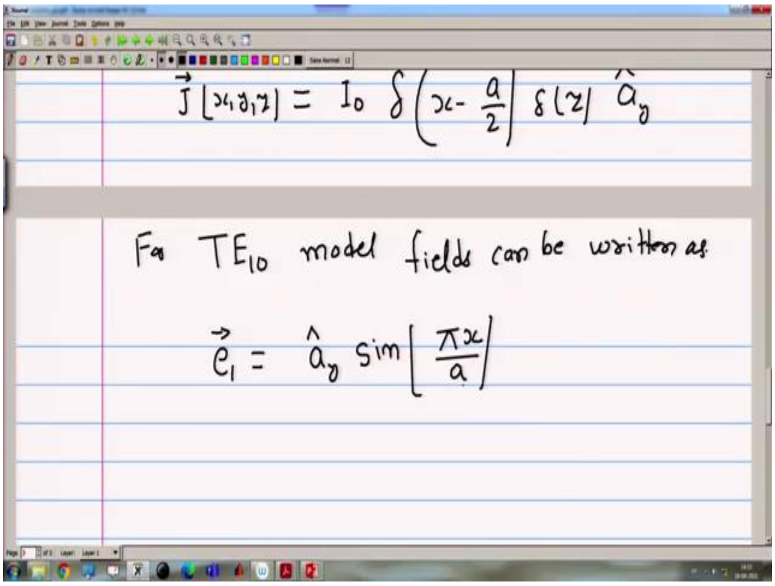
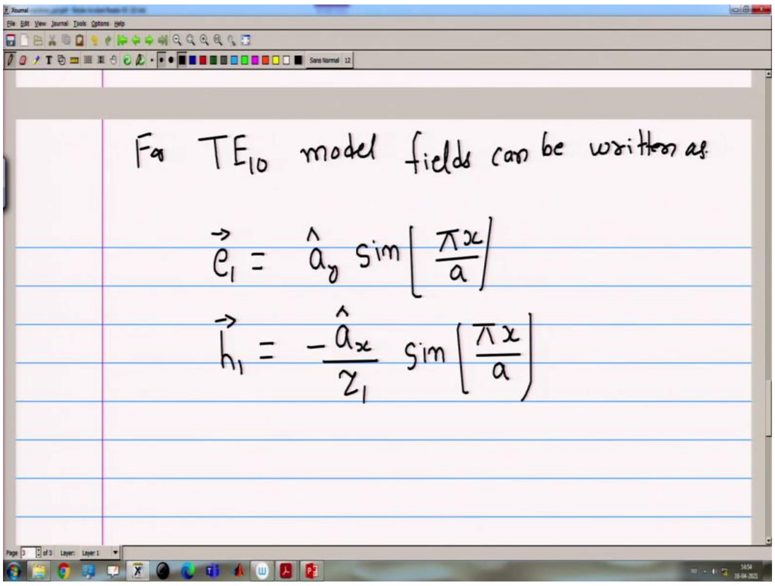
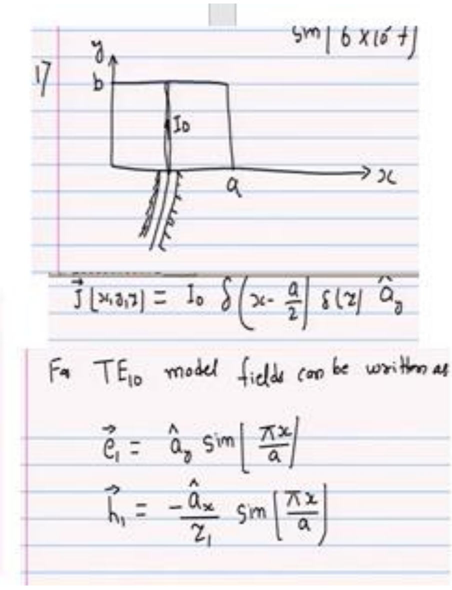






So, here is the waveguide,



For only, z directed current we can write, surface current on the probe surface as **J**.

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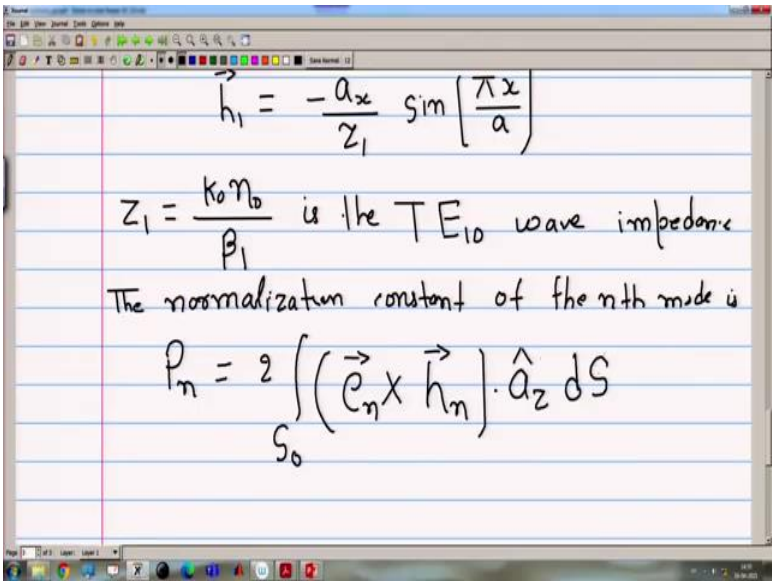
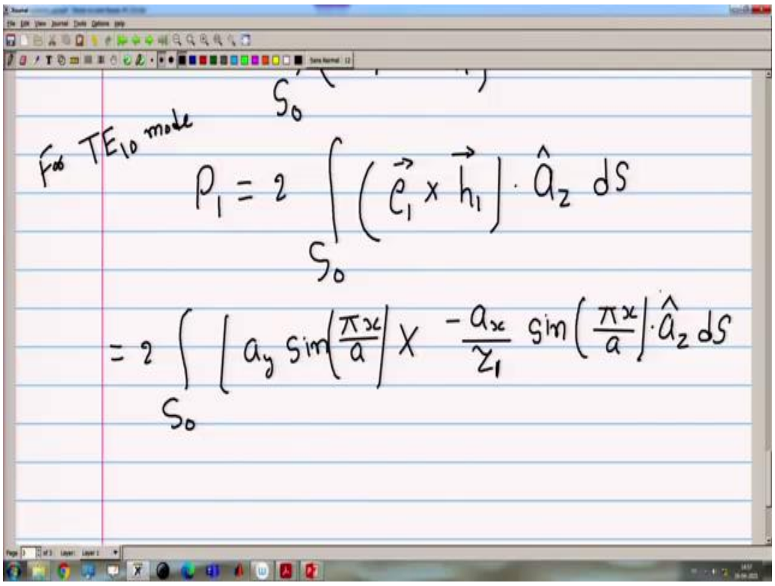


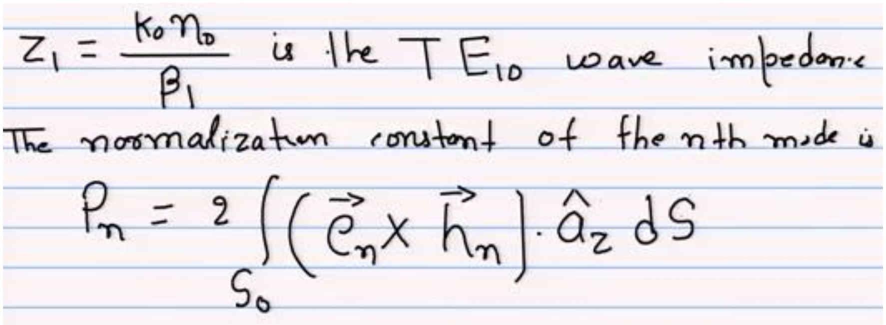


For TE_{10} mode

$$P_1 = 2\int_{S_0} \left(\vec{e}_1 \times \vec{h}_1\right)\cdot \hat{a}_z \, dS$$

$$= 2\int_{S_0} \left[a_y \sin\left(\frac{\pi x}{a}\right) \times \frac{-a_x}{Z_1} \sin\left(\frac{\pi x}{a}\right)\right]\cdot \hat{a}_z \, dS$$

If z1 is the TE_{10} wave impedance then

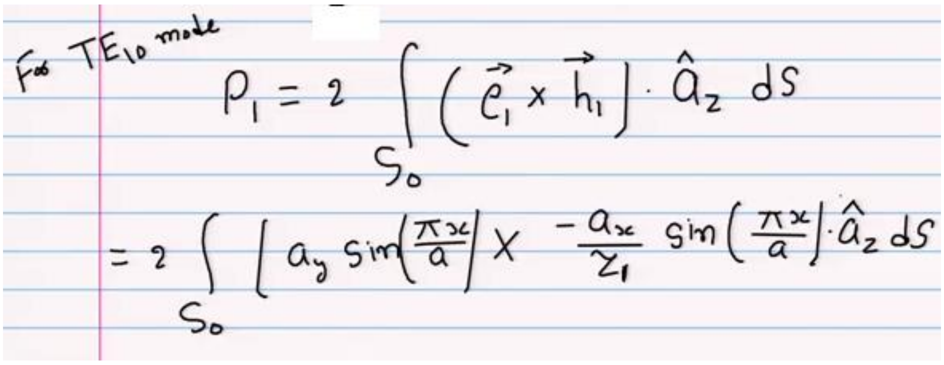


$Z_1 = \frac{k_0 \eta_0}{\beta_1}$ is the TE_{10} wave impedance

The normalization constant of the nth mode is

$$P_n = 2\int_{S_0} \left(\vec{e}_n \times \vec{h}_n\right)\cdot \hat{a}_z \, dS$$

And for the TE_{10} mode,



For TE_{10} mode

$$P_1 = 2\int_{S_0} \left(\vec{e}_1 \times \vec{h}_1\right)\cdot \hat{a}_z \, dS$$

$$= 2\int_{S_0} \left[a_y \sin\left(\frac{\pi x}{a}\right) \times \frac{-a_x}{Z_1} \sin\left(\frac{\pi x}{a}\right)\right]\cdot \hat{a}_z \, dS$$

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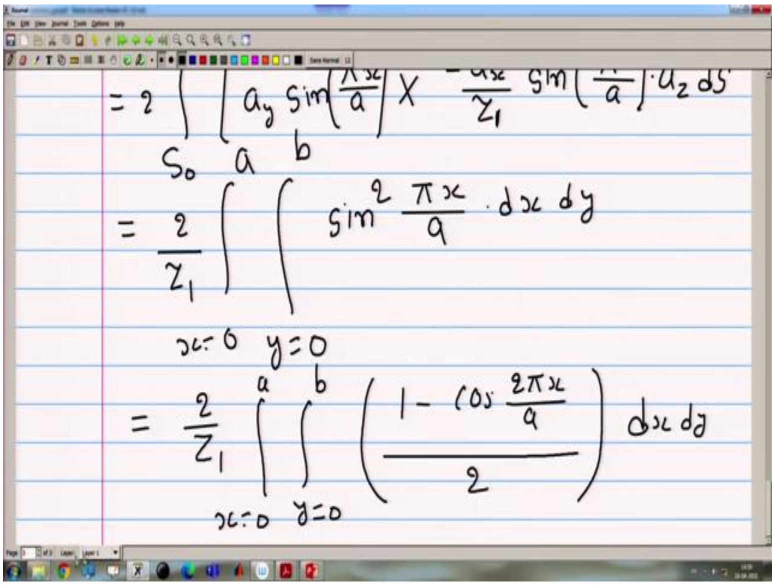

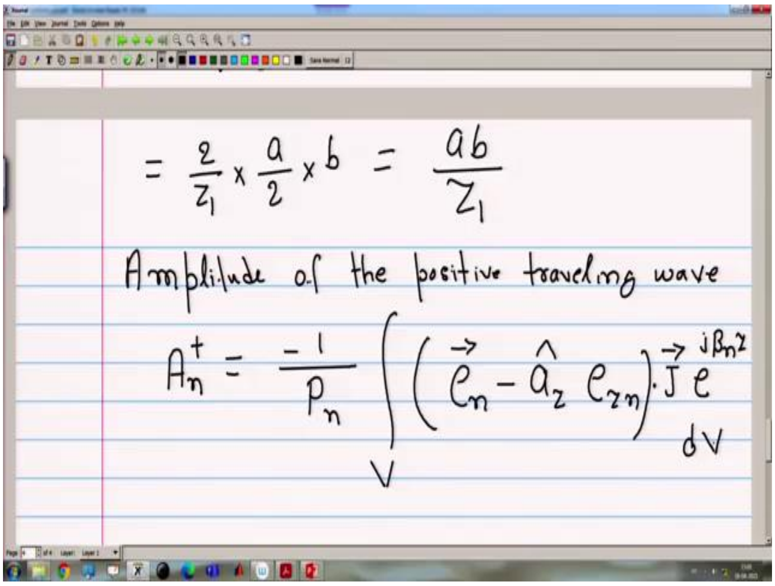

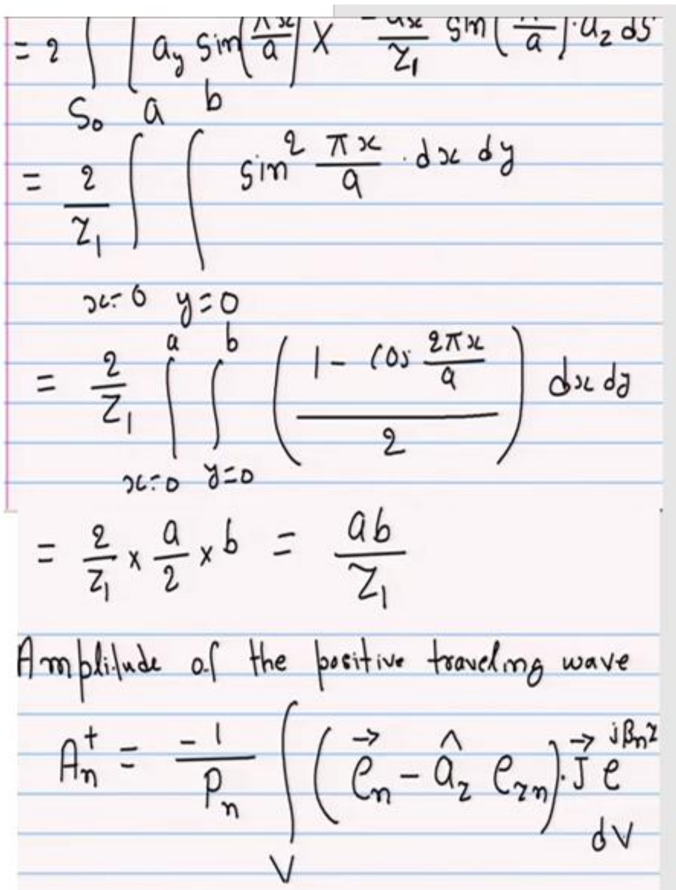

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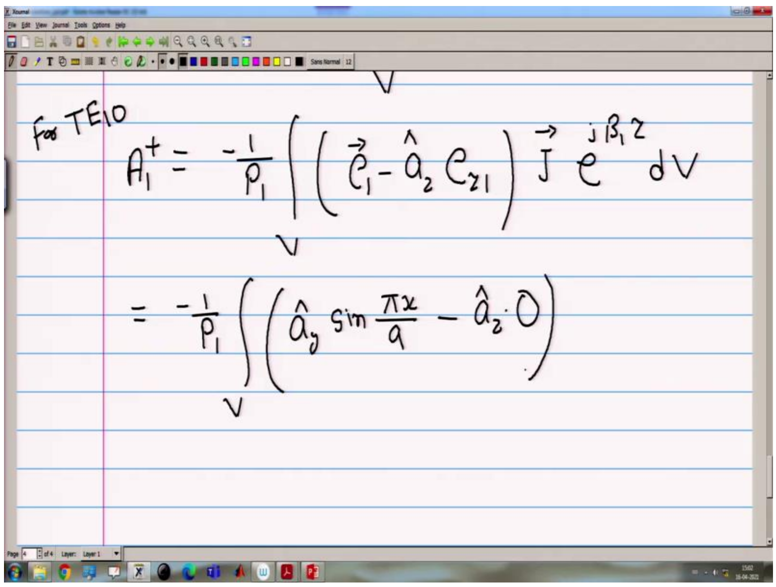
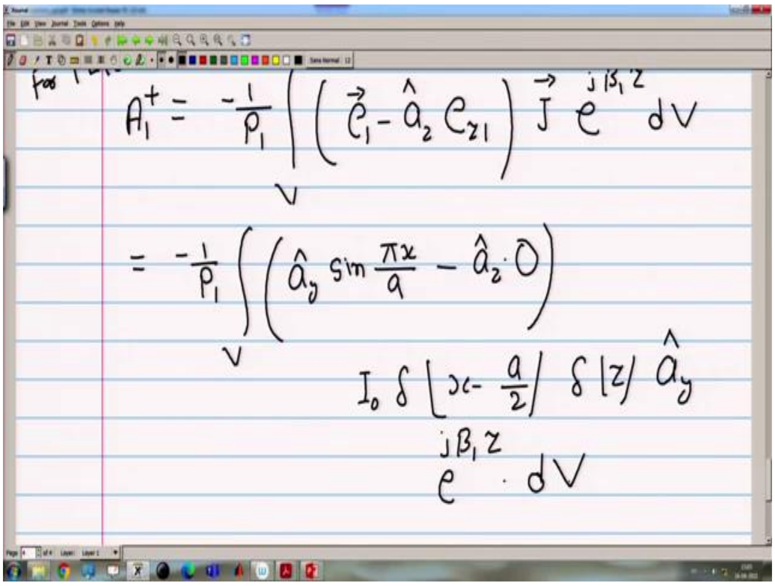
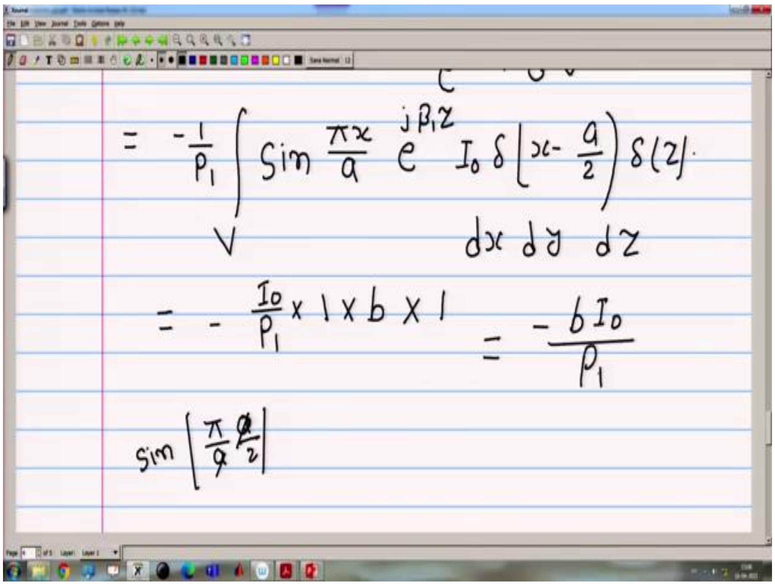
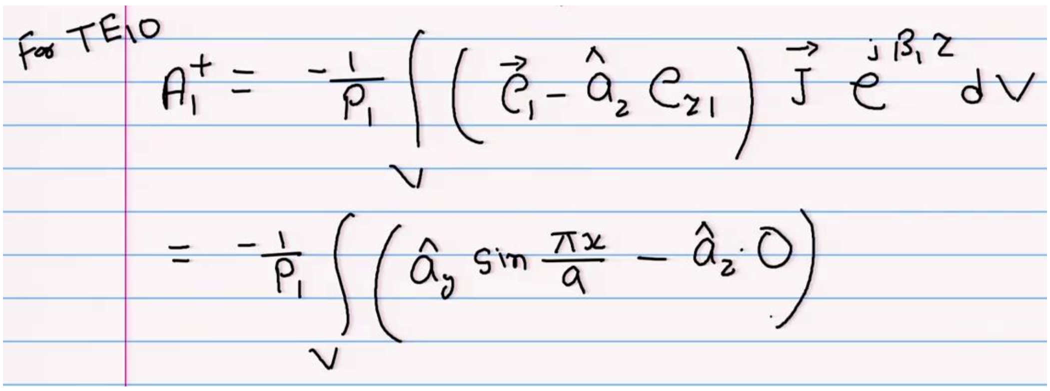
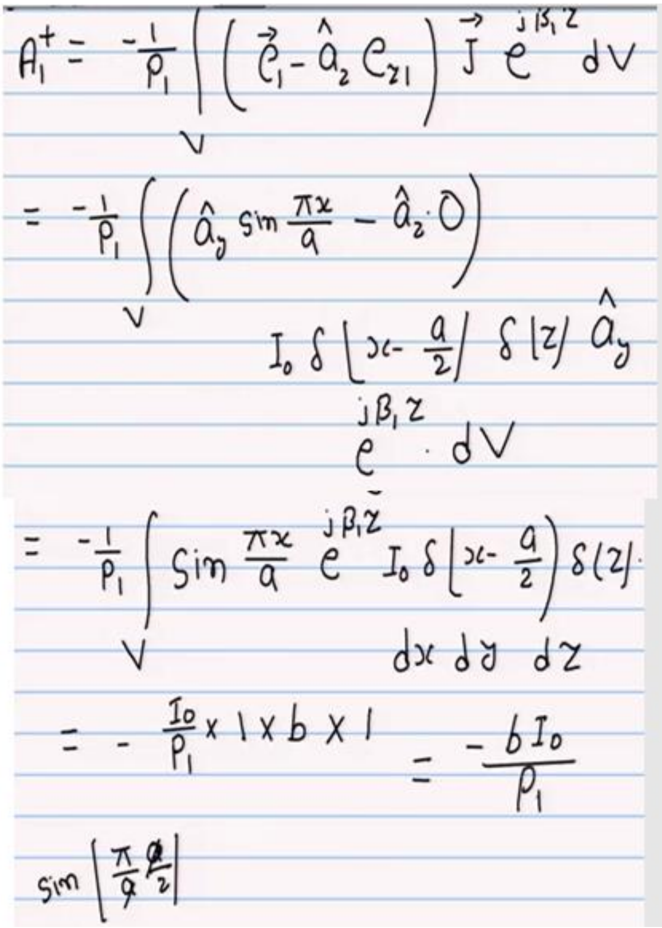






So, for the TE_{10} mode, it can be written as,





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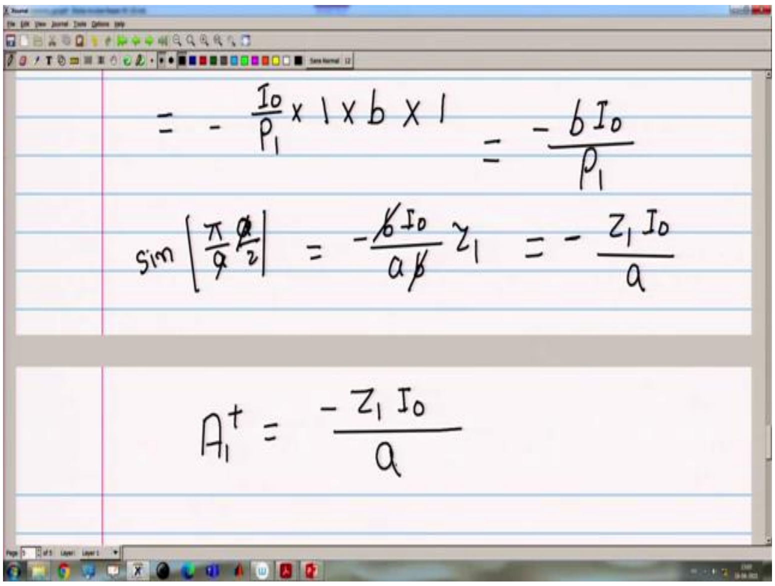

$$\sin\left[\frac{\pi}{a}\frac{a}{2}\right] = -\frac{\beta I_0}{a\beta} Z_1 = -\frac{Z_1 I_0}{a}$$

$$A_1^+ = \frac{-Z_1 I_0}{a}$$

so this is for the positive traveling wave, similar to this we can calculate the amplitude of the negative traveling wave so, that we will calculate in the next class, thank you.