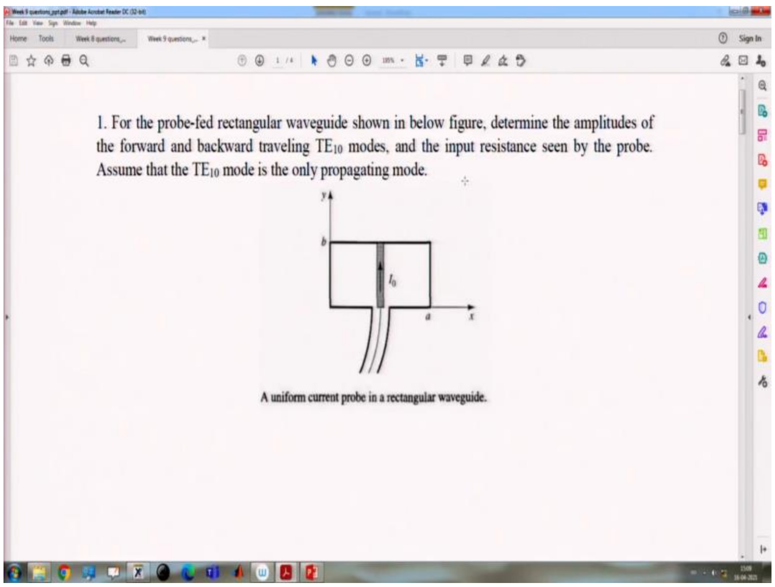
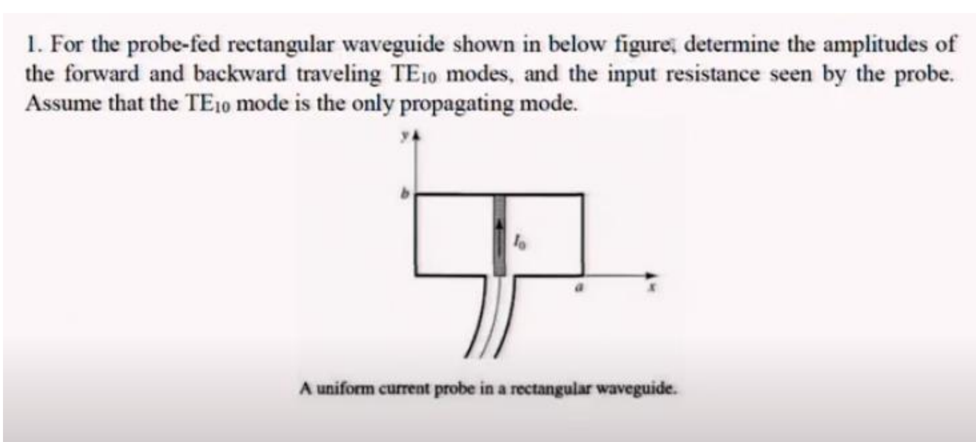
Advanced Microwave Guided-Structure and Analysis
Professor Bratin Ghosh
Department of E & ECE
Indian Institute of Technology, Kharagpur
Lecture 43
The Reciprocity Theorem, Computation of Amplitudes Tutorials

Welcome to the next tutorial class. So, we are calculating the forward and backward amplitude in the rectangular waveguide due to the excitation of some electric source that is placed inside the rectangular waveguide.

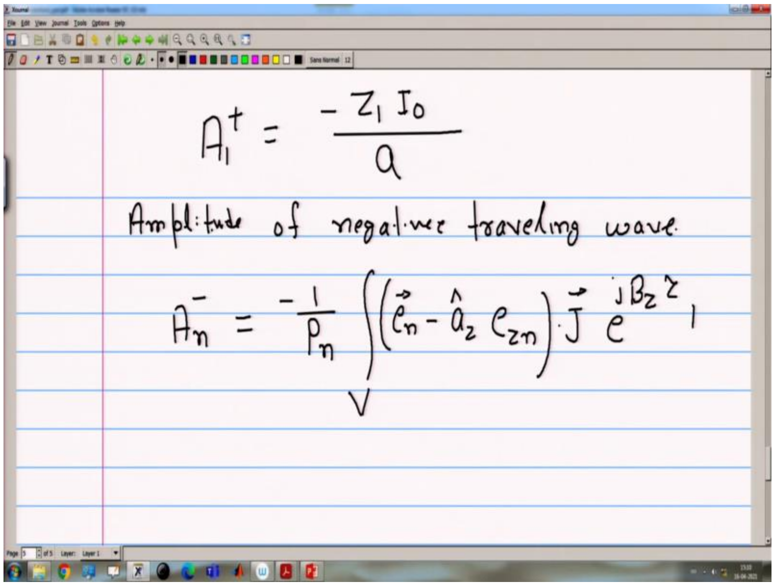
(Refer Slide Time: 00:35)

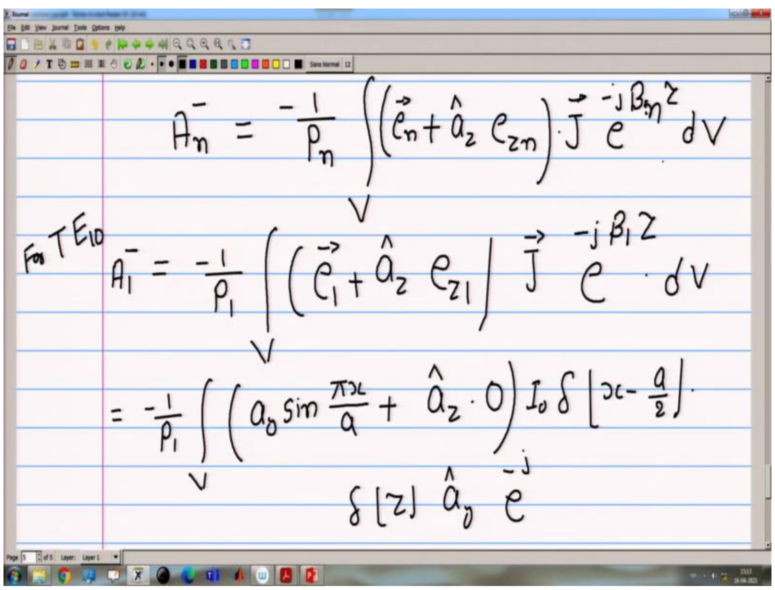




We have to calculate the forward amplitude and backward amplitude; and the input resistance that we are feeding at z is equal to 0. And it is touching at z is equal to b.

(Refer Slide Time: 01:02)





So, we have already calculated this forward amplitude;

$$A_1^+ = \frac{-Z_1 I_0}{a}$$

Amplitude of negative traveling wave

$$A_n^- = \frac{-1}{P_n} \int_V \left(\vec{e}_n - \hat{a}_z e_{zn}\right) \cdot \vec{J} \, e^{j\beta_z z},$$

$$A_n^- = \frac{-1}{P_n} \int_V \left(\vec{e}_n + \hat{a}_z e_{zn}\right) \cdot \vec{J} \, e^{-j\beta_n z} \, dV$$

For TE_{10}

$$A_1^- = \frac{-1}{P_1} \int_V \left(\vec{e}_1 + \hat{a}_z e_{z1}\right) \vec{J} \, e^{-j\beta_1 z} \cdot dV$$

$$= \frac{-1}{P_1} \int_V \left(a_y \sin\frac{\pi x}{a} + \hat{a}_z \cdot 0\right) I_0 \delta\left[x - \frac{a}{2}\right] \cdot \delta[z] \, \hat{a}_y \, e^{-j}$$

$$= \frac{-1}{P_1} \int_V \left(a_y \sin\frac{\pi x}{a} + \hat{a}_z \cdot 0\right) I_0 \delta\left[x - \frac{a}{2}\right] \cdot \delta[z] \, \hat{a}_y \, e^{-j\beta_1 z} \cdot dV$$

$$= \frac{-1}{P_1} \int_V \sin\frac{\pi x}{a} \, I_0 \, \delta\left[x - \frac{a}{2}\right] \delta(z) \, e^{-j\beta_1 z} \cdot dx \, dy \, dz$$

$$= -\frac{1}{P_1} \times 1 \times b \times 1 \times I_0 = \frac{-b I_0}{P_1}$$

$$= -\frac{1}{P_1} \times 1 \times b \times 1 \times I_0 = \frac{-b I_0}{P_1}$$

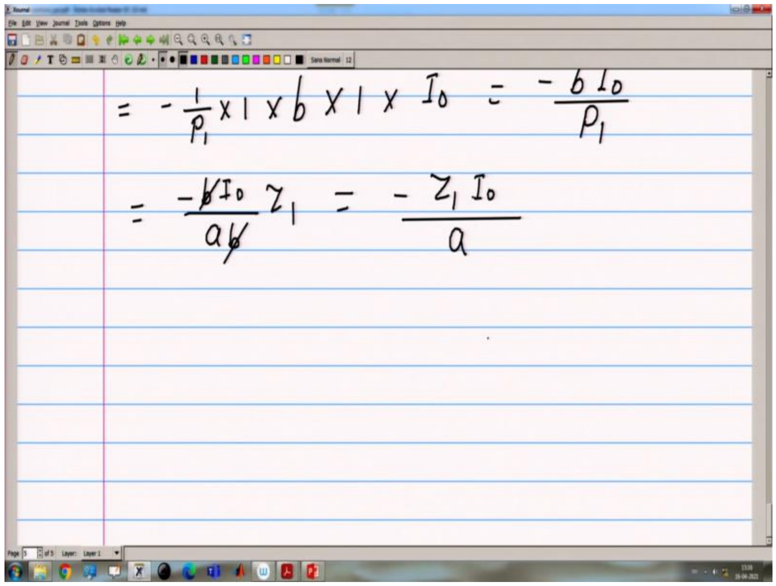
$$= \frac{-b I_0}{a b} Z_1 = \frac{-Z_1 I_0}{a}$$

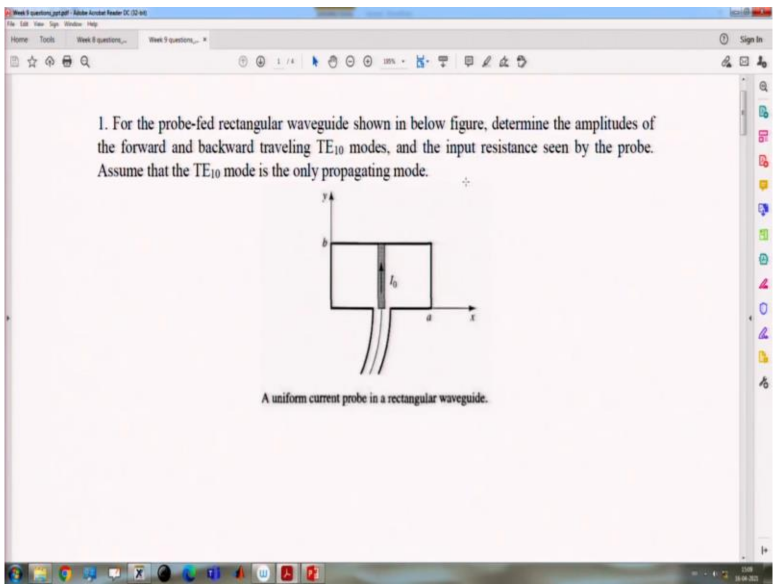
(Refer Slide Time: 05:48)

$$= -\frac{1}{\beta_1}\int_V \left(a_y \sin\frac{\pi x}{a} + \hat{a}_z \cdot 0\right) I_0\, \delta\left[x - \frac{a}{2}\right] \cdot \delta[z]\, \hat{a}_y\, e^{-j\beta_1 z} \cdot dV$$

$$= -\frac{1}{\beta_1}\int_V \sin\frac{\pi x}{a}\, I_0\, \delta\left[x - \frac{a}{2}\right] \delta(z)\, e^{-j\beta_1 z} \cdot dx\, dy\, dz$$

$$= -\frac{1}{\beta_1} \times 1 \times b \times 1 \times I_0 = \frac{-b I_0}{\beta_1}$$





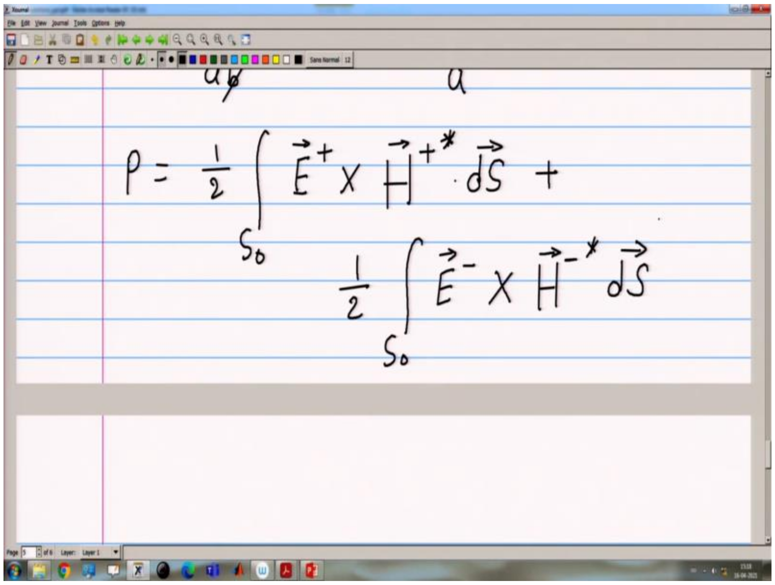
So, this is a forward and backward, so both are same; that amplitude of the forward and backward; after this next problem is calculate the input resistance seen by the probe. Now, we have to calculate the input resistance seen by probe. To compute the input resistance we have to first use the power. So, power we know means, so that will be.

$$P = \frac{1}{2}\int_{S_0} \vec{E}^{+} \times \vec{H}^{+*} \cdot \vec{dS} + \frac{1}{2}\int_{S_0} \vec{E}^{-} \times \vec{H}^{-*} \vec{dS}$$

$$= \int_{S_0} \left[\vec{E}^{+} \times \vec{H}^{+*}\right] \vec{dS}$$

$$= \int_{x=0}^{a}\int_{y=0}^{b} \frac{|A_1^{+}|^2}{Z_1} \sin^2\left(\frac{\pi x}{a}\right) dx\, dy$$

(Refer Slide Time: 08:50)

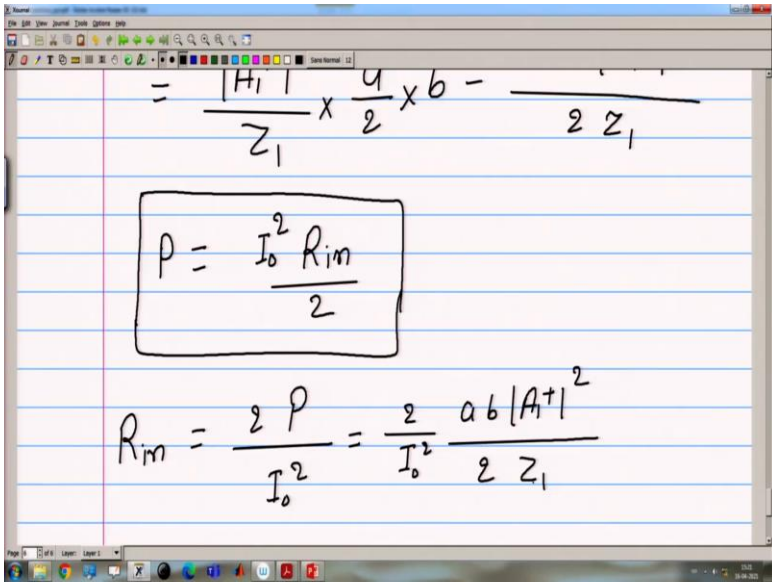


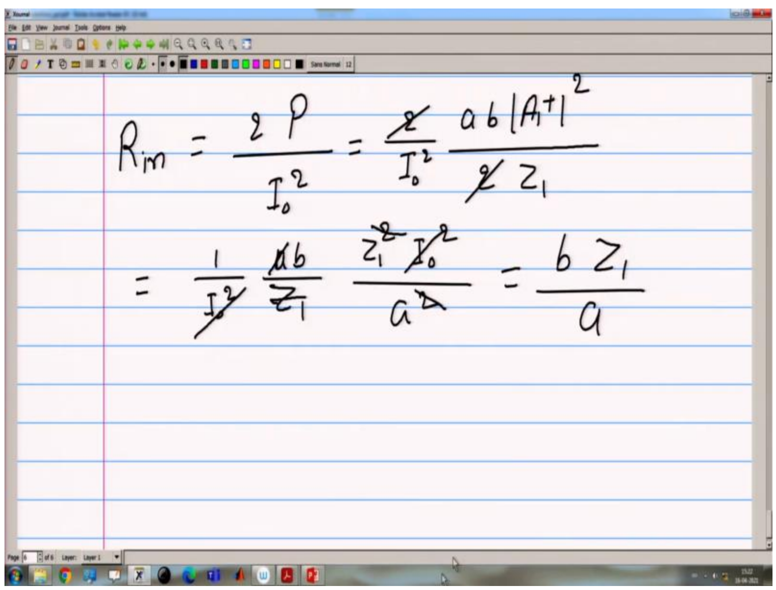
$$= \int_{S_0} \left[\vec{E}^{+} \times H^{+*} \right] dS$$

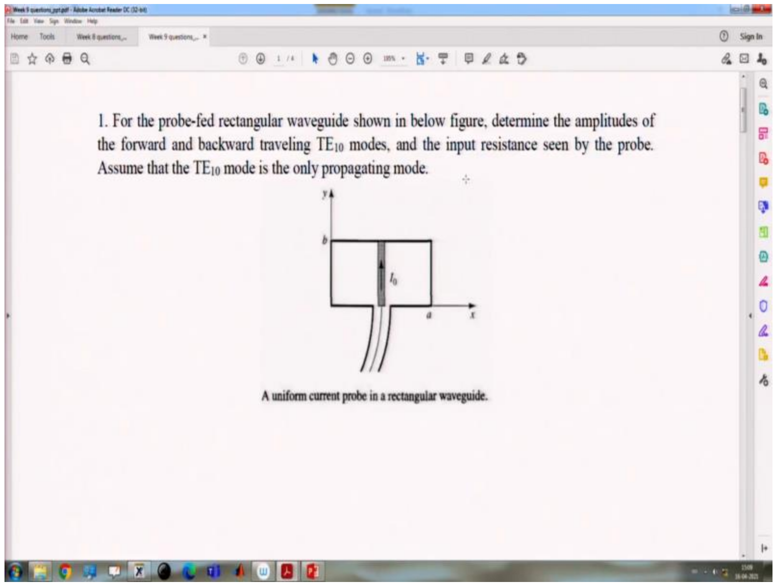
$$= \int_{x=0}^{a} \int_{y=0}^{b} \frac{|A_1^{+}|^2}{Z_1} \sin^2\left(\frac{\pi x}{a}\right) dx\, dy$$

$$= \frac{|A_1^{+}|^2}{Z_1} \times \frac{a}{2} \times b = \frac{a b\, |A_1^{+}|^2}{2 Z_1}$$

(Refer Slide Time: 11:45)







1. For the probe-fed rectangular waveguide shown in below figure, determine the amplitudes of the forward and backward traveling TE_{10} modes, and the input resistance seen by the probe. Assume that the TE_{10} mode is the only propagating mode.

A uniform current probe in a rectangular waveguide.

From this formula we will get the input resistance:

$$P = \frac{I_0^2 R_{in}}{2}$$

$$R_{in} = \frac{2P}{I_0^2} = \frac{2}{I_0^2} \frac{ab|A^+|^2}{2 Z_1}$$

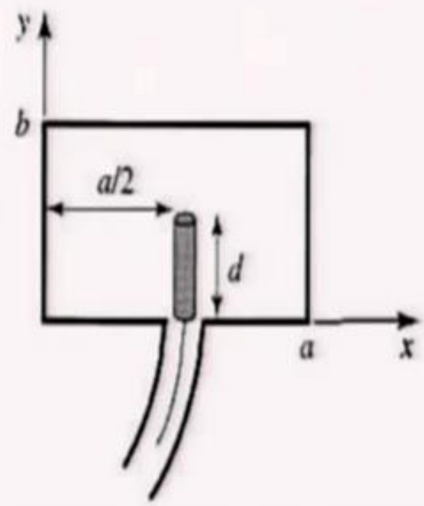
$$R_{in} = \frac{2P}{I_0^2} = \frac{2}{I_0^2} \frac{ab|A^+|^2}{2 Z_1}$$

$$= \frac{1}{I_0^2} \frac{ab}{Z_1} \frac{Z_1^2 I_0^2}{a^2} = \frac{b Z_1}{a}$$

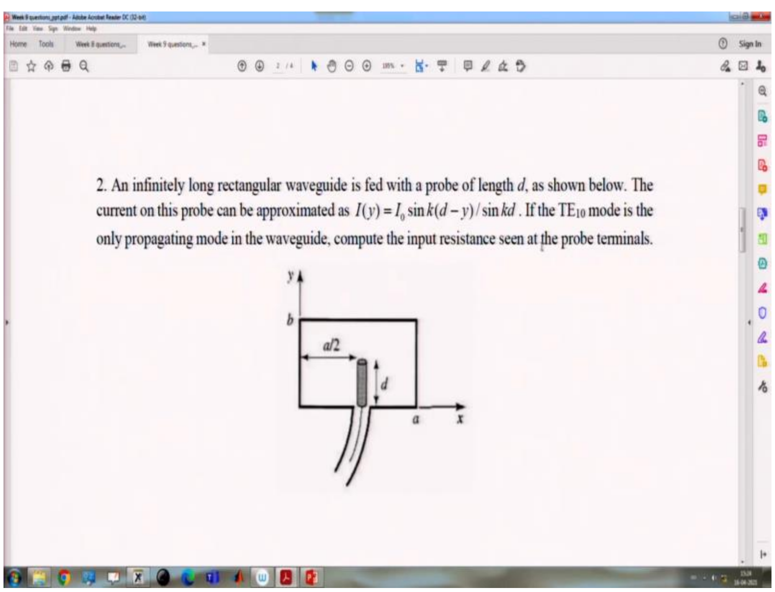
So, this is the input resistance seen by the when will seen from the probe side; so, b Z1 upon a.

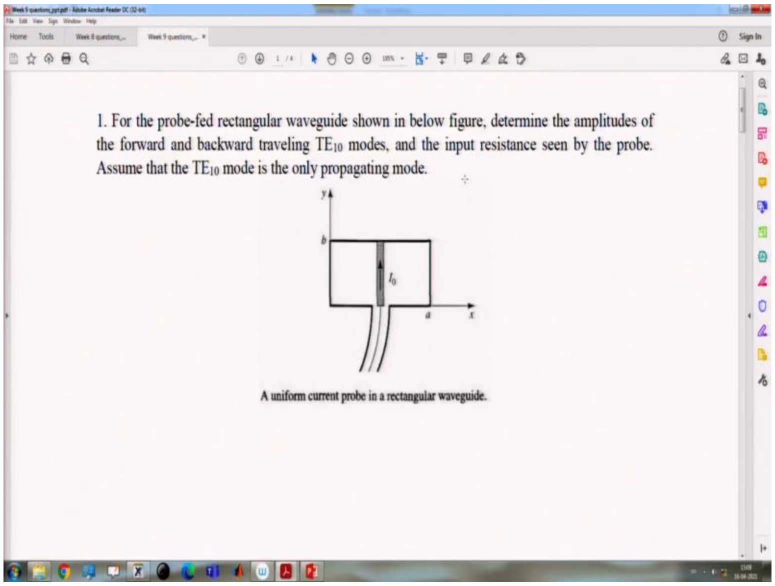
So, this is the complete solution for the first problem; so we had calculated the input resistance seen by the probe. Now, we can move to the second problem.

2. An infinitely long rectangular waveguide is fed with a probe of length d, as shown below. The current on this probe can be approximated as $I(y) = I_0 \sin k(d - y) / \sin kd$. If the TE_{10} mode is the only propagating mode in the waveguide, compute the input resistance seen at the probe terminals.



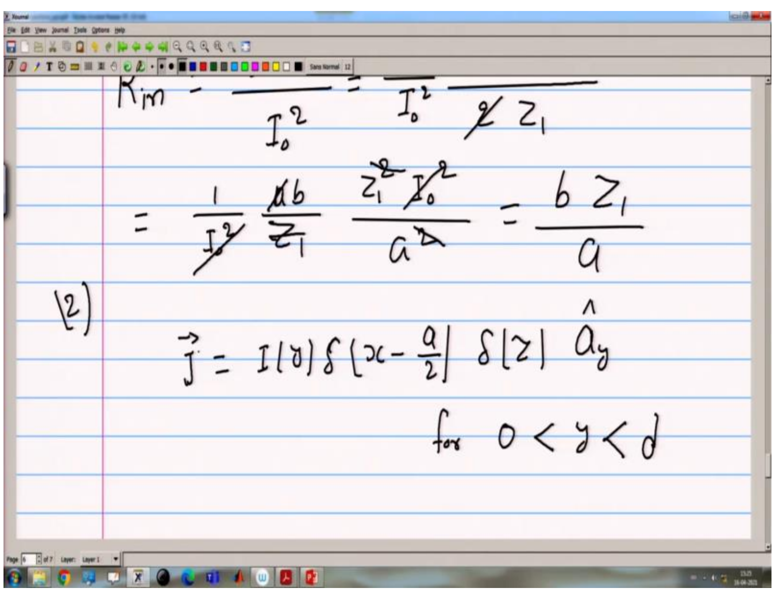
(Refer Slide Time: 14:41)

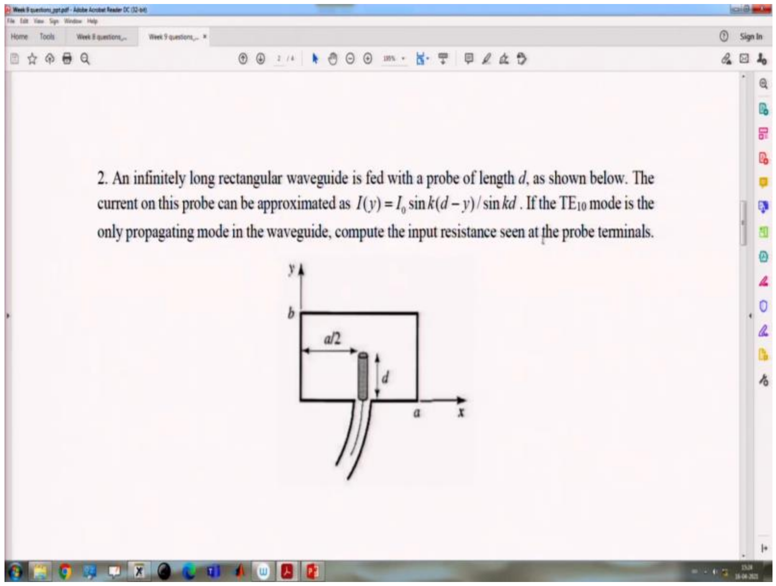




So, different from the means last problem is that in the last problem, it was touching at y is equal to b. But, here it is, it is not touching that y is equal to b; and it is finite in length. It is finite means it is just in the middle of up to d length; so, length of the probe is d, so, we can do the solution for this.

(Refer Slide Time: 16:15)





2. An infinitely long rectangular waveguide is fed with a probe of length d, as shown below. The current on this probe can be approximated as $I(y) = I_0 \sin k(d-y)/\sin kd$. If the TE_{10} mode is the only propagating mode in the waveguide, compute the input resistance seen at the probe terminals.

So, for this case, means J is given like surface current is approximated as

(2)

$$\vec{J} = I(y)\,\delta\left(x - \frac{a}{2}\right)\delta(z)\,\hat{a}_y \quad \text{for } 0 < y < d$$

$$\vec{e}_1 = \hat{a}_y \sin\left[\frac{\pi x}{a}\right],$$

$$\vec{h}_1 = \frac{-\hat{a}_x}{Z_1}\sin\left[\frac{\pi x}{a}\right], \quad Z_1 = \frac{k_0 \eta_0}{\beta_1}$$

(Refer Slide Time: 17:39)

$$\text{for } 0 < y < d$$

$$\vec{e}_1 = \hat{a}_y \sin\left[\frac{\pi x}{a}\right],$$

$$\vec{h}_1 = \frac{-\hat{a}_x}{Z_1} \sin\left[\frac{\pi x}{a}\right], \quad Z_1 = \frac{k_0 \eta_0}{\beta_1}$$

Here z_1 is the wave impedance. So, again we have to first calculate the normalization constant of the for the TE_{10} mode.

For TE_{10},

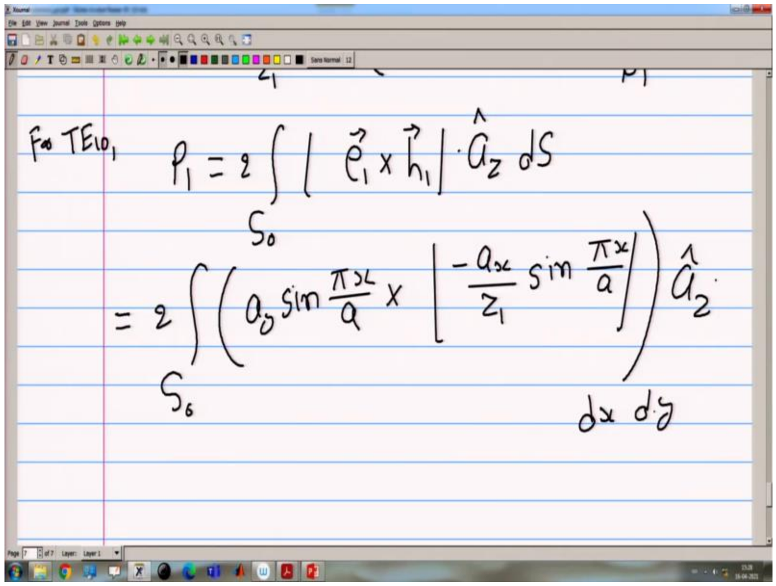
$$P_1 = 2\int_{S_0} \left[\vec{e}_1 \times \vec{h}_1\right] \cdot \hat{a}_z \, dS$$

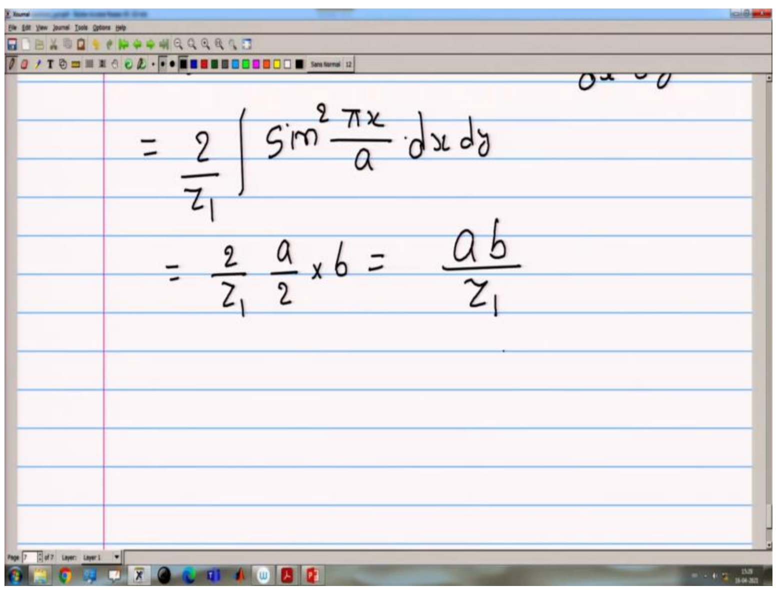
$$= 2\int_{S_0} \left(a_y \sin\frac{\pi x}{a} \times \left[\frac{-a_x}{Z_1} \sin\frac{\pi x}{a}\right]\right) \hat{a}_z \, dx\, dy$$

$$= \frac{2}{Z_1} \int \sin^2\frac{\pi x}{a} \, dx\, dy$$

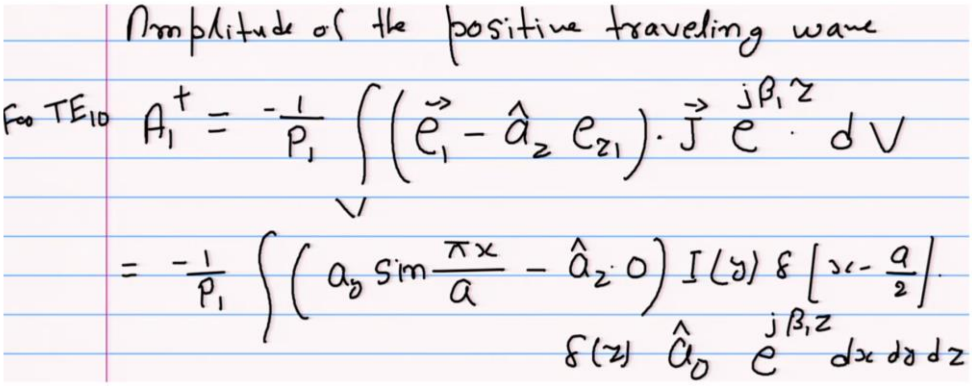
$$= \frac{2}{Z_1} \frac{a}{2} \times b = \frac{ab}{Z_1}$$

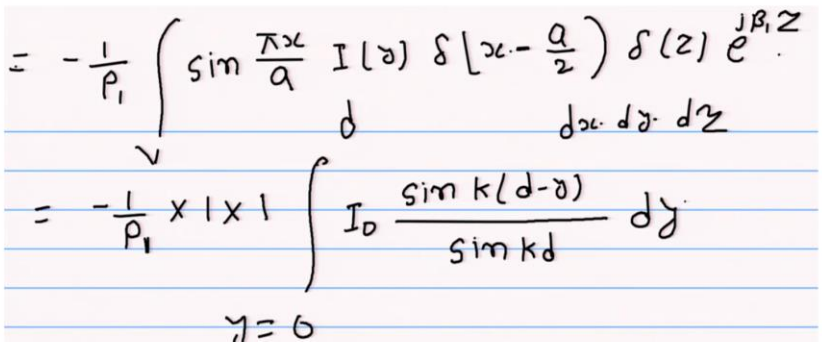
(Refer Slide Time: 18:36)



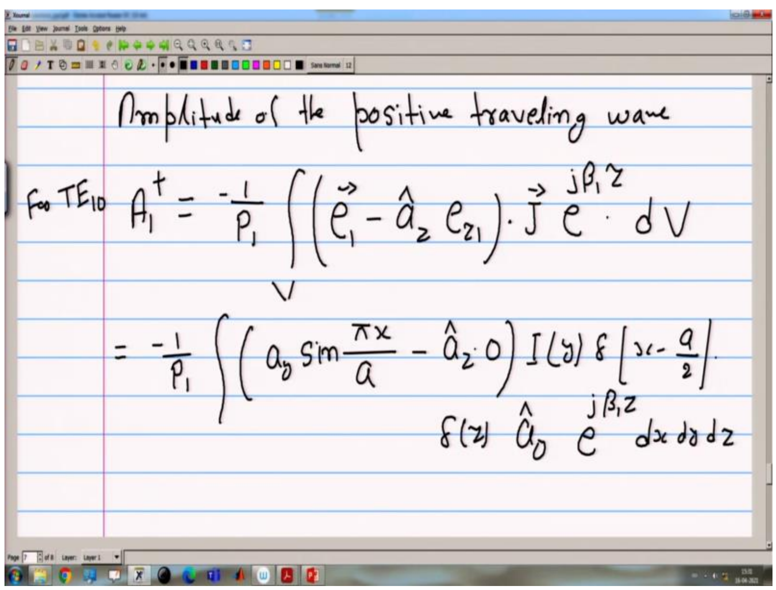


Now, we can calculate the amplitude of the positive traveling wave.





(Refer Slide Time: 20:59)



(Refer Slide Time: 23:30)

$$= -\frac{1}{P_1}\int_v \left(\sin\frac{\pi x}{a}\, I(y)\, \delta\left[x-\frac{a}{2}\right)\delta(z)\, e^{j\beta_1 z}\; dx\, dy\, dz\right.$$

$$= -\frac{1}{P_1}\times 1\times 1\int_{y=0}^{d} I_0\,\frac{\sin k(d-y)}{\sin kd}\, dy$$

Again, let

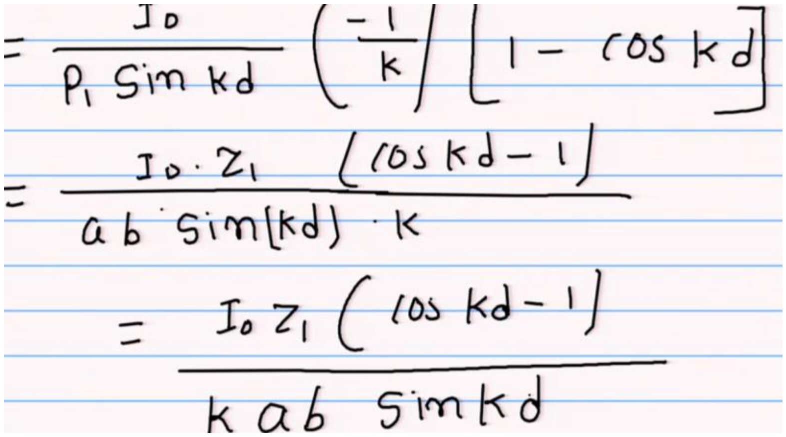
let $w = d-y$ $\quad dw = -dy$

$y=0,\ w=d, \quad y=d,\ w=0$

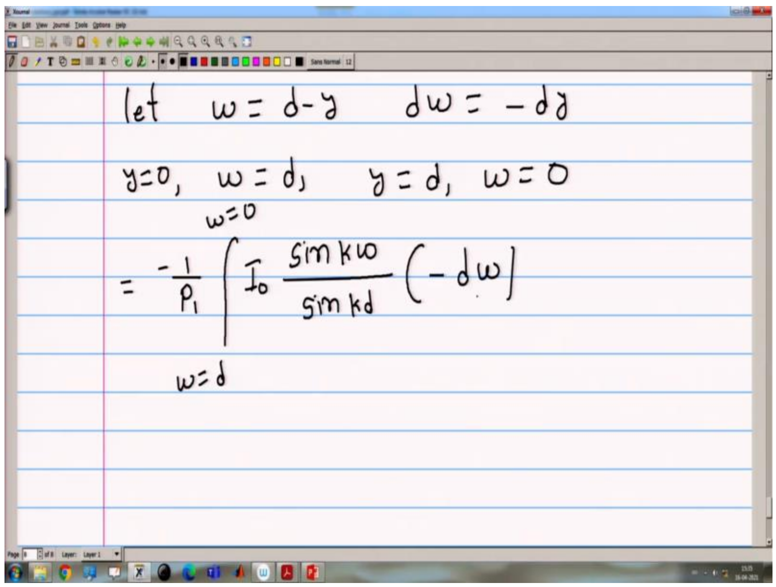
$$= -\frac{1}{P_1}\int_{w=d}^{w=0} I_0\,\frac{\sin kw}{\sin kd}\,(-dw)$$

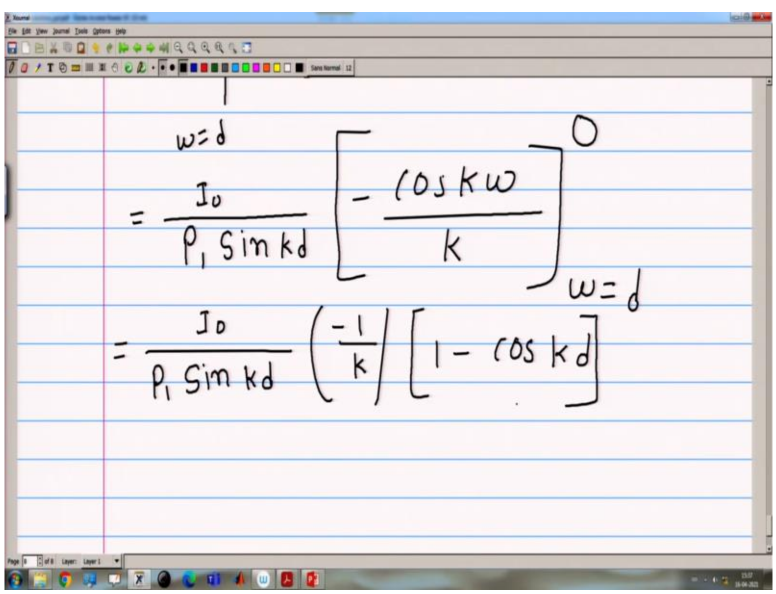
$$= \frac{I_0}{P_1 \sin kd}\left[-\frac{\cos kw}{k}\right]_{w=d}^{0}$$

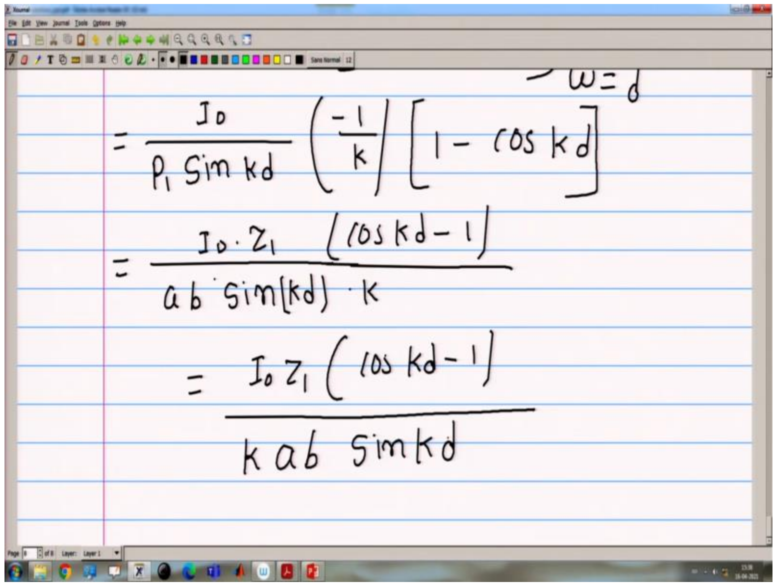
$$= \frac{I_0}{P_1 \sin kd}\left(\frac{-1}{k}\right)\left[1-\cos kd\right]$$



(Refer Slide Time: 26:05)

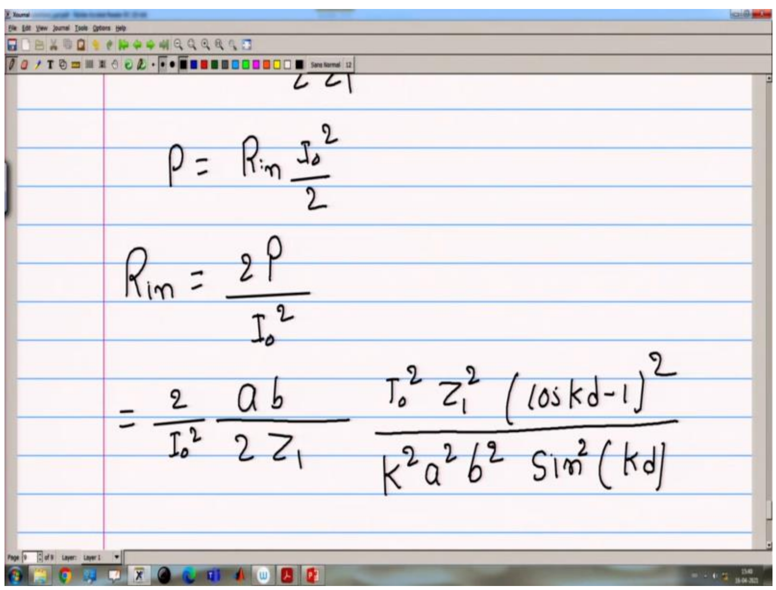


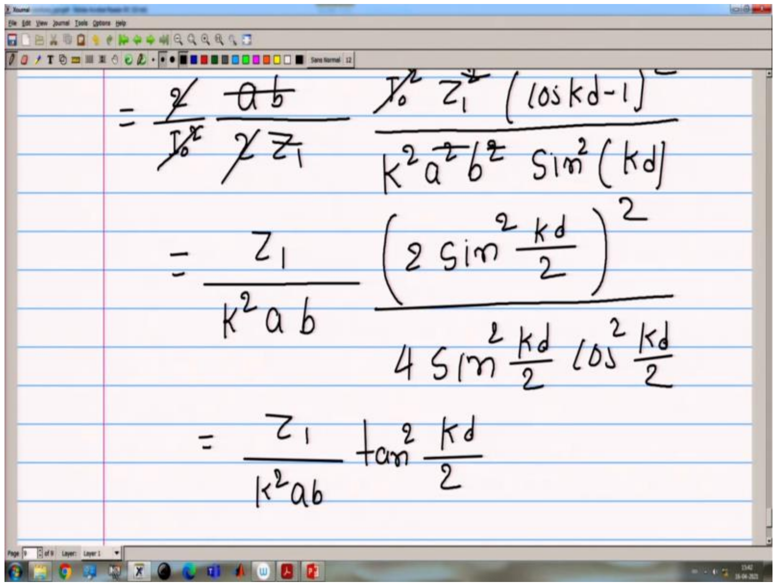


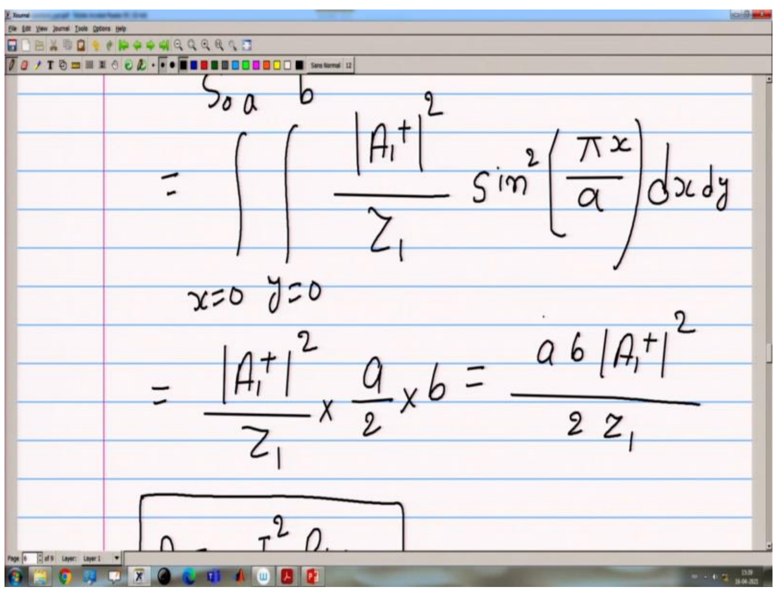


Because this is a symmetric means in the forward direction and in the backward direction; this is symmetrical. Due to that, the amplitude in the forward direction and amplitude in the backward direction, both will be same.

(Refer Slide Time: 30:25)







so we can mention total power flow will be

$$P = R_{in}\frac{I_0^2}{2}$$

$$R_{in} = \frac{2P}{I_0^2}$$

$$= \frac{2}{I_0^2}\,\frac{ab}{2Z_1}\,\frac{I_0^2 Z_1^2 (\cos kd - 1)^2}{k^2a^2b^2 \sin^2(kd)}$$

So, this is the expression to calculate the input resistance;

$$= \frac{2}{I_0^2}\,\frac{ab}{2Z_1}\,\frac{I_0^2 Z_1^2 (\cos kd - 1)^2}{k^2a^2b^2 \sin^2(kd)}$$

$$= \frac{Z_1}{k^2ab}\,\frac{\left(2\sin^2\frac{kd}{2}\right)^2}{4\sin^2\frac{kd}{2}\cos^2\frac{kd}{2}}$$

$$= \frac{Z_1}{k^2ab}\tan^2\frac{kd}{2}$$

So, this is the input resistance for the finite of length d probe, means for the finite probe distance d that length d. Thank you.