Advanced Microwave Guided-Structure and Analysis

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Lecture – 44

The Reciprocity Theorem, Computation of Amplitudes Tutorials (Contd.)

Welcome to the next tutorial class. So, we are doing calculation of amplitude in the rectangular waveguide in the forward and backward direction; and also calculating the input resistance. So, we have already completed this problem. Now, we can go to the next problem.

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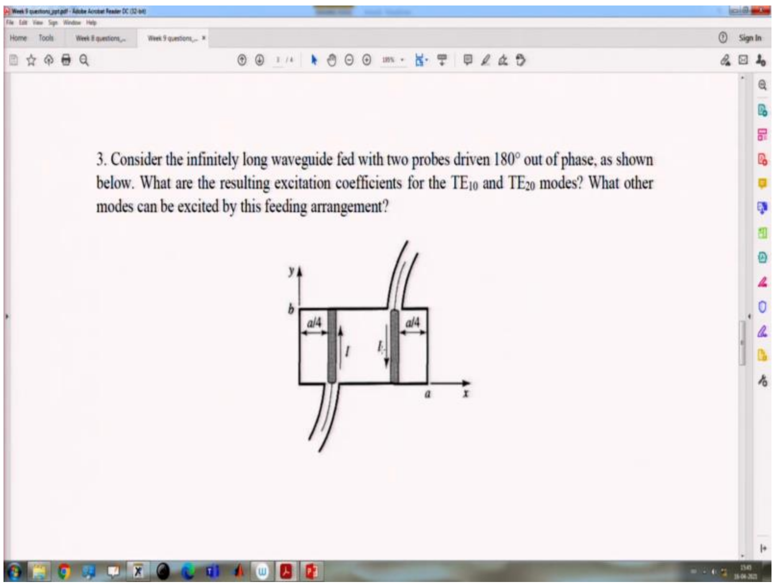

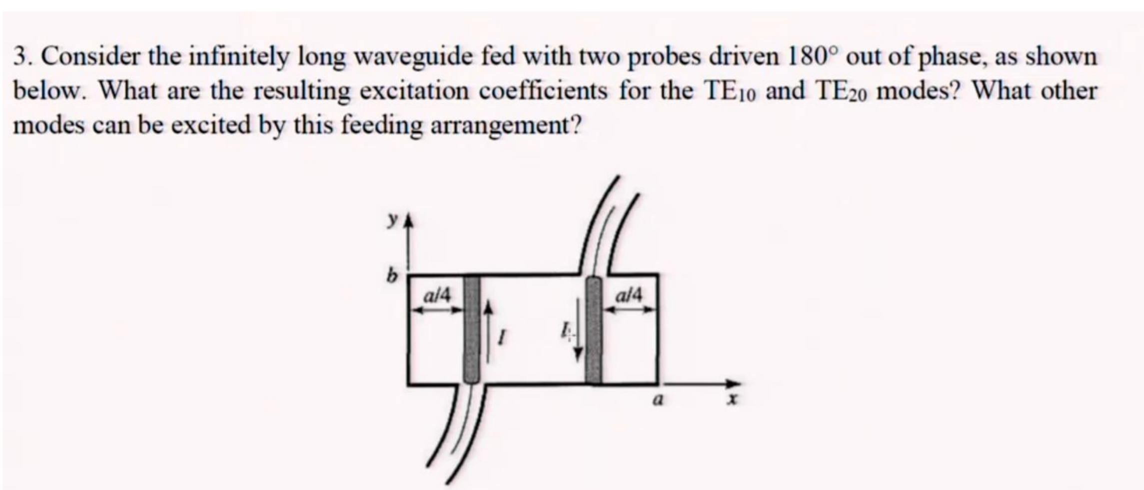

So, for this, we will calculate first the resulting excitation coefficient for the TE_{10} and TE_{20}. So, here two probes that is in the 180 degree out of phase and one is at x is equal to a/4; and other distance is from the left right side, it is from the a distance that is a/4. That means at x is equal to 3a/4; so this is a third problem.

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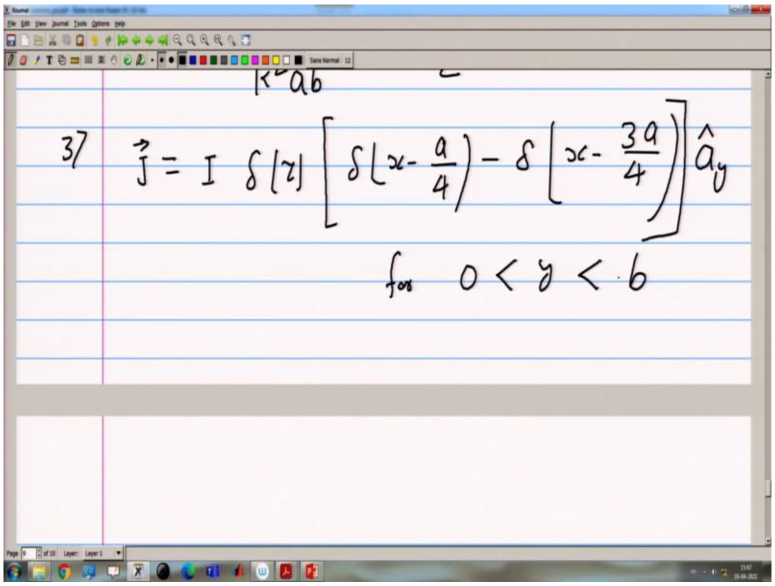

So, expression for that surface current density that can be written as J equal to

$$\vec{J} = I\,\delta(z)\left[\delta\left(x-\frac{a}{4}\right)-\delta\left(x-\frac{3a}{4}\right)\right]\hat{a}_y$$
$$\text{for } 0 < y < b$$

So, it is directed along a_y and this is valid between 0 to b.

So, now model field means that normalized field for TE_{10} mode.

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$$TE_{10} \quad \vec{e}_1 = \hat{a}_y \sin\frac{\pi x}{a}, \quad \vec{h}_1 = -\frac{a_x}{Z_1}\sin\frac{\pi x}{a}, \quad Z_1 = \frac{k_0\eta_0}{\beta_1}$$

$$TE_{20} \quad \vec{e}_2 = \hat{a}_y \sin\frac{2\pi x}{a}, \quad \vec{h}_2 = -\frac{a_x}{Z_2}\sin\frac{2\pi x}{a}, \quad Z_2 = \frac{k_0\eta_0}{\beta_2}$$

$$TE_{10}, \quad P_1 = 2\int_{S_0}\left(\vec{e}_1 \times \vec{h}_1\right]\hat{a}_z\, dS$$

$$TE_{10}, \quad P_1 = 2\int_{S_0}\left(\vec{e}_1 \times \vec{h}_1\right] a_z\, dS$$

$$= 2\int_{S_0}\left[a_y \sin\frac{\pi x}{a} \times \left[\frac{-a_x}{Z_1}\right]\sin\frac{\pi x}{a}\right]\hat{a}_z \cdot dx\, dy$$

$$= 2\cdot\frac{a}{2}\cdot\frac{1}{Z_1}\, b = \frac{ab}{Z_1}$$

For TE_{10} it can be written as,

$$TE_{10} \quad \vec{e}_1 = \hat{a}_y \sin\frac{\pi x}{a}, \quad \vec{h}_1 = -\frac{a_x}{Z_1}\sin\frac{\pi x}{a}, \quad Z_1 = \frac{k_0\eta_0}{\beta_1}$$

$$TE_{20} \quad \vec{e}_2 = \hat{a}_y \sin\frac{2\pi x}{a}, \quad \vec{h}_2 = -\frac{a_x}{Z_2}\sin\frac{2\pi x}{a}, \quad Z_2 = \frac{k_0\eta_0}{\beta_2}$$

And after this we have to calculate the normalization constant that P1 and P2.

$$TE_{10}, \quad P_1 = 2\int_{S_0} \left(\vec{e}_1 \times h_1 \right) a_z \, dS$$

$$= 2\int_{S_0} \left[a_y \sin\frac{\pi x}{a} \times \left(\frac{-a_x}{Z_1} \right) \sin\frac{\pi x}{a} \right] \cdot \hat{a}_z \, dx\, dy$$

$$= 2 \cdot \frac{a}{2} \cdot \frac{1}{Z_1} b = \frac{ab}{Z_1}$$

This is the P1 for that TE_{10} mode; for the TE_{20} mode it will be

$$TE_{20} \quad P_2 = 2\int \left(e_2 \times h_2 \right) a_z \, dS$$

$$= 2\int_{x=0}^{a}\int_{y=0}^{b} \sin^2\frac{2\pi x}{a} \cdot \frac{1}{Z_2} \cdot dx\, dy$$

$$= 2 \times \frac{1}{Z_2} \times \frac{a}{2} b = \frac{ab}{Z_2}$$

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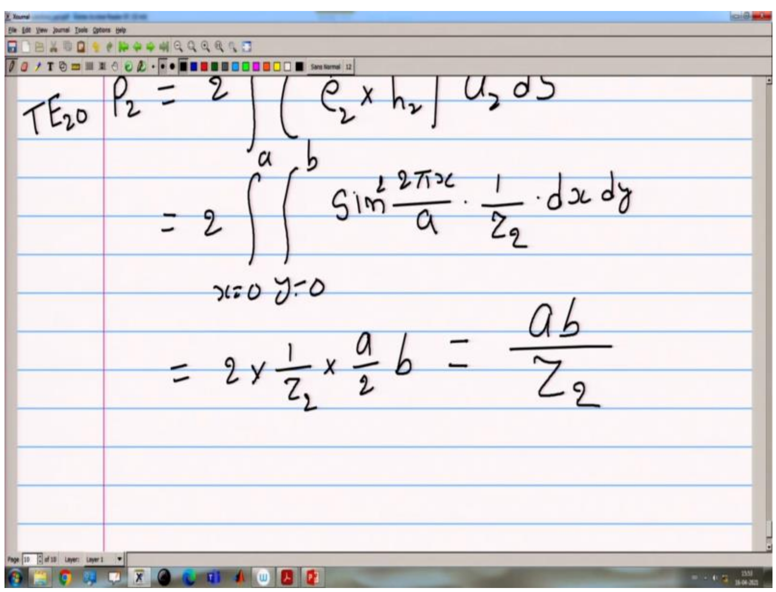

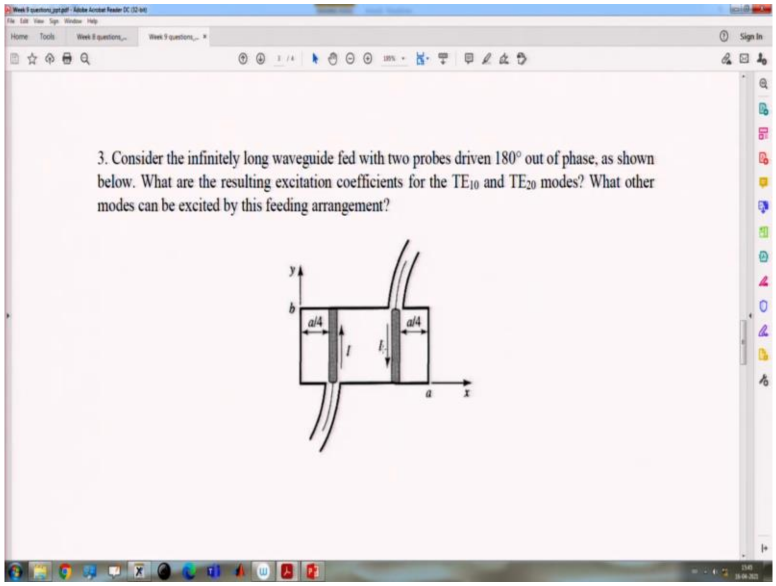

So, these are the normalization factor, and after this we have to calculate the amplitude.

For TE_{10}

$$A_1^+ = -\frac{1}{P_1}\int_V \left[\vec{e}_1 - \hat{a}_z e_{z1}\right] \cdot \vec{J}\, e^{j\beta_1 z}\, dV$$

$$= -\frac{1}{P_1}\int_V a_y \sin\frac{\pi x}{a}\, I\,\delta(z)\left[\delta\left(x-\frac{a}{4}\right) - \delta\left(x-\frac{3a}{4}\right)\right]\hat{a}_y\, e^{j\beta_1 z}\cdot dx\,dy\,dz$$

$$= -\frac{I}{P_1}\left[\sin\frac{\pi}{4} - \sin\frac{3\pi}{4}\right]\cdot 1\cdot b$$

$$= -\frac{bI}{P_1}\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right] = 0$$

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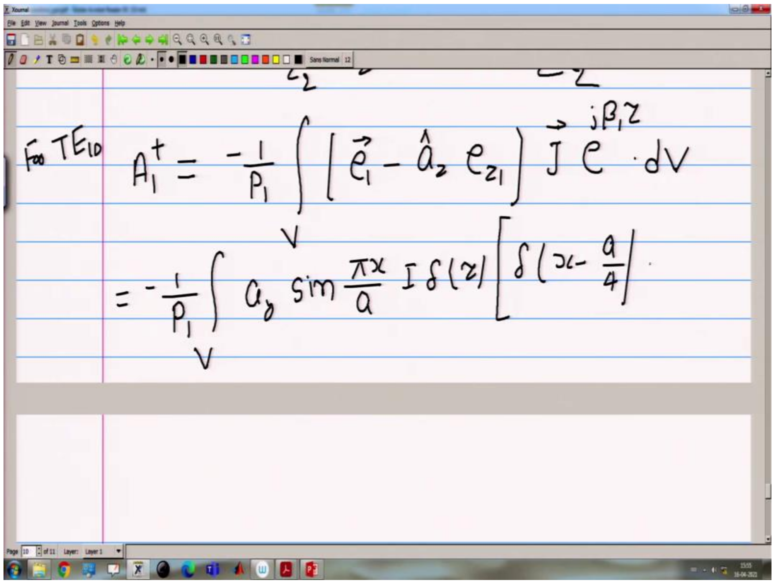

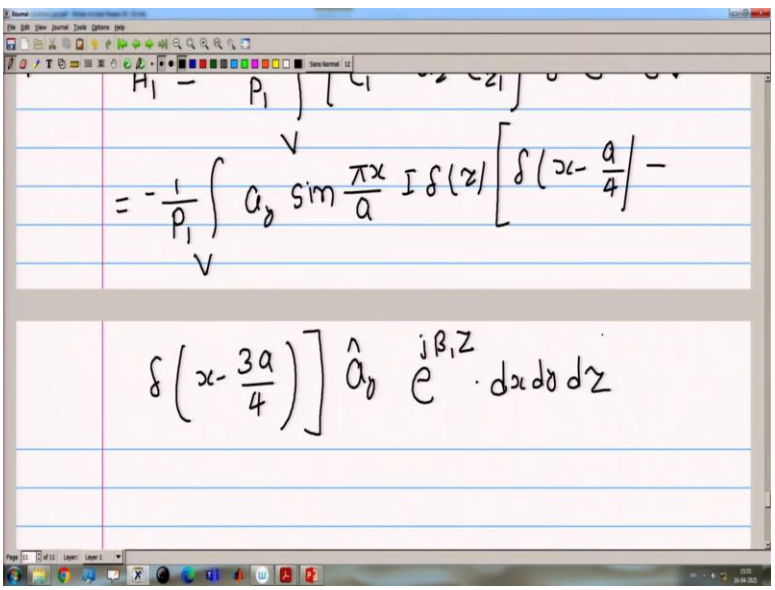

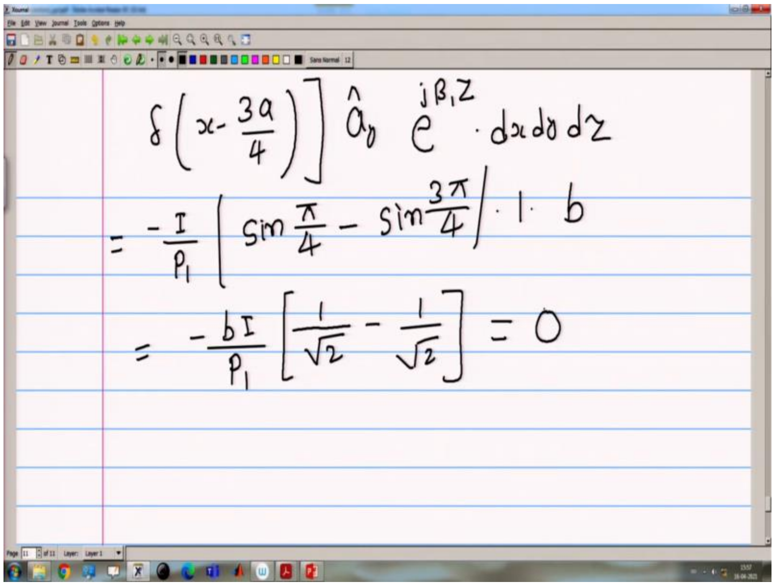

And now similarly we can calculate the A_2 plus for the TE_{20}.

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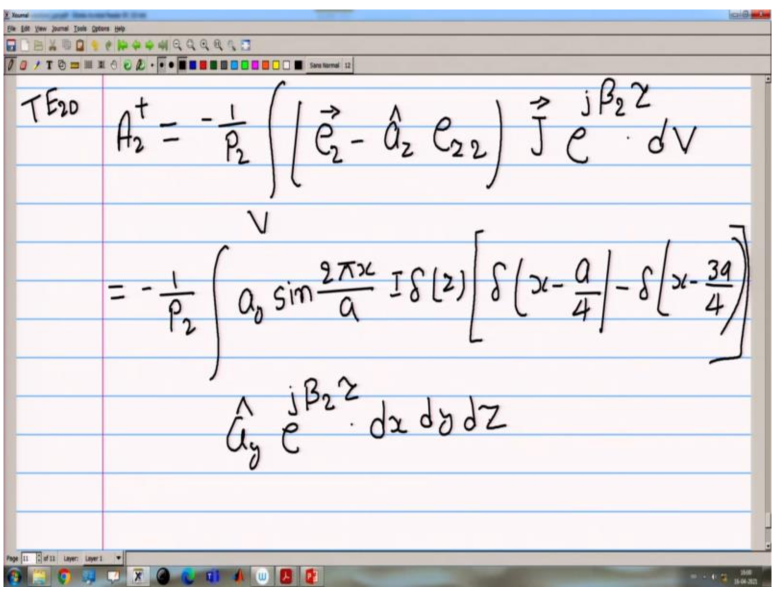

So, for TE_{20} that will be

TE_{20}

$$A_2^+ = -\frac{1}{P_2} \int_V \left(\vec{e}_2 - \hat{a}_z e_{z2} \right) \vec{J} e^{j\beta_2 z} \cdot dV$$

$$= -\frac{1}{P_2} \int a_0 \sin\frac{2\pi x}{a} \, I \delta(z) \left[\delta\left(x - \frac{a}{4}\right) - \delta\left(x - \frac{3a}{4}\right) \right] \hat{a}_y \, e^{j\beta_2 z} \cdot dx \, dy \, dz$$

$$= \frac{-I}{P_2} \left(\sin\frac{\pi}{2} - \sin\frac{3\pi}{2} \right) \cdot 1 \cdot b$$

$$= -\frac{I}{P_2} \left(1 - (-1) \right) b = \frac{-2Ib}{P_2}$$

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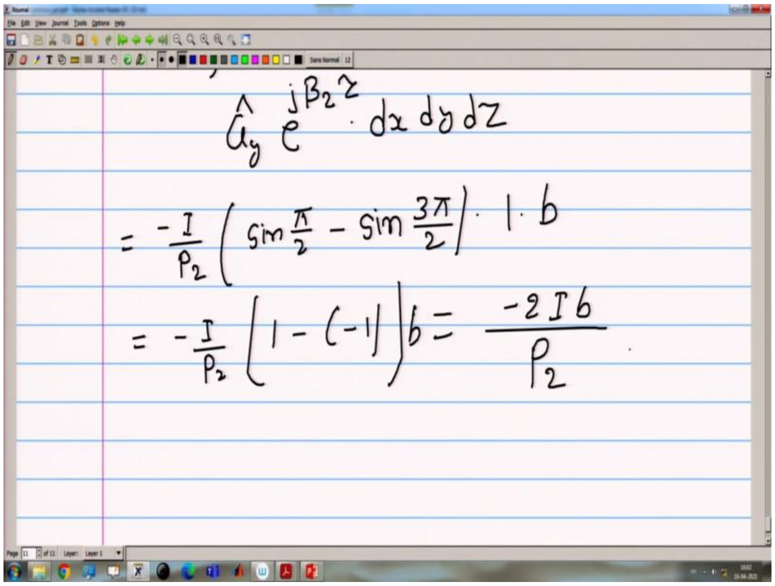

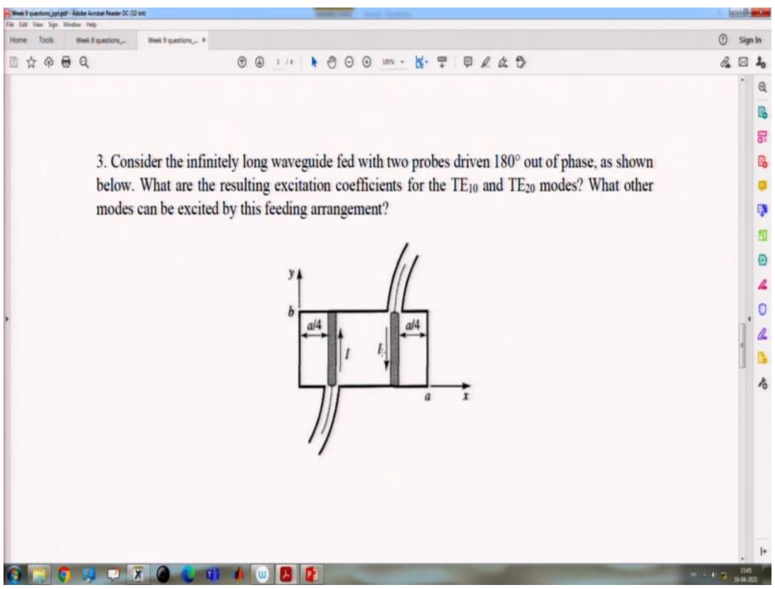

So, we can see that since the excitation has an odd symmetric about the center of the probe. So, it will only excite that mode that have electric field, with an odd symmetry about x is equal to a/2. So, in this case we can see that TE_{20} mode is excited that has finite amplitude. But, when means for that TE_{10} case, that has the amplitude 0; so, that we have to keep in the mind.

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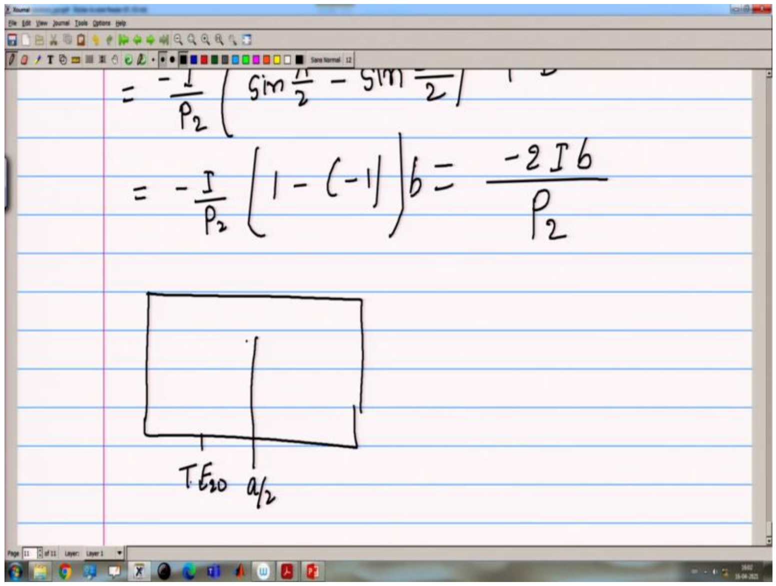

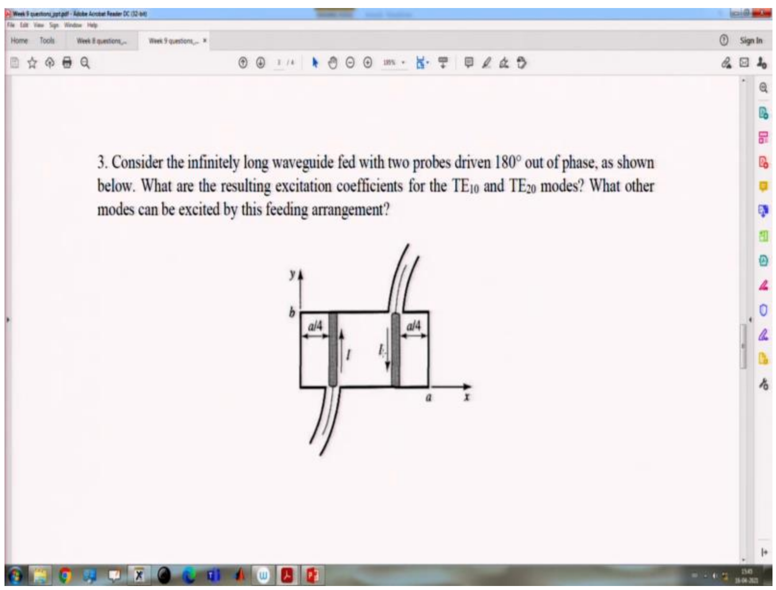

So, for that TE_{10} mode excitation when feeding at the center, then it has the finite amplitude of the forward and backward. But for this case when it is at a/2, when it will at a/4, for that case it will excite on the TE_{20} mode. So, next question we can do. So, what other modes can be excited by this feeding arrangement? So, that will be the odd asymmetric what x is equal to a/2; that will be only excited.

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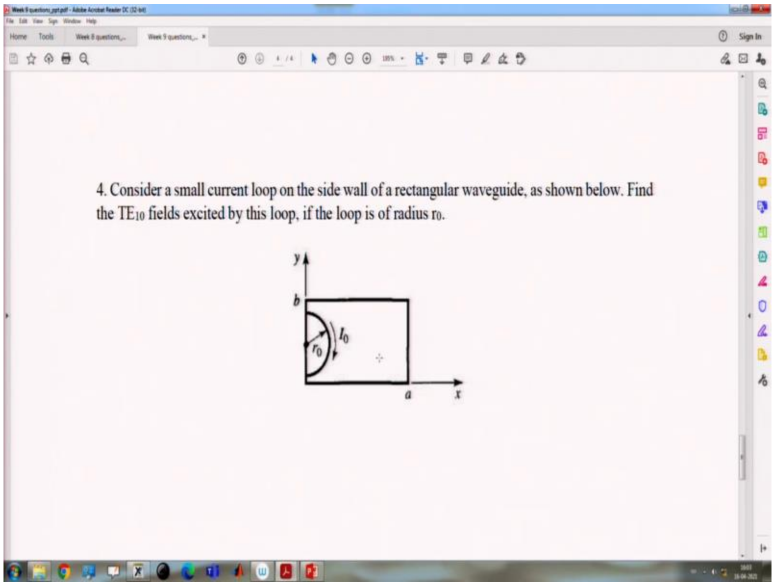
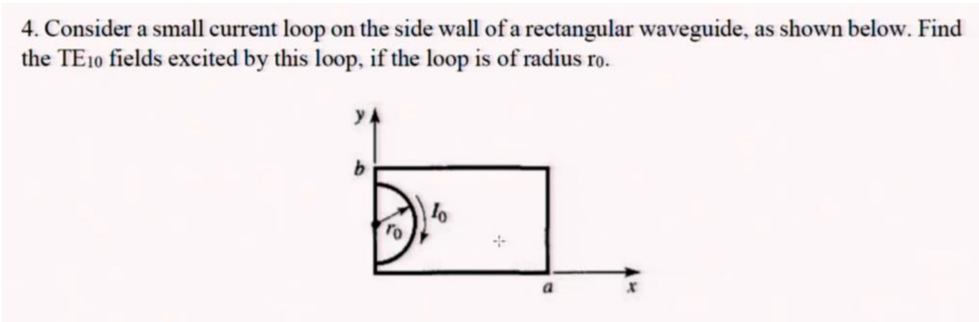


So, next question is



And by removing side wall, it will become full loop; and for the full loop we can write the magnetic equivalent source. So, that can be written as so Pm, and Pm is the infinitesimal magnetic polarization current. So, for the loop case that will excite if it is electric loop so, that is current I_0.

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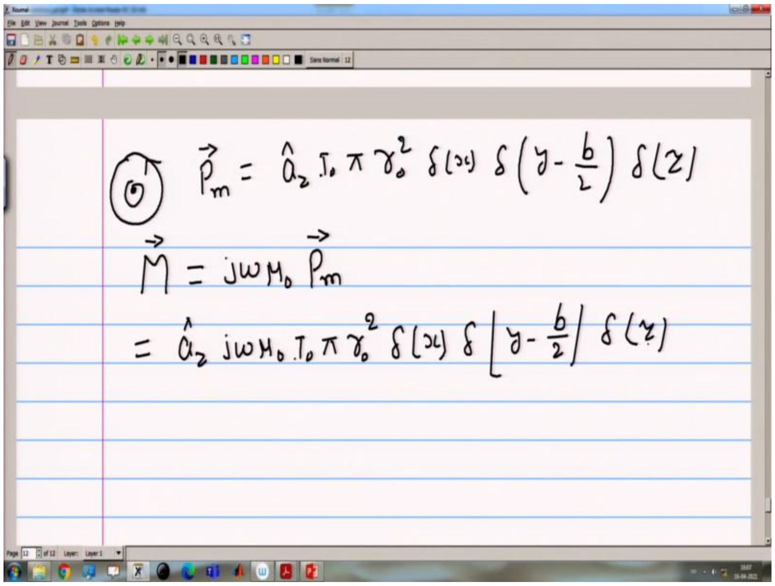

So, if it is loop, so magnetic dipole that will be out of the loop. It is out of the loop, so that will be the magnetic equivalent for this loop. For this case, Pm can be written as

$$\vec{P}_m = \hat{a}_z I_0 \pi r_0^2 \delta(x) \delta\left(y - \frac{b}{2}\right) \delta(z)$$

$$\vec{M} = j\omega\mu_0 \vec{P}_m$$

$$= \hat{a}_z j\omega\mu_0 I_0 \pi r_0^2 \delta(x) \delta\left[y - \frac{b}{2}\right] \delta(z)$$

So, like this Pm that magnetic polarization current can be written; and by equivalent magnetic source will be equivalent magnetic source. As we know that J is the electric source, but M is the magnetic source.

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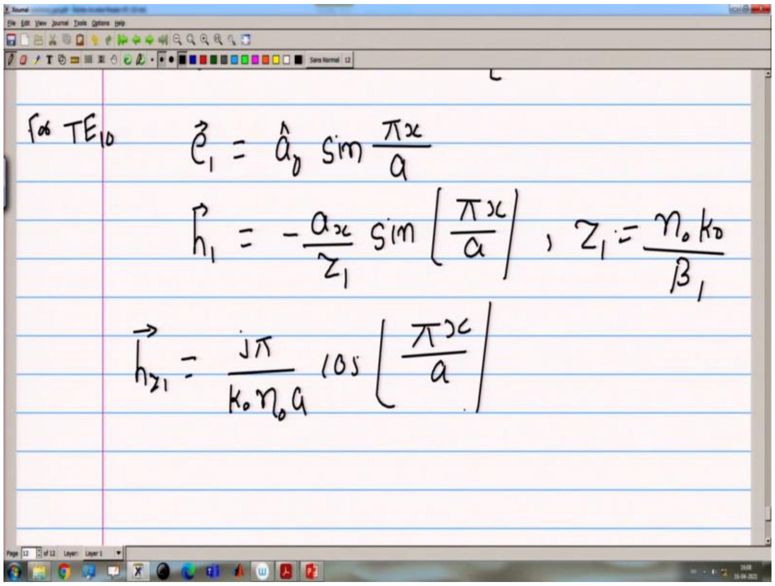
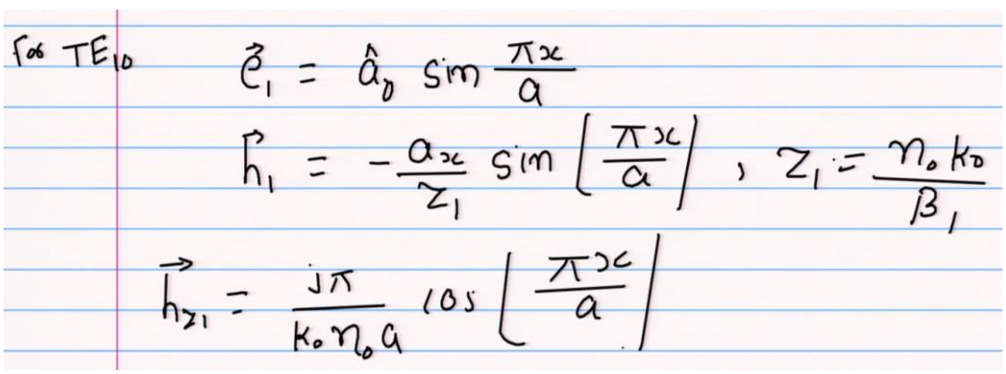


So, for the TE10 mode,



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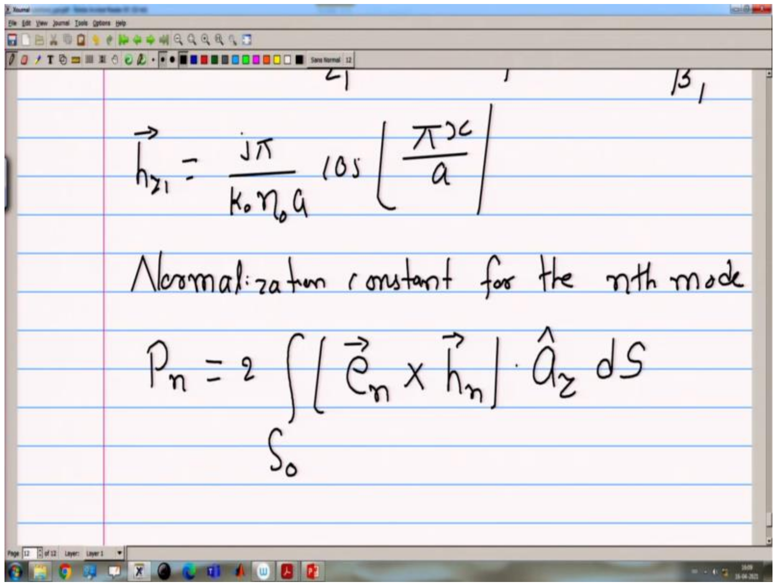
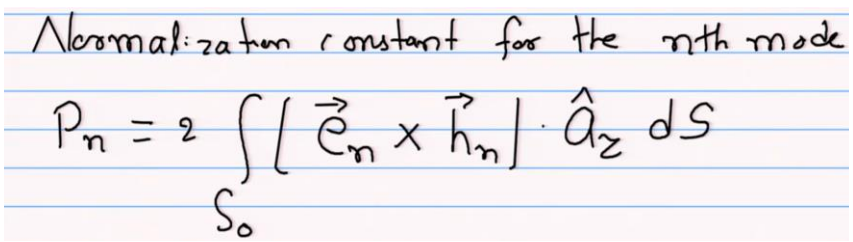


Now, normalization constant for the nth mode, for the for the nth mode.



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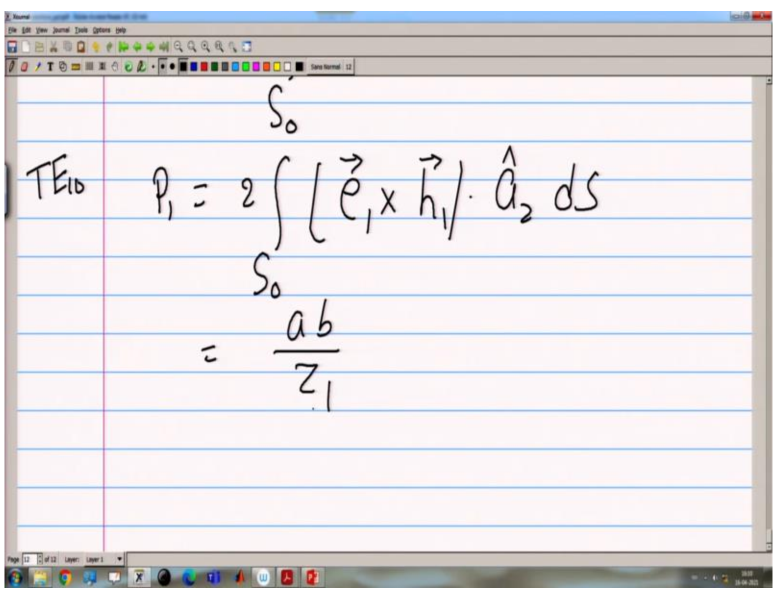

$$\vec{M} = j\omega\mu_0 I_m$$

$$= \hat{a}_z \, j\omega\mu_0 I_0 \pi r_0^2 \, \delta(x)\, \delta\left[y - \frac{b}{2}\right] \delta(z)$$

For TE_{10}

$$\vec{e}_1 = \hat{a}_y \sin\frac{\pi x}{a}$$

$$\vec{h}_1 = -\frac{a_x}{Z_1}\sin\left[\frac{\pi x}{a}\right], \quad Z_1 = \frac{\eta_0 k_0}{\beta_1}$$

$$\vec{h}_{z1} = \frac{j\pi}{k_0 \eta_0 a}\cos\left[\frac{\pi x}{a}\right]$$

So, for TE10 this will be P1,

TE_{10}

$$P_1 = 2\int_{S_0} \left(\vec{e}_1 \times \vec{h}_1\right)\cdot \hat{a}_z \, dS$$

$$= \frac{ab}{Z_1}$$

Now, for the amplitude of the positive travelling wave, that will be A1 plus for the TE_{10} mode; so that we can write and for the nth mode. So, for the nth mode we know

For nth

$$A_n^+ = \frac{1}{P_n}\int_V \left(-\vec{h}_n + \hat{a}_z h_{zn}\right)\cdot \vec{M} e^{j\beta_n z} \cdot dV$$

$$A_1^+ = \frac{1}{P_1}\int_V \left[-h_1 + \hat{a}_z h_{z1}\right] \vec{M} e^{j\beta_1 z} \cdot dV$$

$$= -\frac{1}{P_1}\int_V \left(+\frac{\hat{a}_x}{Z_1}\sin\frac{\pi x}{a} + \hat{a}_z \frac{j\pi}{k_0\eta_0 a}\cos\frac{\pi x}{a}\right)$$

$$\hat{a}_z \, j\omega\mu_0 I_0 \pi r_0^2 \, \delta(x)\, \delta\left[y - \frac{b}{2}\right) \delta(z)$$

$$= \frac{1}{P_1} \frac{j\pi}{k_0 \eta_0 a} 1 \cdot j\omega\mu_0 I_0 \pi r_0^2$$

$$= \frac{-\pi^2 \omega \mu_0 I_0 r_0^2 \cdot z_1}{k_0 \eta_0 a \; ab}$$

$$= \frac{-\pi^2 \cancel{\omega} \cancel{\mu_0} I_0 r_0^2 z_1 \cancel{\sqrt{\epsilon_0}}}{\cancel{\omega} \sqrt{\cancel{\mu_0} \cancel{\epsilon_0}} \sqrt{\cancel{\mu_0}} a^2 b}$$

$$A_1^+ = \frac{-\pi^2 z_1 I_0^2 r_0^2}{a^2 b}$$

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For nth $A_n^+ = \frac{1}{P_n} \int_V \left(-\vec{h}_n + \hat{a}_z h_{zn}\right) \cdot \vec{M} e^{j\beta_n z} \cdot dV$

$$A_1^+ = \frac{1}{P_1} \int_V \left[-h_1 + \hat{a}_z h_{z1}\right] \vec{M} e^{j\beta_1 z} \cdot dV$$

$$= -\frac{1}{P_1}\int_V \left(+\frac{\hat{a}_x}{Z_1}\sin\frac{\pi x}{a} + \hat{a}_z \frac{j\pi}{k_0\eta_0 a}\cos\frac{\pi x}{a}\right)$$

$$\hat{a}_z\, j\omega\mu_0 I_0 \pi r_0^2\, \delta(x)\,\delta\left(y-\frac{b}{2}\right)\delta(z)$$

$$\hat{a}_z\, j\omega\mu_0 I_0 \pi r_0^2\, \delta(x)\,\delta\left(y-\frac{b}{2}\right)\delta(z)$$

$$= \frac{1}{P_1}\frac{j\pi}{k_0\eta_0 a}\, 1\cdot j\omega\mu_0 I_0 \pi r_0^2$$

$$= \frac{-\pi^2\omega\mu_0 I_0 r_0^2 \cdot Z_1}{k_0\eta_0 a \; ab}$$

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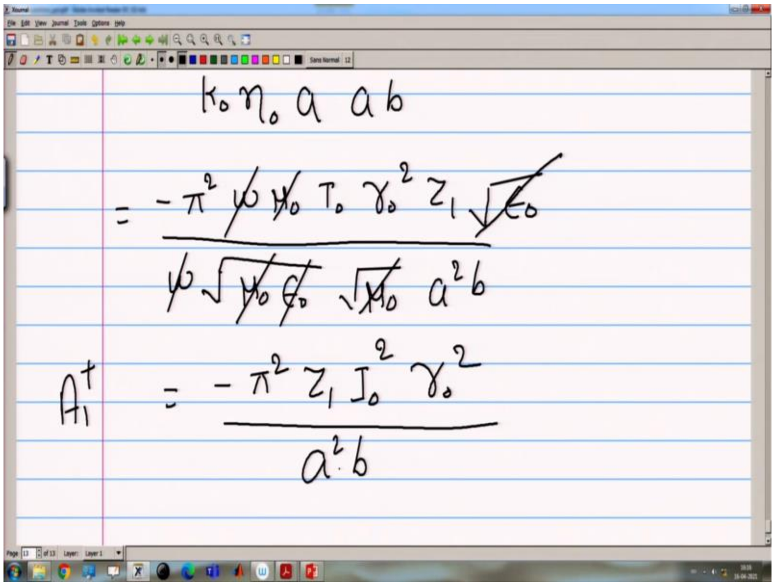

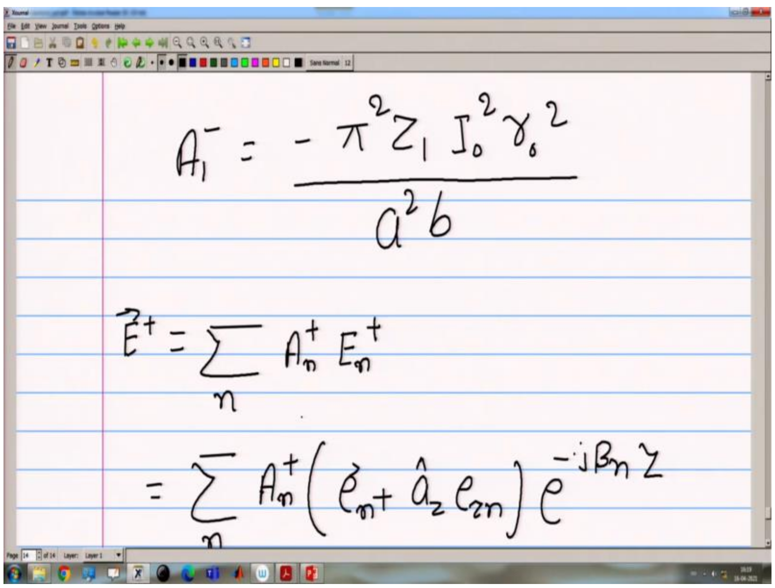

So, this is the amplitude for the positive travelling wave.

Similarly, for the negative travelling wave amplitude will be for A1 minus that we can calculate

$$A_1^- = \frac{-\pi^2 z_1 I_0^2 \gamma_0^2}{a^2 b}$$

And because this will be symmetric in both the direction; so it will give the same amplitude in the negative travelling wave also. For this case also if you will do the calculation.

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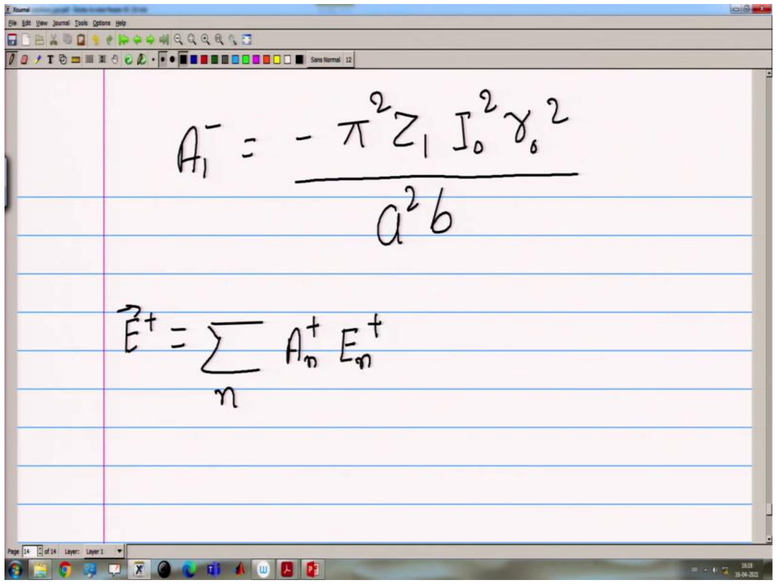

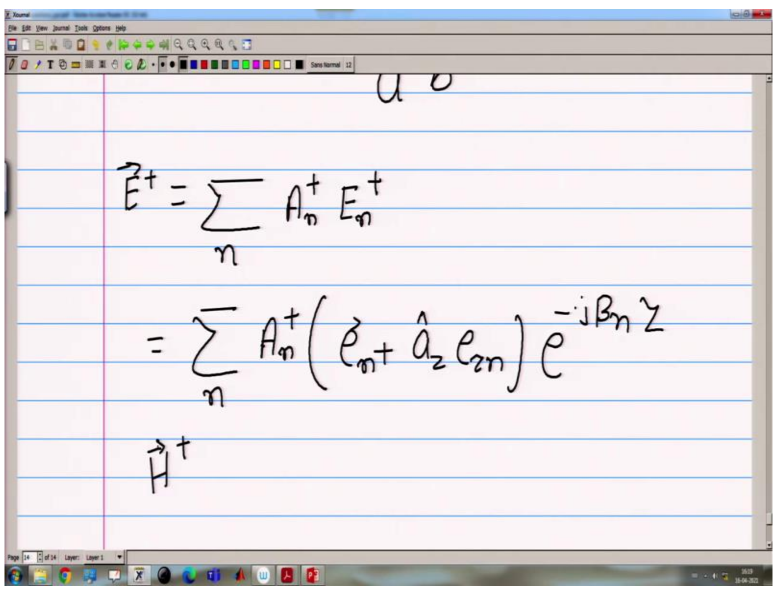

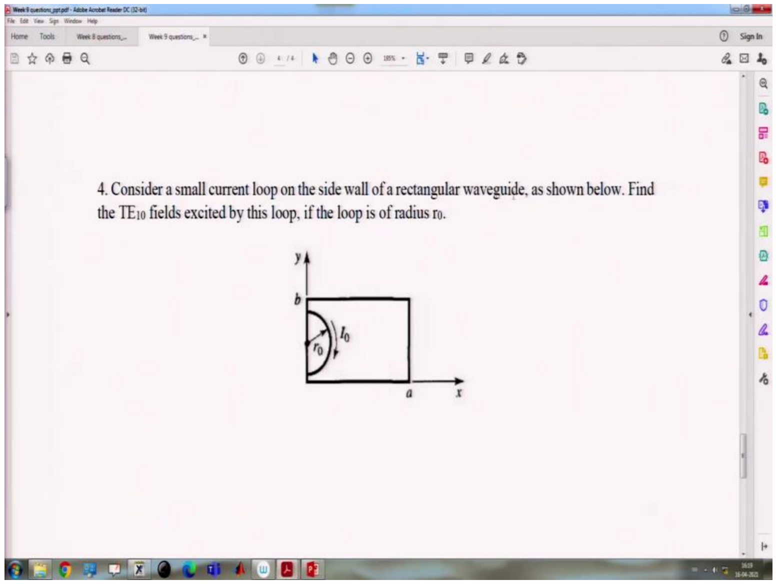

So, we know model field for the positive means how to write that fields in the positive and negative direction. So, that expression is E plus that it can be written as a summation of all the modes inside the waveguide;

$$\vec{E}^{+} = \sum_{n} A_n^{+} E_n^{+}$$

$$= \sum_{n} A_n^{+} \left(\vec{e}_n + \hat{a}_z e_{zn} \right) e^{-j\beta_n z}$$

So, like this in the similar way for the magnetic field, it can be vector H can be written. So, find that TE_{10} field excited by the loop; so, these are the amplitude. So, from this amplitude we can find out the fields; so, these are the A1 plus and A1 minus, so both are equal. So, like this amplitude can be calculated. Thank you.