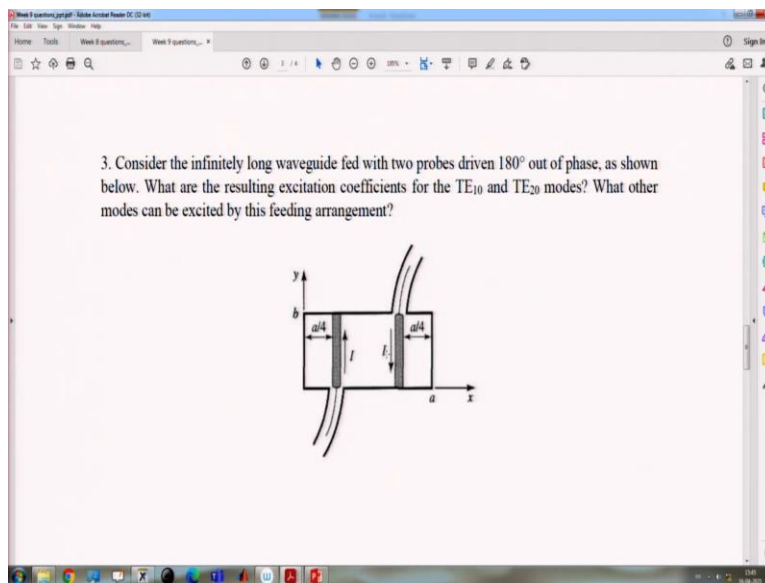


Advanced Microwave Guided-Structure and Analysis
Professor Bratin Ghosh
Department of Electronic & Electronic Communication Engineering
Indian Institute of Technology, Kharagpur
Lecture – 44

The Reciprocity Theorem, Computation of Amplitudes Tutorials (Contd.)

Welcome to the next tutorial class. So, we are doing calculation of amplitude in the rectangular waveguide in the forward and backward direction; and also calculating the input resistance. So, we have already completed this problem. Now, we can go to the next problem.

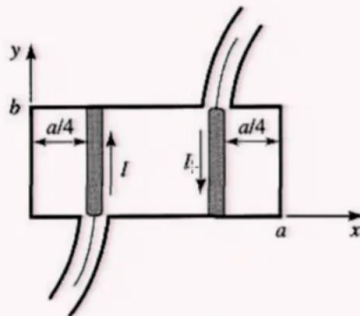
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3. Consider the infinitely long waveguide fed with two probes driven 180° out of phase, as shown below. What are the resulting excitation coefficients for the TE_{10} and TE_{20} modes? What other modes can be excited by this feeding arrangement?

The diagram shows a rectangular waveguide with width a and height b . Two probes are inserted into the waveguide, each of length $a/4$. The probes are driven 180° out of phase, with currents I and $-I$ flowing in opposite directions. The coordinate system (x, y) is shown with the x -axis along the length of the waveguide and the y -axis along the height.

3. Consider the infinitely long waveguide fed with two probes driven 180° out of phase, as shown below. What are the resulting excitation coefficients for the TE_{10} and TE_{20} modes? What other modes can be excited by this feeding arrangement?



So, for this, we will calculate first the resulting excitation coefficient for the TE₁₀ and TE₂₀. So, here two probes that is in the 180 degree out of phase and one is at x is equal to a/4; and other distance is from the left right side, it is from the a distance that is a/4. That means at x is equal to 3a/4; so this is a third problem.

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A screenshot of a digital whiteboard showing a handwritten equation for surface current density \vec{J}_s . The equation is:
$$\vec{J}_s = I \delta(z) \left[\delta\left(x - \frac{a}{4}\right) - \delta\left(x - \frac{3a}{4}\right) \right] \hat{a}_y$$
 Below the equation, it is noted that this is valid for $0 < y < b$. The whiteboard interface includes a toolbar with various drawing tools and a Windows taskbar at the bottom.

So, expression for that surface current density that can be written as J equal to

A close-up of the handwritten equation from the previous slide:
$$\vec{J}_s = I \delta(z) \left[\delta\left(x - \frac{a}{4}\right) - \delta\left(x - \frac{3a}{4}\right) \right] \hat{a}_y$$
 Below the equation, it is noted that this is valid for $0 < y < b$.

So, it is directed along a_y and this is valid between 0 to b.

So, now model field means that normalized field for TE₁₀ mode.

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$$\begin{aligned} \text{TE}_{10} \quad \vec{E}_1 &= \hat{a}_y \sin \frac{\pi x}{a}, \quad \vec{h}_1 = -\frac{a_{zc}}{z_1} \sin \frac{\pi x}{a}, \quad z_1 = \frac{k_0 \eta_0}{\beta_1} \\ \text{TE}_{20} \quad \vec{E}_2 &= \hat{a}_y \sin \frac{2\pi x}{a}, \quad \vec{h}_2 = -\frac{a_{zc}}{z_2} \sin \frac{2\pi x}{a}, \quad z_2 = \frac{k_0 \eta_0}{\beta_2} \\ \text{TE}_{10}, P_1 &= 2 \int_{S_0} (\vec{E}_1 \times \vec{h}_1) \cdot \hat{a}_z \, dS \end{aligned}$$

$$\begin{aligned} \text{TE}_{10}, P_1 &= 2 \int_{S_0} (\vec{E}_1 \times \vec{h}_1) \cdot \hat{a}_z \, dS \\ &= 2 \int_{S_0} \left[\hat{a}_y \sin \frac{\pi x}{a} \times \left[\frac{-a_{zc}}{z_1} \sin \frac{\pi x}{a} \right] \right] \cdot \hat{a}_z \, dx \, dy \\ &= 2 \cdot \frac{a}{2} \cdot \frac{1}{z_1} \cdot b = \frac{ab}{z_1} \end{aligned}$$

For TE₁₀ it can be written as,

$$\begin{aligned} \text{TE}_{10} \quad \vec{E}_1 &= \hat{a}_y \sin \frac{\pi x}{a}, \quad \vec{h}_1 = -\frac{a_{zc}}{z_1} \sin \frac{\pi x}{a}, \quad z_1 = \frac{k_0 \eta_0}{\beta_1} \\ \text{TE}_{20} \quad \vec{E}_2 &= \hat{a}_y \sin \frac{2\pi x}{a}, \quad \vec{h}_2 = -\frac{a_{zc}}{z_2} \sin \frac{2\pi x}{a}, \quad z_2 = \frac{k_0 \eta_0}{\beta_2} \end{aligned}$$

And after this we have to calculate the normalization constant that P1 and P2.

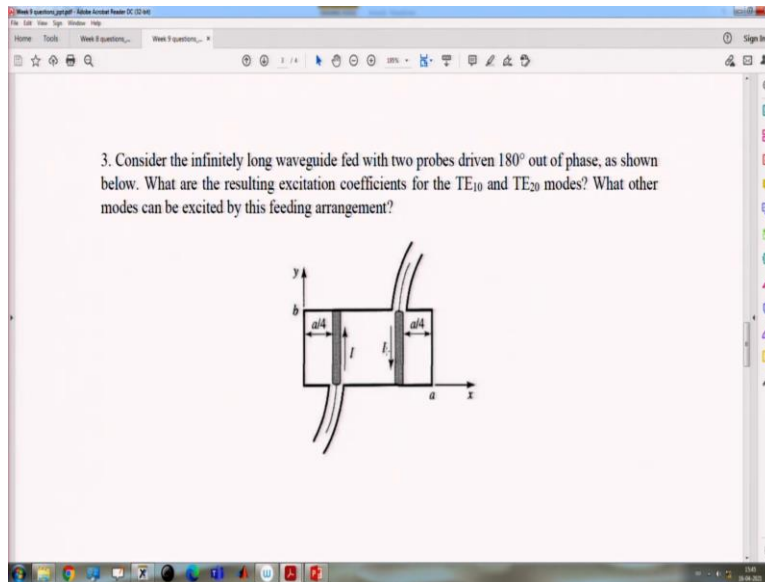
$$\begin{aligned}
 \text{TE}_{10}, P_1 &= 2 \int_{S_0} (\vec{e}_1 \times \vec{h}_1) \cdot \hat{a}_z \, dS \\
 &= 2 \int_{S_0} \left[a_0 \sin \frac{\pi x}{a} \times \left[\frac{-a_z}{z_1} \sin \frac{\pi x}{a} \right] \right] \cdot \hat{a}_z \, dx \, dy \\
 &= 2 \cdot \frac{a}{2} \cdot \frac{1}{z_1} b = \frac{ab}{z_1}
 \end{aligned}$$

This is the P1 for that TE₁₀ mode; for the TE₂₀ mode it will be

$$\begin{aligned}
 \text{TE}_{20}, P_2 &= 2 \int_a^b \int_{x=0}^a (\vec{e}_2 \times \vec{h}_2) \cdot \hat{a}_z \, dx \, dy \\
 &= 2 \int_{x=0}^a \int_{y=0}^b \sin^2 \frac{2\pi x}{a} \cdot \frac{1}{z_2} \cdot dx \, dy \\
 &= 2 \times \frac{1}{z_2} \times \frac{a}{2} b = \frac{ab}{z_2}
 \end{aligned}$$

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$$\begin{aligned}
 \text{TE}_{20}, P_2 &= 2 \int_a^b \int_{x=0}^a (\vec{e}_2 \times \vec{h}_2) \cdot \hat{a}_z \, dx \, dy \\
 &= 2 \int_{x=0}^a \int_{y=0}^b \sin^2 \frac{2\pi x}{a} \cdot \frac{1}{z_2} \cdot dx \, dy \\
 &= 2 \times \frac{1}{z_2} \times \frac{a}{2} b = \frac{ab}{z_2}
 \end{aligned}$$



So, these are the normalization factor, and after this we have to calculate the amplitude.

$$\begin{aligned}
 A_{TE_{10}}^+ &= -\frac{1}{P_1} \int_V [\vec{e}_1 - \hat{a}_z e_{z1}] \vec{J} e^{j\beta_1 z} \cdot dV \\
 &= -\frac{1}{P_1} \int_V \hat{a}_y \sin \frac{\pi x}{a} I \delta(z) \left[\delta(x - \frac{a}{4}) - \delta(x - \frac{3a}{4}) \right] \hat{a}_y e^{j\beta_1 z} \cdot dx dy dz \\
 &= -\frac{I}{P_1} \left[\sin \frac{\pi}{4} - \sin \frac{3\pi}{4} \right] \cdot 1 \cdot b \\
 &= -\frac{bI}{P_1} \left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right] = 0
 \end{aligned}$$

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$$\begin{aligned} \text{For TE}_{10} \quad A_1^+ &= -\frac{1}{P_1} \int_V (\vec{e}_1 - \hat{a}_z e_{z1}) \vec{J} e^{j\beta_1 z} \cdot dV \\ &= -\frac{1}{P_1} \int_V a_y \sin \frac{\pi x}{a} I \delta(z) \left[\delta\left(x - \frac{a}{4}\right) \right. \end{aligned}$$

$$\begin{aligned} H_1 &= P_1 \int_V (\vec{e}_1 - \hat{a}_z e_{z1}) \delta(z) \left[\delta\left(x - \frac{a}{4}\right) - \right. \\ &= -\frac{1}{P_1} \int_V a_y \sin \frac{\pi x}{a} I \delta(z) \left[\delta\left(x - \frac{a}{4}\right) - \right. \\ &\quad \left. \delta\left(x - \frac{3a}{4}\right) \right] \hat{a}_y e^{j\beta_1 z} \cdot dx dy dz \end{aligned}$$

$$\delta\left(x - \frac{3a}{4}\right) \hat{a}_y e^{j\beta_1 z} \cdot dx dy dz$$

$$= -\frac{I}{P_1} \left[\sin \frac{\pi}{4} - \sin \frac{3\pi}{4} \right] \cdot 1 \cdot b$$

$$= -\frac{bI}{P_1} \left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right] = 0$$

And now similarly we can calculate the A_2 plus for the TE_{20} .

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$$TE_{20} \quad A_2^+ = -\frac{1}{P_2} \int_V (\vec{E}_2 - \hat{a}_z E_{z2}) \cdot \vec{J} e^{j\beta_2 z} \cdot dV$$

$$= -\frac{1}{P_2} \int_V a_0 \sin \frac{2\pi x}{a} I \delta(z) \left[\delta\left(x - \frac{a}{4}\right) - \delta\left(x - \frac{3a}{4}\right) \right] \hat{a}_y e^{j\beta_2 z} \cdot dx dy dz$$

So, for TE_{20} that will be

$$\begin{aligned}
 TE_{20} \quad A_2^+ &= -\frac{1}{P_2} \int_V (\vec{e}_2 - \hat{a}_z e_{z2}) \vec{J} e^{j\beta_2 z} dV \\
 &= -\frac{1}{P_2} \int_V \hat{a}_y \sin \frac{2\pi x}{a} I \delta(z) \left[\delta\left(x - \frac{a}{4}\right) - \delta\left(x - \frac{3a}{4}\right) \right] \\
 &\quad \hat{a}_y e^{j\beta_2 z} dx dy dz
 \end{aligned}$$

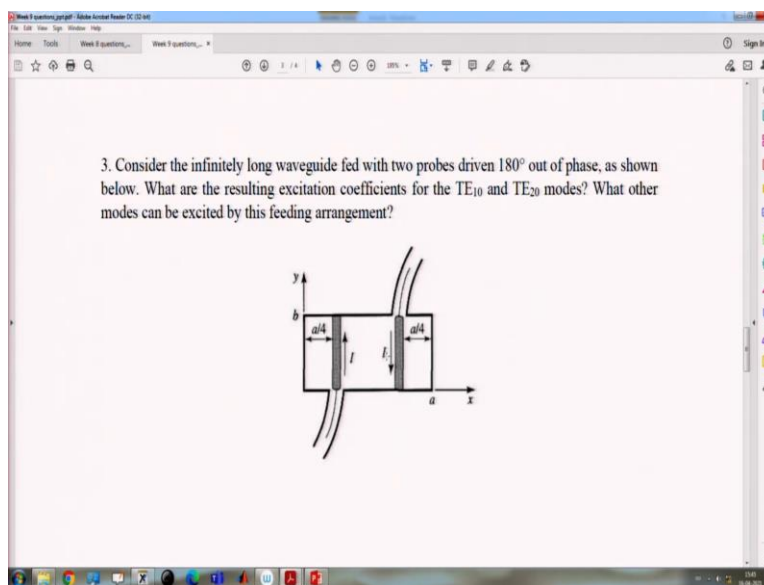
$$\begin{aligned}
 &= -\frac{I}{P_2} \left(\sin \frac{\pi}{2} - \sin \frac{3\pi}{2} \right) \cdot 1 \cdot b \\
 &= -\frac{I}{P_2} (1 - (-1)) b = \frac{-2Ib}{P_2}
 \end{aligned}$$

(Refer Slide Time: 16:30)

$$\hat{A}_y e^{j\beta_2 z} \cdot dx dy dz$$

$$= -\frac{I}{P_2} \left(\sin \frac{\pi}{2} - \sin \frac{3\pi}{2} \right) \cdot 1 \cdot b$$

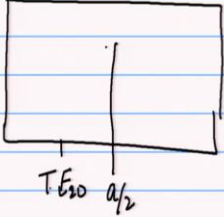
$$= -\frac{I}{P_2} (1 - (-1)) b = \frac{-2Ib}{P_2}$$

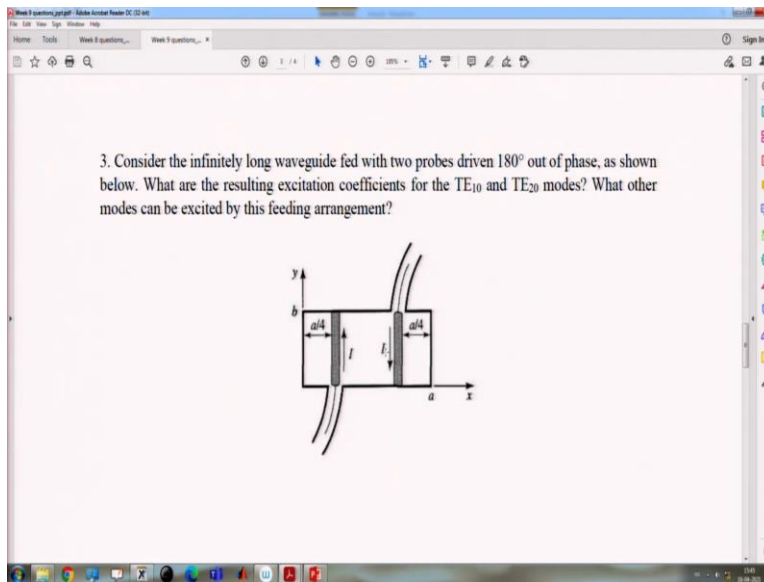


So, we can see that since the excitation has an odd symmetry about the center of the probe. So, it will only excite that mode that have electric field, with an odd symmetry about x is equal to $a/2$. So, in this case we can see that TE₂₀ mode is excited that has finite amplitude. But, when means for that TE₁₀ case, that has the amplitude 0; so, that we have to keep in the mind.

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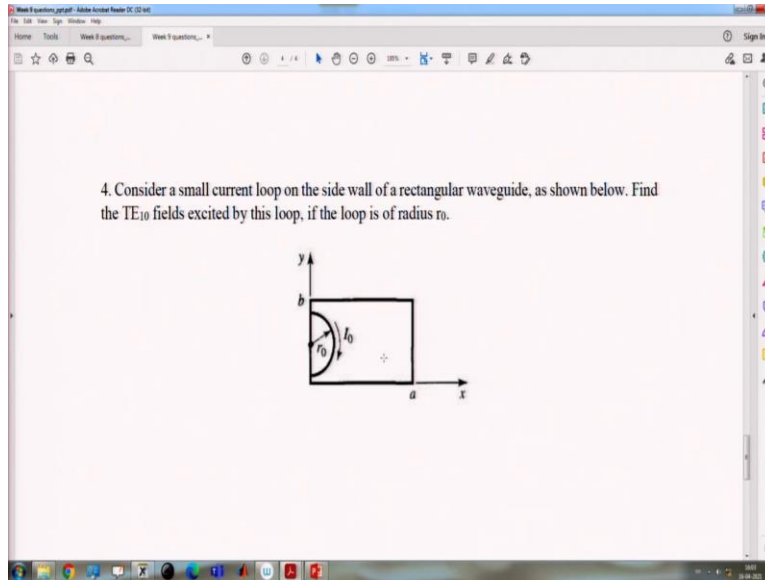
$$= -\frac{I}{P_2} \left(\sin \frac{\pi}{2} - \sin \frac{\pi}{2} \right)$$

$$= -\frac{I}{P_2} (1 - (-1)) b = \frac{-2Ib}{P_2}$$


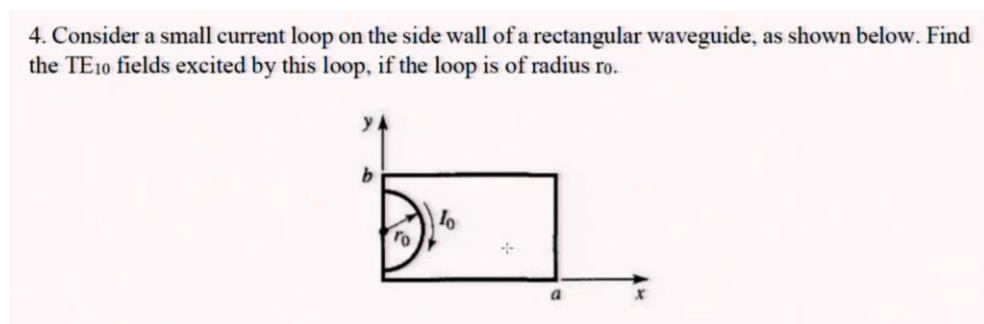


So, for that TE_{10} mode excitation when feeding at the center, then it has the finite amplitude of the forward and backward. But for this case when it is at $a/4$, for that case it will excite on the TE_{20} mode. So, next question we can do. So, what other modes can be excited by this feeding arrangement? So, that will be the odd asymmetric what x is equal to $a/2$; that will be only excited.

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So, next question is



And by removing side wall, it will become full loop; and for the full loop we can write the magnetic equivalent source. So, that can be written as so P_m , and P_m is the infinitesimal magnetic polarization current. So, for the loop case that will excite if it is electric loop so, that is current I_0 .

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$$\vec{P}_m = \hat{a}_z I_0 \pi r_0^2 \delta(r-a) \delta\left(y - \frac{b}{2}\right) \delta(z)$$
$$\vec{M} = j\omega M_0 \vec{P}_m$$
$$= \hat{a}_z j\omega M_0 I_0 \pi r_0^2 \delta(r-a) \delta\left[y - \frac{b}{2}\right] \delta(z)$$

So, if it is loop, so magnetic dipole that will be out of the loop. It is out of the loop, so that will be the magnetic equivalent for this loop. For this case, P_m can be written as

$$\vec{P}_m = \hat{a}_z I_0 \pi r_0^2 \delta(r-a) \delta\left(y - \frac{b}{2}\right) \delta(z)$$
$$\vec{M} = j\omega M_0 \vec{P}_m$$
$$= \hat{a}_z j\omega M_0 I_0 \pi r_0^2 \delta(r-a) \delta\left[y - \frac{b}{2}\right] \delta(z)$$

So, like this P_m that magnetic polarization current can be written; and by equivalent magnetic source will be equivalent magnetic source. As we know that J is the electric source, but M is the magnetic source.

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Handwritten equations for the TE₁₀ mode in a rectangular waveguide. The equations are written on a digital whiteboard with a blue grid background. The text "For TE₁₀" is written on the left side of the first line. The equations are:

$$\vec{E}_1 = \hat{a}_y \sin \frac{\pi x}{a}$$
$$\vec{H}_1 = -\frac{a_{zc}}{Z_1} \sin \left[\frac{\pi x}{a} \right], \quad Z_1 = \frac{\eta_0 k_0}{\beta_1}$$
$$\vec{H}_{z1} = \frac{j\pi}{k_0 \eta_0 a} \cos \left[\frac{\pi x}{a} \right]$$

So, for the TE₁₀ mode,

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$$\vec{H}_{z1} = \frac{j\pi}{k_0 \eta_0 a} \cos \left[\frac{\pi x}{a} \right]$$

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$$\vec{h}_{z1} = \frac{j\pi}{k_0 \eta_0 a} \cos\left(\frac{\pi z}{a}\right)$$

Normalization constant for the nth mode

$$P_n = 2 \int_{S_0} [\vec{E}_n \times \vec{h}_n] \cdot \hat{a}_z dS$$

Now, normalization constant for the nth mode, for the for the nth mode.

Normalization constant for the nth mode

$$P_n = 2 \int_{S_0} [\vec{E}_n \times \vec{h}_n] \cdot \hat{a}_z dS$$

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$$P_1 = 2 \int_{S_0} [\vec{E}_1 \times \vec{h}_1] \cdot \hat{a}_z dS$$

$$= \frac{ab}{Z_1}$$

$$\begin{aligned}
 |I| &= j\omega \mu_0 I_m \\
 &= \hat{a}_z j\omega \mu_0 I_0 \pi \gamma_0^2 \delta(x) \delta\left(y - \frac{b}{2}\right) \delta(z)
 \end{aligned}$$

For TE₁₀

$$\begin{aligned}
 \vec{E}_1 &= \hat{a}_y \sin \frac{\pi x}{a} \\
 \vec{h}_1 &= -\frac{a_{zc}}{z_1} \sin \left[\frac{\pi x}{a} \right], \quad z_1 = \frac{\eta_0 k_0}{\beta_1} \\
 \vec{h}_{z1} &= \frac{j\pi}{k_0 \eta_0 a} \cos \left[\frac{\pi x}{a} \right]
 \end{aligned}$$

So, for TE₁₀ this will be P₁,

$$\begin{aligned}
 \text{TE}_{10} \quad P_1 &= 2 \int_{S_0} (\vec{E}_1 \times \vec{h}_1) \cdot \hat{a}_z \, dS \\
 &= \frac{ab}{z_1}
 \end{aligned}$$

Now, for the amplitude of the positive travelling wave, that will be A₁ plus for the TE₁₀ mode; so that we can write and for the nth mode. So, for the nth mode we know

$$\begin{aligned}
 \text{For } n^{\text{th}} \quad A_n^+ &= \frac{1}{P_n} \int_V (-\vec{h}_n + \hat{a}_z h_{zn}) \cdot \vec{M} e^{j\beta_n z} \, dV \\
 A_1^+ &= \frac{1}{P_1} \int_V [-h_1 + \hat{a}_z h_{z1}] \vec{M} e^{j\beta_1 z} \, dV \\
 &= \frac{1}{P_1} \int_V \left(+\frac{\hat{a}_{zc}}{z_1} \sin \frac{\pi x}{a} + \hat{a}_z \frac{j\pi}{k_0 \eta_0 a} \cos \frac{\pi x}{a} \right) \\
 &\quad \hat{a}_z j\omega \mu_0 I_0 \pi \gamma_0^2 \delta(x) \delta\left(y - \frac{b}{2}\right) \delta(z) \, dV
 \end{aligned}$$

$$= \frac{1}{P_1} \frac{j\pi}{k_0 \eta_0 a} | \cdot j\omega \mu_0 I_0 \pi \gamma_0^2$$

$$= \frac{-\pi^2 \omega \mu_0 I_0 \gamma_0^2 \cdot Z_1}{k_0 \eta_0 a a b}$$

$$= \frac{-\pi^2 \cancel{\mu_0} \cancel{\mu_0} I_0 \gamma_0^2 Z_1 \cancel{\sqrt{\epsilon_0}}}{\cancel{\mu_0} \cancel{\sqrt{\mu_0} \epsilon_0} \cancel{\sqrt{\mu_0}} a^2 b}$$

$$A_1^+ = \frac{-\pi^2 Z_1 I_0^2 \gamma_0^2}{a^2 b}$$

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The screenshot shows a digital whiteboard with two equations. The first equation is for the magnetic vector potential A_m^+ for the n th mode, and the second is for A_1^+ . Both equations involve a volume integral over a region V of the product of a vector function and a magnetic field vector $\vec{M} e^{j\beta_n z}$.

$$\text{For } n\text{th } A_m^+ = \frac{1}{P_n} \int_V (-\hat{h}_n + \hat{a}_z h_{zn}) \cdot \vec{M} e^{j\beta_n z} \cdot dV$$

$$A_1^+ = \frac{1}{P_1} \int_V [-h_1 + \hat{a}_z h_{z1}] \vec{M} e^{j\beta_1 z} \cdot dV$$

$$= -\frac{1}{P_1} \int \left(+\hat{A}_2 \frac{\sin \frac{\pi x}{a}}{z_1} + \hat{A}_2 \frac{j\pi}{k_0 \eta_0 a} \cos \frac{\pi x}{a} \right)$$

$$\hat{A}_2 j\omega \mu_0 I_0 \pi \gamma_0^2 \delta(z_1) \delta\left(y - \frac{b}{2}\right) \delta(z)$$

$$\hat{A}_2 j\omega \mu_0 I_0 \pi \gamma_0^2 \delta(z_1) \delta\left(y - \frac{b}{2}\right) \delta(z)$$

$$= \frac{1}{P_1} \frac{j\pi}{k_0 \eta_0 a} \cdot j\omega \mu_0 I_0 \pi \gamma_0^2$$

$$= \frac{-\pi^2 \omega \mu_0 I_0 \gamma_0^2 \cdot z_1}{k_0 \eta_0 a a b}$$

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$$k_0 \eta_0 a b$$

$$= \frac{-\pi^2 \mu_0 \epsilon_0 T_0 \gamma_0^2 z_1 \sqrt{\epsilon_0}}{\mu_0 \sqrt{\mu_0 \epsilon_0} \sqrt{\mu_0} a^2 b}$$

$$A_1^+ = \frac{-\pi^2 z_1 I_0^2 \gamma_0^2}{a^2 b}$$

$$A_1^- = \frac{-\pi^2 z_1 I_0^2 \gamma_0^2}{a^2 b}$$

$$\vec{E}^+ = \sum_n A_n^+ E_n^+$$

$$= \sum_n A_n^+ (\hat{e}_n + \hat{a}_z e_{zn}) e^{-j\beta_n z}$$

So, this is the amplitude for the positive travelling wave.

Similarly, for the negative travelling wave amplitude will be for A1 minus that we can calculate

$$A_1^- = \frac{-\pi^2 z_1 I_0^2 \gamma_0^2}{a^2 b}$$

And because this will be symmetric in both the direction; so it will give the same amplitude in the negative travelling wave also. For this case also if you will do the calculation.

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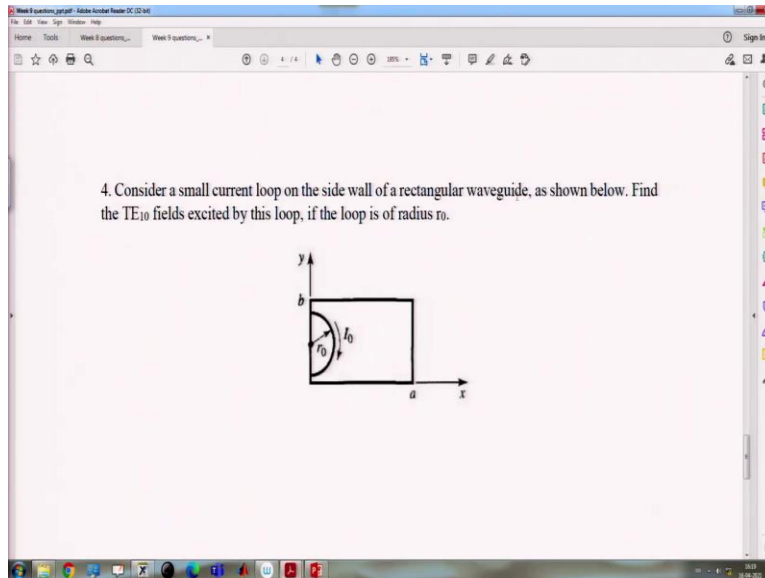
The image shows a presentation slide with a white background and blue horizontal lines. At the top, the equation $A_1^- = \frac{-\pi^2 Z_1 I_0^2 \gamma_0^2}{a^2 b}$ is written in black ink. Below it, the equation $\vec{E}^+ = \sum_n A_n^+ E_n^+$ is written. The slide is part of a presentation window with a toolbar at the top and a status bar at the bottom.

$$A_1^- = \frac{-\pi^2 Z_1 I_0^2 \gamma_0^2}{a^2 b}$$
$$\vec{E}^+ = \sum_n A_n^+ E_n^+$$

The image shows a presentation slide with a white background and blue horizontal lines. At the top, the letter 'a' is written. Below it, the equation $\vec{E}^+ = \sum_n A_n^+ E_n^+$ is written. Below that, the equation $= \sum_n A_n^+ (\hat{e}_n^+ \hat{a}_z e_{zn}) e^{-j\beta_n z}$ is written. At the bottom, the equation \vec{H}^+ is written. The slide is part of a presentation window with a toolbar at the top and a status bar at the bottom.

a

$$\vec{E}^+ = \sum_n A_n^+ E_n^+$$
$$= \sum_n A_n^+ (\hat{e}_n^+ \hat{a}_z e_{zn}) e^{-j\beta_n z}$$
$$\vec{H}^+$$



So, we know model field for the positive means how to write that fields in the positive and negative direction. So, that expression is E plus that it can be written as a summation of all the modes inside the waveguide;

$$\vec{E}^+ = \sum_n A_n^+ E_n^+$$

$$= \sum_n A_n^+ (\hat{e}_n + \hat{a}_z e_{zn}) e^{-j\beta_n z}$$

So, like this in the similar way for the magnetic field, it can be vector H can be written. So, find that TE₁₀ field excited by the loop; so, these are the amplitude. So, from this amplitude we can find out the fields; so, these are the A1 plus and A1 minus, so both are equal. So, like this amplitude can be calculated. Thank you.