Advanced Microwave Guided-Structures and Analysis
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Lecture 45
Analysis of Guided Structures

So, welcome to this session of lecture on Analysis of Guided Structures. We are going to start this session of lecture with the analysis of the partially filled rectangular waveguide. Now, we had already found out the propagation characteristics of rectangular waveguides filled with homogeneous material. We found that the TE to Z and TM to Z modes, adequately describe the mode spectrum inside rectangular waveguides filled with a homogeneous medium. However, unfortunately the same strategy does not work, if the rectangular waveguide is partially filled.

So, the same strategy means the same decomposition of the fields into the TE to Z and the TM to Z modes will not work simply because of the fact, that we cannot apply, or satisfy the boundary condition at the dielectric air interface using the TE to Z, or TM to Z modes. So, this is an interesting situation in which we are in the quest for alternate mode sets. So, that we can satisfy the boundary conditions of this, you know like this relatively complex problem. Let us go to the slides. Let us go to the analysis of such structures. Let us go to the slides.

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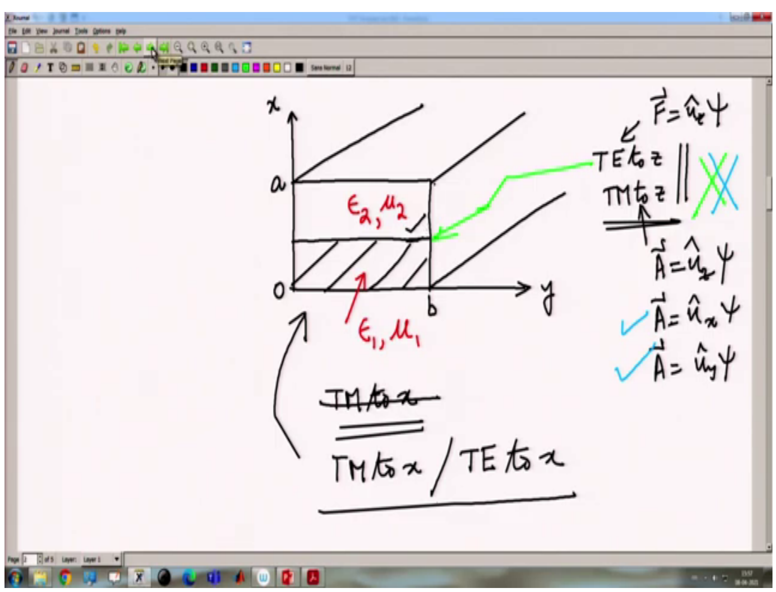

Let us draw firstly the structure of a partially filled dielectric waveguide, for our axis, we will be retaining our original axis, which is x axis up, y axis in the horizontal direction, the waveguide is here, 0 to a along the x direction, 0 to b along the y direction. So, for the start, we are going to stick with the axis nomenclature, we use previous t. So, this is the x axis, this is the y axis, this is my waveguide cross section, this is my wave guide, along the x axis the dimensions 0 to a, along the y axis the dimension 0 to b. This is my dielectric strip, its media properties are epsilon 1 and mu 1, and the media properties here are epsilon 2, mu 2.

So, this is my geometry norm. In this case if I apply the fields corresponding to TE to Z, and TM two Z. We will find that we cannot satisfy the boundary condition at this interface. So, they cannot be used to satisfy the boundary conditions at this interface. And therefore, these are not valid mode sets for the problem.

The question is if these are not valid mode sets for the problem, how do we crack this problem, how do we solve this problem? The first step as we all realized, when we went through our potential function analysis, and when we treated the case of finding the fields from the, you know like radiated by source, or finding the modes inside the waveguide, we did one thing and that is we started from the potential functions.

And then we found out the fields, we applied the expressions of the fields, or we tried to satisfy the boundary conditions of the problem using those expressions of the fields. If we could satisfy the boundary conditions of the problem, then those potential functions, which we selected are valid potential functions. If not if the boundary conditions cannot be satisfied, we have to redo our homework that means our original choice of potential functions is wrong for the boundary condition under investigation.

So, what we have to do? We have to choose an alternate potential function. So, there are in the rectangular coordinate system, it gives me three choices of potential functions, for these choices we selected **A** equal to uz psi try, that is how we came up, or landed with the TM to Z modes. And when we chose **F** equal to uz psi, we discovered that the fields were of the TE to Z form.

But one can also choose as we said that time, that **A** equal to ux psi, or **A** equal to uy psi. And we could use the same expressions of the electric and magnetic fields, which we found out due to the radiating current source in a homogeneous medium. Those same two sets of equations which link up the electric and magnetic fields with the magnetic vector potential, or the electric vector potential. we can use those two same, two equations to find out the electric and magnetic fields in these two cases. And that is exactly what we are going to do.

So, first of all let us start with an alternative mode set. So, if this does not yield me the proper solution. So, we should be starting from another kind of mode set. So, let us take the mode set, which is TM to x, let us consider the TM to x mode and find out whether the TM to x, TE to x mode sets, allow me to satisfy the boundary conditions in this problem.

So, let us take the mode set TM to x and TE to x and use these mode sets to satisfy the boundary conditions of this problem. If this mode sets allow me to satisfy the boundary conditions on the dielectric air interface, which is this interface, our mode set is a valid mode set. So, let us see whether the TM to x, or TE to x mode sets allow me to do that. So, first of all let us write down the expressions for the TM to x and the TE to x, electric and magnetic fields.

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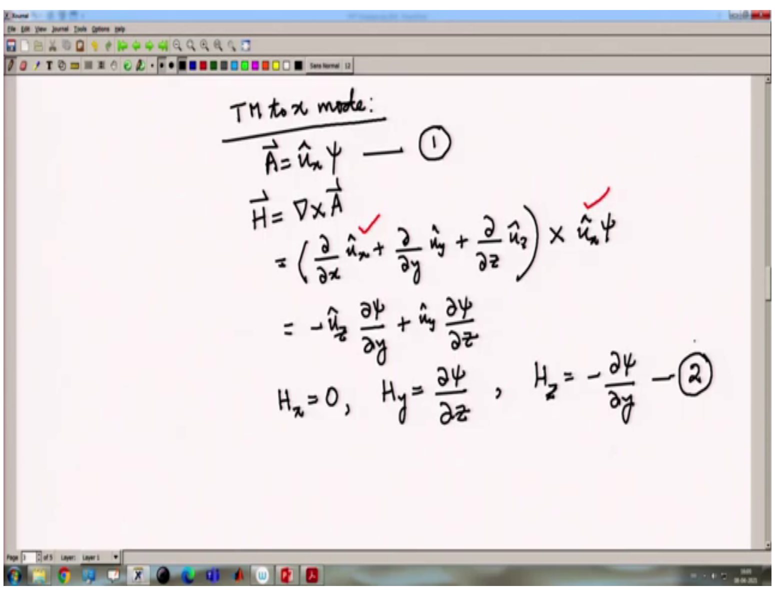

So, for the TM to x mode, we have **A** equal to ux psi, just like for the TM to Z, mode we started with **A** equal to uz psi. We call this equation 1. So, this is our starting point, for deriving the field expressions for the TM to x mode, TM to x mode is transverse magnetic to x, that means there will be no component of the magnetic field along the x direction, because the field is transverse magnetic to x, just like for the transverse magnetic to Z. There is no component of the magnetic field along the z direction.

So, therefore for the transverse magnetic to x, there is no component of the magnetic field along the x direction. So, we will discover that, if **A** equal to ux psi, we indeed find out, or we indeed land up in a situation, where each x component is 0. So, let us find that out, we can derive the magnetic field from **H** is equal to curl of **A**, which we, which we already discussed before. And that becomes equal to del del x of ux plus del del y of uy, plus del del z of uz, times ux psi. And that will be yielding ultimately minus uz del psi del y, plus uy del psi del z. Note that the contribution of this and this will be 0. So, therefore we find Hx to be 0, Hy to be del psi del z, and Hz to be minus del psi del y. We call this equation 2.

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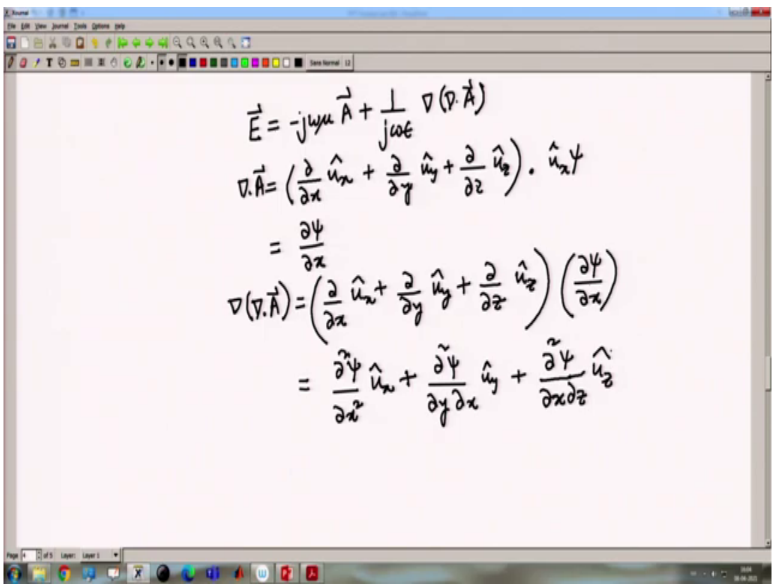

Next, we find the electric fields. So, our familiar equation, which we know before is **E** is equal to minus j omega mu **A** plus 1 by j omega epsilon grad of divergence **A**. So, divergence **A** is del del x, ux plus del del y, uy plus del del z, uz dot ux psi, which is **A**. So, that becomes equal to del psi del x. Look at the other terms go to 0, because uy cross uy dot ux is 0, uz dot ux is 0. So, next divergence of grad **A**, becomes del del x, ux plus del del y, uy plus del del z, uz, times divergence of **A**, which is del psi del x. So, that is del square psi del x square ux plus del square psi del y del x, uy plus del square psi del x del z uz.

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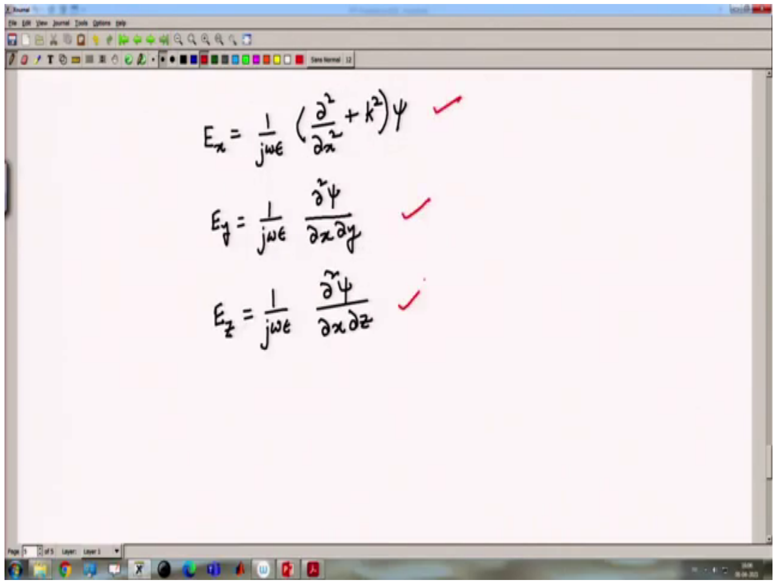

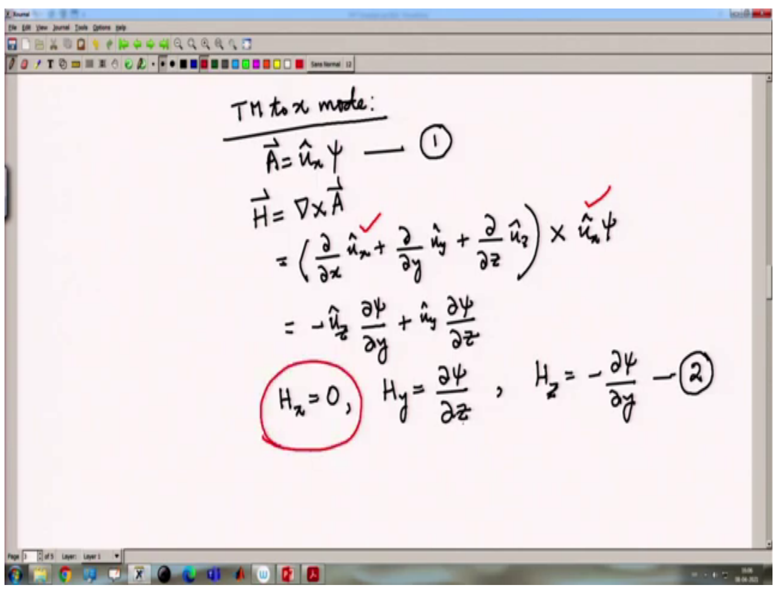

So, my electric field is evaluated from these to be Ex is 1 by j omega epsilon, del square del x square plus k square psi, Ey 1 by j omega epsilon, del square psi, del x del y, and E z is 1 by j omega epsilon, del square psi, del x del z. Similarly, so therefore we see that the mode is TM to x, because Hx is 0, all the other components of the magnetic field are there. And also, the other components of the electric field are also there.

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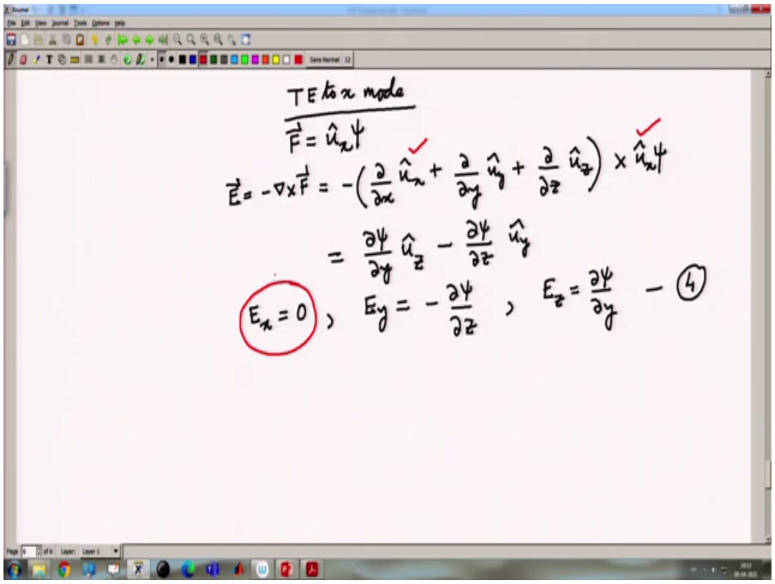

Now, we come to the TE to x mode, for which **F** equal to ux psi, just like for the TE to Z mode, we chose **F** equal to, or the electric vector potential to be equal to uz psi. So, for the TE to x mode, we would have **F** equal to ux psi. And TE to x means Transverse electric to x, that means there will be no Ex component of the field, the x component of the electric field will be 0. Let us see, let us compute the electric and magnetic fields.

So, **E** is given by minus curl of **F**, it is given by minus del del x, ux plus del del y, uy plus del del z, uz, times ux psi, that is del psi del y, uz minus del psi del z, uy. Because these two terms are not going to contribute anything, ux cross ux will be 0. So, therefore Ex is 0, Ey is minus del psi del z, and Ez equal to del psi, and Ez equal to del psi del y. Let us call this equation 4. So, you see now that Ex is 0, corresponding to the transverse electric to x mode.

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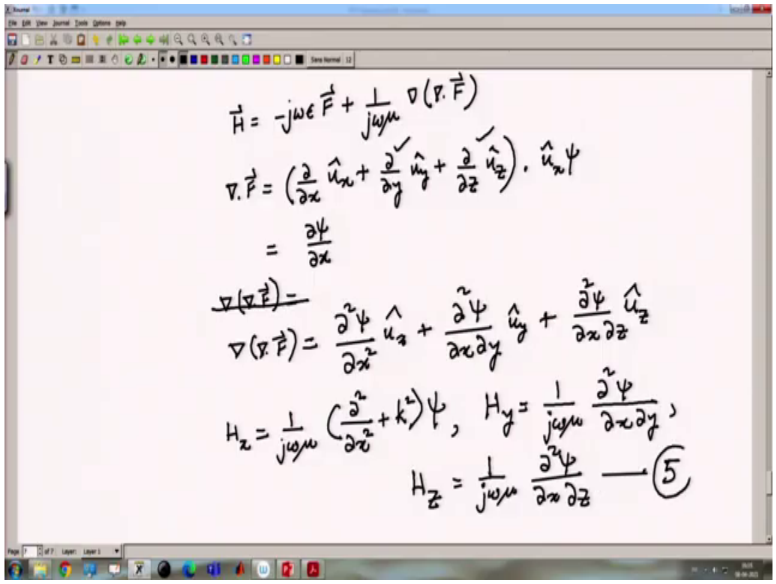

So, for computing the **H** fields, we use **H** equal to minus j omega epsilon **F** plus 1 by j omega mu, grad of divergence **F**. So, divergence **F** is del del x, ux plus del del y, uy plus del del z, uz dot ux psi, that is del psi del x, because these two terms will not contribute anything, uy dot ux is 0, uz dot ux is 0.

So, now grad of divergence **F** will be del square psi, del x square ux plus del square psi del x del y, uy plus del square psi del x del z, uz, del square psi del x del z, uz. So, from here, my magnetic field components work out to be Hx equal to 1 by j omega mu, del square del x square plus k square psi, Hy equal to 1 by j omega mu, del square psi del x del y, and Hz is 1 by j omega mu, del square psi del x del z. So, let us call this equation 5. Thank you we will be continuing in the next lecture.