Advanced Microwave Guided – Structures and Analysis
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Lecture 47
Analysis of Guided Structures (continued)

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So welcome to the continuation session again, for the analysis of the partially filled rectangular waveguide. So, we had learnt before how the TM to x and TE to x mode set is constructed, what are their characteristics. Now we are going to use those mode sets in order to find the dispersion characteristics of the partially filled rectangular waveguide, the TE to x and the TM to x mode dispersion characteristics. We will be starting with the TM to x mode characteristics, so let us go to the slides.

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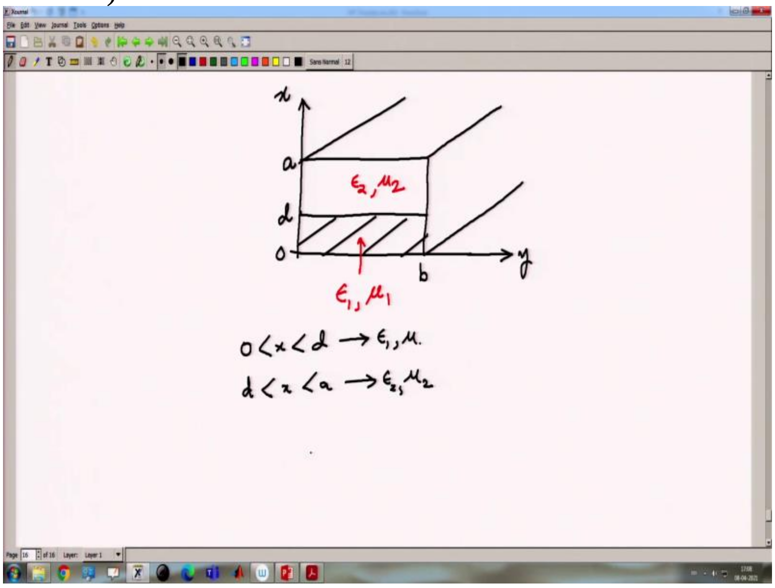
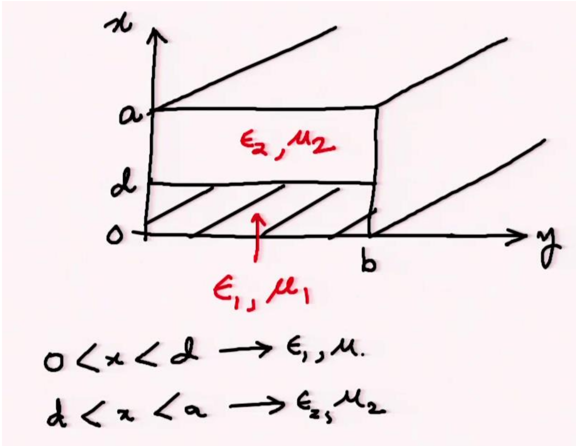


So, let us redraw the partially filled rectangular waveguide once more



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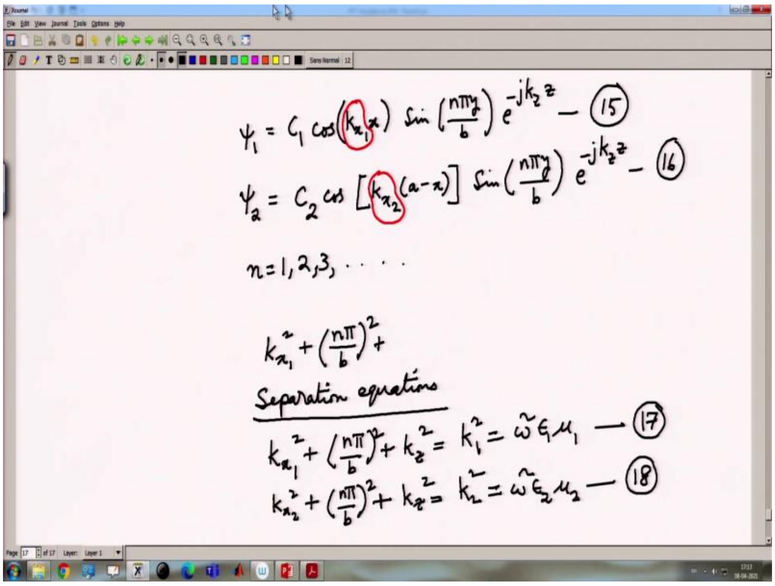

So, and for the fields which are TM to x, we choose the size in each region, region 1 and region 2 to represent the x component of it. So, to satisfy the boundary conditions at the conducting walls, we have

$$\psi_1 = C_1 \cos(k_{x_1} x) \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad \text{— (15)}$$

Note that the phi, the psi function distribution along the y direction is unchanged, because nothing has happened along the y direction, along the y direction there is no discontinuity. However, along the x direction, this wave number has entered, k_{x1} entered here in order to describe the discontinuity which is present at x equal to d, this is due to the discontinuity at x equal to d.

So, let us now write down the expression for psi 2 which is for the region x greater than d. So that is

$$\psi_2 = C_2 \cos\left[k_{x_2}(a-x)\right] \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad — (16)$$

So, again, this wave number k_{x2} is due to the discontinuity which is present at x equal to d, so here n equal to 1, 2, and 3 and so on.

k_z has to be the same in each region for matching the tangential, electric and magnetic field at x equal to d. This is called the phase matching condition, so the tangential wave number along the dielectric interface which is kz must match at x equal to d by the phase matching condition. Therefore, the separation equation for the two regions are,

Separation equations

$$k_{x_1}^2 + \left(\frac{n\pi}{b}\right)^2 + k_z^2 = k_1^2 = \omega^2 \epsilon_1 \mu_1 \quad — (17)$$

$$k_{x_2}^2 + \left(\frac{n\pi}{b}\right)^2 + k_z^2 = k_2^2 = \omega^2 \epsilon_2 \mu_2 \quad — (18)$$

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$$k_{x_2}^2 + \left(\frac{n\pi}{b}\right)^2 + k_z^2 = k_2^2 = \omega^2 \epsilon_2 \mu_2 \quad — (18)$$

From (3), (15) and (16):

$$E_{y_1} = \frac{1}{j\omega\epsilon} \frac{\partial^2 \psi}{\partial x \partial y}$$

$$=$$

$$E_{y_1} = \frac{1}{j\omega\epsilon_1} \frac{\partial^2 \psi_1}{\partial x \partial y}$$

$$= \frac{1}{j\omega\epsilon_1} \frac{\partial^2}{\partial x \partial y}\left[C_1 \cos(k_{x_1}x) \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z}\right]$$

$$= \frac{-1}{j\omega\epsilon_1} C_1 k_{x_1} \left(\frac{n\pi}{b}\right) \sin(k_{x_1}x) \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad — (19)$$

Now we can find out the electric fields from the equations 3, 15 and 16, we can find out the electric fields corresponding to the TM to x mode in region 1 and region 2. Let us find out the electric fields E_{y1}, so let us find out the tangential electric fields E_y and E_z, which can be matched across region 1 and region 2.

So, for the field Ey1 which is in region 1 that can be written as

From (3), (15) and (16):

$$E_{y_1} = \frac{1}{j\omega\epsilon}\frac{\partial^2\psi}{\partial x\partial y}$$

$$=$$

$$E_{y_1} = \frac{1}{j\omega\epsilon_1}\frac{\partial^2\psi_1}{\partial x\partial y}$$

$$= \frac{1}{j\omega\epsilon_1}\frac{\partial^2}{\partial x\partial y}\left[C_1\cos(k_{x_1}x)\sin\left(\frac{n\pi y}{b}\right)e^{-jk_z z}\right]$$

$$= \frac{-1}{j\omega\epsilon_1}C_1 k_{x_1}\left(\frac{n\pi}{b}\right)\sin(k_{x_1}x)\cos\left(\frac{n\pi y}{b}\right)e^{-jk_z z} \quad \text{— (19)}$$

let us call this equation 19. So, this is the Ey component in region 1, the electric field along the y direction.

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$$E_{y_2} = \frac{1}{j\omega\epsilon_2}\frac{\partial^2\psi_2}{\partial x\partial y}$$

$$= \frac{1}{j\omega\epsilon_2}\frac{\partial^2}{\partial x\partial y}\left[C_2\cos\{k_{x_2}(a-x)\}\sin\left(\frac{n\pi y}{b}\right)e^{-jk_z z}\right]$$

$$= \frac{1}{j\omega\epsilon_2}C_2 k_{x_2}\left(\frac{n\pi}{b}\right)\sin[k_{x_2}(a-x)]\cos\left(\frac{n\pi y}{b}\right)e^{-jk_z z} \quad \text{— (20)}$$

$$E_{z_1} = \frac{1}{j\omega\epsilon_1}\frac{\partial^2\psi_1}{\partial x\partial z}$$

$$= \frac{1}{\omega\epsilon_1}C_1 k_{x_1}k_z\sin(k_{x_1}x)\sin\left(\frac{n\pi y}{b}\right)e^{-jk_z z} \quad \text{— (21)}$$

Now, we can similarly find the electric field along the y direction in region 2. So Ey2 that will be given by 1

$$E_{y_2} = \frac{1}{j\omega\epsilon_2}\frac{\partial^2\psi_2}{\partial x\partial y}$$

$$= \frac{1}{j\omega\epsilon_2}\frac{\partial^2}{\partial x\partial y}\left[C_2\cos\{k_{x_2}(a-x)\}\sin\left(\frac{n\pi y}{b}\right)e^{-jk_z z}\right]$$

$$= \frac{1}{j\omega\epsilon_2}C_2 k_{x_2}\left(\frac{n\pi}{b}\right)\sin[k_{x_2}(a-x)]\cos\left(\frac{n\pi y}{b}\right)e^{-jk_z z} \quad \text{— (20)}$$

let us call this equation 20.

So similarly, Ez1 can be obtained which is the z component of the electric field in region 1 that is given by 1 by

$$E_{z_1} = \frac{1}{j\omega\epsilon_1} \frac{\partial^2 \psi_1}{\partial x \partial z}$$
$$= \frac{1}{\omega\epsilon_1} C_1\, k_{x_1} k_z \sin(k_{x_1} x) \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad \text{—} \; (21)$$

let us call this 21.

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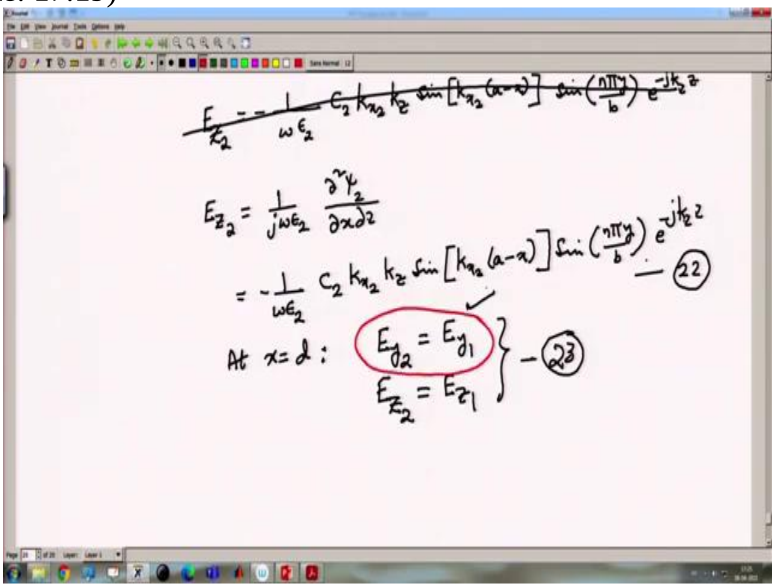

And the Ez in medium 2 which is Ez2 is given by

$$E_{z_2} = \frac{1}{j\omega\epsilon_2} \frac{\partial^2 \psi_2}{\partial x \partial z}$$
$$= -\frac{1}{\omega\epsilon_2} C_2\, k_{x_2} k_z \sin[k_{x_2}(a-x)] \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad \text{—} \; (22)$$

$$\text{At } x = d: \quad \left.\begin{aligned} E_{y_2} &= E_{y_1} \\ E_{z_2} &= E_{z_1} \end{aligned}\right\} \; \text{—} \; (23)$$

let us call this equation 22.

Now however by the continuity of the electric fields at x equal to d, we know that E_y and E_z components of the fields which are the tangential field components at x equal to d must be continuous at x equal to d. So therefore, at x equal to d we can write E_{y2} equal to E_{y1} and E_{z2} and equal to E_{z1}. So, we call this equation 23.

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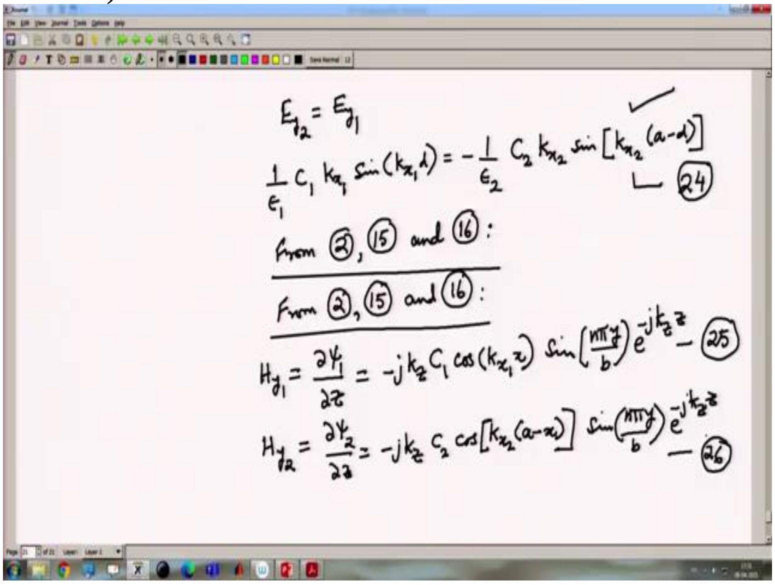

Now, applying this equation which is E_{y2} equal to E_{y1} and substituting the values for E_{y2} equal to E_{y1}, we will get the equation,

$$E_{y2} = E_{y1}$$

$$\frac{1}{\epsilon_1} C_1 k_{x_1} \sin(k_{x_1} d) = -\frac{1}{\epsilon_2} C_2 k_{x_2} \sin[k_{x_2}(a-d)] \quad \text{—} \ (24)$$

So let us call this equation 24. This is by E_{y2} equal to E_{y1}.

If we apply the boundary conditions that the other tangential component of the electric field E_{z2} that is equal to E_{z1} the same equation as 24 will be obtained. Now we compute the tangential components of the magnetic fields Hy and Hz. So, from 2, 15 and 16, we can compute the magnetic fields, the tangential magnetic fields corresponding to the TM to x mode.

So, we particularly have H_{y1} and H_{y2}

From (2), (15) and (16):

$$H_{y_1} = \frac{\partial \psi_1}{\partial z} = -jk_z C_1 \cos(k_{x_1} x) \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad \text{—} \ (25)$$

$$H_{y_2} = \frac{\partial \psi_2}{\partial z} = -jk_z C_2 \cos[k_{x_2}(a-x)] \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad \text{—} \ (26)$$

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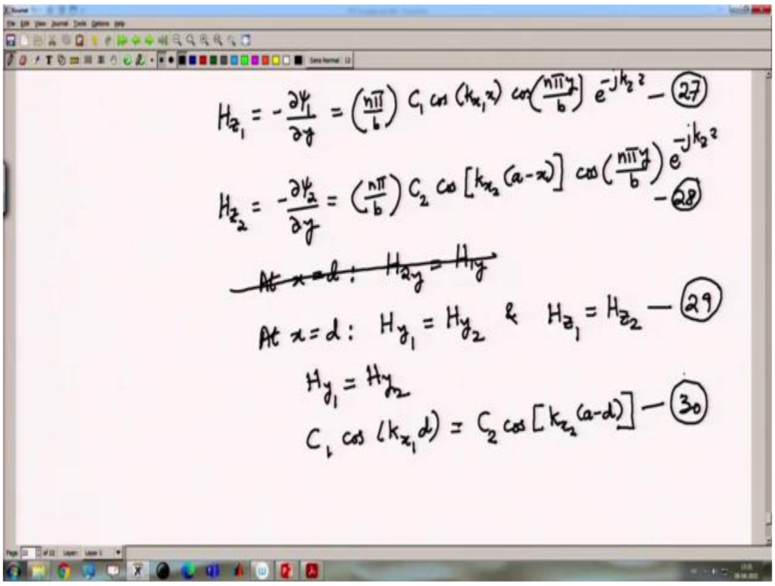

And in a similar manner we can also obtain H_{z1} and H_{z2},

$$H_{z_1} = -\frac{\partial \psi_1}{\partial y} = \left(\frac{n\pi}{b}\right) C_1 \cos(k_{x_1} x) \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad (27)$$

$$H_{z_2} = -\frac{\partial \psi_2}{\partial y} = \left(\frac{n\pi}{b}\right) C_2 \cos[k_{x_2}(a-x)] \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad (28)$$

Now, we know that at x equal to d, the tangential magnetic field components must be continuous, so H_{2y} equal to H_{1y}.

So, we obviously know at x equal to d the tangential magnetic field components must be continuous, so we have at x equal to d, Hy1 equal to Hy2 and Hz1 must also be equal to Hz2,

$$\text{At } x = d: \quad H_{y_1} = H_{y_2} \quad \& \quad H_{z_1} = H_{z_2} \quad (29)$$

$$H_{y_1} = H_{y_2}$$

$$C_1 \cos(k_{x_1} d) = C_2 \cos[k_{x_2}(a-d)] \quad (30)$$

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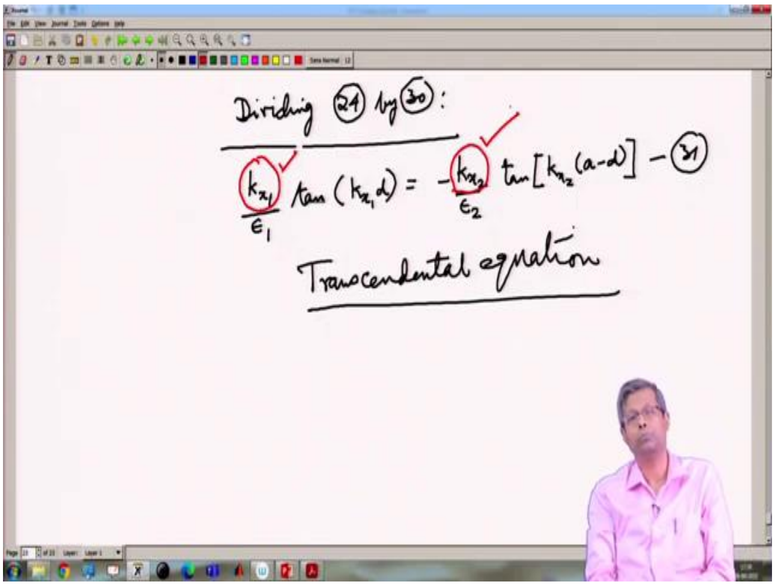
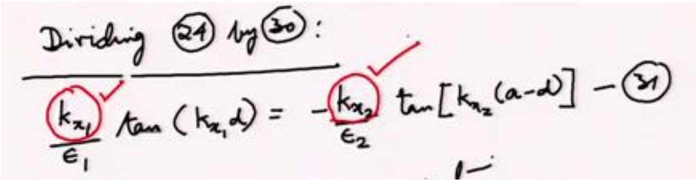


Now if we divide 24 and 30, we get



So, replacing kx1 and kx2 by kz will yield the root for kz from this equation.

So, this is the transcendental equation, for determining the k_z values or the allowable k_z values. If k_z is known then from 17 and 18 which are the separation equations k_{x1} and k_{x2}, these two values will also be known, so all the eigen values of the problem are known. This completes the dispersion solution of the TM to x mode in the partially field dielectric waveguide. We will next discuss the TE to x mode solution; let us stop here for now, we will continue.