**Advanced Microwave Guided – Structures and Analysis**
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**Lecture 48**
**Analysis of Guided Structures (continued)**

So welcome to this session on the analysis of the partially filled rectangular waveguide. This is essentially a continuation of the previous session in which we studied the analysis or the dispersion characteristics of the transverse magnetic to x modes. So, in this session we are going to study the dispersion characteristics of the transverse electric to x modes.

So, just to remind you that this kind of guided wave structure does not support the TM to z or the TE to Z which we already discussed and the mode characteristics which are essentially supported for this kind of structure in order to satisfy the boundary conditions at the dielectric to air interface are the TM to x and the TE to x.

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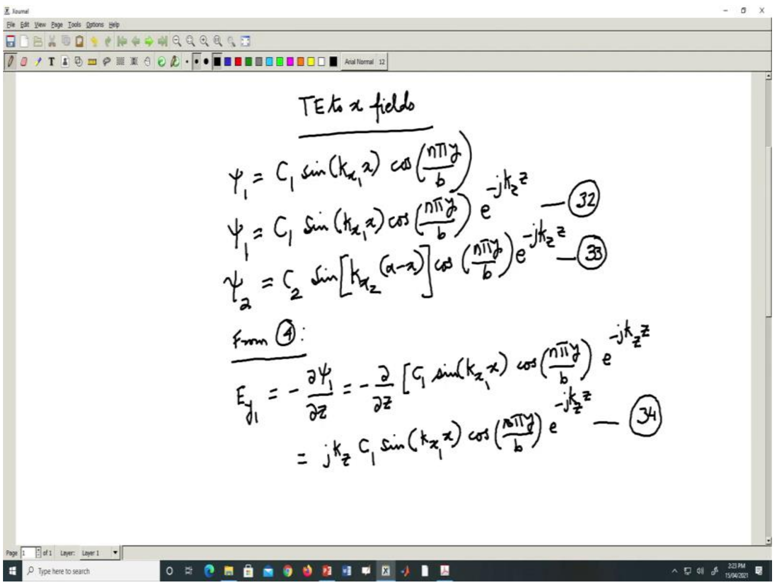


Let us go to the lecture. So for the fields which are TE to x, we choose the size in each region to represent the x component of f that is going to give me the TE to x modes. To satisfy the boundary conditions and the conducting walls we take

$$\underline{\text{TE to } x \text{ fields}}$$

$$\psi_1 = C_1 \sin(k_{x_1} x) \cos\left(\frac{n\pi y}{b}\right)$$

$$\psi_1 = C_1 \sin(k_{x_1} x) \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad \text{— (32)}$$

Similarly, psi 2 will be

$$\psi_2 = C_2 \sin\left[k_{x_2}(a-x)\right] \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad \text{— (33)}$$

so we call this equation 33

Please refer to the diagram which we had in the last class for the partially filled rectangular waveguide from which these notations will be cleared.

Now equation 4 if you look at equation 4 it gives me the electric field components of the TE to x mode in terms of the psi function. We can evaluate the component, the tangential component Ey from equation 4 in region 1 and region 2. So from equation 4 we

$$\underline{\text{From (4):}}$$

$$E_{y_1} = -\frac{\partial \psi_1}{\partial z} = -\frac{\partial}{\partial z}\left[C_1 \sin(k_{x_1} x) \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z}\right.$$

$$= jk_z C_1 \sin(k_{x_1} x) \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad \text{— (34)}$$

so that becomes equation number 34.

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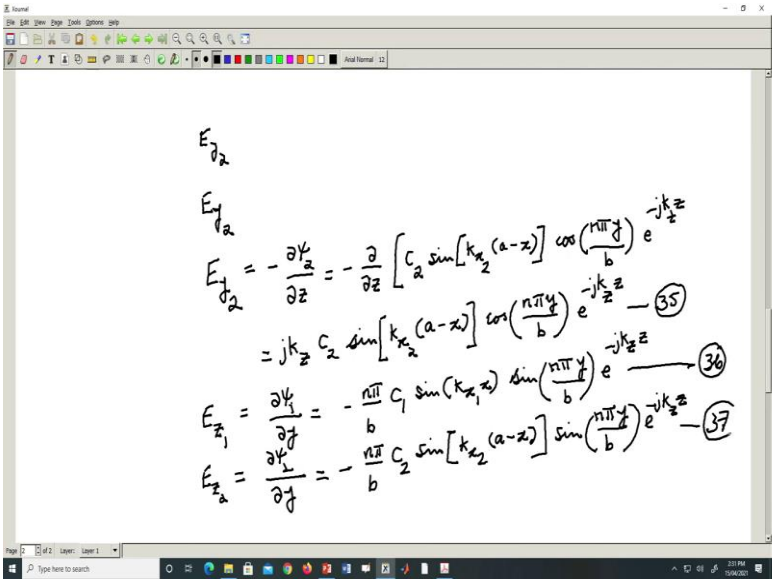

Similarly, the Ey component in region 2 becomes

$$E_{y_2} = -\frac{\partial \psi_2}{\partial z} = -\frac{\partial}{\partial z}\left[C_2 \sin[k_{x_2}(a-x)] \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z}\right]$$

$$= jk_z C_2 \sin[k_{x_2}(a-x)] \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad (35)$$

so let us call this equation 35.

Similarly

$$E_{z_1} = \frac{\partial \psi_1}{\partial y} = -\frac{n\pi}{b} C_1 \sin(k_{x_1} x) \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \longrightarrow (36)$$

$$E_{z_2} = \frac{\partial \psi_2}{\partial y} = -\frac{n\pi}{b} C_2 \sin[k_{x_2}(a-x)] \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad (37)$$

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From (5):

$$H_{y_1} = \frac{1}{j\omega\mu_1}\frac{\partial^2\psi_1}{\partial x\,\partial y} = -\frac{1}{j\omega\mu_1} k_{x_1}\left(\frac{n\pi}{b}\right) C_1 \cos(k_{x_1} x)\sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad (38)$$

$$H_{y_2} = \frac{1}{j\omega\mu_2}\frac{\partial^2\psi_2}{\partial x\,\partial y} = \frac{1}{j\omega\mu_2} k_{x_2}\left(\frac{n\pi}{b}\right) C_2 \cos\left[k_{x_2}(a-x)\right]\sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad (39)$$

$$H_{z_1} = \frac{1}{j\omega\mu_1}\frac{\partial^2\psi_1}{\partial x\,\partial z} = \frac{1}{j\omega\mu_1} k_{x_1}(-jk_z) C_1 \cos(k_{x_1} x)\cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad (40)$$

$$H_{z_2} = k$$

Similarly we can obtain the TE to x tangential magnetic field components for the structure from equation 5. We write from 5,

From (5):

$$H_{y_1} = \frac{1}{j\omega\mu_1}\frac{\partial^2\psi_1}{\partial x\,\partial y} = -\frac{1}{j\omega\mu_1} k_{x_1}\left(\frac{n\pi}{b}\right) C_1 \cos(k_{x_1} x)\sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad (38)$$

$$H_{y_2} = \frac{1}{j\omega\mu_2}\frac{\partial^2\psi_2}{\partial x\,\partial y} = \frac{1}{j\omega\mu_2} k_{x_2}\left(\frac{n\pi}{b}\right) C_2 \cos\left[k_{x_2}(a-x)\right]\sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad (39)$$

$$H_{z_1} = \frac{1}{j\omega\mu_1}\frac{\partial^2\psi_1}{\partial x\,\partial z} = \frac{1}{j\omega\mu_1} k_{x_1}(-jk_z) C_1 \cos(k_{x_1} x)\cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad (40)$$

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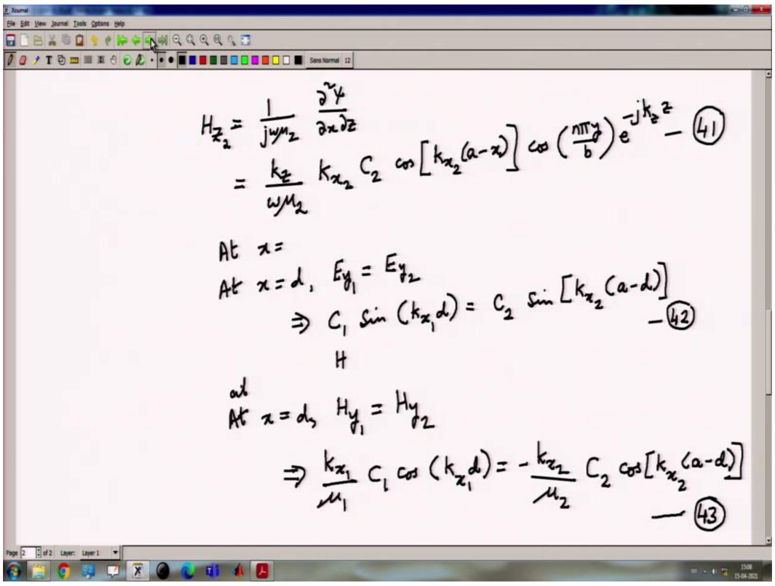

Similarly Hz2 can be written as 1

$$H_{z_2} = \frac{1}{j\omega\mu_2}\frac{\partial^2\psi}{\partial x\,\partial z} = \frac{k_z}{\omega\mu_2} k_{x_2} C_2 \cos\left[k_{x_2}(a-x)\right] \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad \text{— } (41)$$

Now we apply the continuity of the tangential electric and the tangential magnetic fields same as for the TM to x mode at x equal to d, so at the dielectric air interface which is at x equal to d.

So at the dielectric air interface which is at x equal to d, we have $E_{y1}$ equal to $E_{y2}$,

At $x = d$, $E_{y_1} = E_{y_2}$

$$\Rightarrow C_1 \sin(k_{x_1} d) = C_2 \sin\left[k_{x_2}(a-d)\right] \quad \text{— } (42)$$

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At $x = d$, $H_{y_1} = H_{y_2}$

$$\Rightarrow \frac{k_{x_1}}{\mu_1} C_1 \cos(k_{x_1} d) = -\frac{k_{x_2}}{\mu_2} C_2 \cos\left[k_{x_2}(a-d)\right] \quad \text{— } (43)$$

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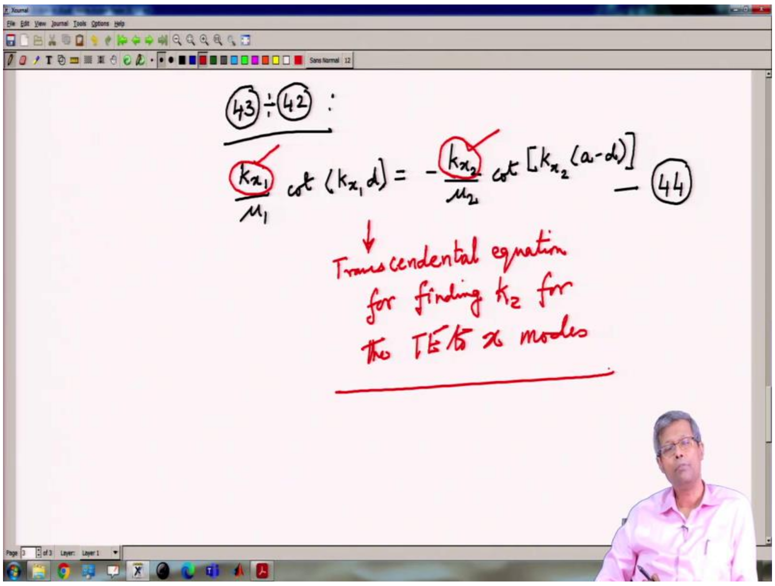
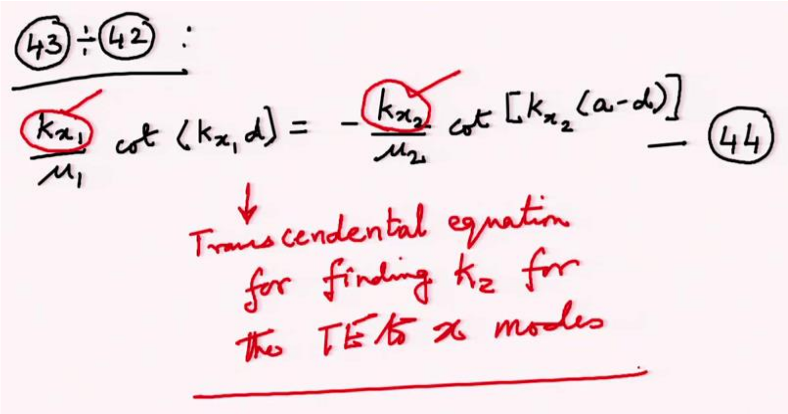


Division of 43 by 42, if I divide 43 by 42 we will obtain




Therefore 44 is a transcendental equation for determining the kz for the TE to x modes, so this is the transcendental equation for finding kz for the TE to x modes.

Therefore, we have found out how to compute the propagation constant kz for both the TM to x and the TE to x modes and we have applied the boundary condition at the dielectric air interface in order to find out the propagation constants. Therefore, these are the two valid mode sets which can be used to evaluate the propagation constants in the partially filled dielectric waveguide for the TM to x and the TE to x modes.

This completes the analysis of the dispersion characteristics of the partially filled dielectric waveguide we will be continuing in our next lecture on dielectric slab guides, we stop here.