

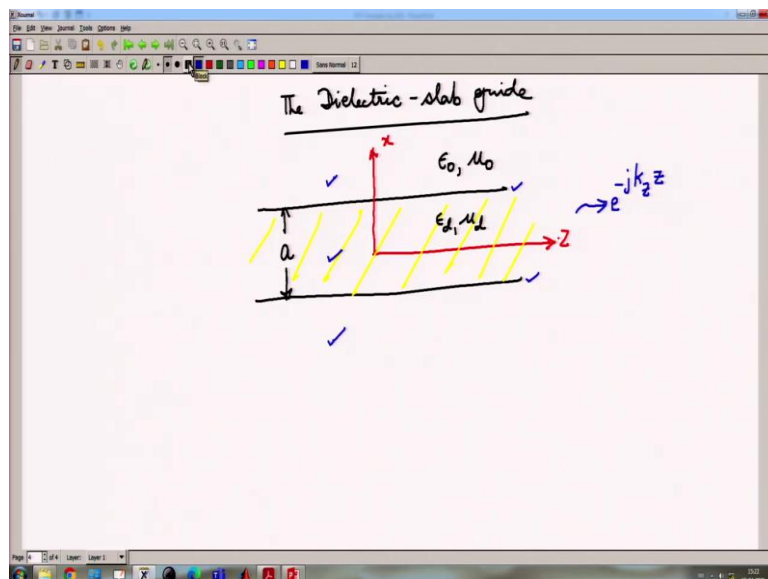
**Advanced Microwave Guided – Structures and Analysis**  
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**Lecture 49**  
**Analysis of Guided Structures (continued)**

So welcome to this session of the lecture on dielectric slab guides. So, this is a very important structure which is used for all substrate based circuits and antennas, because we all know the importance of the dielectric substrate in antenna design and microwave circuit design. For antenna design for example, we are extremely concerned with things like surface wave modes, how much power does the surface wave extract which is directly revealed or directly manifested in the back radiation characteristics of the antenna.

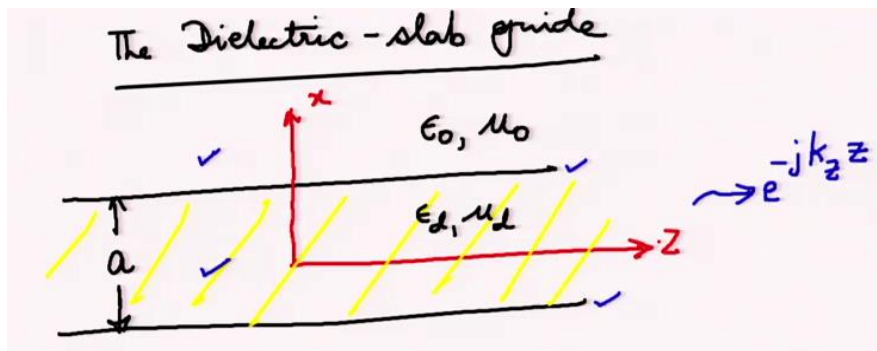
In many cases we want to suppress surface wave modes by using engineered substrates, but before all these things we need a physical understanding as to what is going on inside the substrate, what are particularly the modes which are excited in the substrate, number one. How to calculate the propagation characteristics of such modes?

What are the cutoff frequencies of such modes? And particularly what happens so to say below cut off and above cut off, for the dielectric slab. So this will lead to a very important and a very comprehensive and detailed understanding about the behavior of a dielectric slab which is almost only present in any kind of planar microwave circuit design be it be antennas or be it be microwave circuits. Therefore, let us go to the analysis of such structures.

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This is the dielectric slab guide



So first of all it is important to understand that we do not always need conductors for the guidance of electromagnetic waves. Dielectric slabs perform also the function of guidance of electromagnetic waves but with the difference because in addition to the waves being guided in this region the waves will also be radiated in such a region or it will exist in that region, because these 2 surfaces are dielectric to air interfaces.

We shall consider this problem to be a 2 dimensional problem, allowing no variation with the y coordinate. Therefore, along the y direction we will assume that there is no variation of electromagnetic field, so it is desired now to find the z travelling waves that is the waves travelling along the z direction with  $e^{-jk_z z}$ .

So the variation in this direction will be of the form e to the power minus j, so the waves in this direction are going to have a variation of  $e^{-jk_z z}$ . So modes either TE or TM to either x or z can be found, so modes TE and TM to either x or z can be found and we will choose the TE and TM to z modes, because the propagation is along the z direction.

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$$\begin{aligned} & \text{TM to } z \\ & \underline{\text{TM to } z}: \\ & \vec{A} = \hat{u}_z \psi \quad \text{--- (1)} \\ & H_x = \frac{\partial \psi}{\partial y}, \quad H_y = -\frac{\partial \psi}{\partial x}, \quad H_z = 0 \quad \text{--- (2)} \\ & E_x = \frac{1}{j\omega\epsilon} \frac{\partial^2 \psi}{\partial x \partial z}, \quad E_y = \frac{1}{j\omega\epsilon} \frac{\partial^2 \psi}{\partial y \partial z}, \\ & E_z = \frac{1}{j\omega\epsilon} \left( \frac{\partial^2}{\partial z^2} + k^2 \right) \psi \quad \text{--- (3)} \end{aligned}$$

So for the modes which are TM to z let us write down for modes which are TM to z we already know for the modes which are TM to z, we already know its field distribution because we have studied such modes in the case of the rectangular waveguide.

$$\begin{aligned} & \underline{\text{TM to } z}: \\ & \vec{A} = \hat{u}_z \psi \quad \text{--- (1)} \\ & H_x = \frac{\partial \psi}{\partial y}, \quad H_y = -\frac{\partial \psi}{\partial x}, \quad H_z = 0 \quad \text{--- (2)} \\ & E_x = \frac{1}{j\omega\epsilon} \frac{\partial^2 \psi}{\partial x \partial z}, \quad E_y = \frac{1}{j\omega\epsilon} \frac{\partial^2 \psi}{\partial y \partial z}, \\ & E_z = \frac{1}{j\omega\epsilon} \left( \frac{\partial^2}{\partial z^2} + k^2 \right) \psi \quad \text{--- (3)} \end{aligned}$$

So this equation also we have studied before, so all these are the field distribution of the TM to z mode, but for the current case we have del del y equal to zero.

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The screenshot shows a presentation slide with the following handwritten content:

$$\frac{\partial}{\partial y} = 0 \quad \checkmark$$
$$\frac{\partial}{\partial z} = -jk_z \quad \checkmark$$

②, ③ :

$$E_x = -\frac{k_z}{\omega\epsilon} \frac{\partial \psi}{\partial x}$$
$$E_z = \frac{1}{j\omega\epsilon} (k^2 - k_z^2) \psi$$
$$H_y = -\frac{\partial \psi}{\partial x}$$

Two vertical red lines are drawn to the right of the equations.

So for the current case we have no variation along the y, so we have

The image shows the following handwritten equations:

$$\frac{\partial}{\partial y} = 0 \quad \checkmark$$
$$\frac{\partial}{\partial z} = -jk_z \quad \checkmark$$

Therefore, equation number 2 and 3 can be written now as

The image shows the following handwritten equations:

②, ③ :

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Two vertical red lines are drawn to the right of the equations.

These are the 3 field components under under these two conditions, so these are the 3 field components under these two conditions pertaining to the dielectric slab.

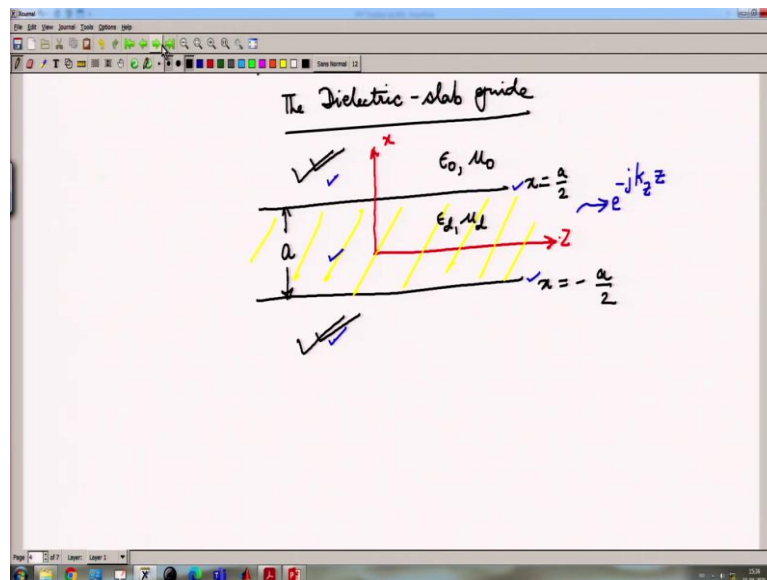
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(1)  $\psi$  is an odd function of  $x$ , denoted by  $\psi^o$   
 (2)  $\psi$  " " even " " " " " "  $\psi^e$

$\psi_d^o = A \sin(ux) e^{-jk_z z} \quad |x| < \frac{a}{2} \text{ --- (5)}$   
 (dielectric region)

Airy region:

$\psi_a^o = B e^{vx} e^{-jk_z z}, \quad x > \frac{a}{2}$   
 $\psi_a^o = -B e^{-vx} e^{-jk_z z}, \quad x < -\frac{a}{2}$



Now we shall consider two cases separately

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$\psi_d^o = A \sin(ux) e^{-jk_z z} \quad |x| < \frac{a}{2} \text{ --- (5)}$   
 (dielectric region)

In the air region we have

Air region:

$$\psi_a^0 = B e^{-vx} e^{-jk_z z}, \quad x > \frac{a}{2}$$

$$\psi_a^0 = -B e^{vx} e^{-jk_z z}, \quad x < -\frac{a}{2}$$

The change in sign of  $v$  is to also ensure that the fields decay down to zero when  $x$  equal to minus infinity and here also for  $x$  greater than  $a/2$  it is  $e$  to the power minus  $vx$ , because we have to ensure that the fields decay down to zero at  $x$  equal to plus infinity. Therefore, these 2 signs have to obey the behavior of the waves at  $x$  equal to plus infinity and  $x$  equal to minus infinity respectively.

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Separation equations:

$$\begin{aligned} u^2 + k_z^2 = k_x^2 &= \omega^2 \epsilon_d \mu_d \\ -v^2 + k_z^2 = k_0^2 &= \omega^2 \epsilon_0 \mu_0 \end{aligned} \quad \left. \vphantom{\begin{aligned} u^2 + k_z^2 = k_x^2 \\ -v^2 + k_z^2 = k_0^2 \end{aligned}} \right\} \textcircled{7}$$

$$\frac{e^{-vx}}{e} = e^{-j(jv)x}$$

$$\frac{e^{-vx}}{e} = e^{-j(-jv)x}$$

(1)  $\psi$  is an odd function of  $x$ , denoted by  $\psi^o$   
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Air region:

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} --- (6)

Now we can also write down the separation equation in each region, so the separation equations become

Separation equations:

$$u^2 + k_z^2 = k_d^2 = \omega^2 \epsilon_d \mu_d \quad \text{--- (7)}$$

$$-v^2 + k_z^2 = k_0^2 = \omega^2 \epsilon_0 \mu_0$$

$$e^{-vx} = e^{j(jv)x}$$

$$e^{-vx} = e^{-j(-jv)x}$$

So  $u$  is the eigen number along the  $x$  direction,  $k_z$  is along the  $z$  direction, there is no variation along the  $y$  direction, so the third eigen number is zero or the third eigen value is zero and we have  $k_d^2$  which is the wave number inside the dielectric, the wave number squared inside the dielectric so to say.

Now we need to evaluate the electric field components tangential to the air dielectric interface, because we have to find out the values of  $a$  and  $b$ . In order to find out the values of the unknown constants  $a$  and  $b$  we need to evaluate the tangential electric field components and the tangential magnetic field components at the dielectric air interface.

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$$\text{For } |x| < \frac{a}{2} :$$

$$E_z = \frac{A}{j\omega\epsilon_d} (k_d^2 - k_z^2) \sin(ux) e^{-jk_z z} \quad (\text{from (4), (5)})$$

$$= \frac{A}{j\omega\epsilon_d} u^2 \sin(ux) e^{-jk_z z} \quad \text{--- (8)}$$

$$(\because k_d^2 - k_z^2 = u^2 \text{ from (7)})$$

$$H_y = -\frac{\partial \psi_d^0}{\partial x} = -A u \cos(ux) e^{-jk_z z} \quad \text{--- (9)}$$

So for that purpose we write

$$\text{For } |x| < \frac{a}{2} :$$

$$E_z = \frac{A}{j\omega\epsilon_d} (k_d^2 - k_z^2) \sin(ux) e^{-jk_z z} \quad (\text{from (4), (5)})$$

$$= \frac{A}{j\omega\epsilon_d} u^2 \sin(ux) e^{-jk_z z} \quad \text{--- (8)}$$

$$(\because k_d^2 - k_z^2 = u^2 \text{ from (7)})$$

$$H_y = -\frac{\partial \psi_d^0}{\partial x} = -A u \cos(ux) e^{-jk_z z} \quad \text{--- (9)}$$

Similarly, the tangential field component  $H_y$  in the dielectric or inside the dielectric for the odd mode becomes we call this equation 9. This completes the fields inside the dielectric, so we will continue next with the fields in the air region, let us pause here.