Advanced Microwave Guided-Structure and Analysis

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Lecture No. 05

Scattering Matrix Concepts Tutorials

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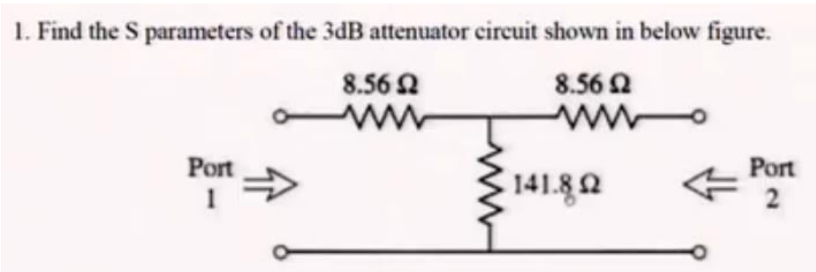
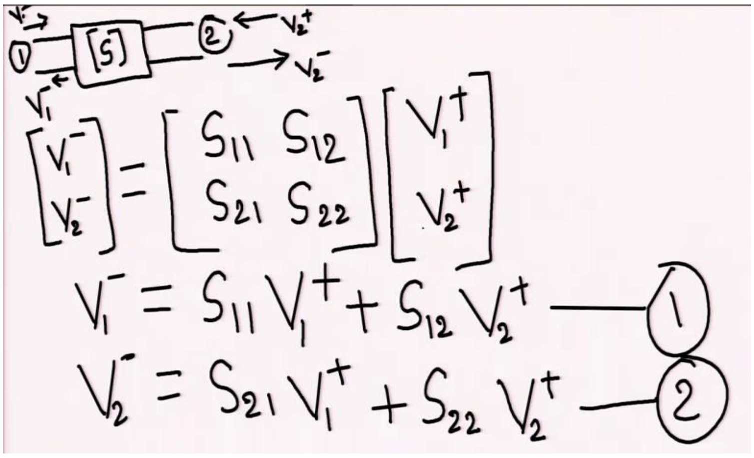




Here, for port 1, V_1^+ is incoming and V_1^- is outgoing waves. Similarly, for port 2, V_2^+ is incoming and V_2^- is outgoing.

The S parameter can be defined as

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

From this matrix situation we can write

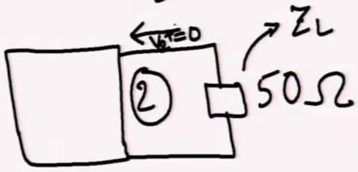
$V_1^- = S_{11} V_1^+ + S_{12} V_2^+$ and

$V_2^- = S_{21} V_1^+ + S_{22} V_2^+$.

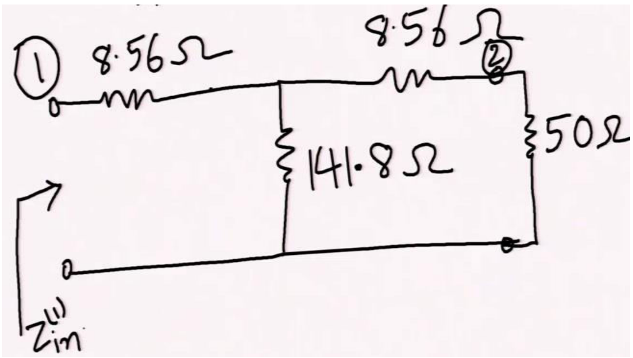
To find out the S_{11}, we can write

$S_{11} = V_1^- / V_1^+$ when $V_2^+ = 0$.

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0}$$



$$\Gamma^{(1)} = \frac{Z_{in}^{(1)} - Z_0}{Z_{in}^{(1)} + Z_0}$$



$$Z_{in}^{(1)} = 8.56 + \frac{141.8 \times (8.56+50)}{141.8 + 8.56 + 50}$$
$$= 50\Omega$$

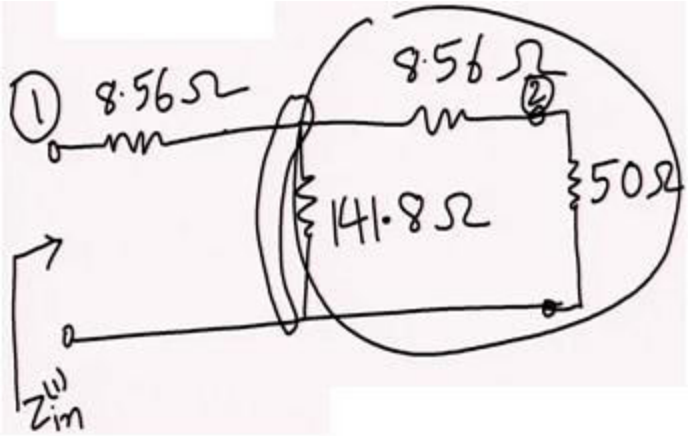
$$S_{11} = 0, \; S_{22} = 0$$

Because this network is symmetrical S_{22} be equal to 0.

After this we have to find out the remaining parameter that is S_{12} and S_{21}. So, again means S_{11}, S_{21} and S_{12} will be same, because this is symmetrical. And after this we can see that S_{21} can be defined as

$$S_{21} = \frac{V_2^-}{V_1^+}\bigg|_{V_2^+=0}$$

Using the voltage division and we can see that



$$V_1^+ = V_1,\ V_2^- = V_2$$

$$V_2^- = V_2 = V_1 \times \frac{41.44}{41.44+8.56} \times \frac{50}{50+8.56} = 0.707\, V_1$$

$$S_{21} = S_{12} = 0.707 = \frac{1}{\sqrt{2}}$$

$$\frac{|V_1^+|^2}{2Z_0} \mapsto \text{input power}$$

$$\text{Output power} = \frac{|V_2^-|^2}{2Z_0} = \frac{|S_{21}V_1^+|^2}{2Z_0}$$

$$= \frac{|S_{21}|^2}{2Z_0}|V_1^+|^2 = \frac{|V_1^+|^2}{4Z_0}$$

$$[S] = \begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix}$$

2. A two-port network is known to have the following scattering matrix

$$[S] = \begin{bmatrix} 0.15\angle 0^\circ & 0.85\angle -45^\circ \\ 0.85\angle 45^\circ & 0.2\angle 0^\circ \end{bmatrix}$$

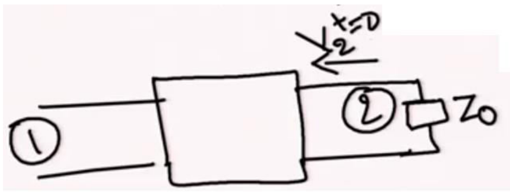
Determine if the network is reciprocal is reciprocal, and lossless. If port two is terminated with a matched load, what is the return loss seen at port 1? If port two is terminated with a short circuit, what is the return loss seen at port 1?

We can see that S_{11} is not equal to S_{22} and also S_{12} is not equal to S_{21} and this is not symmetric. So, the network is not reciprocal So, this network is not reciprocal.

Since S is not symmetric, the network is not reciprocal. These S parameters should be lossless so, it should satisfy the unitary condition. So, for the to satisfy the unitary condition we have to go for

$$|S_{11}|^2 + |S_{21}|^2 = 0.15^2 + 0.85^2 = 0.745 \neq 1$$

So, this is not a lossless network. Now next part is if the two-port is terminated with matched load what is the return loss seen at port 1.

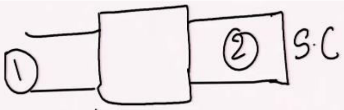


$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = 0.15$$

$$\Gamma = S_{11} = 0.15$$

$$\text{Return loss} = -20\log|\Gamma| = -20\log|0.15| = 16.5\,\text{dB}$$

Now, in the next part, port 2 is terminated with short circuit.



$$V_2^+ = -V_2^-$$

$$V_1^- = S_{11} V_1^+ + S_{12} V_2^+$$
$$= S_{11} V_1^+ - S_{12} V_2^-$$

$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+$$
$$= S_{21} V_1^+ - S_{22} V_2^-$$

$$V_2^- = \frac{S_{21}}{1+S_{22}} V_1^+$$

$$\frac{V_1^-}{V_1^+} = S_{11} - S_{12} \frac{V_2^-}{V_1^+}$$

$$= S_{11} - S_{12} \frac{S_{21}}{1+S_{22}}$$

$$\Gamma = \frac{V_1^-}{V_1^+} = S_{11} - \frac{S_{12} S_{21}}{1+S_{22}}$$

$$= 0.15 - \frac{0.85\angle -45^\circ \times 0.85\angle 45^\circ}{1+0.2}$$
$$= -0.452$$

$$RL = -20 \log |\Gamma|$$
$$= -20 \log |0.452|$$

= 6.9 dB