Advanced Microwave Guided-Structure and Analysis
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Lecture – 50
Analysis of Guided Structures (cont.)

So, welcome to this session which is a continuation of our treatment of the dielectrics lab guide. We had previously found out the field components inside the dielectric slab. Now, we investigate the field components in the air region.

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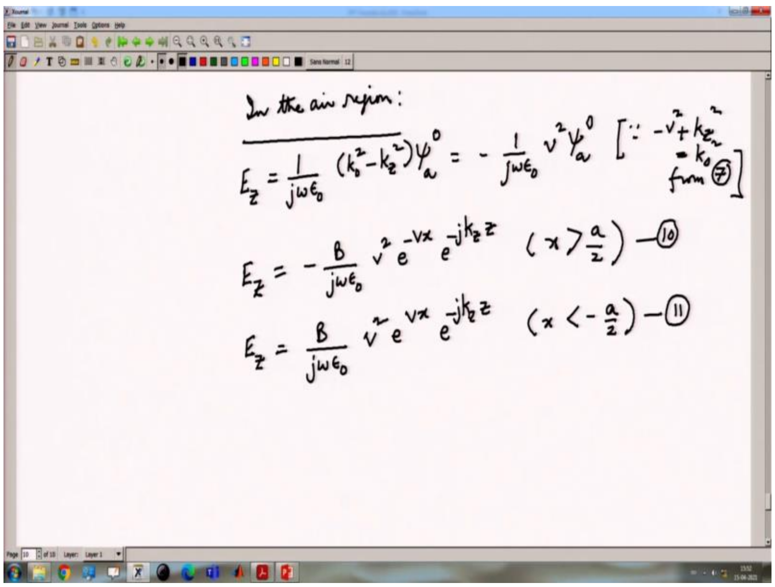
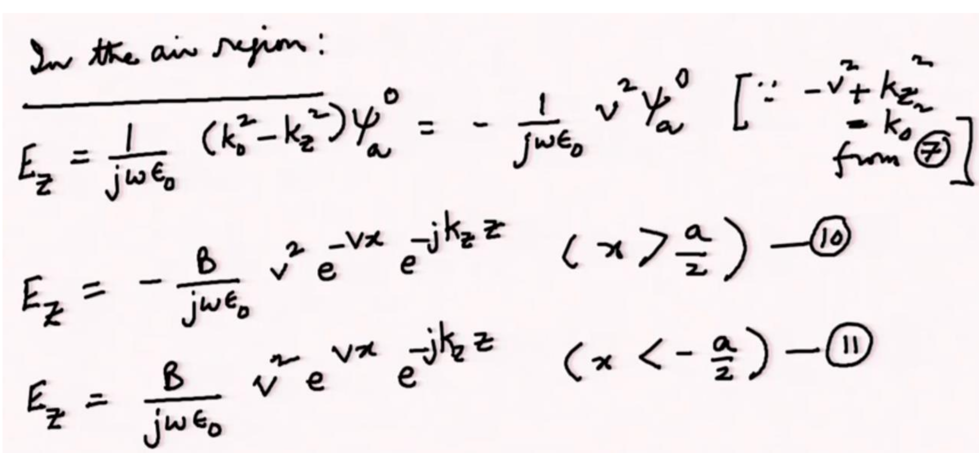


So, we write in the air region. We have Ez given by



we call this as equation-10 and equation-11.

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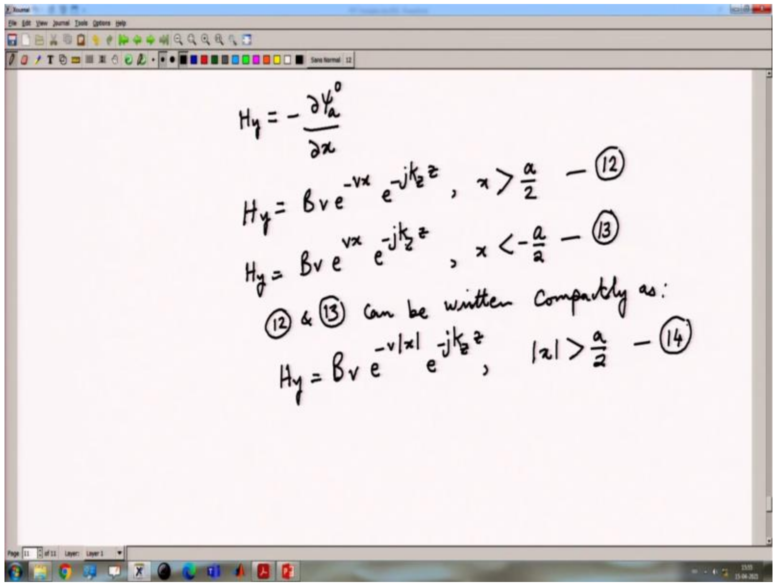
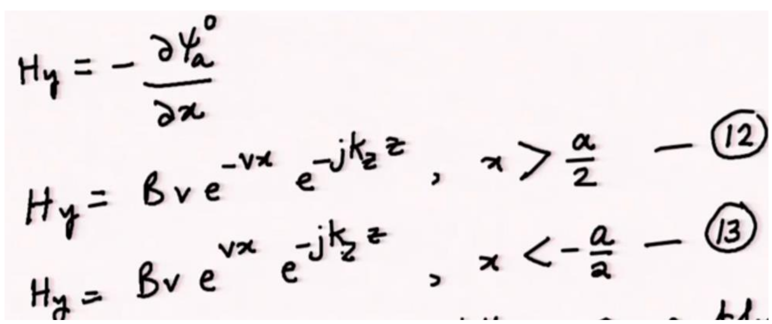
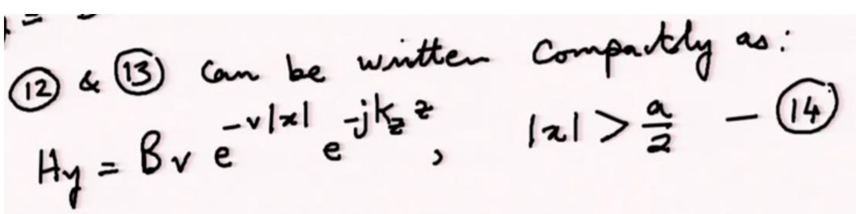


So, similarly we can write Hy in the air region as,



we call this equation-12 and equation-13.

So, 12 and 13 can be compactly written; it can be written compactly as Hy equal to



we call this 14. Now, that we have found out Ez and Hy in the dielectric and air regions. We can just apply the continuity of the fields at the dielectric air interface, in order to find out the unknown constants A and B. So, what we do is, we say.

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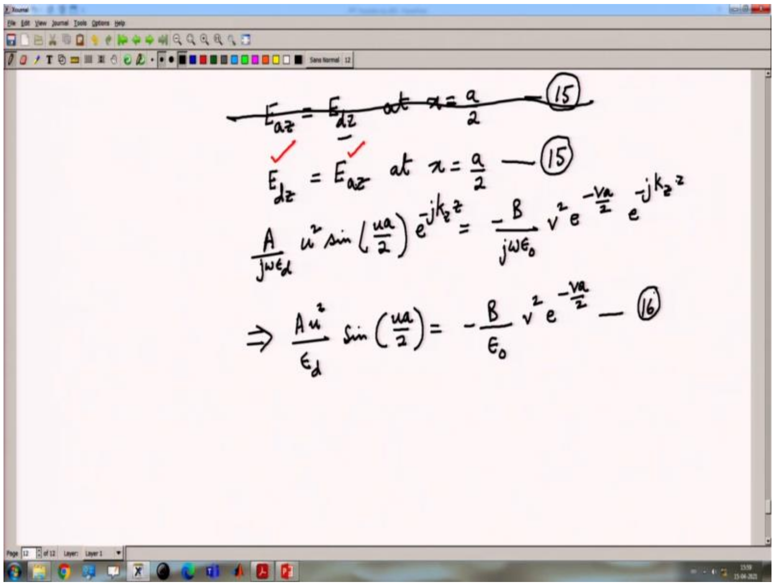
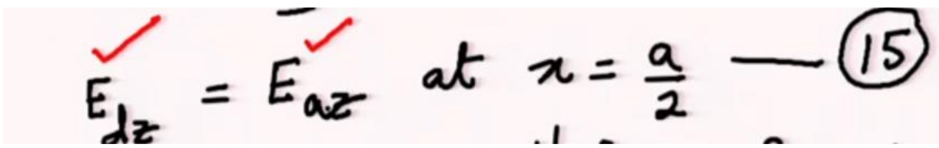
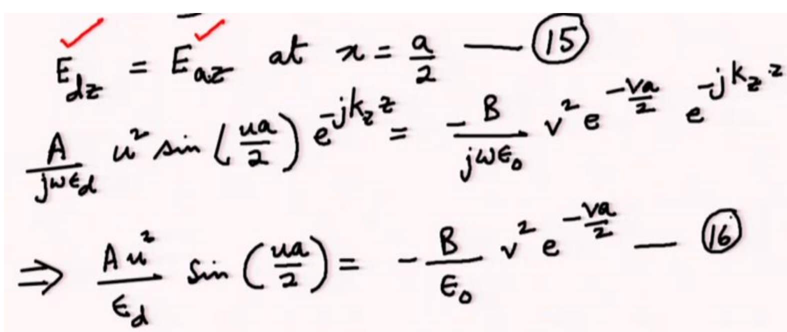


So, what we do is, we say Eaz equal to Edz at x equal to a/2; we call this 15.



Where, E_{dz} is the z component of the electric field in the dielectric region; and E_{az} is the z component of the electric field in the air region. Because they are tangential electric fields at the air dielectric interface, which is x equal to a/2; they are equated at x equal to a/2.

And when we write down the explicit forms of E_{dz} and E_{az}, which we have already obtained; we get



we call this equation-16. Similarly, we can equate the tangential magnetic field Hy at the dielectric air interface.

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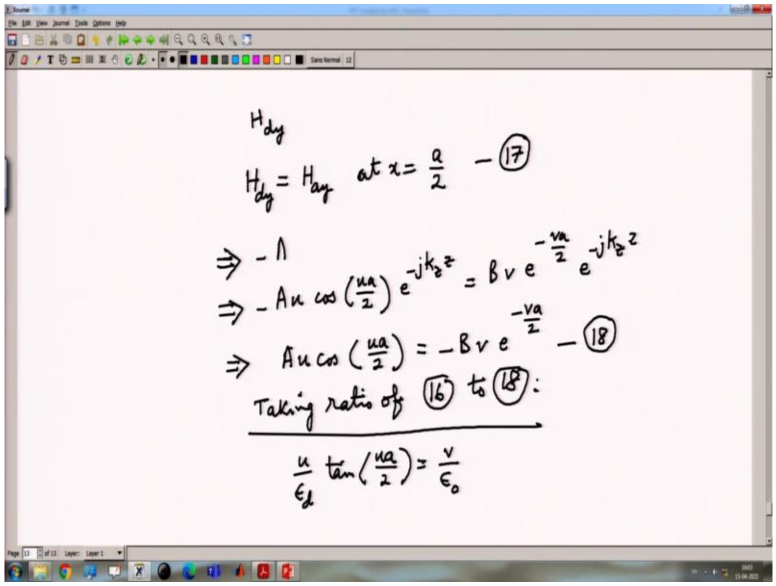

So, we write H_{dy} equal to H_{ay} at x equal to a/2;

$$H_{dy} = H_{ay} \quad \text{at } x = \frac{a}{2} \qquad -(17)$$

so we call this equation-17.

So, writing the down the explicit forms of H_{dy}, which is the y component of the magnetic field inside the dielectric; and H_{ay} which is the y component of the magnetic field in the air region. Both evaluated at x equal to a/2.

We obtained this as

$$\Rightarrow -Au\cos\left(\frac{ua}{2}\right)e^{-jk_z z} = Bve^{-\frac{va}{2}}e^{-jk_z z}$$

$$\Rightarrow Au\cos\left(\frac{ua}{2}\right) = -Bve^{-\frac{va}{2}} \qquad -(18)$$

so this is equation-18. So, now we all have to do is to take the ratio of equation number-16 and 18.

16 was obtained by equating the tangential electric field E_z across the air dielectric interface. 18 was obtained by applying the continuity of H_y across the air dielectric interface. So, if I take the ratio of 16 to 18 taking ratio of 16 to 18 in order to drive out A and B. When we take the ratio 16 to 18, we remove A and B, the unknown constants. So, we get

Taking ratio of (16) to (18):

$$\frac{u}{\epsilon_d} \tan\left(\frac{ua}{2}\right) = \frac{v}{\epsilon_0}$$

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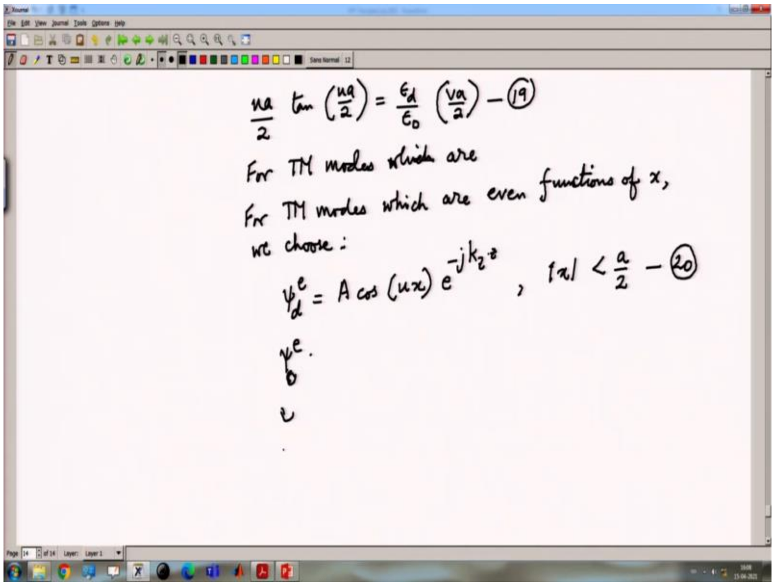

Or, we can write this equation as equation-19.

$$\frac{ua}{2} \tan\left(\frac{ua}{2}\right) = \frac{\epsilon_d}{\epsilon_0}\left(\frac{va}{2}\right) \quad -(19)$$

So, we can use the functional form of x tan x. So you, if you look at equation number-19 and you look back at equation number7; equation-7 links u and v to k_z. So, if I replace u and v in equation-19 with k_z obtained from equation-7; we will obtain the characteristic equation for determining the k_z, and the cut off frequencies for the odd TM modes.

So, replacing u and v by the separation equation and expressing it in terms of k_z; we need a single equation in terms of k_z from which k_z can be obtained. And thereafter we can find the cut off frequencies for the odd TM modes. Now, for the TM modes which are even functions of x, a similar procedure can be invoked to find out the propagation constants and the cut off frequencies of the corresponding TM to z even modes.

For TM modes which are even functions of x, a similar procedure can be invoked in order to find out the propagation constant k_z, and the cut off frequencies for the TM even modes. So, we first of all choose the corresponding psi function, which will be a even distribution with respect to the x direction. So, psi d even which is

For TM modes which are even functions of x, we choose:

$$\psi_d^e = A\cos(ux)\, e^{-jk_z z}, \quad |x| < \frac{a}{2} \quad \text{—} \; (20)$$

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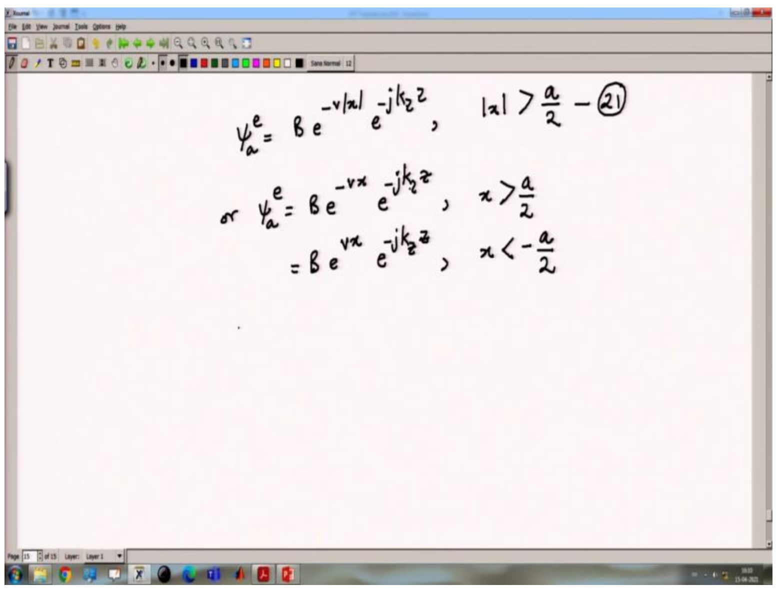

And psi $_a$ even is given by

$$\psi_a^e = B e^{-v|x|} e^{-jk_z z}, \quad |x| > \frac{a}{2} \quad \text{—} \; (21)$$

$$\text{or} \quad \psi_a^e = B e^{-vx} e^{-jk_z z}, \quad x > \frac{a}{2}$$

$$= B e^{vx} e^{-jk_z z}, \quad x < -\frac{a}{2}$$

So, now we can compute the electric and magnetic fields in the dielectric region and the air region.

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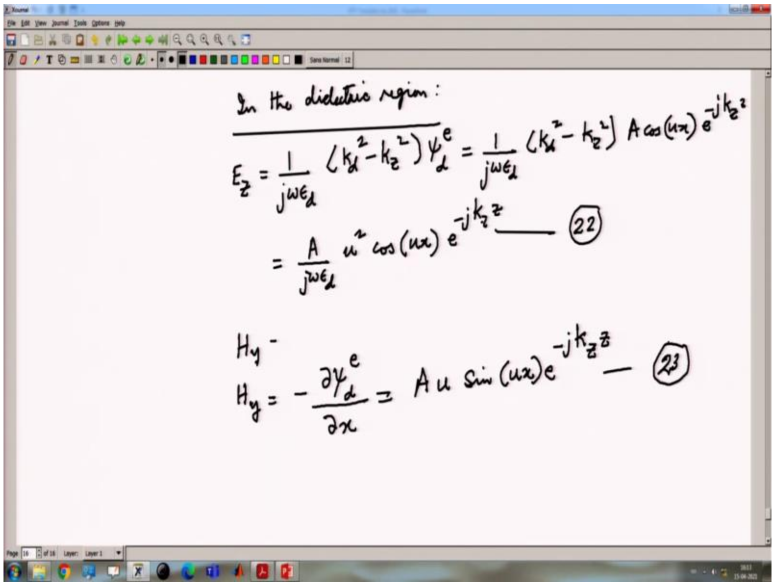

So, in the dielectric region, we have Ez

In the dielectric region :

$$E_z = \frac{1}{j\omega\epsilon_d}\left(k_d^2 - k_z^2\right)\psi_d^e = \frac{1}{j\omega\epsilon_d}\left(k_d^2 - k_z^2\right) A\cos(ux)\, e^{-jk_z z}$$

$$= \frac{A}{j\omega\epsilon_d} u^2 \cos(ux)\, e^{-jk_z z} \quad \text{——} \quad (22)$$

we can call this equation-22.

Similarly, we find H_y in the dielectric region; And that is given

$$H_y = -\frac{\partial \psi_d^e}{\partial x} = A u \sin(ux) e^{-jk_z z} \quad \text{—} \quad (23)$$

so this is equation-23. So, this completes the electric and magnetic fields in the dielectric region, for the TM even mode. So, we will continue for the fields inside the air region in our next lecture; let us stop here.