**Advanced Microwave Guided-Structure and Analysis**
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**Lecture – 51**
**Analysis of Guided Structures (cont.)**

Welcome to this session of the lecture on the dielectric slab which is a continuation of the previous lecture. In the last lecture we discussed the electric and magnetic fields inside the dielectric region for the TM even mode. We will continue from there and we will find out the electric and magnetic fields in the air region for the TM even mode, for subsequent matching of the electric and magnetic fields across the dielectric air interface, in order to evaluate the propagation constant of the TM even mode.

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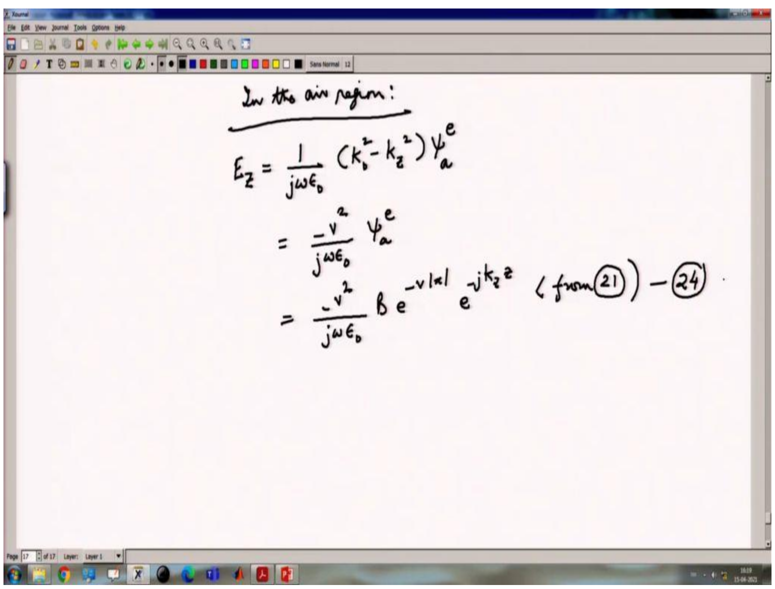
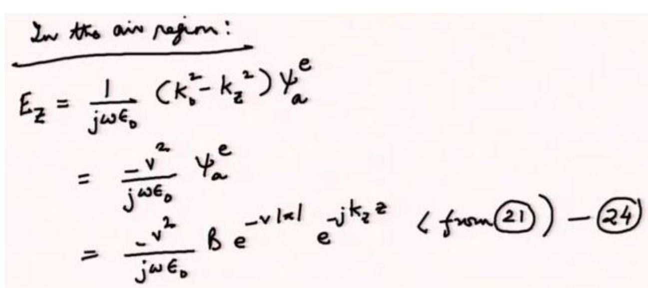



So, in the air region, electric field $E_z$ is given by



we call this equation-24. Similarly, we can find out Hy, the tangential magnetic field at the in the air region.

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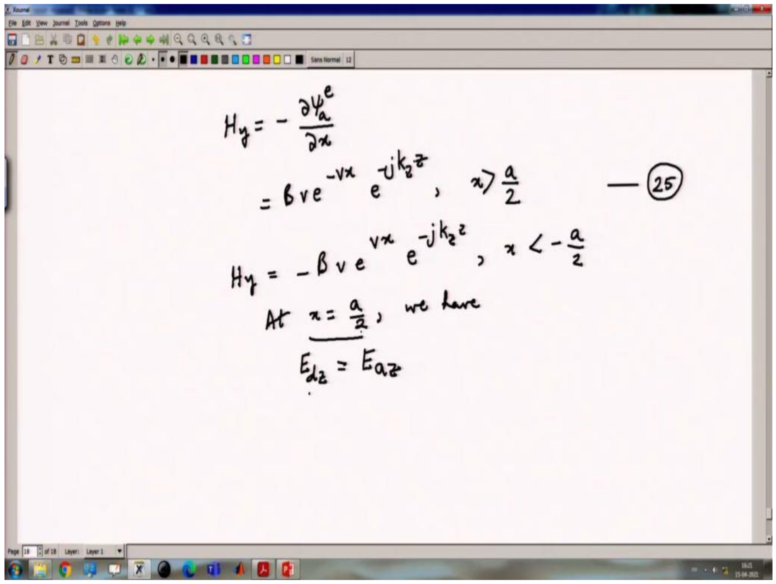


So, $H_y$ is given by

$$H_y = -\frac{\partial \psi_a^e}{\partial x} = \beta v e^{-vx} e^{-jk_z z}, \quad x > \frac{a}{2} \qquad \text{— (25)}$$

we call this equation-25. So, now we equate the fields at x equal to a/2. So, at x equal to a a/2, we have $E_{dz}$ equal to $E_{az}$; so the electric field is not in the dielectric, must be equal to the electric field Ez in the air region at x equal to a/2. So, therefore we can write expanding $E_{dz}$ and $E_{az}$.

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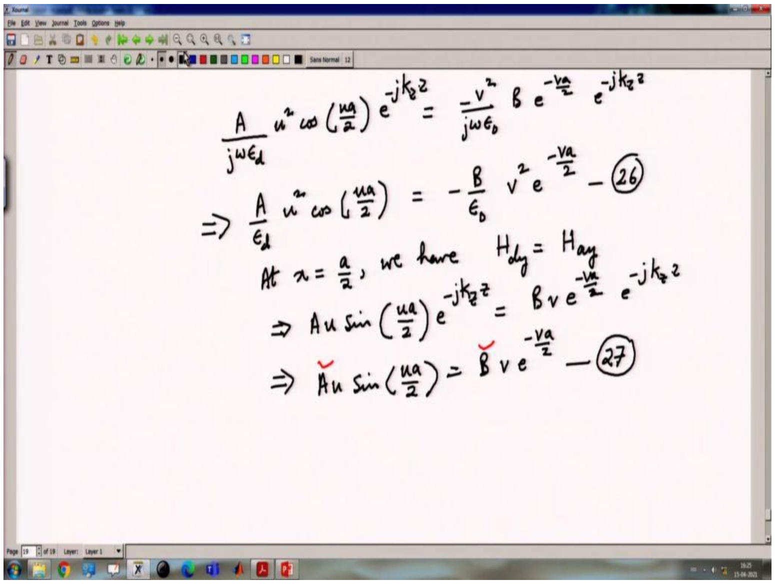

$$\frac{A}{j\omega\epsilon_d} u^2 \cos\left(\frac{ua}{2}\right) e^{-jk_z z} = \frac{-v^2}{j\omega\epsilon_D} B e^{-\frac{va}{2}} e^{-jk_z z}$$

$$\Rightarrow \frac{A}{\epsilon_d} u^2 \cos\left(\frac{ua}{2}\right) = -\frac{B}{\epsilon_D} v^2 e^{-\frac{va}{2}} \quad \text{— (26)}$$

so this is equation-26.

At x equal to a/2 we also have the equality of the tangential magnetic field components $H_y$. So, we write at x equal to a/2, we have

At $x = \frac{a}{2}$, we have $H_{dy} = H_{ay}$

$$\Rightarrow A u \sin\left(\frac{ua}{2}\right) e^{-jk_z z} = B v e^{-\frac{va}{2}} e^{-jk_z z}$$

$$\Rightarrow A u \sin\left(\frac{ua}{2}\right) = B v e^{-\frac{va}{2}} \quad \text{— (27)}$$

let us call this equation-27.

So, we have equated the tangential electric field $E_z$ at x equal to a/2; and the tangential magnetic field $H_y$ at x equal to a/2. In order to drive out these unknown constants A and B; we will again take the ratio between 26 and 27.

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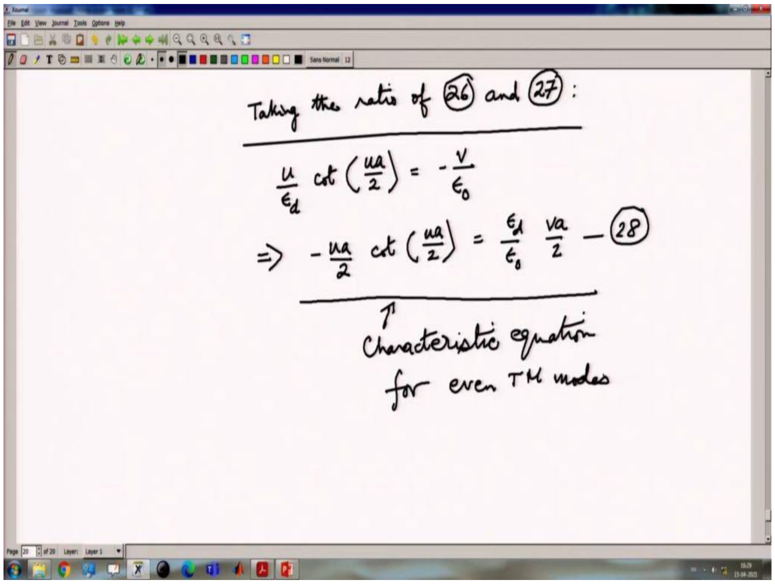
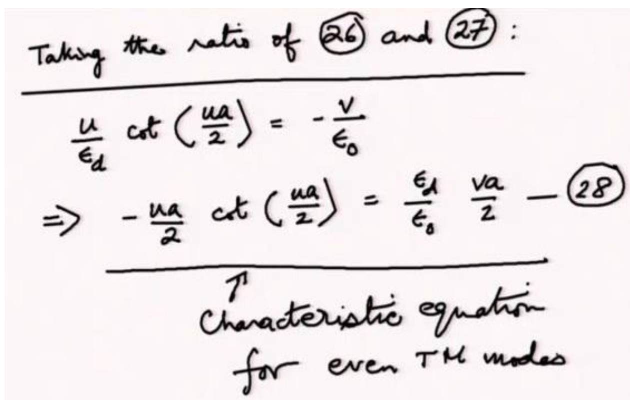



So, taking the ratio of 26 and 27, we obtain




call this equation-28, in order to use the form of the function, the generic form of the function x cot x.

So, this is the characteristic equation for determining the $k_z$ and cut-off frequencies of the even TM modes; because u and v can be expressed in terms of $k_z$ from the separation equation. So, equation-28 can be recast totally in terms of $k_z$ replacing u and v; that will yield a single equation in $k_z$, from which $k_z$ can be obtained.

So, this is the characteristic equation for determining the $k_z$, and the cut-off frequencies for the TM even modes. So, there is a complete duality between the TM and TE modes of the slab waveguide. So, the characteristic equations of the TE modes are just dual to those of the TM modes.

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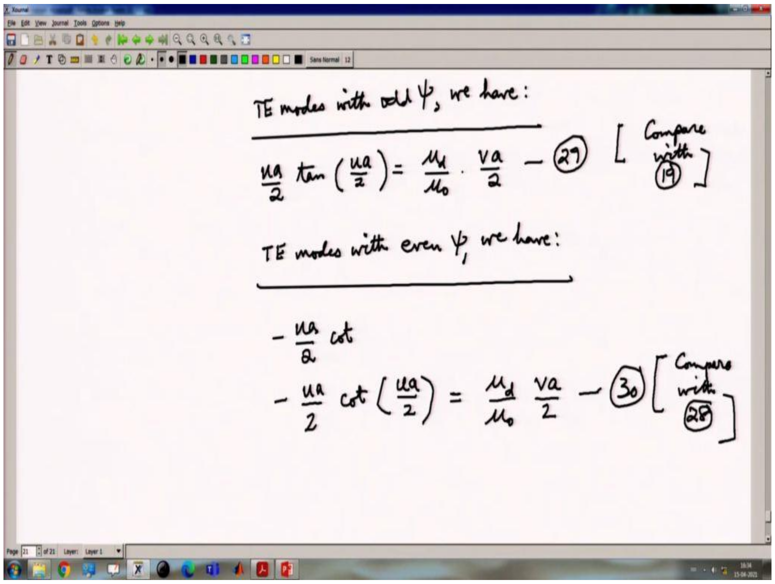
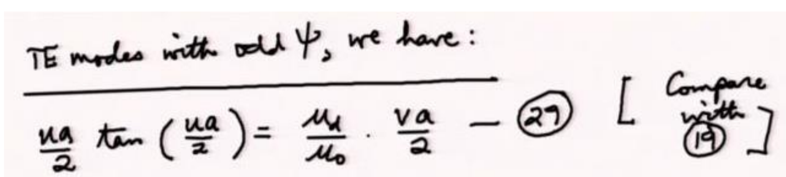



We can write down the characteristic equation for the TE modes with odd psi, which can be evaluated in exactly the same manner as we did for the TM modes with odd psi. So, for the TE modes with odd psi, we have




we marked this as equation-29.

Compare with 19, which is the characteristic equation for the odd TM modes. So, we see that $\varepsilon_d$ is replaced by $\mu_d$ and $\varepsilon_0$ by $\mu_0$. So, for the TE modes with even psi, after performing exactly the same analysis as we did for the TM modes with even psi, we will obtain the characteristic equation.

TE modes with even $\psi$, we have:

$$-\frac{ua}{a}\cot$$

$$-\frac{ua}{2}\cot\left(\frac{ua}{2}\right) = \frac{\mu_d}{\mu_0}\frac{va}{2} \quad \text{— (30)} \quad \left[\text{Compare with (28)}\right]$$

so this is equation-30.

Again compare this with equation-28; so, for the even TM modes. We see that $\varepsilon_d$ is replaced by mu d, and $\varepsilon_0$ by $\mu_0$.

We should also note that odd functions generating the TE modes, or the odd potential functions generating the TE modes, are given by the same equations as the TM modes; which are equations 5 and 6. And similarly the even functions generating the TE modes, are given by exactly the same equations from which the TM even modes are obtained.

The same potential functions from which the TM even modes are obtained; and those are equations 20 and 21. So, the fields are obtained from the psi by equations, which are dual to equation-4. So, for the TE modes, the fields are obtained from the psi by the equations which are exactly dual to equations 4; that are relevant for the TM modes.

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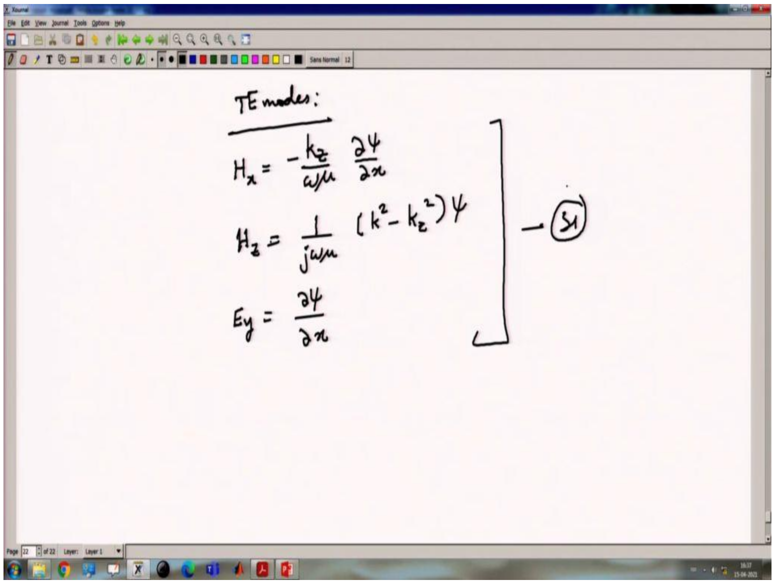
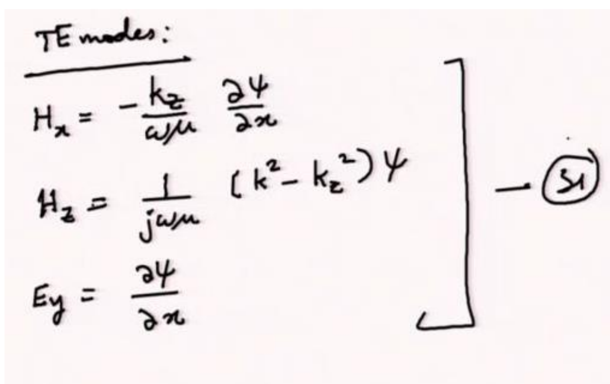



So, for the TE modes, the fields are obtained from the psi functions as




let us call these sets of equations as 31.

Now, because the dielectric guide is an open structure, the concept of cut-off frequency for such guides is somewhat different compared to metallic guides; where the fields do not escape from inside the metal. So, what is happening here? So, we have the fields which are sinusoidally or co-sinusoidally dependent on the x coordinate inside the dielectric slab.

And which are exponentially decaying above and below the dielectric slab. So, under such a condition the guidance takes place inside the slab. So, we say that above the cut-off frequency, the dielectric guide propagates a mode unattenuated; that means $k_z$ which is the propagation constant along the z direction is real.

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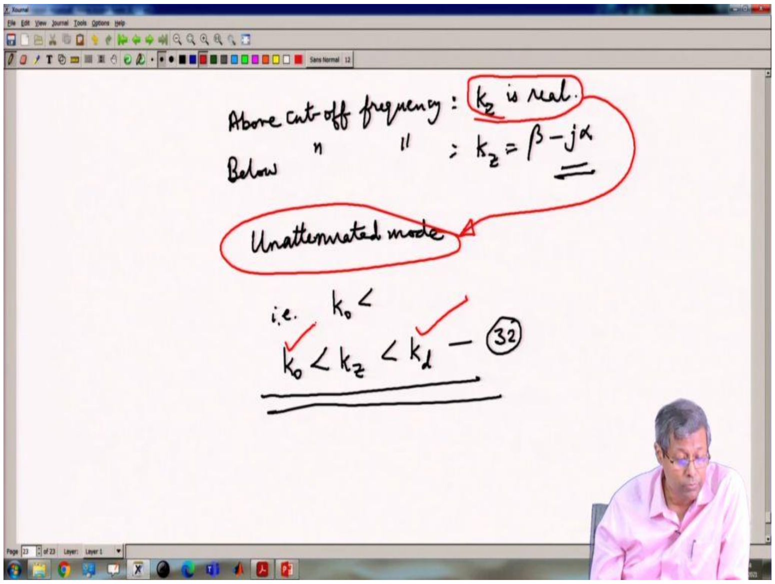


So, we write above the cut-off frequency $k_z$ is real. However, below the cut-off frequency below the cut-off frequency, $k_z$ becomes equal to beta minus j alpha. But, since the dielectric is loss-free, this attenuation of the wave which is the complex part of $k_z$ is due to the radiation of energy in the air region as the wave propagates inside the dielectric.

So, as the wave propagates inside the dielectric is it progressively loses energy in the air region. This condition is what characterizes the wave below cut-off. So, this is saliently different from metallic waveguide as we saw, where there is no radiation of electromagnetic waves; because the waves are only confined between the metallic walls of the guide. So, therefore the dielectric guides operating below cut-off can be used as antennas; because they are radiating energy to free space.

And therefore, I can make the same dielectric strip or dielectric guide operate as a guided wave medium with $k_z$ real above cut-off; which are called essentially called surface waves inside the dielectric. Or, I can make it work as an antenna, so that these waves have complex propagation constant. And therefore, as they proceed or as they progress inside the dielectric slab, they progressively lose energy in the air region.

This in fact, marks the below cut-off region of the dielectric slab, where the dielectric slab can be used as an antenna. It is very important to understand, therefore that a lossless dielectric slab can have an imaginary propagation constant. This is a very important thing to appreciate that the dielectric strip though perfectly assume lossless can have an imaginary propagation constant.

And this imaginary propagation constant is due to the radiation of electromagnetic waves in the air region, as the wave the TE or TM mode progresses inside the dielectric slab. Now, the question is what demarcates below cut-off and above cut-off? So, the phase constant of an unattenuated mode.

So, if the mode is unattenuated, which means that kz is real for this condition which is an unattenuated mode. It lies between the phase constant of the dielectric and that of the air region. So, the propagation constant or phase constant of an unattenuated mode will lie between the intrinsic phase constants of the dielectric and the air region.

That is we will have $k_0$ less than $k_z$ less than $k_d$, 32. We go back to equation number-7; we see that $k_z$ is less than $k_d$. Now, if $k_z$ is greater than $k_0$, then v is real. So, therefore if v is real, then the distribution of fields in the air region is exponentially decaying; or it is of the exponentially decaying time. So, have this very clear picture in mind that when a wave propagates inside the dielectric slab above cut-off. Or, when the dielectric slab is being operated above cut-off, the waves in the air region are of the exponentially decaying type, while $k_z$ is real. This condition is mathematically the same as this condition, that $k_z$ will lie between the intrinsic phase constant in the dielectric region, which is $k_d$.

And the intrinsic phase constant in the air region, which is $k_0$. So, the lowest frequency for which the unattenuated propagation exists is called the cutoff frequency. This we have borrowed from the concept of cut-off frequencies in the waveguide. So, the lowest frequency for which $k_z$ is real; we call it the cut-off frequency. So, therefore, cut-off occurs as $k_z$ tends to 0.

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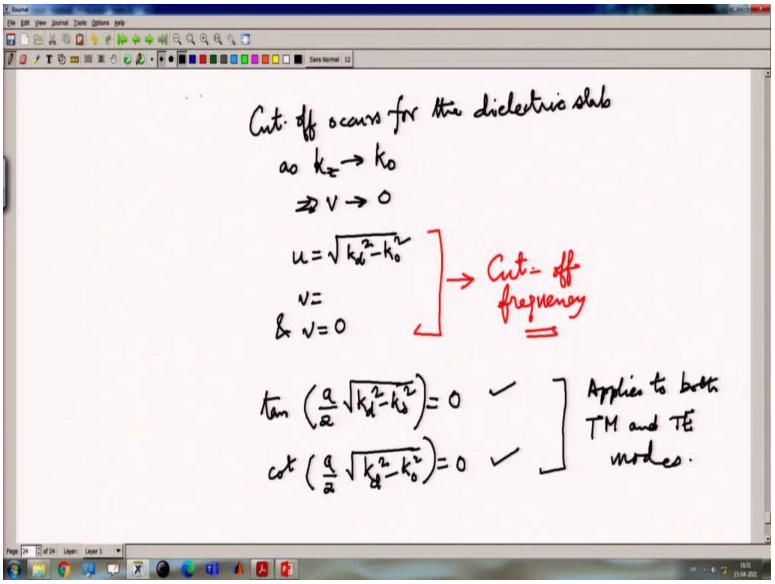
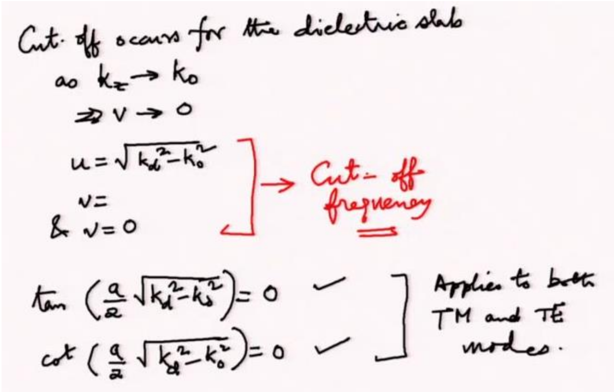



Cut-off occurs for the dielectric slab as $k_z$ tends to $k_0$. Because if $k_z$ becomes lesser than $k_0$, then v or the Eigen number or the Eigen value in the air region is going to be complex; so, $k_z$ is lesser than $k_0$, v or the Eigen number in the air region will become complex. And therefore, we will have propagation in the air region not evanescence in the air region; not exponentially decaying waves in the air region.

We will start to have propagation in the air region; because we will turn out to be complex. So, it will be instead of e to the power minus vx, e to the minus j v prime x; so, there will be propagation in the air region. So, therefore, the dielectric slab is going to radiate in the air region. Therefore, its energy will reduce as it propagates inside the lossless dielectric strip.

This condition or this behavior of the dielectric slab will occur below cutoff, when the dielectric strip will therefore behave like an antenna. So, $k_z$ approaching $k_0$ is the same as v approaching 0. And therefore, the cutoff frequencies can be obtained from the characteristic equations by using or by setting




So, this constitutes the cut-off frequency of the dielectric slab guide. And when we do that when we substitute them in the characteristic equations, we obtained the above equation. So, we can look at equation number-19, and substitute u.

Since, v tends to 0, the right-hand side of 19 tends to 0. And because, so this equation can be obtained from equation-19; when we look at equation-19, we substitute v equal to 0. So, the right-hand side of equation-19 becomes equal to 0, and we obtain the above condition. Similarly, we can look at equation number-28, and from there we will obtain the condition. So, this is for the odd TM mode, this is for the even TM mode; and the same equation holds good for the TE modes; because the characteristic equations are the same; only change is replacing epsilon by mu. So, these apply to both applies to both the TE and TM modes.

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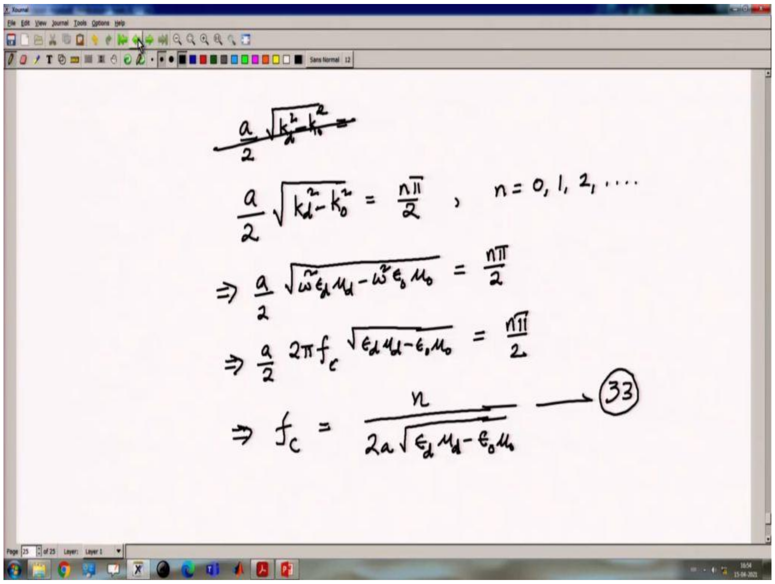

So, these equations are satisfied

$$\frac{a}{2}\sqrt{k_d^2 - k_0^2} = \frac{n\pi}{2}, \quad n = 0, 1, 2, \ldots$$

$$\Rightarrow \frac{a}{2}\sqrt{\omega^2 \epsilon_d \mu_d - \omega^2 \epsilon_0 \mu_0} = \frac{n\pi}{2}$$

$$\Rightarrow \frac{a}{2}\, 2\pi f_c \sqrt{\epsilon_d \mu_d - \epsilon_0 \mu_0} = \frac{n\pi}{2}$$

$$\Rightarrow f_c = \frac{n}{2a\sqrt{\epsilon_d \mu_d - \epsilon_0 \mu_0}} \longrightarrow (33)$$

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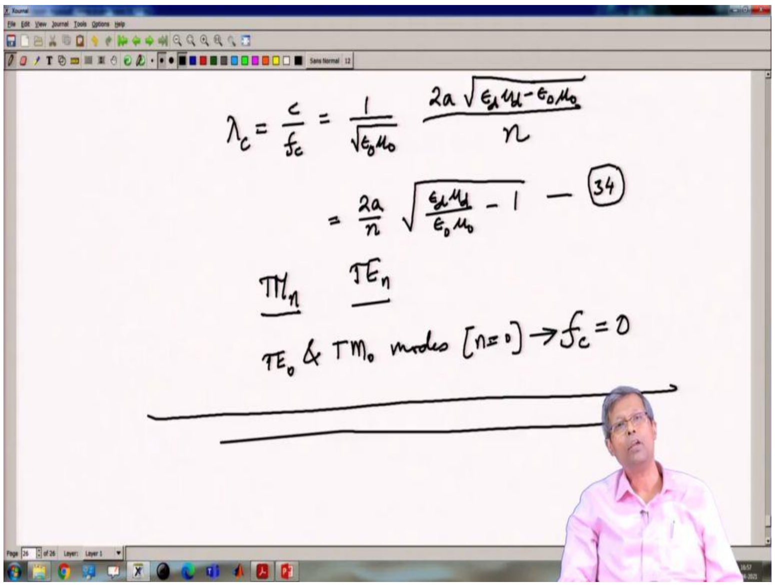


The corresponding cut-off wavelength lambda c becomes equal to

$$\lambda_c = \frac{c}{f_c} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \frac{2a\sqrt{\epsilon_d \mu_d - \epsilon_0 \mu_0}}{n}$$

$$= \frac{2a}{n}\sqrt{\frac{\epsilon_d \mu_d}{\epsilon_0 \mu_0} - 1} \quad - (34)$$

we call this expression or the cut-off wavelength lambda c as equation-34.

So, the modes are ordered as TM n, the modes are TM n and TE n; so these are the modes in the dielectric slab; or called also the dielectric slab modes. So, we should also note that the fc or the cut-off frequency corresponding to the $TE_0$ mode and the $TM_0$ modes. So, for the $TE_0$ and the $TM_0$ modes for n equal to 0; the corresponding cut-off frequency is 0.

So, the $TE_0$ modes and the $TM_0$ modes in the slab possess a zero cut-off frequency; that is the propagate right from dc. So, this concludes the session on the evaluation of propagation constant inside substrate based structures, or the dielectric slab. So, we now have a much more useful a physically meaningful insight for the design of substrate based circuits and antennas.

We have a clear understanding of what is meant by propagation in a dielectrics slab; what is meant by cut-off in a dielectric slab? What happens in propagation? What happens in cut-off? What are the relevant mathematical parameters characterizing the propagation region and the cut-off region, which is invaluable for the understanding of any kind of substrate based circuits? Thank for your attention.