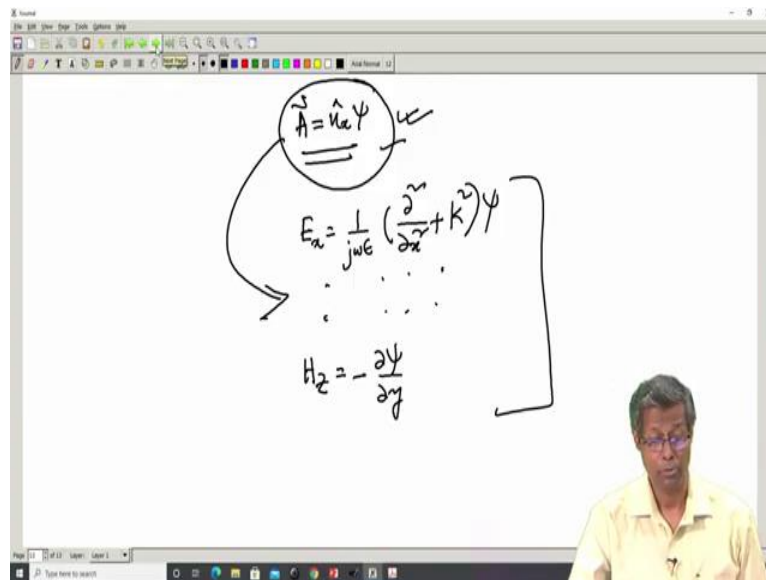


Advanced Microwave Guided-Structures and Analysis
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Lecture 54
Analysis of Guided Structures (cont.)

Welcome to the continuation of the lecture on the non-radiating dielectric guide. So, in this part we are going to find out the explicit field expressions, in the dielectric and air regions for the LSM and the LSE modes of the NRD. And thereafter we are going to move to the characteristic equations for the NRD, which is going to yield a complete a set of equations for characterizing the NRD, and its dispersion characteristics.

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So, in our previous lecture, we had explicitly written down the potential functions for the LSM even and the LSM odd modes, for the NRD, in the air and dielectric regions, from these potential functions as we said that, we can use the expressions, which relate the electric and magnetic fields with the psi function, for this substitution.

So, the electric and magnetic fields resulting out of this condition

$$\vec{A} = \hat{u}_x \Psi$$

$$E_x = \frac{1}{j\omega\epsilon} \left(\frac{\partial^2}{\partial x^2} + k^2 \right) \Psi$$

$$\dots$$

$$H_z = -\frac{\partial \Psi}{\partial y}$$

So, they can be used to find out the electric and magnetic fields of the LSM modes, because this characterizes the LSM modes, so as we said that there is no normal component of the magnetic field to the air dielectric interface. So, it is a TM to x mode.

So, therefore this equation characterizes the LSM modes, and therefore we have already derived the relationship between the psi function and the electric and magnetic fields for this mode, or we have expressed the electric and magnetic field in terms of the psi function for this kind of substitution before. So, we use exactly those electric and magnetic fields resulting out of this condition, of the magnetic vector potential, which characterizes the TM to x mode.

So, we are not going to write those electric and magnetic field equations, because we already know them, what we are you going to use them in order to find out the electric and magnetic fields for the LSM even and LSM odd modes. So, I can just write for reference the electric field. So,

$$\vec{A} = \hat{u}_x \Psi$$

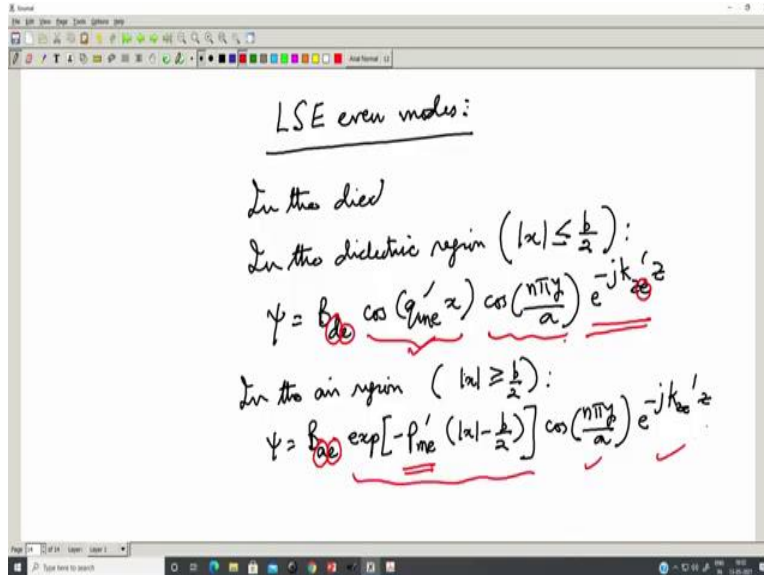
$$E_x = \frac{1}{j\omega\epsilon} \left(\frac{\partial^2}{\partial x^2} + k^2 \right) \Psi$$

$$\dots$$

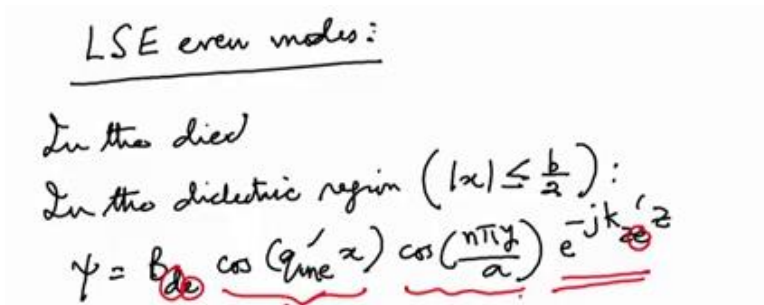
$$H_z = -\frac{\partial \Psi}{\partial y}$$

So, you go through all those sets of equations for this condition, which we derived before. And from those equations we can find out the explicit forms of the electric and magnetic fields for the LSM even and LSM odd modes.

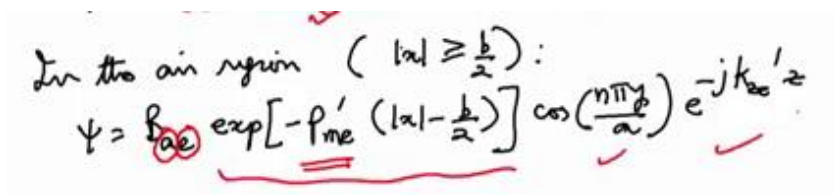
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So, now we come to the potential functions for the LSE even modes. So, we are not going to explain too much here, we are just going to write it down for reference sake, because we are already now familiar with the LSM even and odd modes. So, in the dielectric region,



So, where there is propagation, so this again refers to the even mode. So, similarly in the air region,



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LSE odd modes:

In the dielectric region ($|x| \leq \frac{1}{2}$):

$$\psi = B_{d0}$$

$$\psi = B_{d0} \sin(q'_{m0} x) \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{z0}' z}$$

In the air region ($|x| \geq \frac{1}{2}$):

$$\psi = B_{a0} \exp[-p'_{m0} (|x| - \frac{1}{2})] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{z0}' z}$$

$$= -B_{a0} \exp[-p'_{m0} (|x| - \frac{1}{2})] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{z0}' z}$$

(for $x \geq \frac{1}{2}$)

(for $x \leq -\frac{1}{2}$)

LSE even modes:

In the dielectric region ($|x| \leq \frac{1}{2}$):

$$\psi = B_{d0} \cos(q'_{me} x) \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{z0}' z}$$

In the air region ($|x| \geq \frac{1}{2}$):

$$\psi = B_{a0} \exp[-p'_{me} (|x| - \frac{1}{2})] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{z0}' z}$$

So, again if I write it now for the LSE odd mode, in the dielectric region,

LSE odd modes:

In the dielectric region ($|x| \leq \frac{1}{2}$):

$$\psi = B_{d0}$$

$$\psi = B_{d0} \sin(q'_{m0} x) \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{z0}' z}$$

In the air region,

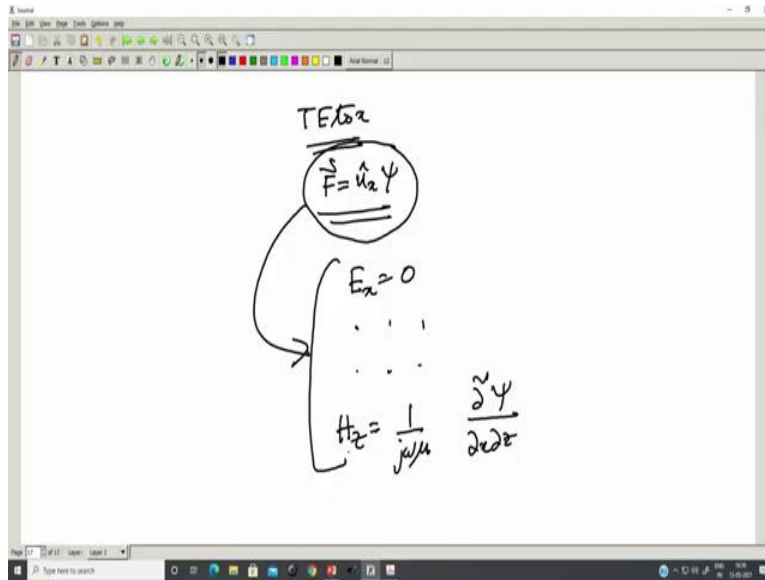
In the air region: ($|x| \geq \frac{b}{2}$):

$$\Psi = B_{a0} \exp[-\rho'_{m0} (|x| - \frac{b}{2})] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{z0}'z} \quad (\text{for } x \geq \frac{b}{2})$$

$$= -B_{a0} \exp[-\rho'_{m0} (|x| - \frac{b}{2})] \cos\left(\frac{n\pi y}{a}\right) e^{+jk_{z0}'z} \quad (\text{for } x \leq -\frac{b}{2})$$

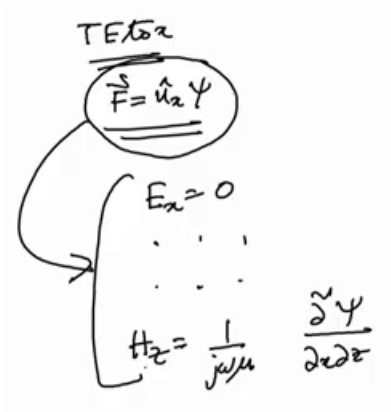
And the reason why for the odd mode these are two different expressions is the same as the LSM odd mode. As to why these are two separate expressions, because it is the odd mode and we have to match the fields on both the regions.

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Now, from these potential functions, because the LSE mode is nothing but the TE to x mode, it is the TE to x mode because there is no electric field along, or no electric field, because there is no electric field normal to the air dielectric interface. So, it is transverse electric to x.

So, therefore it can be obtained from the expression



The expressions for the electric and magnetic fields we obtained for this condition are exactly the same electric and magnetic fields, we would obtain for both the LSE even and LSE odd modes because they are TE to x modes.

So, I do not need to write those equations explicitly, because we have already covered those equations. But for references sake I will just write down the value of E_x . So, E_x was 0, and all the way down if you had gone to H_z , I have already covered all these six sets of equations.

And therefore I will not write them down again. So, using these expressions, the explicit expressions for the electric and magnetic fields for all the modes can be written down. So, we can quickly write down the expressions for the modes.

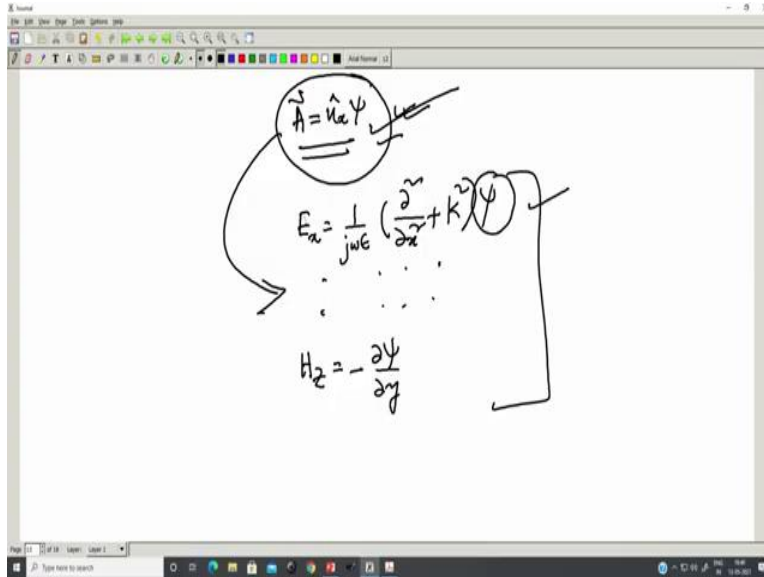
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LSK even mode

In the dielectric region ($|x| \leq \frac{b}{2}$):

$$E_x = \frac{Ade (k_0 \epsilon_r - \gamma_{me}^2)}{j\omega \epsilon_0 \epsilon_r} \cos(\gamma_{me} x) \sin\left(\frac{n\pi y}{a}\right) e^{jk_z z}$$
$$E_y = \frac{-Ade \gamma_{me} \left(\frac{n\pi}{a}\right)}{j\omega \epsilon_0 \epsilon_r} \sin(\gamma_{me} x) \cos\left(\frac{n\pi y}{a}\right) e^{jk_z z}$$
$$E_z = \frac{Ade k_{ze} \gamma_{me}}{\omega \epsilon_0 \epsilon_r} \sin(\gamma_{me} x) \sin\left(\frac{n\pi y}{a}\right) e^{-jk_z z}$$

$$H_x = 0$$
$$H_y = -Ade j k_{ze} \cos(\gamma_{me} x) \sin\left(\frac{n\pi y}{a}\right) e^{-jk_z z}$$
$$H_z = -\frac{Ade n\pi}{a} \cos(\gamma_{me} x) \cos\left(\frac{n\pi y}{a}\right) e^{-jk_z z}$$



So, for the LSM even mode in the dielectric region, which is

LSM even mode

In the dielectric region ($|x| \leq \frac{b}{2}$):

$$E_x = \frac{Ade (k_0^2 \epsilon_r - \gamma_{me}^2)}{j\omega\epsilon_0 \epsilon_r} \cos(\gamma_{me} x) \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{ze} z}$$

$$E_y = \frac{-Ade \gamma_{me} \left(\frac{n\pi}{a}\right)}{j\omega\epsilon_0 \epsilon_r} \sin(\gamma_{me} x) \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{ze} z}$$

$$E_z = \frac{Ade k_{ze} \gamma_{me}}{\omega\epsilon_0 \epsilon_r} \sin(\gamma_{me} x) \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{ze} z}$$

$$H_x = 0$$

$$H_y = -Ade j k_{ze} \cos(\gamma_{me} x) \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{ze} z}$$

$$H_z = -\frac{Ade n\pi}{a} \cos(\gamma_{me} x) \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{ze} z}$$

This completes the set of electric and magnetic fields in the dielectric region.

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In the air region ($|x| \geq \frac{b}{2}$):

$$E_x = \frac{A_{ae} (\rho_{me} + k_0^2)}{j\omega\epsilon_0} \exp\left[-\rho_{me} \left(|x| - \frac{b}{2}\right)\right] \sin\left(\frac{n\pi y}{a}\right) e^{-jk_z z}$$

$$E_y = -\frac{A_{ae} (\rho_{me} n\pi)}{j\omega a \epsilon_0} \exp\left[-\rho_{me} \left(|x| - \frac{b}{2}\right)\right] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_z z}$$

(for $x \geq \frac{b}{2}$)

$$= \frac{A_{ae} (\rho_{me} n\pi)}{j\omega a \epsilon_0} \exp\left[-\rho_{me} \left(|x| - \frac{b}{2}\right)\right] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_z z}$$

(for $x \leq -\frac{b}{2}$)

Now, if you want to write down the fields in the air region,

In the air region ($|x| \geq \frac{b}{2}$):

$$E_x = \frac{A_{ae} (\rho_{me} + k_0^2)}{j\omega\epsilon_0} \exp\left[-\rho_{me} \left(|x| - \frac{b}{2}\right)\right] \sin\left(\frac{n\pi y}{a}\right) e^{-jk_z z}$$

$$E_y = -\frac{A_{ae} (\rho_{me} n\pi)}{j\omega a \epsilon_0} \exp\left[-\rho_{me} \left(|x| - \frac{b}{2}\right)\right] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_z z}$$

(for $x \geq \frac{b}{2}$)

$$= \frac{A_{ae} (\rho_{me} n\pi)}{j\omega a \epsilon_0} \exp\left[-\rho_{me} \left(|x| - \frac{b}{2}\right)\right] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_z z}$$

(for $x \leq -\frac{b}{2}$)

So, the fields change in the two air regions, region 1 and region 3.

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$$E_z = \frac{A_0 e^{k_z z} p_m e^{-p_m (|x| - \frac{b}{2})}}{\omega \epsilon_0} \sin\left(\frac{n\pi y}{a}\right) e^{-jk_z z} \quad (\text{for } x > \frac{b}{2})$$

$$= -\frac{A_0 e^{k_z z} p_m e^{-p_m (|x| - \frac{b}{2})}}{\omega \epsilon_0} \sin\left(\frac{n\pi y}{a}\right) e^{-jk_z z} \quad (\text{for } x < -\frac{b}{2})$$

$$H_x = 0$$

$$H_y = -A_0 e^{jk_z z} e^{-p_m (|x| - \frac{b}{2})} \sin\left(\frac{n\pi y}{a}\right) e^{-jk_z z}$$

$$H_z = -\frac{A_0 n\pi}{a} e^{-p_m (|x| - \frac{b}{2})} \cos\left(\frac{n\pi y}{a}\right) e^{-jk_z z}$$

Similarly, we have

$$E_z = \frac{A_0 e^{k_z z} p_m e^{-p_m (|x| - \frac{b}{2})}}{\omega \epsilon_0} \sin\left(\frac{n\pi y}{a}\right) e^{-jk_z z} \quad (\text{for } x > \frac{b}{2})$$

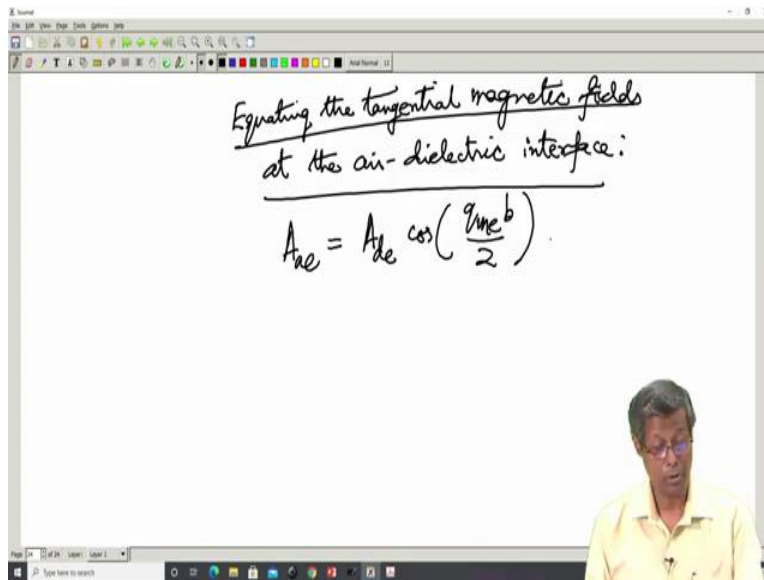
$$= -\frac{A_0 e^{k_z z} p_m e^{-p_m (|x| - \frac{b}{2})}}{\omega \epsilon_0} \sin\left(\frac{n\pi y}{a}\right) e^{-jk_z z} \quad (\text{for } x < -\frac{b}{2})$$

$$H_x = 0$$

$$H_y = -A_0 e^{jk_z z} e^{-p_m (|x| - \frac{b}{2})} \sin\left(\frac{n\pi y}{a}\right) e^{-jk_z z}$$

$$H_z = -\frac{A_0 n\pi}{a} e^{-p_m (|x| - \frac{b}{2})} \cos\left(\frac{n\pi y}{a}\right) e^{-jk_z z}$$

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Now, if we equate the tangential magnetic fields at the air dielectric interface. So, equating the tangential magnetic fields at the air dielectric interface we will get please verify again this expression

Equating the tangential magnetic fields
at the air-dielectric interface:

$$A_{te} = A_{de} \cos\left(\frac{\eta_{me} b}{2}\right)$$

So, let us stop here for now, we will continue with the field expressions for the other modes.