Advanced Microwave Guided-Structures and Analysis
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Lecture 55
Analysis of Guided Structures (cont.)

So, welcome to the remaining part of the lecture on the non-radiating dielectric guide. So, today we are going to finish the explicit field expressions for the remaining modes of the NRD, as well as the discuss the characteristic equation of the NRD, from which the cut off frequencies of all the modes can be obtained.

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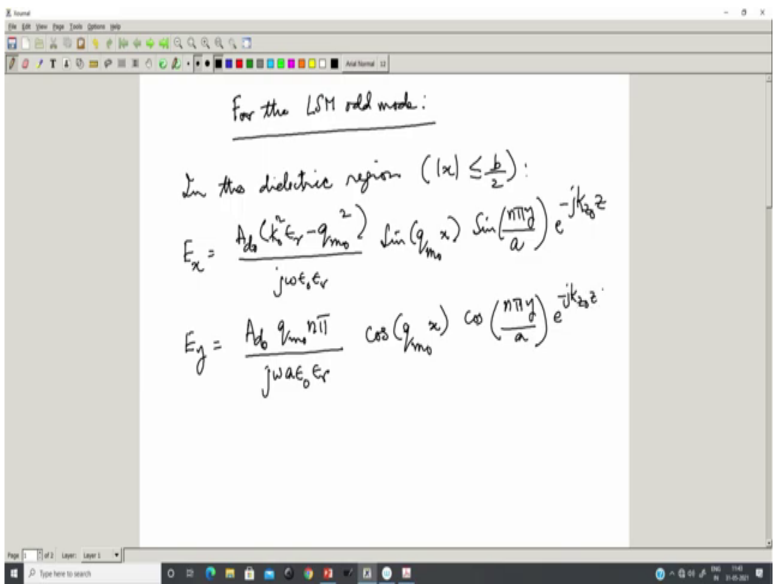

So, the last time we had discuss the explicit field expressions for the LSM even mode. We are going to write down the explicit field expressions for the LSM odd mode, and thereafter the LSE even mode and the LSE odd mode, not a very difficult exercise, but just for reference sake. So, you should verify yourself as you see these expressions.

So, for the LSM odd mode, we write in the dielectric region, For the LSM odd mode:

In the dielectric region ($|x| \le \frac{b}{2}$):

$$E_x = \frac{A_{do}(k_0^2\epsilon_r - q_{mo}^2)}{j\omega\epsilon_0\epsilon_r} \sin(q_{mo}x) \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{zo}z}$$

$$E_y = \frac{A_{do}\, q_{mo} n\pi}{j\omega a\epsilon_0\epsilon_r} \cos(q_{mo}x) \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{zo}z}$$

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$$H_z = -\frac{A_{do} n\pi}{a} \sin(q_{mo}x) \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{zo}z}$$

In the air region ($|x| \ge \frac{b}{2}$):

$$E_x = \frac{A_{ao}(p_{mo}^2+k_0^2)}{j\omega\epsilon_0} \exp\left[-p_{mo}\left(|x|-\frac{b}{2}\right)\right] \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{zo}z} \quad \left(\text{for } x \ge \frac{b}{2}\right)$$

$$= -\frac{A_{ao}(p_{mo}^2+k_0^2)}{j\omega\epsilon_0} \exp\left[-p_{mo}\left(|x|-\frac{b}{2}\right)\right] \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{zo}z} \quad \left(\text{for } x \le -\frac{b}{2}\right).$$

$$H_z = -\frac{A_{do} n\pi}{a} \sin(q_{mo}x) \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{zo}z}$$

In the air region ($|x| \ge \frac{b}{2}$):

$$E_x = \frac{A_{ao}(p_{mo}^2+k_0^2)}{j\omega\epsilon_0} \exp\left[-p_{mo}\left(|x|-\frac{b}{2}\right)\right] \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{zo}z} \quad \left(\text{for } x \ge \frac{b}{2}\right)$$

$$= -\frac{A_{ao}(p_{mo}^2+k_0^2)}{j\omega\epsilon_0} \exp\left[-p_{mo}\left(|x|-\frac{b}{2}\right)\right] \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{zo}z} \quad \left(\text{for } x \le -\frac{b}{2}\right).$$

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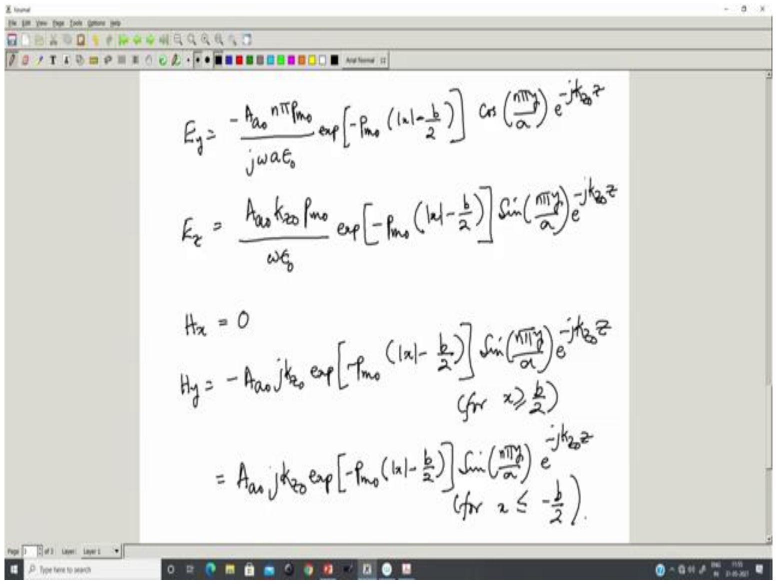

Similarly,

$$E_y = \frac{-A_{a0}\, n\pi p_{m0}}{j\omega a \epsilon_0} \exp\left[-p_{m0}\left(|x|-\frac{b}{2}\right)\right] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{z0}z}$$

$$E_z = \frac{A_{a0} k_{z0} p_{m0}}{\omega \epsilon_0} \exp\left[-p_{m0}\left(|x|-\frac{b}{2}\right)\right] \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{z0}z}$$

$$H_x = 0$$

$$H_y = -A_{a0}\, jk_{z0} \exp\left[-p_{m0}\left(|x|-\frac{b}{2}\right)\right] \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{z0}z} \quad \left(\text{for } x \geq \frac{b}{2}\right)$$

$$= A_{a0}\, jk_{z0} \exp\left[-p_{m0}\left(|x|-\frac{b}{2}\right)\right] \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{z0}z} \quad \left(\text{for } x \leq -\frac{b}{2}\right).$$

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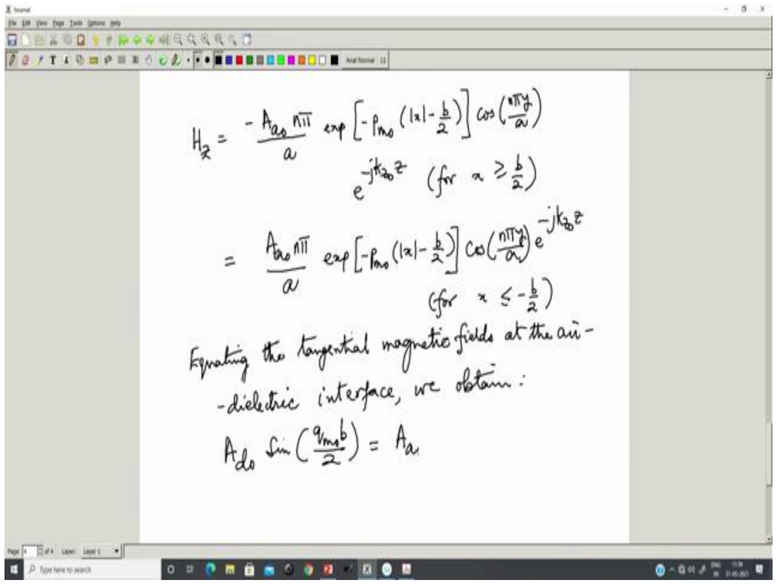

$$H_z = -\frac{A_{a0} n\pi}{a} \exp\left[-p_{m0}\left(|x| - \frac{b}{2}\right)\right] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{z0}z} \quad \left(\text{for } x \geq \frac{b}{2}\right)$$

$$= \frac{A_{a0} n\pi}{a} \exp\left[-p_{m0}\left(|x| - \frac{b}{2}\right)\right] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{z0}z} \quad \left(\text{for } x \leq -\frac{b}{2}\right)$$

So, now equating the tangential magnetic fields at the air dielectric interface will obtain the equation, which we obtained similar to the previous case for the LSM even mode. So, equating the tangential magnetic fields at the air dielectric interface, we obtain the relationship

Equating the tangential magnetic fields at the air-dielectric interface, we obtain:

$$A_{d0} \sin\left(\frac{q_{m0} b}{2}\right) = A_a$$

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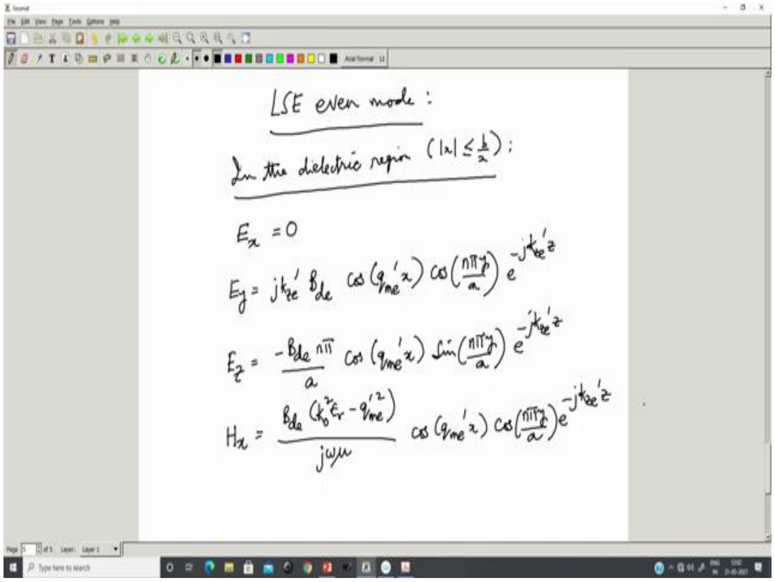

So, now we do the same exercise for the LSE even mode. Let us write down the explicit field equations, in the dielectric and air regions. So, for the LSE even mode, in the dielectric region,

LSE even mode:

In the dielectric region $\left(|x| \le \frac{b}{2}\right)$:

$$E_x = 0$$

$$E_y = jk_{ze}' B_{de} \cos(q_{me}' x) \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{ze}' z}$$

$$E_z = \frac{-B_{de}\, n\pi}{a} \cos(q_{me}' x) \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{ze}' z}$$

$$H_x = \frac{B_{de}\left(k_0^2 \epsilon_r - q_{me}'^2\right)}{j\omega\mu} \cos(q_{me}' x) \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{ze}' z}$$

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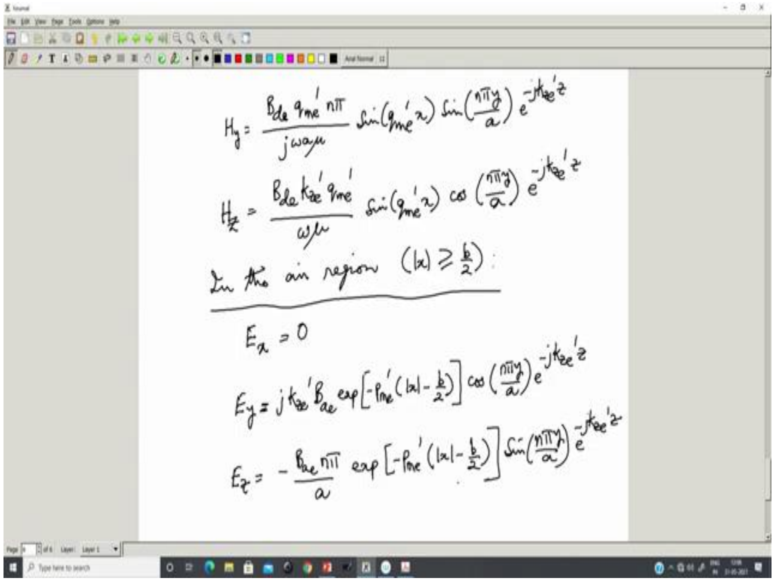

H y is given by

$$H_y = \frac{B_{de} q_{me}' n\pi}{j\omega a\mu} \sin(q_{me}' x) \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{ze}' z}$$

$$H_z = \frac{B_{de} k_{ze}' q_{me}'}{\omega\mu} \sin(q_{me}' x) \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{ze}' z}$$

Now, in the air region,

In the air region $\left(|x| \geq \frac{b}{2}\right)$:

$$E_x = 0$$

$$E_y = j k_{ze}' B_{ae} \exp\left[-p_{me}'\left(|x| - \frac{b}{2}\right)\right] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{ze}' z}$$

$$E_z = -\frac{B_{ae} n\pi}{a} \exp\left[-p_{me}'\left(|x| - \frac{b}{2}\right)\right] \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{ze}' z}$$

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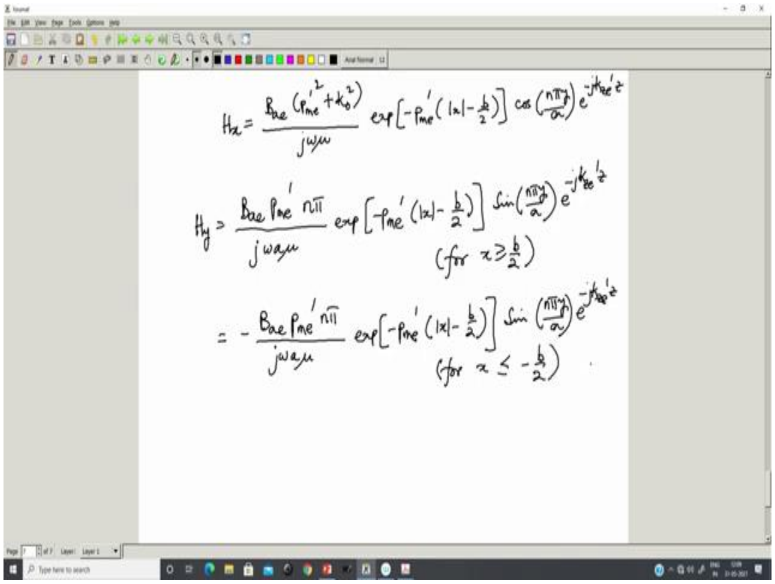

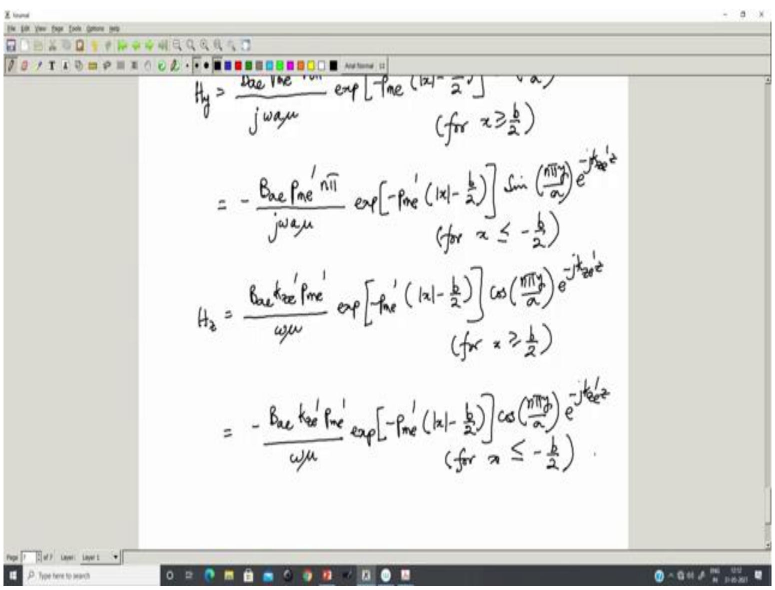

H x is given

$$H_x = \frac{B_{ae}\left(p_{me}'^{2} + k_0^2\right)}{j\omega\mu} \exp\left[-p_{me}'\left(|x| - \frac{b}{2}\right)\right] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{ze}' z}$$

$$H_y = \frac{B_{ae}\, p_{me}'\, n\pi}{j\omega a\mu} \exp\left[-p_{me}'\left(|x| - \frac{b}{2}\right)\right] \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{ze}' z} \quad \left(\text{for } x \geq \frac{b}{2}\right)$$

$$= -\frac{B_{ae}\, p_{me}'\, n\pi}{j\omega a\mu} \exp\left[-p_{me}'\left(|x| - \frac{b}{2}\right)\right] \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{ze}' z} \quad \left(\text{for } x \leq -\frac{b}{2}\right)$$

$$H_y = \frac{B_{ae}\, p_{me}'\, n\pi}{j\omega a\mu} \exp\left[-p_{me}'\left(|x| - \frac{b}{2}\right)\right] \quad \left(\text{for } x \geq \frac{b}{2}\right)$$

$$= -\frac{B_{ae}\, p_{me}'\, n\pi}{j\omega a\mu} \exp\left[-p_{me}'\left(|x| - \frac{b}{2}\right)\right] \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{ze}' z} \quad \left(\text{for } x \leq -\frac{b}{2}\right)$$

$$H_z = \frac{B_{ae}\, k_{ze}'\, p_{me}'}{\omega\mu} \exp\left[-p_{me}'\left(|x| - \frac{b}{2}\right)\right] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{ze}' z} \quad \left(\text{for } x \geq \frac{b}{2}\right)$$

$$= -\frac{B_{ae}\, k_{ze}'\, p_{me}'}{\omega\mu} \exp\left[-p_{me}'\left(|x| - \frac{b}{2}\right)\right] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{ze}' z} \quad \left(\text{for } x \leq -\frac{b}{2}\right)$$

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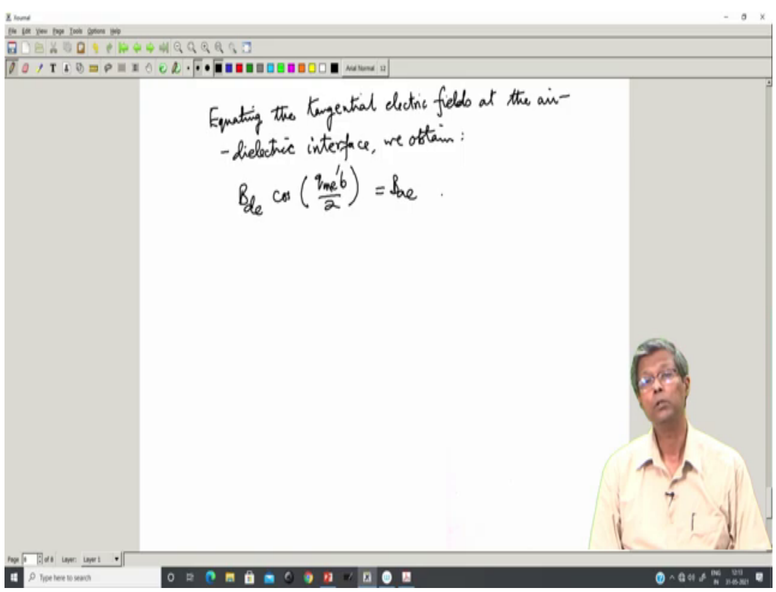

So, in this case we equate the tangential electric fields at the air dielectric interface. In order to find the relationship between the coefficients in the dielectric region and the air region. So, equating the tangential electric fields at the air dielectric interface, we obtain

Equating the tangential electric fields at the air-
-dielectric interface, we obtain:

$$B_{de} \cos\left(\frac{q_{me}' b}{2}\right) = B_{ae} .$$