

Advanced Microwave Guided-Structures and Analysis
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Lecture 56
Analysis of Guided Structures (cont.)

Welcome to the continuation of the lecture of the non-radiating dielectric guides, so we will now look at the LSE odd mode, the explicit field expressions and thereafter continue to the characteristic equations for the guide.

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For the LSE odd mode:
 In the dielectric region ($|x| \leq \frac{a}{2}$):

$$E_x = 0$$

$$E_y = B_{do} j k_0' \sin(q_{mo}' x) \cos\left(\frac{n\pi y}{a}\right) e^{-j k_0' z}$$

$$E_z = -\frac{B_{do} n \pi}{a} \sin(q_{mo}' x) \sin\left(\frac{n\pi y}{a}\right) e^{-j k_0' z}$$

$$H_x = \frac{B_{do} (k_0'^2 \epsilon_r - q_{mo}'^2)}{j \omega \mu} \sin(q_{mo}' x) \cos\left(\frac{n\pi y}{a}\right) e^{-j k_0' z}$$

$$H_y = -\frac{B_{do} q_{mo}' n \pi}{j \omega \mu} \cos(q_{mo}' x) \sin\left(\frac{n\pi y}{a}\right) e^{-j k_0' z}$$

$$H_z = -\frac{B_{do} k_0' q_{mo}'}{\omega \mu} \cos(q_{mo}' x) \cos\left(\frac{n\pi y}{a}\right) e^{-j k_0' z}$$

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in the air region ($|x| \geq \frac{b}{2}$):

$$E_x = 0$$
$$E_y = jk_{z0}' B_{a0} \exp[-\rho_{m0}'(|x| - \frac{b}{2})] \cos(\frac{n\pi y}{a}) e^{-jk_{z0}' z} \quad (\text{for } x \geq \frac{b}{2})$$
$$= -jk_{z0}' B_{a0} \exp[-\rho_{m0}'(|x| - \frac{b}{2})] \cos(\frac{n\pi y}{a}) e^{-jk_{z0}' z} \quad (\text{for } x \leq -\frac{b}{2})$$
$$E_z = -\frac{B_{a0} n\pi}{a} \exp[-\rho_{m0}'(|x| - \frac{b}{2})] \sin(\frac{n\pi y}{a}) e^{-jk_{z0}' z} \quad (\text{for } x \geq \frac{b}{2})$$
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$$= -jk_{z0}' B_{a0} \exp[-\rho_{m0}'(|x| - \frac{b}{2})] \cos(\frac{n\pi y}{a}) e^{-jk_{z0}' z} \quad (\text{for } x \leq -\frac{b}{2})$$
$$E_z = -\frac{B_{a0} n\pi}{a} \exp[-\rho_{m0}'(|x| - \frac{b}{2})] \sin(\frac{n\pi y}{a}) e^{-jk_{z0}' z} \quad (\text{for } x \geq \frac{b}{2})$$
$$= \frac{B_{a0} n\pi}{a} \exp[-\rho_{m0}'(|x| - \frac{b}{2})] \sin(\frac{n\pi y}{a}) e^{-jk_{z0}' z} \quad (\text{for } x \leq -\frac{b}{2})$$

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$$\begin{aligned}
 H_x &= \frac{\rho_{so} (k_{so}^2 + k_z^2)}{j\omega\mu} \exp[-\rho_{so}' (|x| - \frac{b}{2})] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{so}' z} \\
 &\quad \text{(for } x \geq \frac{b}{2}\text{)} \\
 &= -\frac{\rho_{so} (k_{so}^2 + k_z^2)}{j\omega\mu} \exp[-\rho_{so}' (|x| - \frac{b}{2})] \cos\left(\frac{n\pi y}{a}\right) e^{jk_{so}' z} \\
 &\quad \text{(for } x \leq -\frac{b}{2}\text{)} \\
 H_y &= \frac{\rho_{so} \rho_{so}' n\pi}{j\omega\mu} \exp[-\rho_{so}' (|x| - \frac{b}{2})] \sin\left(\frac{n\pi y}{a}\right) e^{-jk_{so}' z} \\
 H_z &= \frac{\rho_{so} k_{so} \rho_{so}'}{\omega\mu} \exp[-\rho_{so}' (|x| - \frac{b}{2})] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{so}' z} \\
 &\quad \text{(for } x \geq \frac{b}{2}\text{)}.
 \end{aligned}$$

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 H_z &= \frac{\rho_{so} k_{so} \rho_{so}'}{\omega\mu} \exp[-\rho_{so}' (|x| - \frac{b}{2})] \cos\left(\frac{n\pi y}{a}\right) e^{-jk_{so}' z} \\
 &\quad \text{(for } x \geq \frac{b}{2}\text{)} \\
 &= -\frac{\rho_{so} k_{so} \rho_{so}'}{\omega\mu} \exp[-\rho_{so}' (|x| - \frac{b}{2})] \cos\left(\frac{n\pi y}{a}\right) e^{jk_{so}' z} \\
 &\quad \text{(for } x \leq -\frac{b}{2}\text{)}
 \end{aligned}$$

Equating the tangential electric fields at the air-dielectric interface, we obtain:

$$\rho_{so} \sin\left(\frac{n\pi b}{2}\right) = \rho_{so}$$

Now,

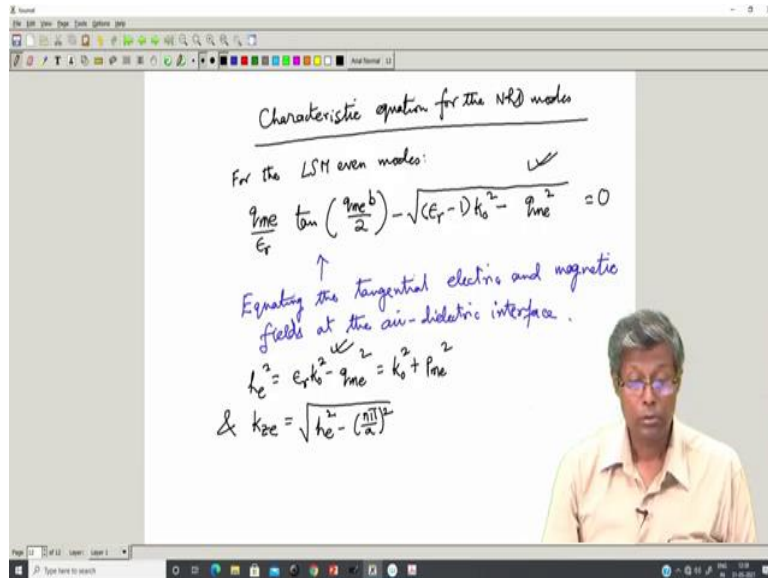
$$\begin{aligned}
H_x &= \frac{B_{ao} (p_{m0}'^2 + k_0'^2)}{j\omega\mu} \exp[-p_{m0}' (|x| - \frac{b}{2})] \cos(\frac{n\pi y}{a}) e^{-jk_0' z} && (\text{for } x \geq \frac{b}{2}) \\
&= -\frac{B_{ao} (p_{m0}'^2 + k_0'^2)}{j\omega\mu} \exp[-p_{m0}' (|x| - \frac{b}{2})] \cos(\frac{n\pi y}{a}) e^{-jk_0' z} && (\text{for } x \leq -\frac{b}{2}) \\
H_y &= \frac{B_{ao} p_{m0}' n\pi}{j\omega\mu} \exp[-p_{m0}' (|x| - \frac{b}{2})] \sin(\frac{n\pi y}{a}) e^{-jk_0' z} \\
H_z &= \frac{B_{ao} k_0' p_{m0}'}{\omega\mu} \exp[-p_{m0}' (|x| - \frac{b}{2})] \cos(\frac{n\pi y}{a}) e^{-jk_0' z} && (\text{for } x \geq \frac{b}{2}) \\
&= -\frac{B_{ao} k_0' p_{m0}'}{\omega\mu} \exp[-p_{m0}' (|x| - \frac{b}{2})] \cos(\frac{n\pi y}{a}) e^{-jk_0' z} && (\text{for } x \leq -\frac{b}{2})
\end{aligned}$$

That is for x lesser than or equal to $-b/2$, so as before equating the tangential electric fields at the air dielectric interface, we will obtain the relationship between the coefficients in the dielectric and the air regions. So, getting the tangential electric fields at the air dielectric interface, we obtain

Equating the tangential electric fields at the air-dielectric interface, we obtain:

$$B_{do} \sin\left(\frac{n\pi b}{2}\right) = B_{ao}$$

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So following this, we write down the characteristic equations for the NRD Modes, they are obtained by equating the tangential electric and magnetic fields at the air dielectric interface and using the Helmholtz equation in the dielectric and air regions.

So, please verify them from the previous given field expressions so equate the tangential electric and magnetic fields at the air dielectric interface, and use the Helmholtz equations which will be written. So, that will give the, yield the characteristic equation for the NRD Modes.

So, for the LSM even modes,

$$\begin{aligned}
 & \text{Characteristic equation for the NRD modes} \\
 & \text{For the LSM even modes:} \\
 & \frac{\gamma_{me}}{\epsilon_r} \tan\left(\frac{\gamma_{me} b}{2}\right) - \sqrt{(\epsilon_r - 1)k_0^2 - \gamma_{me}^2} = 0 \\
 & \uparrow \\
 & \text{Equating the tangential electric and magnetic} \\
 & \text{fields at the air-dielectric interface.} \\
 & h_e^2 = \epsilon_r k_0^2 - \gamma_{me}^2 = k_0^2 + \gamma_{me}^2 \\
 & \& k_{ze} = \sqrt{h_e^2 - \left(\frac{\pi n}{a}\right)^2}
 \end{aligned}$$

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for the LSM odd modes:

$$\frac{q_{mo}}{\epsilon_r} \cot\left(\frac{q_{mo} b}{a}\right) + \sqrt{(\epsilon_r - 1)k_0^2 - q_{mo}^2} = 0$$

$$h_0^2 = \epsilon_r k_0^2 - q_{mo}^2 = k_0^2 + p_{mo}^2$$

$$k_{kzo} = \sqrt{k_0^2 - \left(\frac{n\pi}{a}\right)^2}$$

for the LSE even modes:

$$q_{me}' \tan\left(\frac{q_{me}' b}{a}\right) - \sqrt{(\epsilon_r - 1)k_0^2 - q_{me}'^2} = 0$$

$$h_e'^2 = \epsilon_r k_0^2 - q_{me}'^2 = k_0^2 + p_{me}'^2$$

$$\& k_{kee}' = \sqrt{k_e'^2 - \left(\frac{n\pi}{a}\right)^2}$$

So, then similarly for the LSM odd modes, we will have by the same methodology, by equating the tangential electric and magnetic fields at the air dielectric interface, we will obtain

for the LSM odd modes:

$$\frac{q_{mo}}{\epsilon_r} \cot\left(\frac{q_{mo} b}{a}\right) + \sqrt{(\epsilon_r - 1)k_0^2 - q_{mo}^2} = 0$$

$$h_0^2 = \epsilon_r k_0^2 - q_{mo}^2 = k_0^2 + p_{mo}^2$$

$$k_{kzo} = \sqrt{k_0^2 - \left(\frac{n\pi}{a}\right)^2}$$

For the LSE even modes, we have

for the LSE even modes:

$$q_{me}' \tan\left(\frac{q_{me}' b}{a}\right) - \sqrt{(\epsilon_r - 1)k_0^2 - q_{me}'^2} = 0$$

$$h_e'^2 = \epsilon_r k_0^2 - q_{me}'^2 = k_0^2 + p_{me}'^2$$

$$\& k_{kee}' = \sqrt{k_e'^2 - \left(\frac{n\pi}{a}\right)^2}$$

And the propagation constant along the z direction for the LSE even mode, given by k_{ze}' . So, similarly if you set k_{ze}' equal to 0, we will obtain the cut off frequency for the LSE even mode.

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For the LSE odd modes:

$$q_{m0}' \cot\left(\frac{q_{m0}' b}{a}\right) + \sqrt{(\epsilon_r - 1)k_0^2 - q_{m0}'^2} = 0$$

$$h_0'^2 = \epsilon_r k_0^2 - q_{m0}'^2 = k_0^2 + p_{m0}'^2$$

$$\& k_{z0}' = \sqrt{k_0^2 - \left(\frac{n\pi}{a}\right)^2}$$

Now, for the LSE odd modes, we have

For the LSE odd modes:

$$q_{m0}' \cot\left(\frac{q_{m0}' b}{a}\right) + \sqrt{(\epsilon_r - 1)k_0^2 - q_{m0}'^2} = 0$$

$$h_0'^2 = \epsilon_r k_0^2 - q_{m0}'^2 = k_0^2 + p_{m0}'^2$$

And the propagation constant along the z direction, for the LSE odd Mode k_{z0}' being given by

$$\& k_{z0}' = \sqrt{k_0^2 - \left(\frac{n\pi}{a}\right)^2}$$

So, this completes our treatment of the NRD guide including the field expressions and the characteristic equations for the modes, together with the cutoff conditions, for the hybrid modes of the NRD, so thank you.