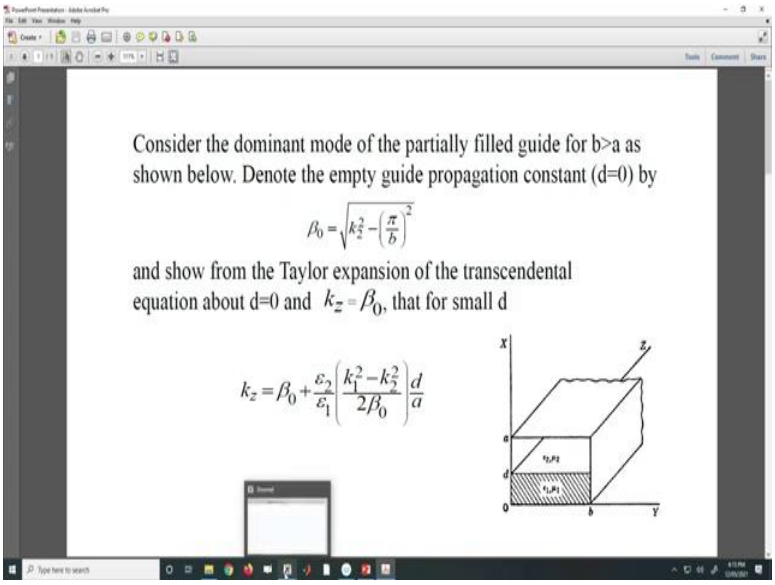
**Advanced Microwave Guided-Structures and Analysis**
**Professor Bratin Ghosh**
**Department of Electronics & Electrical Communication Engineering**
**Indian Institute of Technology Kharagpur**
**Lecture 57**
**Analysis of Guided Structures Tutorials**

Hello everyone, today we will solve some numerical problems on partially filled waveguide.

(Refer Slide Time: 00:20)



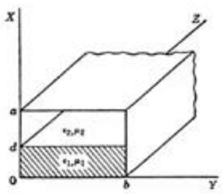

So to start with this is our first numerical problem, and it states that,

Consider the dominant mode of the partially filled guide for b>a as shown below. Denote the empty guide propagation constant (d=0) by

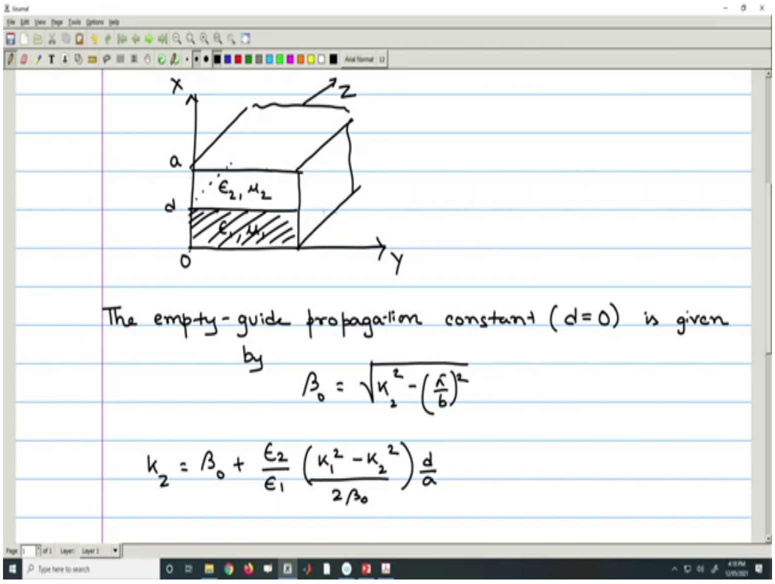
$$\beta_0 = \sqrt{k_2^2 - \left(\frac{\pi}{b}\right)^2}$$

and show from the Taylor expansion of the transcendental equation about d=0 and $k_z = \beta_0$, that for small d

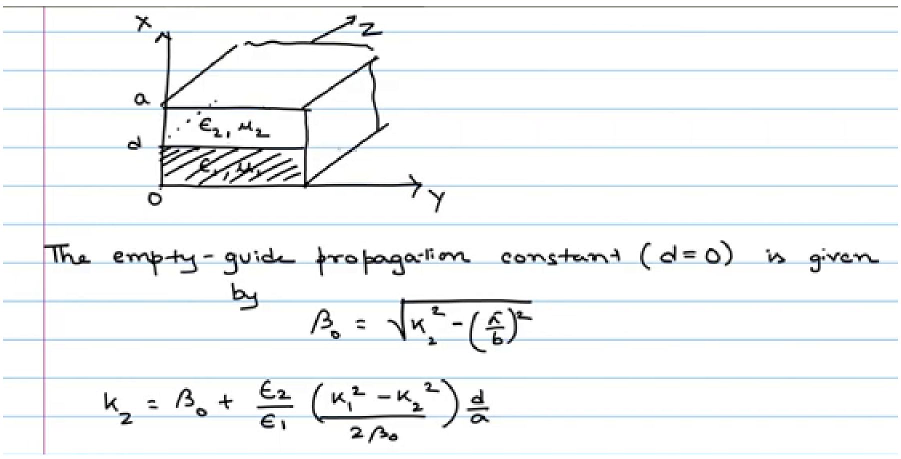
$$k_z = \beta_0 + \frac{\varepsilon_2}{\varepsilon_1}\left(\frac{k_1^2 - k_2^2}{2\beta_0}\right)\frac{d}{a}$$



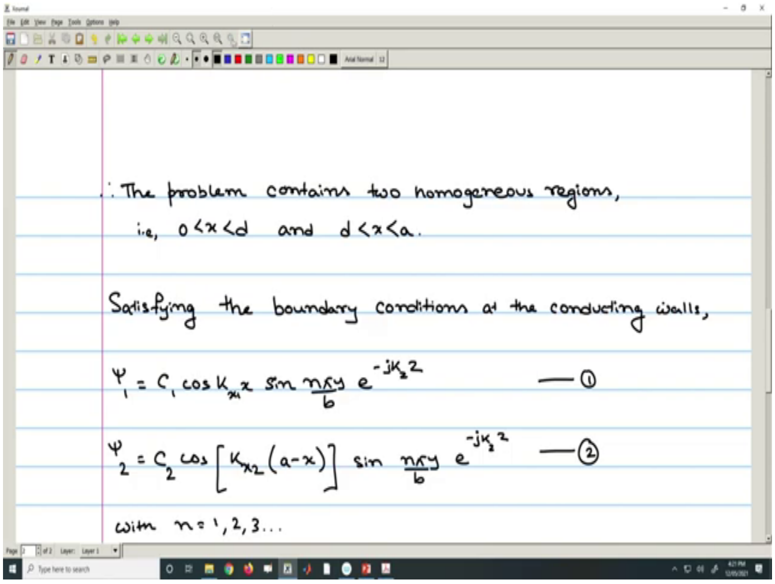
(Refer Slide Time: 01:23)




So, we will start solving this problem, so at first in the given question,

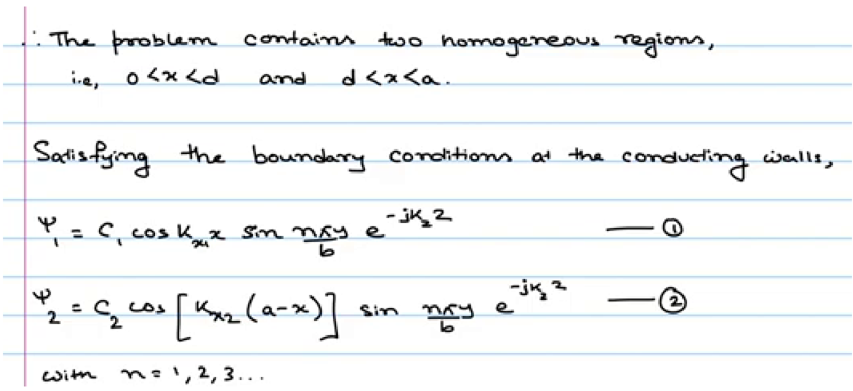


(Refer Slide Time: 05:05)

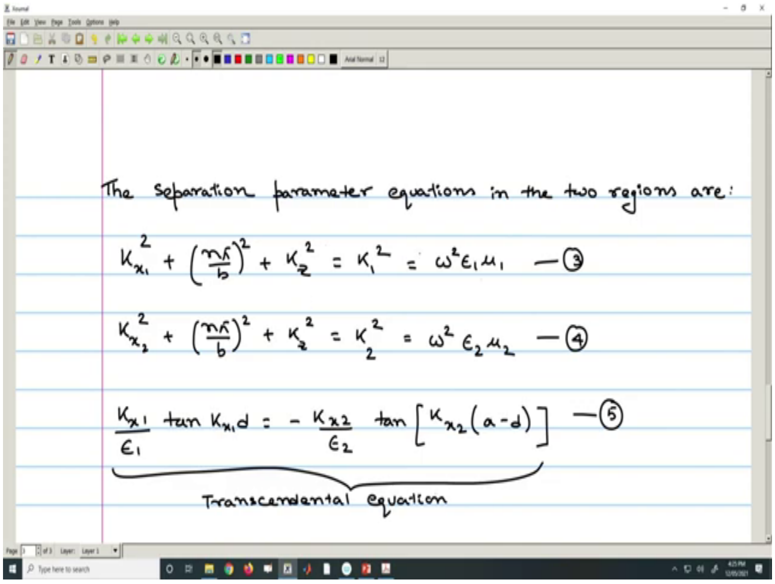



So, therefore we can write like this, that the problem therefore, the problem contains, therefore the problem contains, two homogeneous regions that is, from one from 0 to d where x is ranging from 0 to d and the other one when, x is ranging from d to a. Now, such problems can be solved by finding solutions in each of the regions, such that the tangential components of each are continuous across the common boundary.

So, therefore satisfying the boundary conditions at the conducting walls, we can write down,



(Refer Slide Time: 08:00)




Now, it has been anticipated that

The separation parameter equations in the two regions are:

$$K_{x_1}^2 + \left(\frac{n\pi}{b}\right)^2 + K_z^2 = K_1^2 = \omega^2\epsilon_1\mu_1 \quad —(3)$$

$$K_{x_2}^2 + \left(\frac{n\pi}{b}\right)^2 + K_z^2 = K_2^2 = \omega^2\epsilon_2\mu_2 \quad —(4)$$

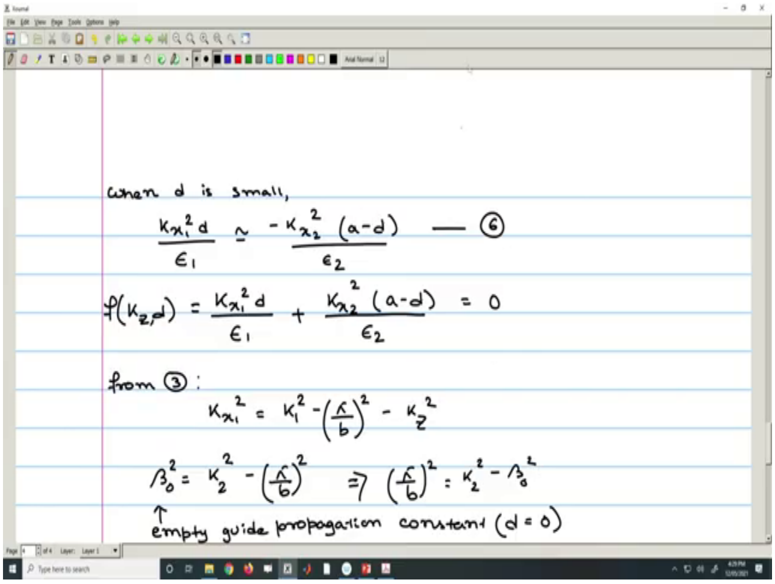
So, we can denote these two equations as number 3 and equation number 4.

Now, then we can find out the fields and after finding out the fields an imposing the boundary conditions and the matching at the boundary we can write

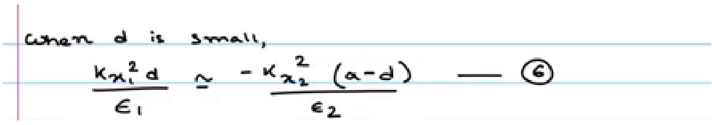
$$\underbrace{\frac{K_{x1}}{\epsilon_1}\tan K_{x_1}d = -\frac{K_{x2}}{\epsilon_2}\tan\left[K_{x_2}(a-d)\right]}_{\text{Transcendental equation}} \quad —(5)$$

this one can be written down as, transcendental equation and this is used for determining all the possible $k_z$ values, where $k_z$ is the mode propagation constant.

(Refer Slide Time: 11:40)

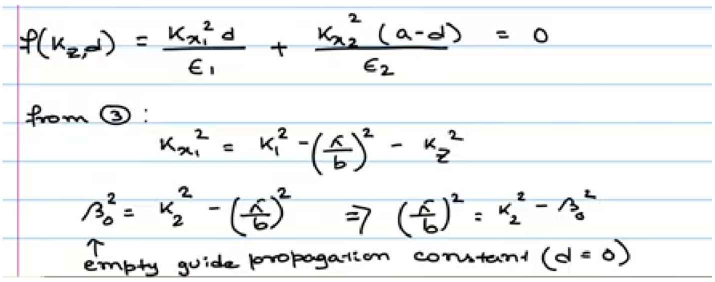



Now, when d is small, then the above equation can be approximated as

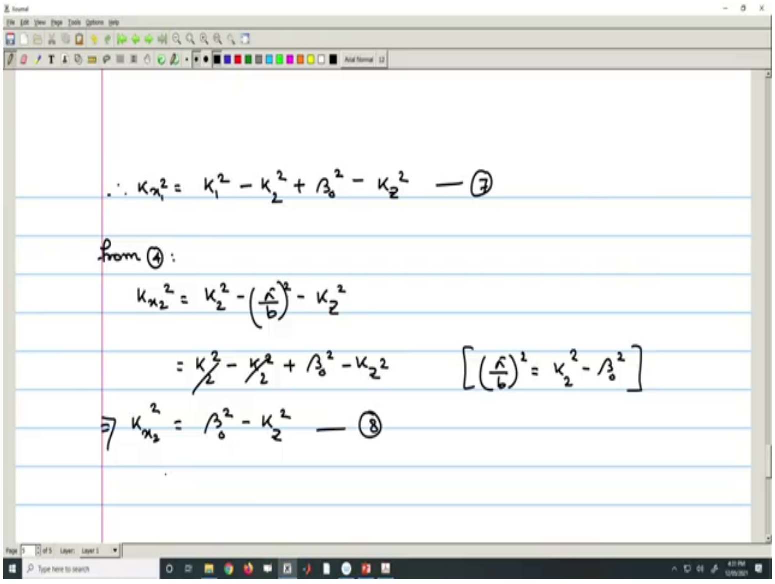



so let us denote this equation as equation number 6.

Now, what we will do, we will write, this equation by denoting it as a function of $k_z$ and d so we can tell



(Refer Slide Time: 15:03)




Therefore, we can write

$$\therefore K_{x_1}^2 = K_1^2 - K_2^2 + \beta_0^2 - K_z^2 \quad \text{— (7)}$$

Let us give this as equation number 7.

from (4):

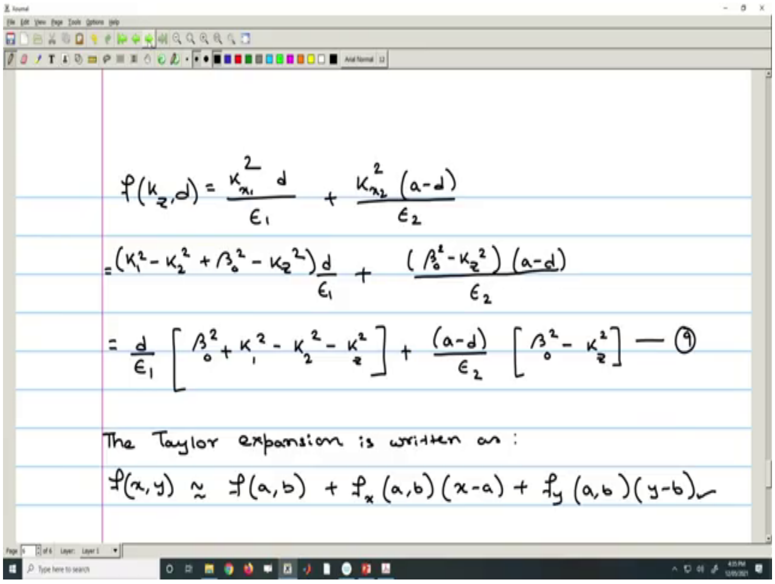
$$K_{x_2}^2 = K_2^2 - \left(\frac{\pi}{b}\right)^2 - K_z^2$$

$$= \cancel{K_2^2} - \cancel{K_2^2} + \beta_0^2 - K_z^2 \qquad \left[\left(\frac{\pi}{b}\right)^2 = K_2^2 - \beta_0^2\right]$$

$$\Rightarrow K_{x_2}^2 = \beta_0^2 - K_z^2 \quad \text{— (8)}$$

So, this is equation number 8. Now what we will do, we will substitute this value.

(Refer Slide Time: 17:37)



$$f(k_z, d) = \frac{K_{x_1}^2 \, d}{\epsilon_1} + \frac{K_{x_2}^2 (a-d)}{\epsilon_2}$$

$$= (K_1^2 - K_2^2 + \beta_0^2 - K_z^2)\frac{d}{\epsilon_1} + \frac{(\beta_0^2 - K_z^2)(a-d)}{\epsilon_2}$$

$$= \frac{d}{\epsilon_1}\left[\beta_0^2 + K_1^2 - K_2^2 - K_z^2\right] + \frac{(a-d)}{\epsilon_2}\left[\beta_0^2 - K_z^2\right] \quad — (9)$$

So, let us denote this equation as equation number 9.

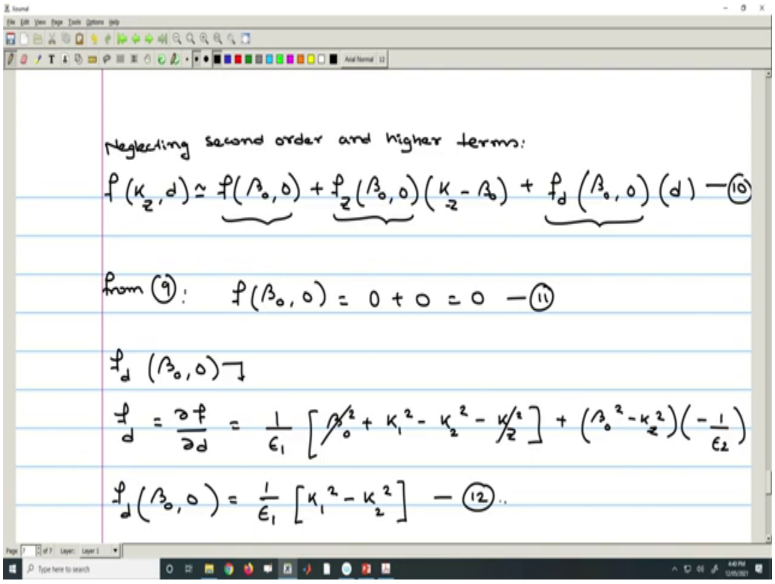
Now what we will do, we will write down the Taylor expansion,

The Taylor expansion is written as :

$$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

So, here we have neglected the second order and all the higher terms, so now, in this general formula we will, place.

(Refer Slide Time: 21:41)




Neglecting second order and higher terms:

$$f(K_z,d) \simeq \underbrace{f(\beta_0,0)} + \underbrace{f_z(\beta_0,0)}(K_z-\beta_0) + \underbrace{f_d(\beta_0,0)}(d) \quad - (10)$$

so this is equation number 10. Here, we have neglected second order and higher terms.

from (9): $f(\beta_0,0) = 0 + 0 = 0 \quad - (11)$

so this is equation number 11.

Now, we will find out

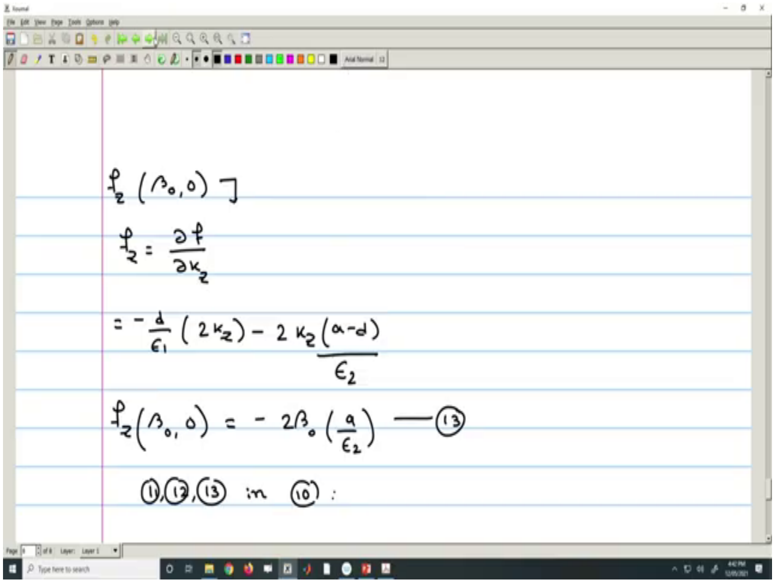
$$f_d(\beta_0,0)$$

$$f_d = \frac{\partial f}{\partial d} = \frac{1}{\epsilon_1}\left[\cancel{\beta_0^2} + K_1^2 - K_z^2 - \cancel{K_z^2}\right] + (\beta_0^2 - K_z^2)\left(-\frac{1}{\epsilon_2}\right)$$

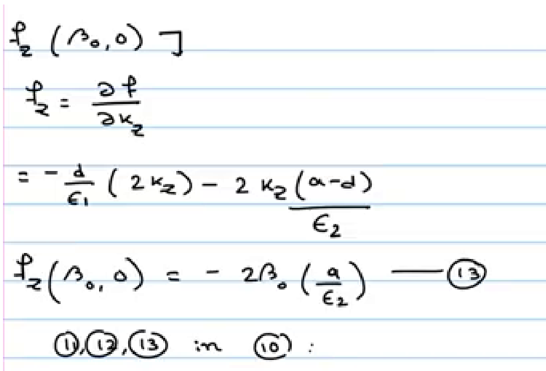
$$f_d(\beta_0,0) = \frac{1}{\epsilon_1}\left[K_1^2 - K_z^2\right] \quad - (12)$$

so this is equation number 12.

(Refer Slide Time: 26:27)



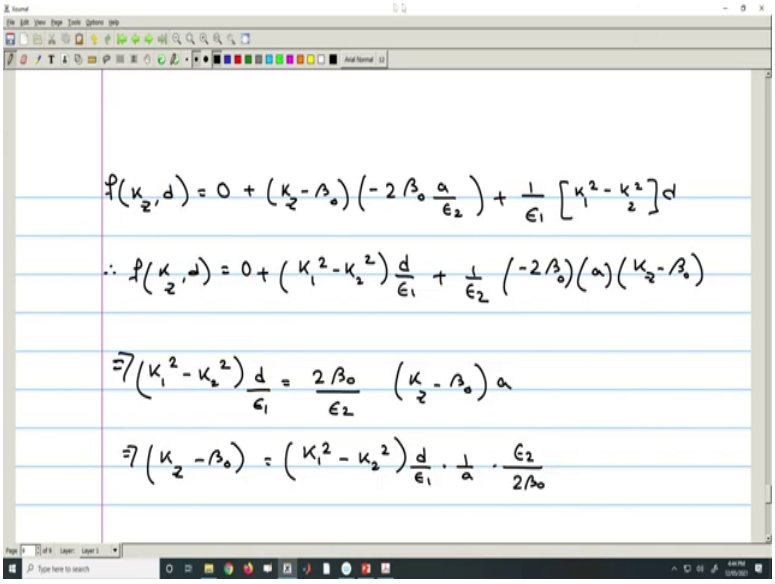

Now, we will




So, let give this as equation 13.

Now, we will substitute, all these values that is equation number 11, 12 and 13 in equation number 10, so substituting 11, 12 and 13 in 10.

(Refer Slide Time: 28:21)




We will get,

$$f(k_z, d) = 0 + (k_z - \beta_0)\left(-2\beta_0 \frac{a}{\epsilon_2}\right) + \frac{1}{\epsilon_1}\left[k_1^2 - k_2^2\right] d$$

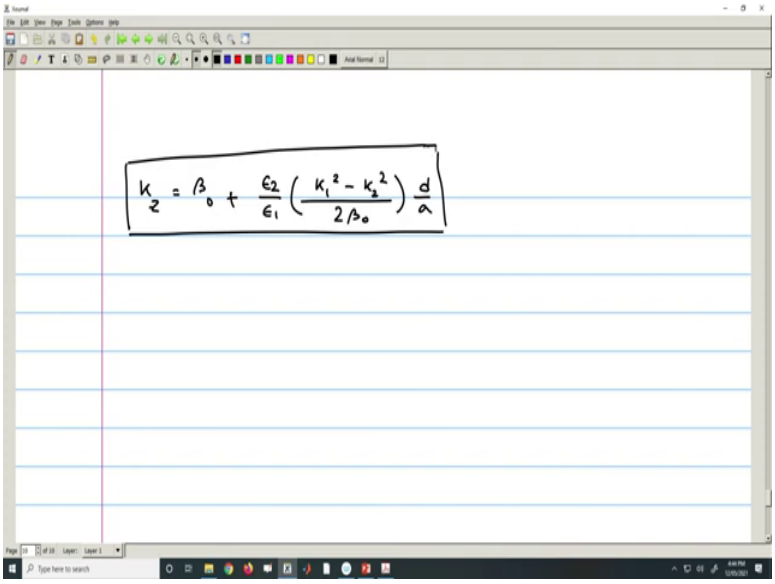
$$\therefore f(k_z, d) = 0 + (k_1^2 - k_2^2)\frac{d}{\epsilon_1} + \frac{1}{\epsilon_2}(-2\beta_0)(a)(k_z - \beta_0)$$

$$\Rightarrow (k_1^2 - k_2^2)\frac{d}{\epsilon_1} = \frac{2\beta_0}{\epsilon_2}(k_z - \beta_0) a$$

$$\Rightarrow (k_z - \beta_0) = (k_1^2 - k_2^2)\frac{d}{\epsilon_1} \cdot \frac{1}{a} \cdot \frac{\epsilon_2}{2\beta_0}$$

$$\boxed{k_z = \beta_0 + \frac{\epsilon_2}{\epsilon_1}\left(\frac{k_1^2 - k_2^2}{2\beta_0}\right)\frac{d}{a}}$$

(Refer Slide Time: 30:36)




So, thank you on next class we will gain solve a some few more problems on partially filled guides, thank you.