Advanced Microwave Guided-Structures and Analysis
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Lecture 58
Analysis of Guided Structures Tutorials (cont.)

Hello everyone, today we will solve numerical problems based on partially filled waveguide.

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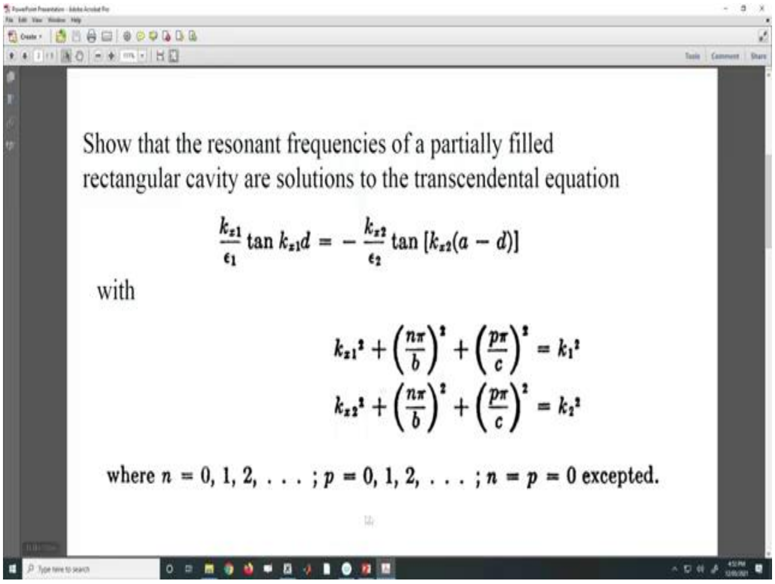

The first one is show that the resonant frequencies of a partially filled rectangular cavity are solutions to the transcendental equation given by $\frac{k_{x1}}{\varepsilon_1}\tan\left(k_{x1}d\right) = \frac{k_{x2}}{\varepsilon_2}\tan\left(k_{x2}\left(a-d\right)\right)$ with, $k_{x1}^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2 = k_1^2$ and

$k_{x2}^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2 = k_2^2$.

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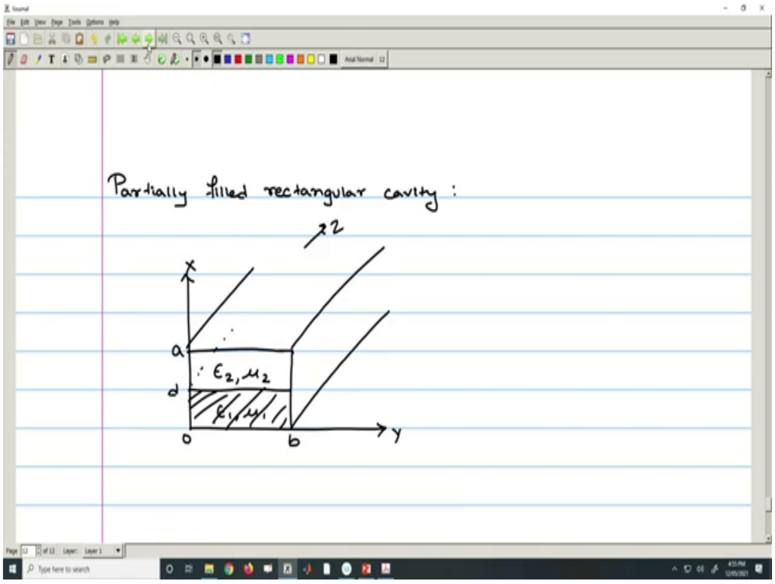

So, we will start solving the problem, so in the question we have partially filled rectangular cavity. We can draw this as, so this will be x axis, this is y and it is a rectangular cavity. So, in this we have said, and since this is partially filled so this is the origin 0 and then this will be a, this is b. Let us denote this point as d so this one is filled with epsilon 2, Mu 2 and this is epsilon 1 Mu 1.

So, this is partially filled rectangular cavity so we will have additional conductors covering the z equals to 0 and z equals to c. So, at both the ends we are having at z equals to 0 and, as z equals to c we are having conductors placed. So, for this we can write the potential functions as.

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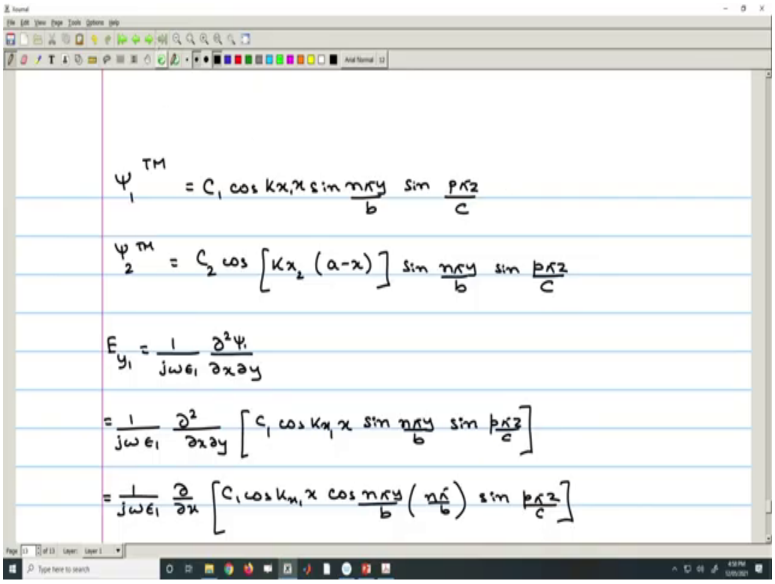

To start with, we will start from writing ψ_1^{TM} will be $c_1 \cos(k_{x1}x)\sin\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{c}\right)$.

Similarly, ψ_2^{TM} will $c_2 \cos\left(k_{x2}(a-x)\right)\sin\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{c}\right)$. So, these are the psi functions and now, what we will do, we will find out all the fields and then from the continuity equation we will try to find out the transcendental one.

So, we can write **E1** as E_{y1} will be $\frac{1}{j\omega\varepsilon_1}\frac{\partial^2\psi_1}{\partial x\partial y}$. So, we can write $\frac{1}{j\omega\varepsilon_1}\frac{\partial^2}{\partial x\partial y}\left[c_1\cos(k_{x1}x)\sin\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{c}\right)\right]$.

Which gives us $\frac{1}{j\omega\varepsilon_1}\frac{\partial}{\partial x}\left[c_1\cos(k_{x1}x)\cos\left(\frac{n\pi y}{b}\right)\left(\frac{n\pi}{b}\right)\sin\left(\frac{p\pi z}{c}\right)\right]$.

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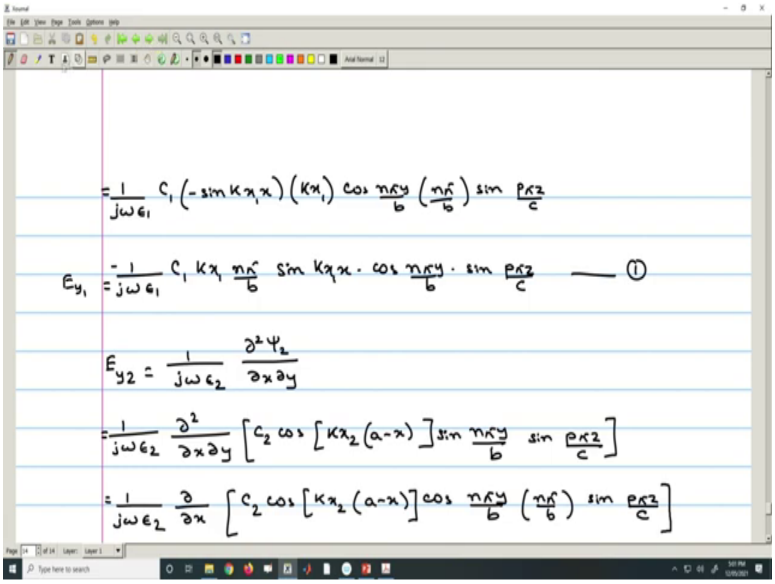

So, now we will differentiate with respect to x, so it will be $-\frac{1}{j\omega\varepsilon_1}\left[c_1k_{x1}\left(\frac{n\pi}{b}\right)\sin\left(k_{x1}x\right)\cos\left(\frac{n\pi y}{b}\right)s\,\text{in}\left(\frac{p\pi z}{c}\right)\right]$. So, this is E_{y1}.

Now, E_{y2} so E_{y2} will be $\frac{1}{j\omega\varepsilon_2}\frac{\partial^2\psi_2}{\partial x\partial y}$, so again in the similar manner we will write, $\frac{1}{j\omega\varepsilon_2}\frac{\partial^2}{\partial x\partial y}\left[c_2\cos\left(k_{x2}\left(a-x\right)\right)\sin\left(\frac{n\pi y}{b}\right)s\,\text{in}\left(\frac{p\pi z}{c}\right)\right]$. Now, first we will differentiate this with respect to y, so we will write $\frac{1}{j\omega\varepsilon_2}\frac{\partial}{\partial x}\left[c_2\cos\left(k_{x2}\left(a-x\right)\right)\cos\left(\frac{n\pi y}{b}\right)\left(\frac{n\pi}{b}\right)s\,\text{in}\left(\frac{p\pi z}{c}\right)\right]$

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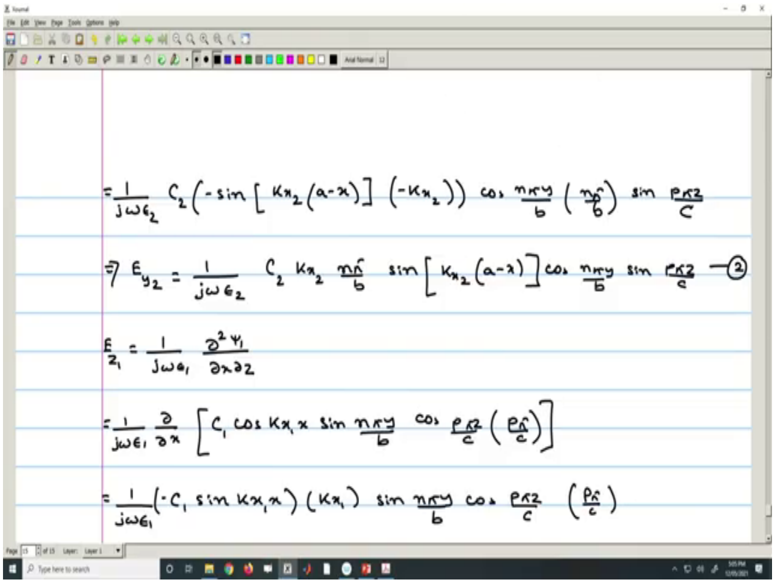

So now, we will differentiate the above with respect to x. So, we will have $\frac{1}{j\omega\varepsilon_2}\left[c_2 - \sin\left(k_{x2}\left(a-x\right)\right)\left(-k_{x2}\right)\cos\left(\frac{n\pi y}{b}\right)\left(\frac{n\pi}{b}\right)s\,\text{in}\left(\frac{p\pi z}{c}\right)\right]$. Therefore, we can write E_{y2} is equal to $\frac{1}{j\omega\varepsilon_2}\left[c_2 k_{x2}\left(\frac{n\pi}{b}\right)\sin\left(k_{x2}\left(a-x\right)\right)\cos\left(\frac{n\pi y}{b}\right)s\,\text{in}\left(\frac{p\pi z}{c}\right)\right]$. We can give this as equation number 2. Now, E_{z1}, so, E_{z1} will be $\frac{1}{j\omega\varepsilon_1}\frac{\partial^2\psi_1}{\partial x\partial z}$.

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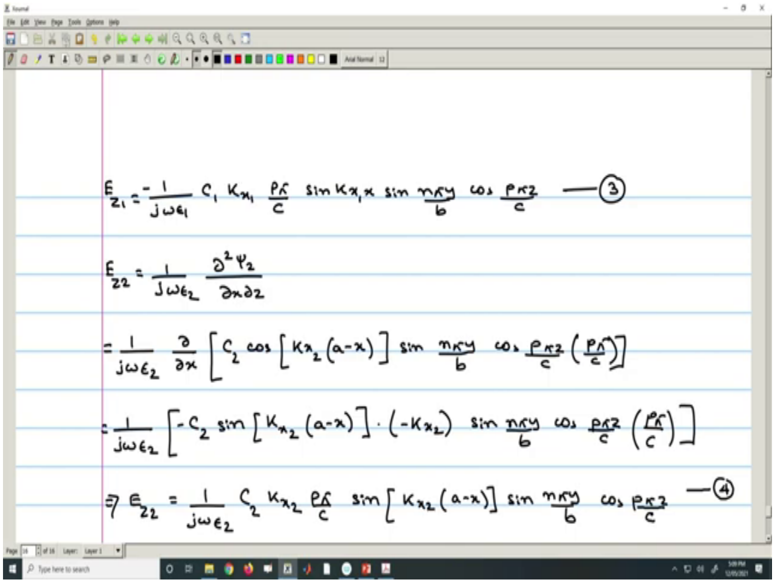

So, therefore, we can write Ez1 as $-\frac{1}{j\omega\varepsilon_1}\left[c_1 k_{x1}\left(\frac{p\pi}{c}\right)\sin\left(k_{x1}x\right)\sin\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{c}\right)\right]$, so this will give us equation number 3. Now, we will find out Ez2, so Ez2 will be $\frac{1}{j\omega\varepsilon_2}\frac{\partial^2\psi_2}{\partial x\partial z}$.

So, that is equal to $\frac{1}{j\omega\varepsilon_2}\left[c_2 k_{x2}\left(\frac{p\pi}{c}\right)\sin\left(k_{x2}\left(a-x\right)\right)\sin\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{c}\right)\right]$. So we will give this equation as equation number 4. Therefore, so now we have Ey1, Ey2, Ez1 and Ez2.

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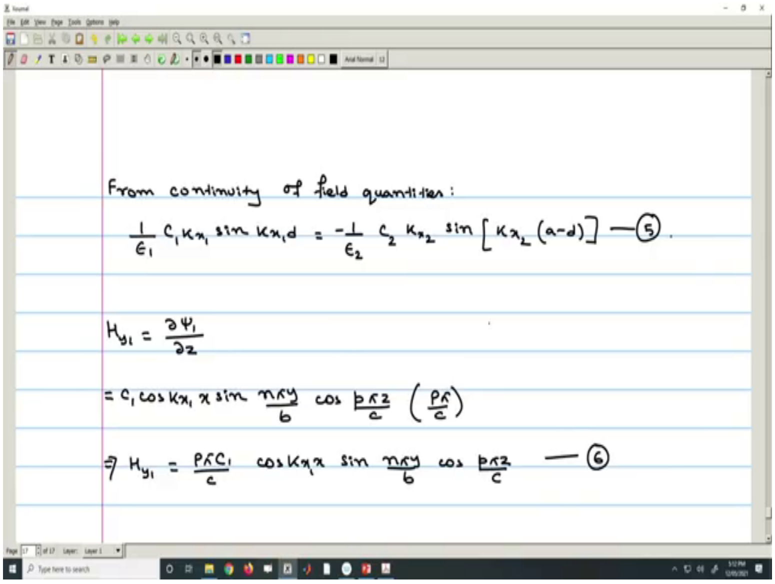

Now, from continuity of field quantities, we can write so from continuity of field quantities, we can write $\frac{1}{\varepsilon_1}\left[c_1 k_{x1}\sin\left(k_{x1}d\right)\right]=\frac{-1}{\varepsilon_2}\left[c_2 k_{x2}\sin\left(k_{x2}\left(a-d\right)\right)\right]$ that is, we are equating, Ey1 and Ey2. So, we will get this. So, this equation we get from the continuity of field quantities.

Now, we will find out Hy1, Hy2, Hz1 and Hz2. So, we know Hy1, we can write it down like $\frac{\partial\psi_1}{\partial z}$. So, we will have just one differentiation with respect to z, so we can write it like, $\frac{p\pi c_1}{c}\cos\left(k_{x1}x\right)\sin\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{c}\right)$. Let us denote this as equation number 6. So we have now Hy1 with us. Now, we will calculate Hy2.

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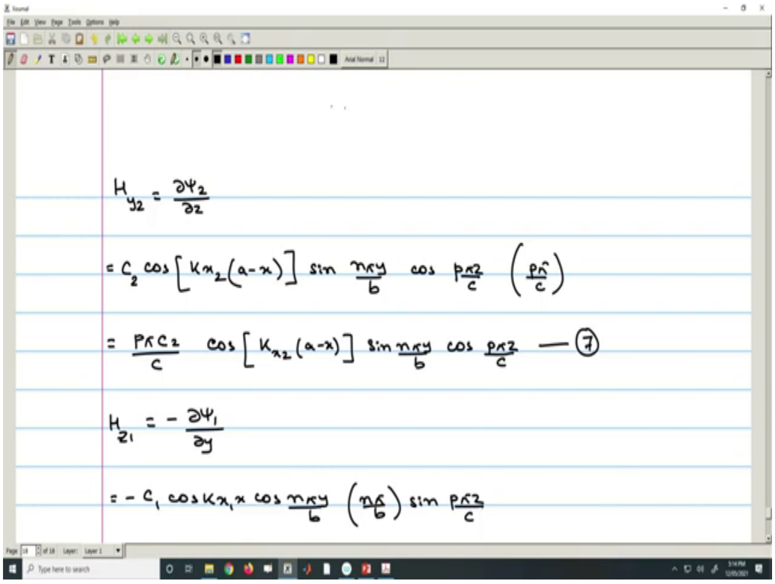

So, Hy2 will be $\frac{\partial \psi_2}{\partial z}$, so this will give us $c_2 \cos\left(k_{x2}\left(a-x\right)\right)\sin\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{c}\right)\left(\frac{p\pi}{c}\right)$. So, therefore we can write this like, so we have $\left(\frac{p\pi c_2}{c}\right)\cos\left(k_{x2}\left(a-x\right)\right)\sin\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{c}\right)\left(\frac{p\pi}{c}\right)$. So, this is Hy2 we will give this as equation 7 so this is equation number 7.

Now, Hz1. So again Hz1 is minus of $\frac{\partial \psi_1}{\partial y}$, so that is equals to $c_1 \cos\left(k_{x1}x\right)\cos\left(\frac{n\pi y}{b}\right)\left(\frac{n\pi}{b}\right)\sin\left(\frac{p\pi z}{c}\right)$.

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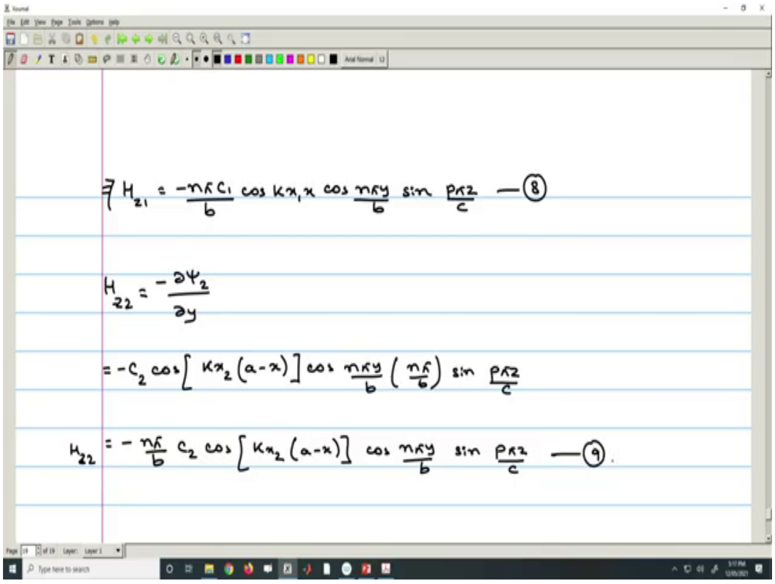

Therefore, we can write Hz1 as minus of $\left(\frac{n\pi c_1}{c}\right)\cos(k_{x1}x)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{c}\right)$. So, this is equation 8. Now, we will find out Hz2, so Hz2 is minus of $\frac{\partial\psi_2}{\partial y}$ that will give us $\left(-\frac{n\pi c_2}{b}\right)\cos\left(k_{x2}\left(a-x\right)\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{c}\right)$. So we can give this as equation number 9.

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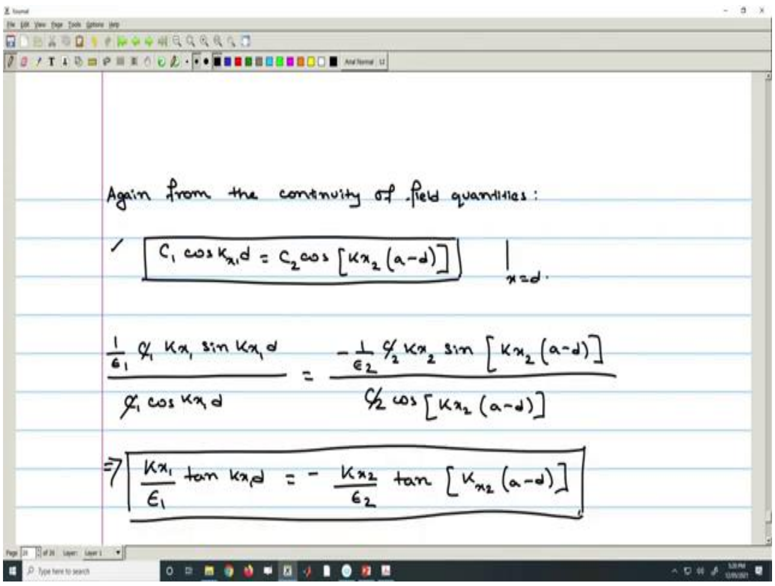

So, again from the continuity of field equations, we can write so again from the continuity of field quantities, we can write $c_1\cos(k_{x1}d)=c_2\cos\left(k_{x2}\left(a-d\right)\right)$. That is at x equals to d, we are checking the continuity of the field equations. So, this is at x equals to d fine, so this is, this one again we are getting from the continuity of field quantities.

Now, we will divide this equation with the equation number, this 5 so we will divide 5 with this equation and we will have $\frac{1}{\varepsilon_1} c_1 k_{x1} \sin\left(\frac{k_{x1} d}{c_1}\right)$ is equal to, there we had $-\frac{1}{\varepsilon_2} c_2 k_{x2} \sin\left(\frac{k_{x2}(a-d)}{c_2}\right) \cos\left(k_{x2}(a-d)\right)$.

So thus, we can see, so c1 c1 will cancel out, this c1, this c1, this c2, c2. So, we will have kx1 upon epsilon 1 so sin kx1 d upon cos kx1 d will give us tan kx1 d is equal to so this is minus so we will have, kx2 upon epsilon 2, tan of again kx2, a minus d. So, thus satisfying the given condition, so that is all for today thank you so much.