Advanced Microwave Guided – Structures and Analysis
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Lecture 63
Circular Cavity

So, in this part of the lecture we are going to use the treatment of the circular waveguide to the analysis of the circular cavity, which is nothing but the circular waveguide capped by two metallic plates at two z- locations. And we are going to find the resonant frequency of the circular cavity, the stored energy in the cavity, the power dissipated in the walls of the cavity, and thereby evaluate the Q- factor of the cavity.

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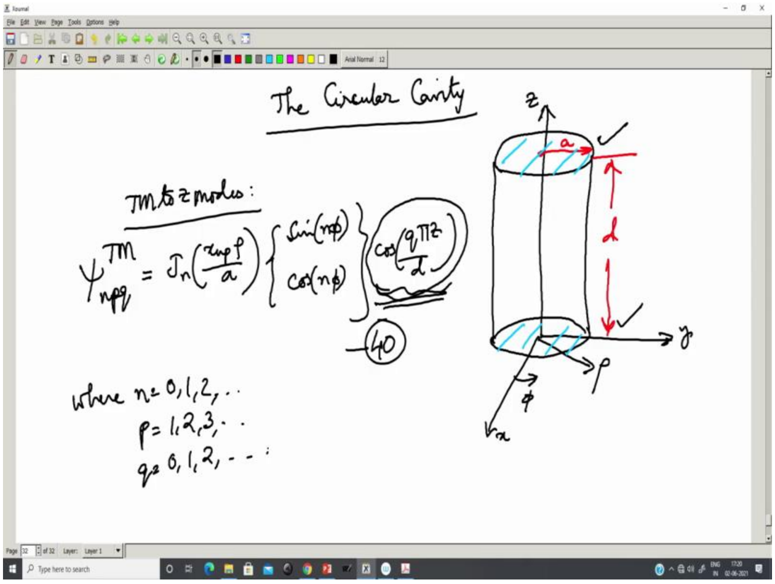

Let us draw the cavity, so this is the z-, this is y-, this is x-, this is phi, this is rho, the radius a of the cavity, and the height d of the cavity. As we said it is a section of the circular waveguide closed by conductors at z equal to 0 and z equal to d. Therefore, it must be satisfying the boundary conditions at z equal to 0 and z equal to d, in addition to the boundary conditions for the circular waveguide, namely the vanishing of the tangential fields at rho is equal to a .

So, the additional boundary conditions must be satisfied at this surface and at this surface, which means the tangential electric field must vanish at z equal to 0 and z equal to a . So, which means the tangential electric fields at z equal to 0 and z equal to a are E_ρ and E_ϕ . They must both vanish at z equal to 0 and z equal to a. So, we will get if we follow these principles, the TM to z- modes, the corresponding ψ function given by ψ_{npq}^{TM} as $j_n\left(\frac{x_{np}\rho}{a}\right)$ because nothing is disturbed along the rho direction and nothing is disturbed along the phi direction, so it remains $\begin{Bmatrix} \sin(n\phi) \\ \cos(n\phi) \end{Bmatrix}$.

However, we have $\cos\left(\frac{q\pi z}{d}\right)$ which is equation 40, $\cos\left(\frac{q\pi z}{d}\right)$ corresponds to standing waves along the z directions. So, this form of function whether it is sin or cos is determined by exactly the same logic as we followed for the rectangular waveguide cavity and I leave it up to you. So, we follow exactly the same reason that the tangential components of **E** has to finish at z equal to 0 and z equal to d, and we come up with the appropriate sin or cos decomposition corresponding to this vanishing of the tangential electric fields at z equal to 0 and z equal to d.

So, whatever wave function along the z direction satisfies this vanishing of the electric fields at z equal to 0 and z equal to d must be chosen and this choice is exactly the same as in the rectangular waveguide or follows the exactly the same principles as we did for the rectangular waveguide. So, n is equal to 0, 1, 2, p equal to 1, 2, 3 and q equal to 0, 1, 2.

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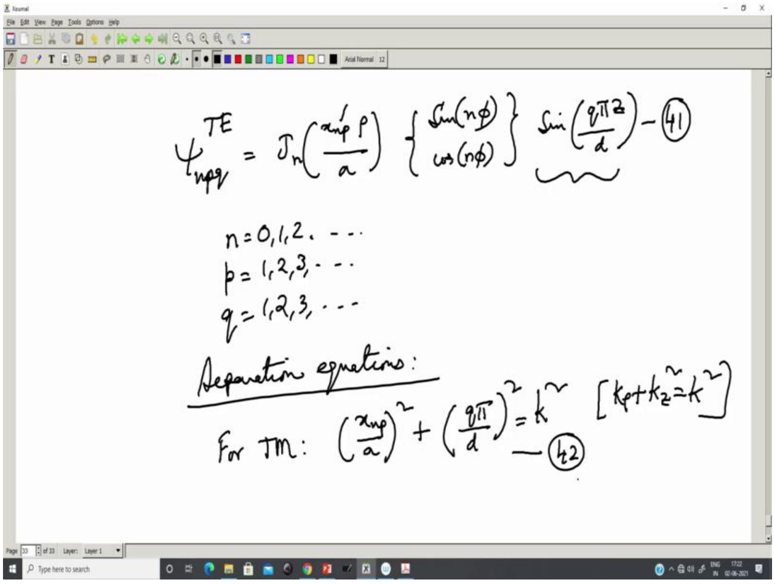

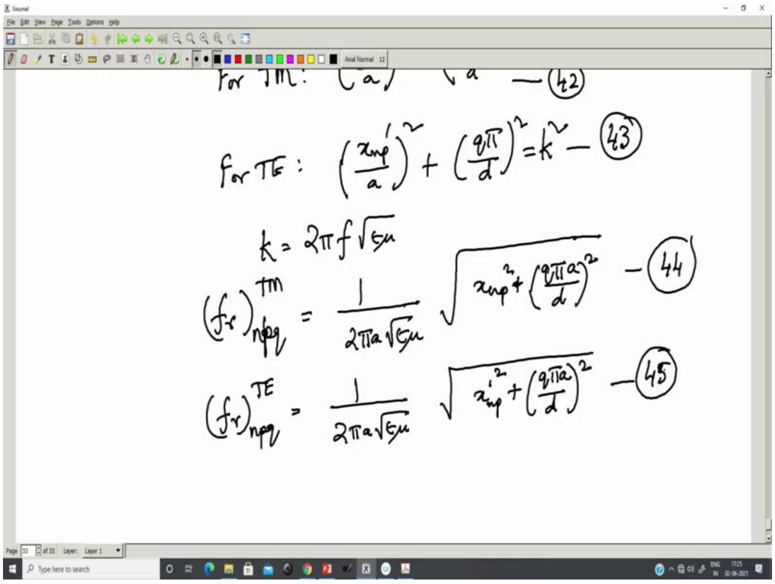

Similarly, the set of modes corresponding to the TE to z modes of the cavity, the potential functions corresponding to the TE to z- modes will be given by ψ_{npq}^{TE} as J $j_n\left(\frac{x_{np}\rho}{a}\right)\begin{Bmatrix}\sin(n\phi)\\ \cos(n\phi)\end{Bmatrix}\sin\left(\frac{q\pi z}{d}\right)$. Again the choice of this sine function should be easy for you to find out and I leave it up to you.

So, n is 0, 1, 2, p equal to 1, 2, 3 and q equal to 1, 2, 3, because it is a sine function. So, the separation equation for the TM and the TE mode becomes, for the TM mode we have $\left(\frac{x_{np}}{a}\right)^2+\left(\frac{q\pi}{d}\right)^2=k^2$. This is nothing but $k_\rho^2+k_z^2=k^2$, so we call this equation 42.

And for the TE case, the same equation $\left(\frac{x'_{np}}{a}\right)^2+\left(\frac{q\pi}{d}\right)^2=k^2$ that is 43. Now, since k is equal to the familiar $2\pi f\sqrt{\varepsilon\mu}$. We can solve for the resonant frequencies of the cavity from these two expressions.

So, fr the resonant frequency of the cavity correspond corresponding to the npq TM mode is given by $(f_r)_{npq}^{TM}=\frac{1}{2\pi a\sqrt{\varepsilon\mu}}\sqrt{x_{np}^2+\left(\frac{q\pi}{d}\right)^2}$, we call this equation 44. And fr npq for the TE mode becomes $(f_r)_{npq}^{TE}=\frac{1}{2\pi a\sqrt{\varepsilon\mu}}\sqrt{x'^2_{np}+\left(\frac{q\pi}{d}\right)^2}$ this becomes 45 corresponding to the npq th TE mode, so these two expressions yield the resonant frequencies of the cavity.

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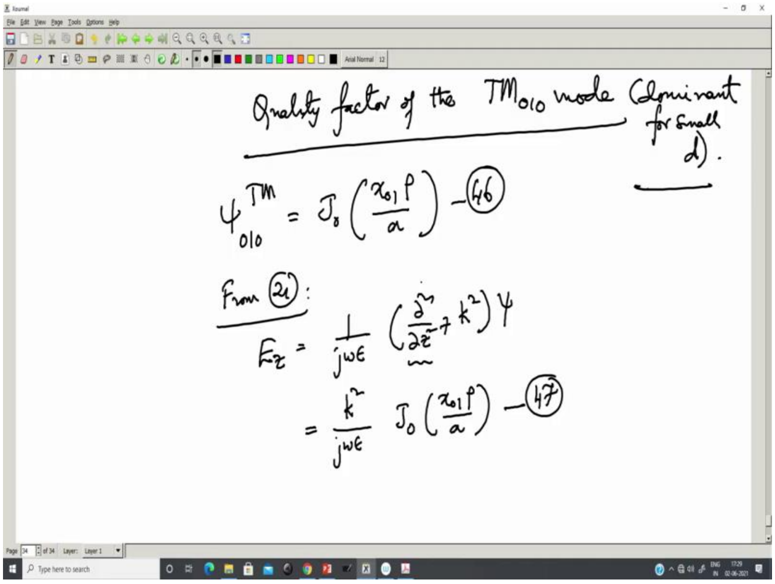

Now in order to find out the Q factor of the cavity, the quality factor of the cavity we specialize to the TM010 mode, so the quality factor of the dominant TM010 mode, dominant for small values of the cavity height. We need to find out the fields, because we need to find out the energy stored. For that we need the fields, and we need the power dissipated on the walls for which we need the fields.

So, first of all we need to find out the ψ function for the TM010 mode. So, the ψ_{010}^{TM} to z mode is given by $j_0\left(\frac{x_{01}\rho}{a}\right)$. So, n is 0 that means uniform variation along the phi direction the azimuthal variation of the field is uniform and the distribution of the field is also uniform along the z direction, because there also we have q as 0.

Only p is 1, so therefore the psi function becomes $j_0\left(\frac{x_{01}\rho}{a}\right)$ and from there we can find out all the field components. From 21 we have Ez given by $\frac{1}{j\omega\varepsilon}\left(\frac{\partial^2}{\partial z^2}+k^2\right)\psi$ that is $\frac{k^2}{j\omega\varepsilon} j_0\left(\frac{x_{01}\rho}{a}\right)$, we call this as equation 47. So, we note that $\frac{\partial}{\partial z}$ is 0 because there is no z variation in 46, so $\frac{\partial}{\partial z}$ this term is 0.

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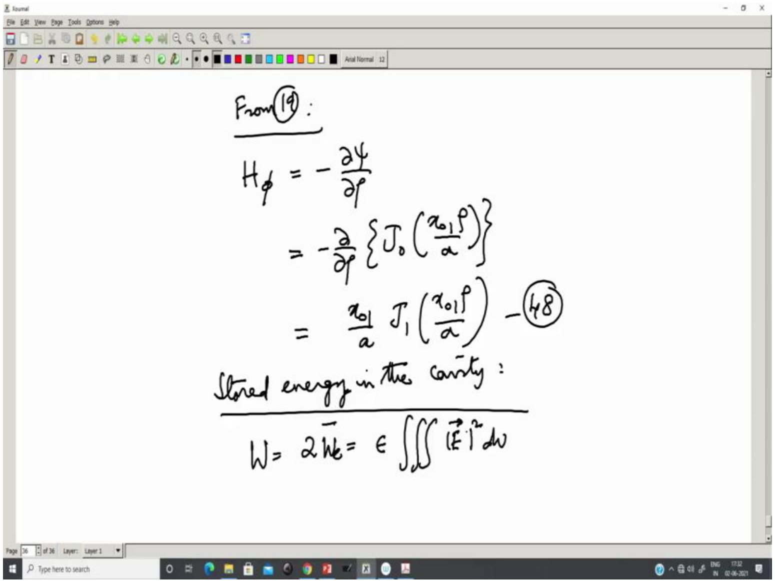

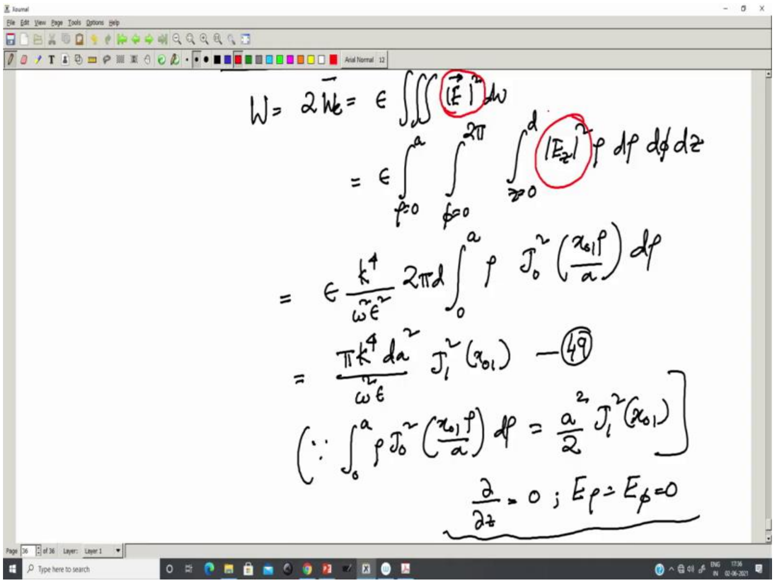

Then from equation 19 we can find out the magnetic field component H_ϕ that is $-\frac{\partial \psi}{\partial \rho}$ that is given by $-\frac{\partial}{\partial \rho}\left(j_0\left(\frac{x_{01}\rho}{a}\right)\right)$ that is given by $\frac{x_{01}}{a}\left(j_0\left(\frac{x_{01}\rho}{a}\right)\right)$, which is 48, so we know Ez and we know Hz. Now we can calculate the stored energy in the cavity, please verify what happens to the other field components; please verify that yourself why we have considered only Ez and Hz.

The store energy in the cavity is given by W is equal to $W = 2\overline{w}^e = \varepsilon \iiint \left|\vec{E}\right|^2 dv$ that is given by $\frac{\pi k^4 da^2}{\omega^2 \varepsilon}\left(j_0\left(x_{01}\right)\right)^2$, let us call this equation number 49.

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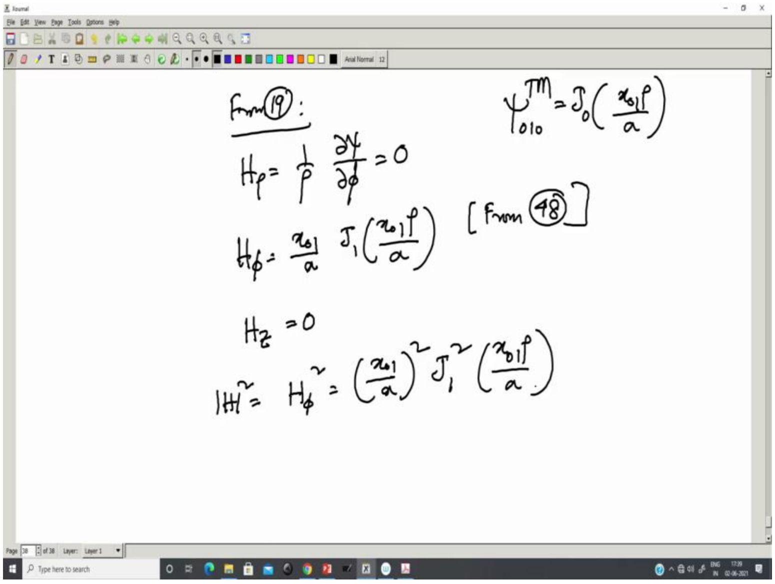

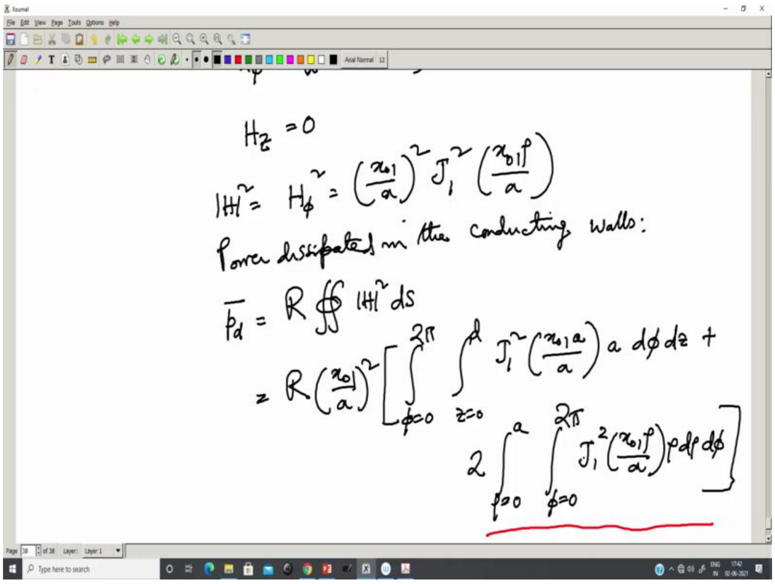

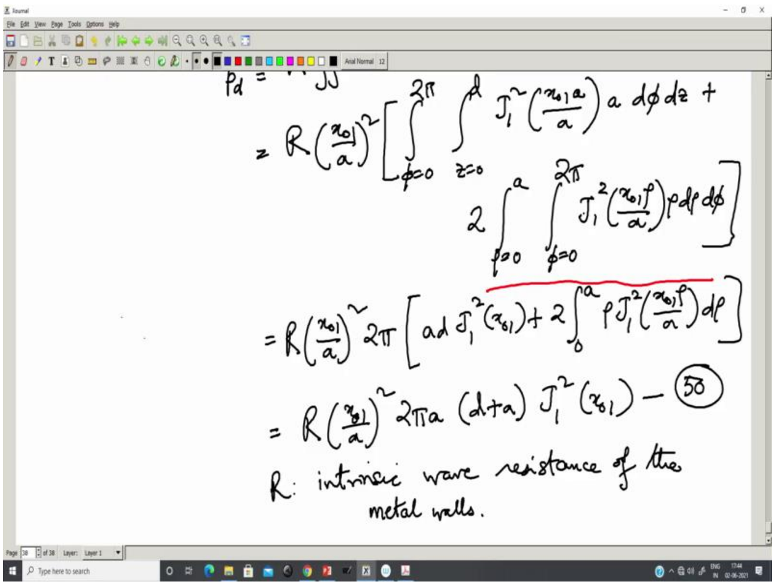

Now in order to find out the power dissipated in the conducting walls of the cavity we need to find the corresponding tangential magnetic fields on the walls of the cavity, so we go to equation number 19 for that TM to z modes and from 19 we obtain H_ρ as $\frac{1}{\rho}\frac{\partial\psi}{\partial\phi}=0$, because psi has no phi variation, again right here ψ_{010}^{TM} was $\left(j_0\left(\frac{x_{01}\rho}{a}\right)\right)$, it has no phi variation, so this is 0, H_ρ is 0.

We have H_ϕ as $\frac{x_{01}}{a}\left(j_0\left(\frac{x_{01}\rho}{a}\right)\right)$ that is from equation 48 and Hz is 0. Therefore, mod **H** square is H_ϕ square which is $\left(\frac{x_{01}}{a}\left(j_0\left(\frac{x_{01}\rho}{a}\right)\right)\right)^2$. So, power dissipated in the conducting walls is given by $\overline{p}_d = R\oiint |H|^2\, ds$ that is over the curved surface area of the cylinder.

And that eventually turns out to be $R\left(\frac{x_{01}}{a}\right)^2 2\pi a\left(d+a\right) j_1^2\left(x_{01}\right)$; that is equation 50, where R is the intrinsic wave resistance of the metal walls, so R is the intrinsic wave resistance of the metal walls.

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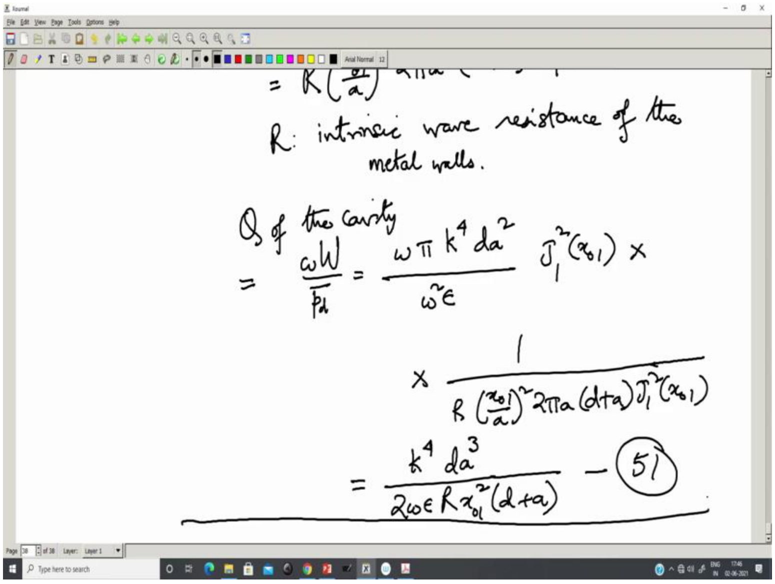

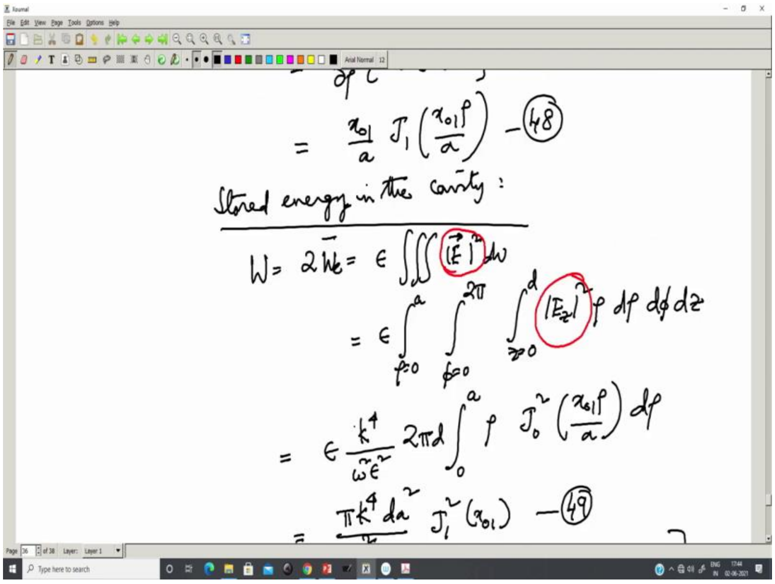

So now, the Q of the cavity is given by $\frac{\omega W}{\bar{p}_d}$ which we already found out that is equal to $\frac{\omega\pi k^4 da^2}{\omega^2\varepsilon}\left(j_0\left(x_{01}\right)\right)^2 \times \frac{1}{R\left(\frac{x_{01}}{a}\right)^2 2\pi a\left(d+a\right)j_1^2\left(x_{01}\right)}$. Then that becomes on simplification $\frac{k^4 da^3}{2\omega\varepsilon R x_{01}^2\left(d+a\right)}$, call this is equation 51.

So, this finally yields the expression of the Q factor of the cylindrical cavity which is very important for characterizing the cavity in terms of the stored energy and the power dissipated on the cylindrical cavity walls together with the end caps. So, this completes our treatment of the cylindrical cavity and the entire chapter on cylindrical wave function circular waveguide and cylindrical cavity. Thank you.