**Advanced Microwave Guided – Structures and Analysis**
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**Lecture 64**
**Cylindrical Wave Functions Tutorials**

Hello everyone. So, in today's session we will solve some numerical problems based on cylindrical wave functions.

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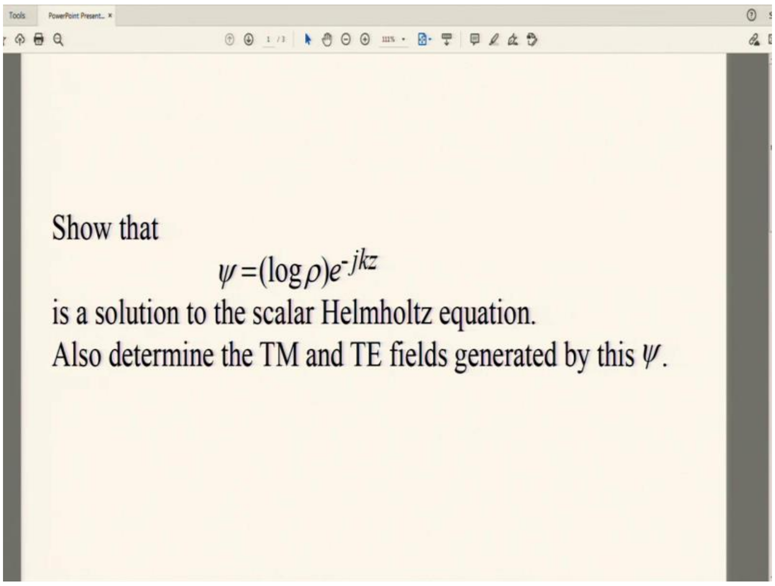


So, the first question is that we need to show $\psi = \log(\rho) e^{-jk_z z}$ is a solution to the scalar Helmholtz equation. Also we need to determine the TM and TE fields generated by this $\psi$.

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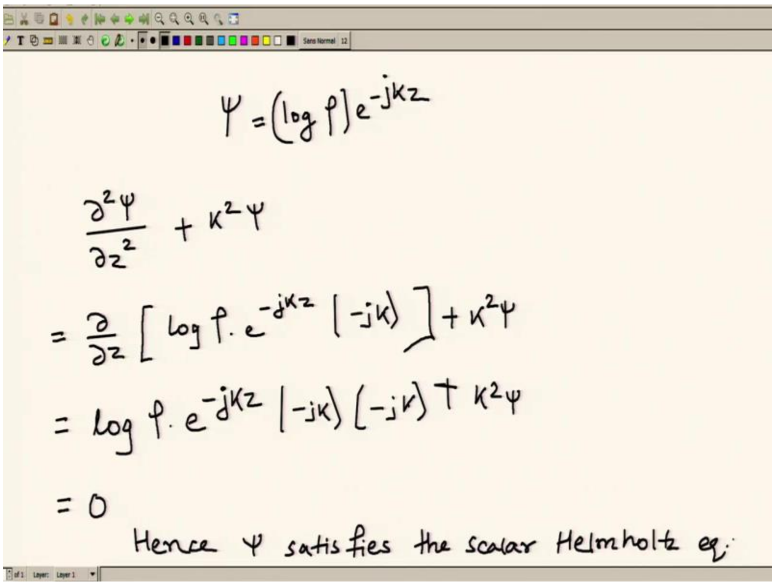


So, to start with we have $\psi = \log(\rho)e^{-jk_z z}$, so this is the psi function that has been provided in the question. So, at first what we need, we need to show that this $\psi$ is a solution to the scalar Helmholtz equation. So, we will find out $\frac{\partial^2\psi}{\partial z^2} + k^2\psi$, so we will perform the derivation with respect to z.

So, it will be $\frac{\partial}{\partial z}\left(\frac{\partial\psi}{\partial z}\right)$, first is $\frac{\partial\psi}{\partial z}$, so we will differentiate this function with respect to z first. So, on differentiating, we get $\log(\rho)e^{-jkz}(-jk)$. Again, we need to perform differentiation with respect to z, so it will be, $\log(\rho)e^{-jkz}(-jk)(-jk)$ plus $k^2\psi$, so this is 0, and hence $\psi$ satisfies the scalar Helmholtz equation. Now what? Now the question says like we need to determine the TM and TE fields generated by this $\psi$.

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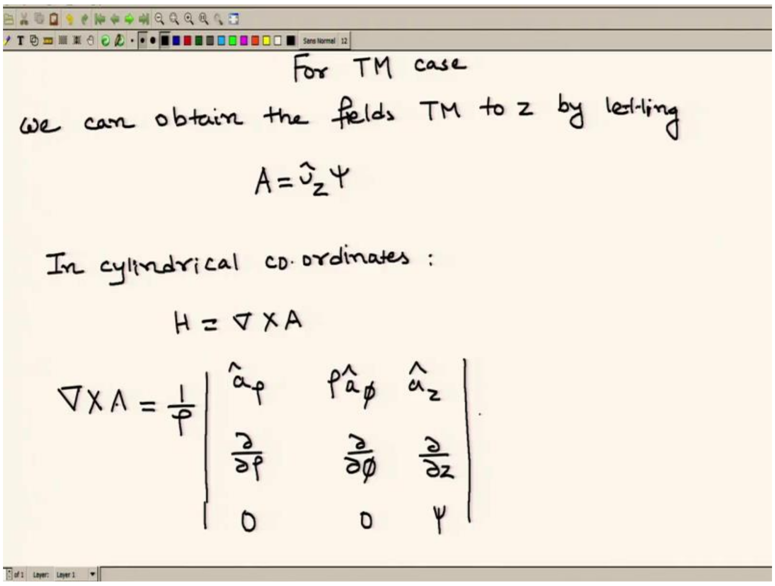


So, to start with will start for the TM case first. We can obtain the fields TM to z by letting **A** as $u_z \psi$ . Now in cylindrical coordinates, we know **H** will be curl of **A**.

We will find out curl of **A** in cylindrical coordinates, so we will have curl of **A** equals to

$$\frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ 0 & 0 & \psi \end{vmatrix}$$

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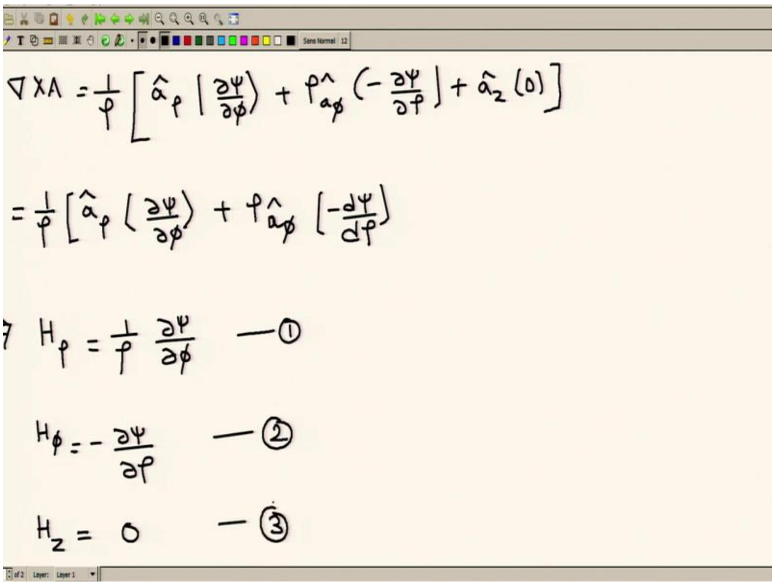

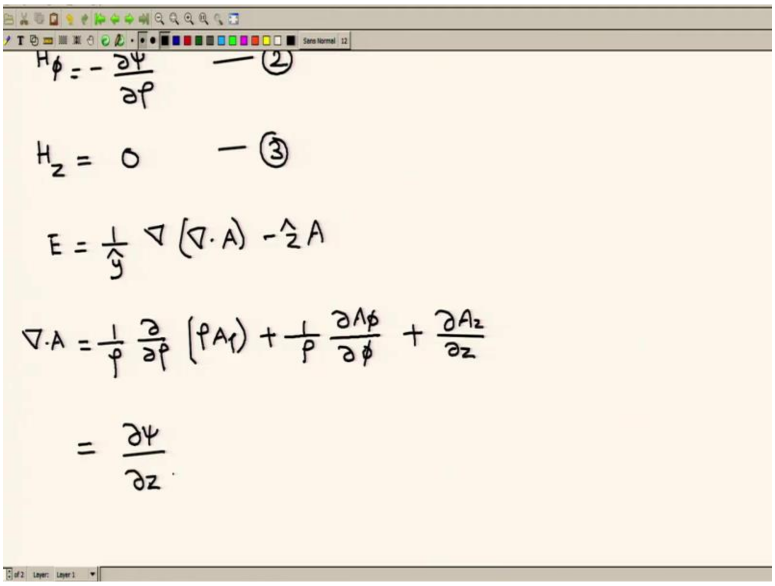


Since **H** is curl of **A**, so now we can write the fields as $H_\rho = \frac{1}{\rho}\frac{\partial\psi}{\partial\phi}$ , $H_\phi = -\frac{\partial\psi}{\partial\rho}$ , $H_z = 0$ . So, let us denote these equations as equation number 1, this is 2 and this is 3.

Now for the **E** fields we will write **E** as $\frac{1}{\hat{y}}\nabla\left(\nabla\cdot\vec{A}\right)-\hat{z}\vec{A}$. Now, we can write $\nabla\cdot\vec{A}$ as in cylindrical coordinates $\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho A_\rho\right)+\frac{1}{\rho}\frac{\partial}{\partial\phi}\left(A_\phi\right)+\frac{\partial}{\partial z}\left(A_z\right)$. So, for this case we are left with, this will be 0 and so we are left with $\frac{\partial}{\partial z}(\psi)$ .

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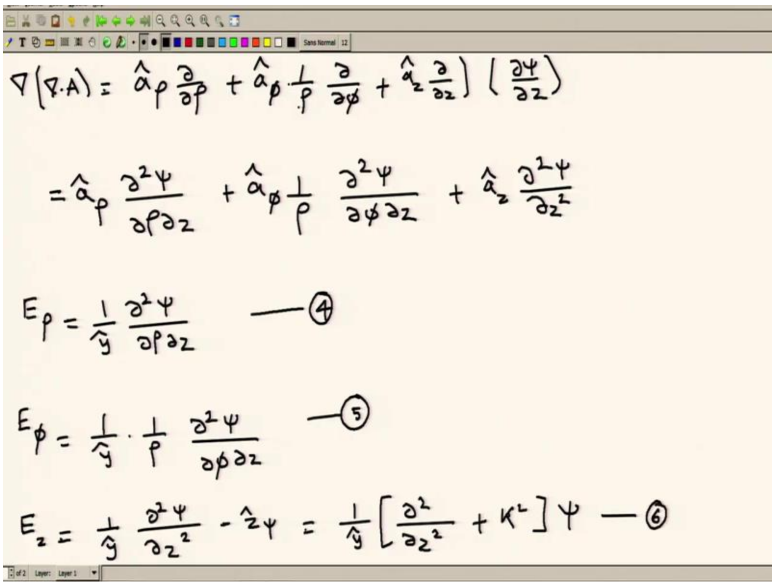

Therefore, we can write $\hat{a}_\rho \frac{\partial^2 \psi}{\partial \rho \partial z} + \hat{a}_\phi \frac{1}{\rho} \frac{\partial^2 \psi}{\partial \phi \partial z} + \hat{a}_z \frac{\partial^2 \psi}{\partial^2 z}$.

Now, we will substitute this and then find out the **E** fields. So, we will get

$$E_\rho = \frac{1}{\hat{y}} \frac{\partial^2 \psi}{\partial \rho \partial z}$$

$$E_\phi = \frac{1}{\hat{y}} \frac{1}{\rho} \frac{\partial^2 \psi}{\partial \phi \partial z}$$

$$E_z = \frac{1}{\hat{y}} \left[ \frac{\partial^2}{\partial^2 z} + k^2 \right] \psi$$

, so we will denote this as 4, this as 5 and this as equation 6.

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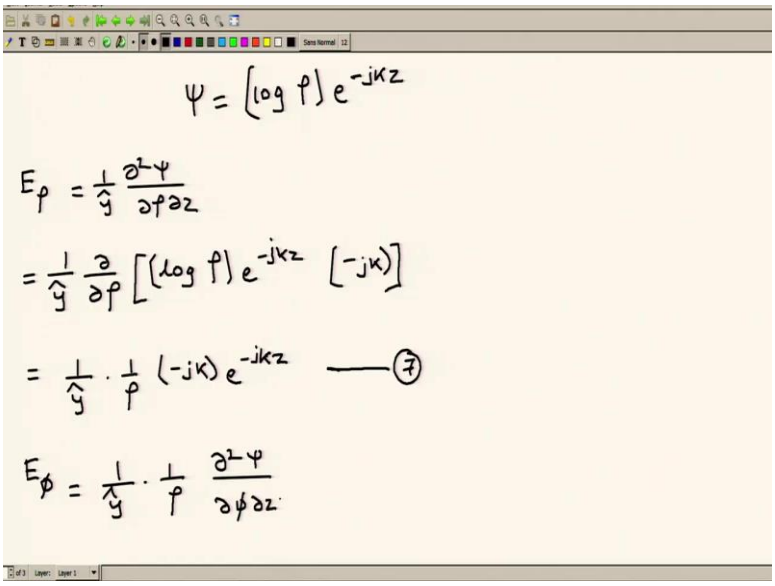


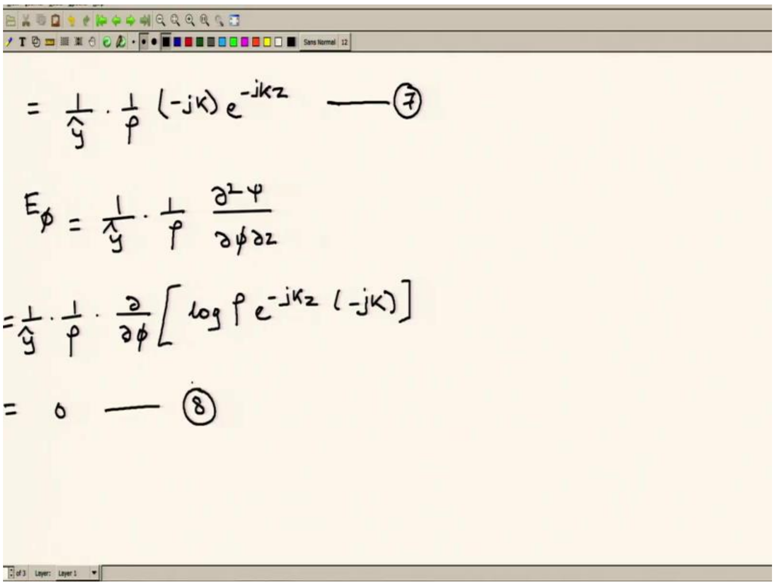

Now in the question we have $\psi = \log(\rho) e^{-jkz}$. So, we will have $E_\rho = \frac{1}{\hat{y}} \frac{\partial^2 \psi}{\partial \rho \partial z}$ that we derived previously. Now we will substitute this $\psi$ in this place, so this will be $\frac{1}{\hat{y}} \frac{1}{\rho} e^{-jkz} (-jk)$. So this is equation 7. Now we will have $E_\phi$, we have $E_\phi$ as 0. This is equation number 8.

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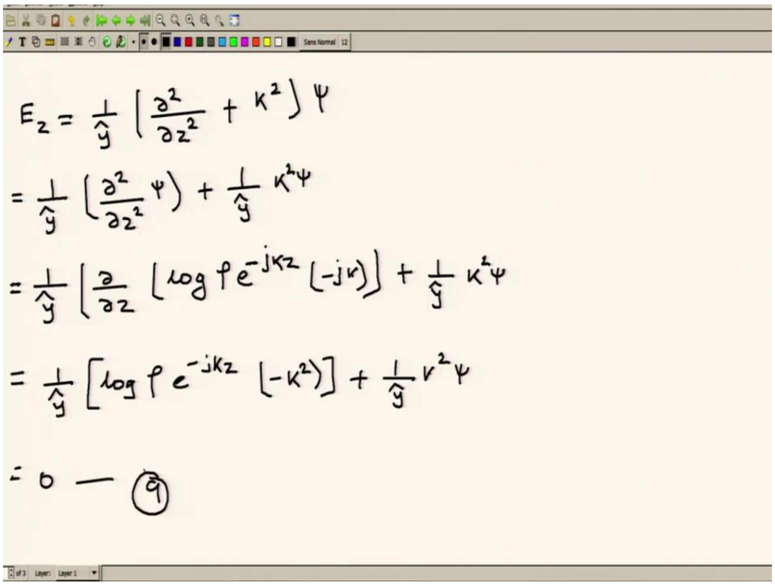

Now for Ez, we can write Ez as $\frac{1}{\hat{y}} \left[ \frac{\partial^2}{\partial^2 z} + k^2 \right] \psi$.

Now, again differentiating with respect to z, we will have 0, so basically this term gets as 0. Ez is also 0, so give this equation number as equation number 9.

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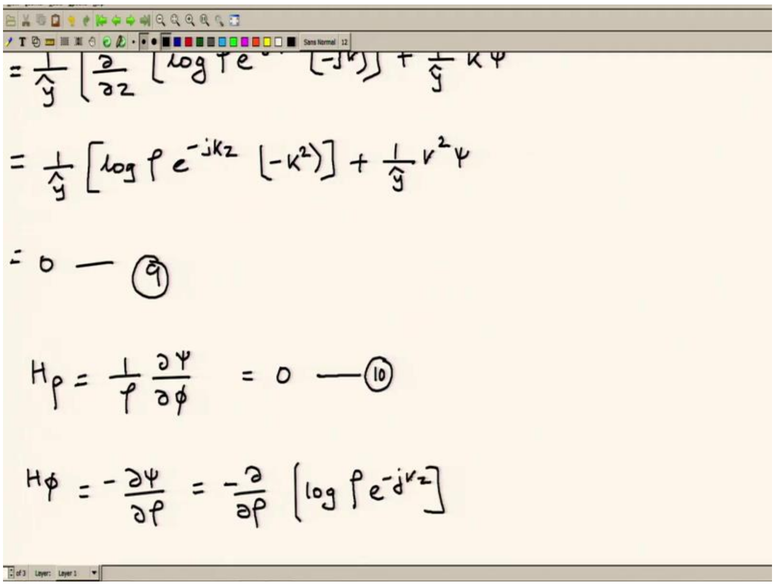


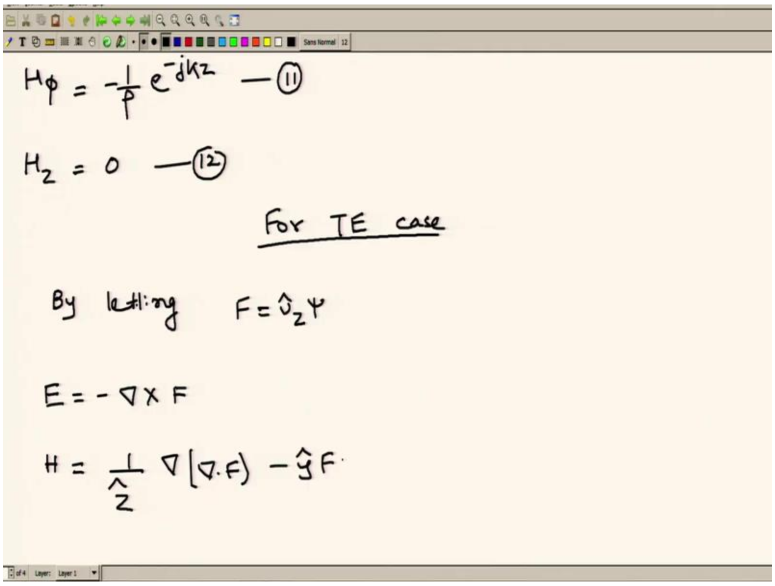


Now, $H_\rho$, $H_\phi$ and, Hz. So, $H_\rho$ we derived it as $H_\rho = \dfrac{1}{\rho}\dfrac{\partial\psi}{\partial\phi}$, so differentiating $\psi$ with respect to $\phi$ will give us 0, so this is equation number 10. Now $H_\phi$, so $H_\phi$ we derived as $-\dfrac{\partial\psi}{\partial\rho}$. So, it comes as minus $\dfrac{1}{\rho}e^{-jkz}$, so this is equation number 11 and Hz was 0, so these are the fields for TM case.

Now similarly for TE case we can obtain the field TE to z by considering **F** as $u_z\psi$. So, we can write **E** as minus curl of **F** and **H** as $\dfrac{1}{\hat{z}}\nabla\left(\nabla\bullet\vec{F}\right)-\hat{y}\vec{F}$. So, in the similar pattern we will find out the fields for the TE case.

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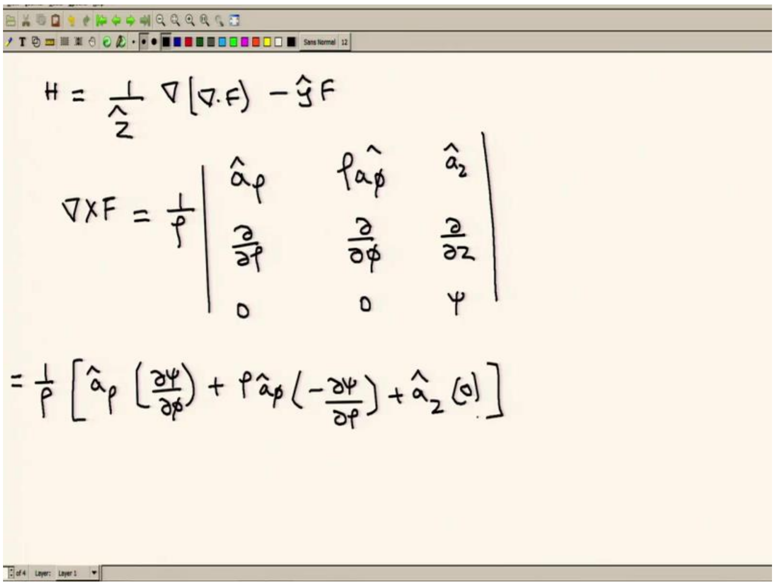


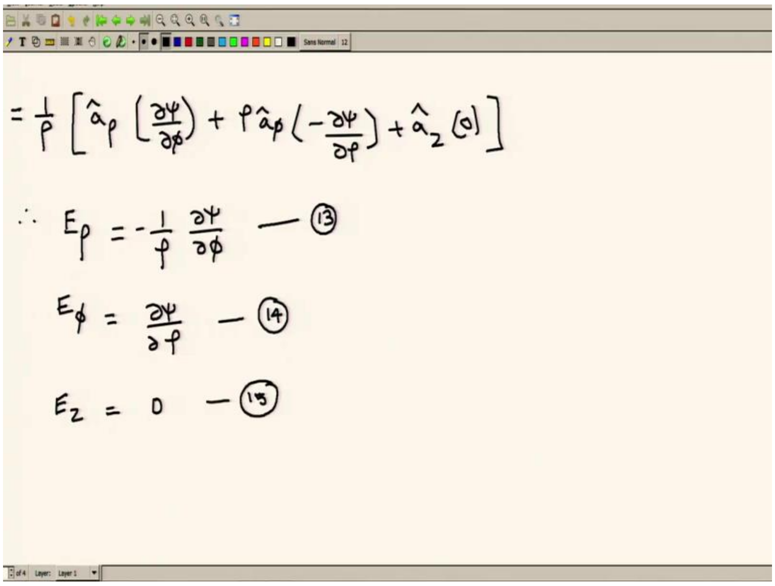


So, we will start from finding out $\nabla \times \vec{F}$ . We can write $\nabla \times \vec{F}$ as $\frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 0 & \psi \end{vmatrix}$. This will give us $\frac{1}{\rho}\hat{a}_\rho \frac{\partial}{\partial \phi}(\psi) + \hat{a}_\phi \frac{\partial}{\partial \rho}(\psi) + \hat{a}_z(0)$.

Therefore, since **E** was minus of curl of **F**, so we can write the fields as $E_\rho = -\frac{1}{\rho}\frac{\partial \psi}{\partial \phi}, E_\phi = \frac{\partial \psi}{\partial \rho}, E_z = 0$. So, let us denote this number as 13, this has equation number 14, and this is equation number 15.

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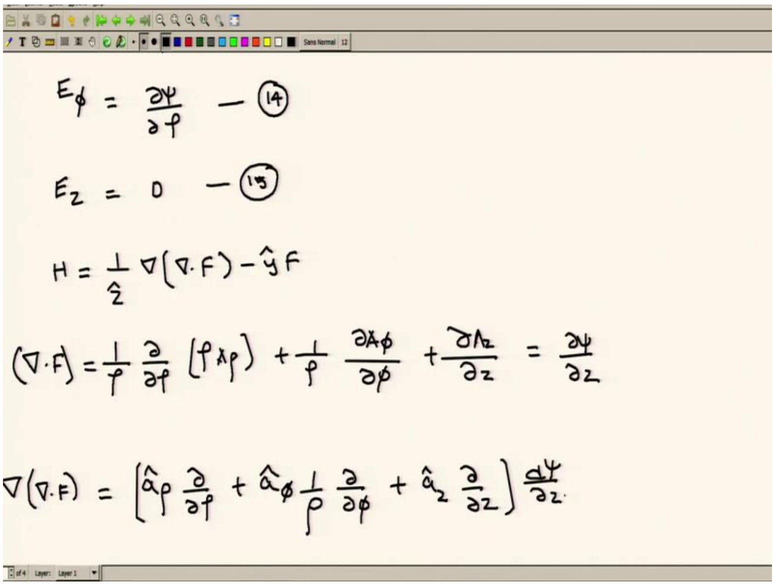


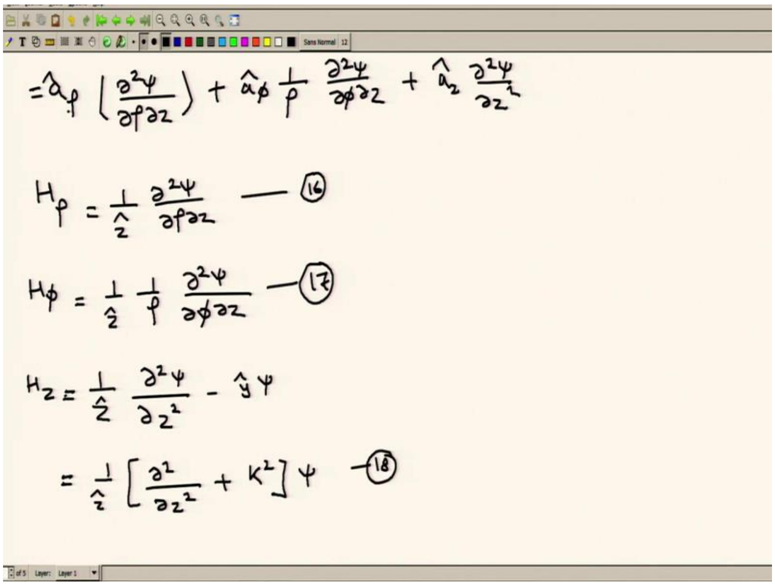


Next for **H** we will find $\frac{1}{\hat{z}}\nabla\left(\nabla\bullet\vec{F}\right)-\hat{y}\vec{F}$. So, we will find $H_\rho = \frac{1}{\hat{z}}\frac{\partial^2\psi}{\partial\rho\partial z}$, H phi we will get $H_\phi = \frac{1}{\hat{z}}\frac{1}{\rho}\frac{\partial^2\psi}{\partial\phi\partial z}$ and Hz we will have $H_z = \frac{1}{\hat{y}}\left[\frac{\partial^2}{\partial^2 z}+k^2\right]\psi$. So, let us denote these equations as equation number 16, this as 17 and this as equation 18.

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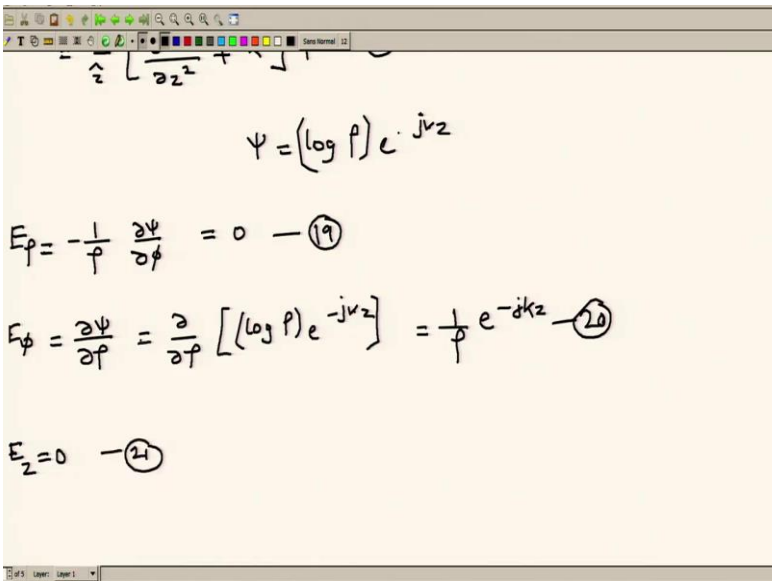


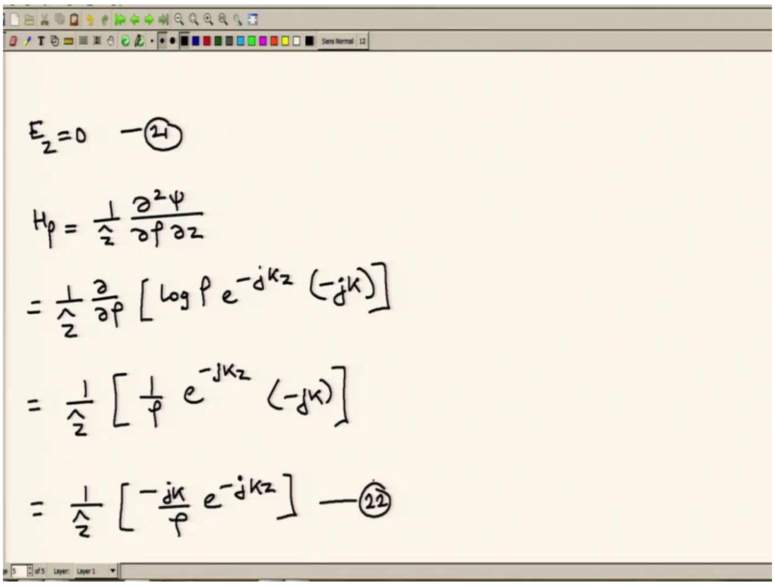


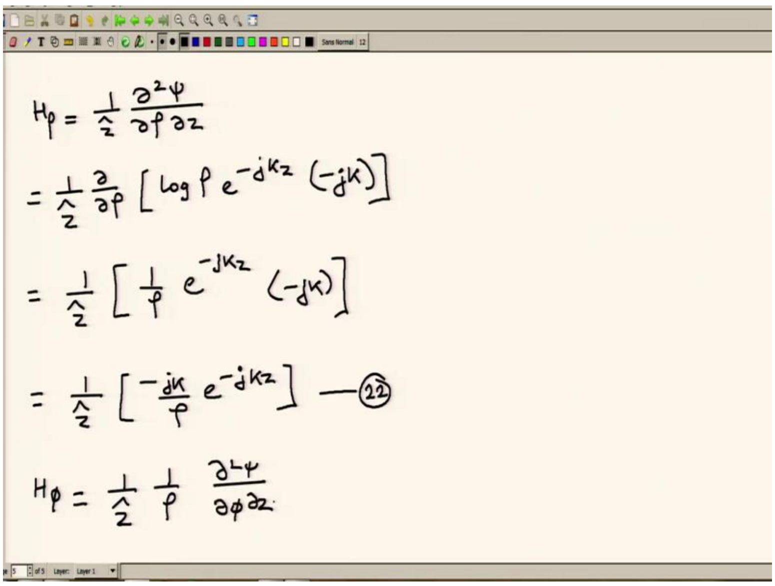

Now in the given question $\psi$ was $\log(\rho)e^{-jkz}$, so we will have $E_\rho$ as 0. Then $E_\phi$ we have $E_\phi = \frac{\partial\psi}{\partial\rho}$. So, we will get $\frac{1}{\rho}e^{-jkz}$ and then we had Ez as 0. So, let us denote this as an equation 19, 20 and 21.

Now the **H** fields. So, we have $H_\rho$ as $\frac{1}{\hat{z}}\frac{\partial^2\psi}{\partial\rho\partial z}$, so it will have $\frac{1}{\hat{z}}\left[\frac{-jk}{\rho}e^{-jkz}\right]$. We will give it as equation number 22. Then we will find out $H_\phi$, so $H_\phi$ is 1 upon z cap 1 upon rho del square psi upon del phi del z.

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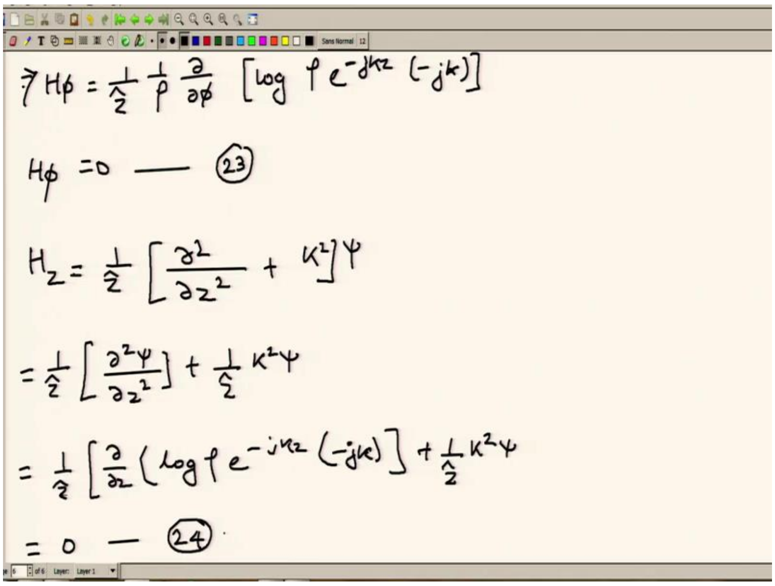


So, $H_\phi$ is also 0, so this is equation 23. Now Hz, so Hz is $\frac{1}{\hat{y}}\left[\frac{\partial^2}{\partial^2 z}+k^2\right]\psi$, we will substitute $\psi$, so we will get that this is also coming as 0, so this is equation number 24. So, these are the TE and TM fields that are generated by this psi function. So, this is all for now, thank you so much.