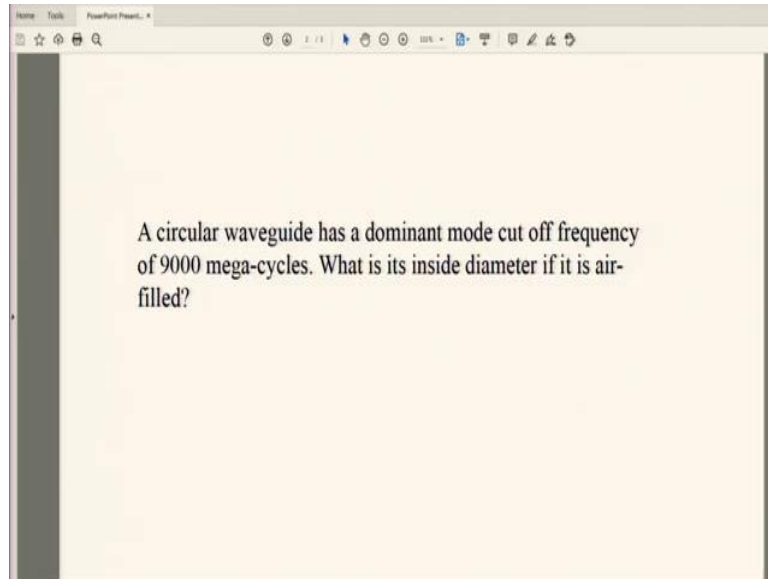


Advanced Microwave Guided-Structures and Analysis
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Lecture 65
Cylindrical Wave Functions Tutorials (cont.)

Hello everyone, so this is the second tutorial session on cylindrical wave functions.

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So, the first question says that we have a circular waveguide and the dominant mode cutoff frequency of 9000 mega cycles. So, what is its inside diameter if it is air filled?

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$$f_c = 9000 \text{ MHz} = 9000 \times 10^6 \text{ Hz}$$

Dominant mode $\rightarrow TE_{11}$

$$\lambda_c^{TE} = \frac{2\pi a}{x'_{np}}$$

for $TE_{11} \rightarrow x'_{np} = 1.841$

$$\lambda_c^{TE} = \frac{2\pi a}{1.841}$$

$$\lambda_c^{TE} = \frac{2\kappa a}{1.841}$$

$$\lambda_c = \frac{3 \times 10^8}{9000 \times 10^6} = \frac{1}{30} \text{ m} = \frac{10}{3} \text{ cm}$$

$$\frac{2\kappa a}{1.841} = \frac{10}{3}$$

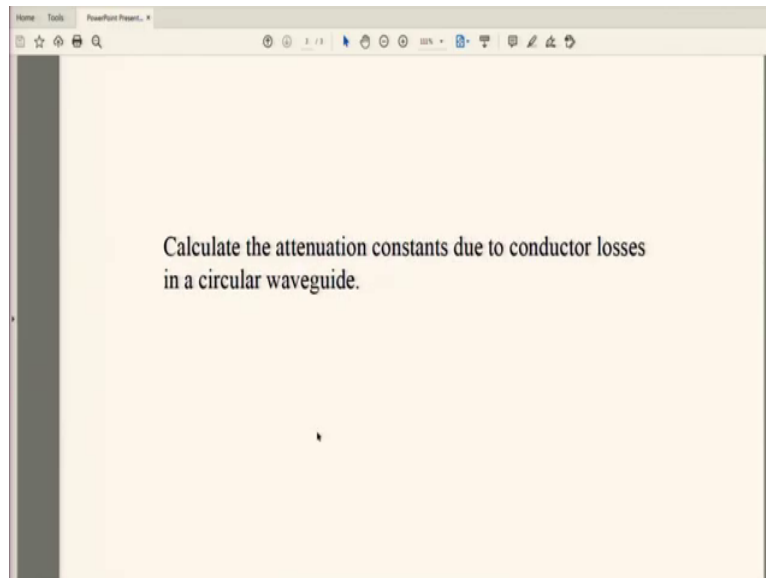
$$\Rightarrow a = \frac{10 \times 1.841}{2\kappa \times 3} = 0.977 \text{ cm}$$

So, the cutoff frequency for the circular waveguide is given as 9000 Mega Hertz that is equal to 9000 into 10 to the power 6 Hertz. Now, in the question it is said that it is in the dominant mode operation. So for circular waveguide the dominant mode is, we know TE₁₁. So, for dominant mode λ_c^{TE} , we can write it down like $\frac{2\pi a}{x'_{n\rho}}$, where a is the unknown that we need to

find out. $x'_{n\rho}$ is given as four 1.841, so we will have then λ_c^{TE} as 2 Pi a upon 1.841.

Now, we can also write λ_c^{TE} as c by f. So f is 9000 into 10 to the power 6 that gives us 1 upon 30 meter. So, that is 10 upon 3 centimeter. Fine so, therefore we can write, 2 Pi a upon 1.841 as 10 upon 3, so then we will have a as 10 multiplied by 1.841 divided by 2 Pi multiplied by 3. So, this will give us 0.977 centimeter.

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For TM modes

$$\Psi^{TM} = J_n(k_p r) \cos n\phi e^{-jk_z z}$$
$$E_r = \frac{1}{\gamma} \frac{\partial^2 \Psi}{\partial r \partial z}$$
$$= \frac{1}{\gamma} \frac{\partial}{\partial r} [J_n(k_p r) \cos n\phi e^{-jk_z z} (-jk_z)]$$
$$= \frac{1}{\gamma} J_n'(k_p r) k_p (-jk_z) \cos n\phi e^{-jk_z z} \quad \text{--- ①}$$

$$= \frac{1}{\gamma} J_n'(k_p r) k_p (-jk_z) \cos n\phi e^{-jk_z z} \quad \text{--- ①}$$
$$E_\phi = \frac{1}{\gamma r} \frac{\partial^2 \Psi}{\partial \phi \partial z}$$
$$= \frac{1}{\gamma r} \frac{\partial}{\partial \phi} [J_n(k_p r) \cos n\phi e^{-jk_z z} (-jk_z)]$$
$$= \frac{1}{\gamma r} J_n(k_p r) \sin n\phi (n) e^{-jk_z z} (-jk_z) \quad \text{--- ②}$$

In the next question, we need to calculate the attenuation constants due to conductor losses in a circular waveguide. So, for finding out the attenuation constant, we will consider both the modes that is for TM modes and TE modes. So, to start with TM modes, we can write, Psi of TM as $j_n(k_\rho \rho) \cos(n\phi) e^{-jk_z z}$. Therefore, from the fields that we had previously found out that

$$E_\rho \text{ is given by } \frac{1}{\hat{y}} \frac{\partial^2 \psi}{\partial \rho \partial z}.$$

So, this will be $\frac{1}{\hat{y}} (j'_n(k_\rho \rho))(k_\rho)(-jk_z) \cos(n\phi) e^{-jk_z z}$. So let us give this equation as equation number 1.

Now, again E_ϕ for TM modes, we have $\frac{1}{\hat{y}\rho} \frac{\partial^2 \psi}{\partial \phi \partial z}$. So, this will give

us $\frac{1}{\hat{y}\rho} (j_n(k_\rho \rho))(n)(-jk_z) \sin(n\phi) e^{-jk_z z}$. This is equation number 2.

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The image shows a handwritten derivation on a digital whiteboard. It starts with the expression for the phi component of the electric field, E_ϕ , which is given by $\frac{1}{\hat{y}\rho} \frac{\partial^2 \psi}{\partial \phi \partial z}$. The wave potential ψ is assumed to be $j_n(k_\rho \rho) \cos(n\phi) e^{-jk_z z}$. The derivation proceeds as follows:

$$= \frac{1}{\hat{y}\rho} J_n(k_\rho \rho) \sin n\phi (n) e^{-jk_z z} (-jk_z) \quad \text{--- (2)}$$

$$H_\rho = \frac{1}{\rho} \frac{\partial \psi}{\partial \phi}$$

$$= \frac{1}{\rho} J_n(k_\rho \rho) (-\sin n\phi) (n) e^{-jk_z z}$$

$$= -\frac{n}{\rho} J_n(k_\rho \rho) \sin n\phi e^{-jk_z z} \quad \text{--- (3)}$$

$$H_\phi = -\frac{\partial \psi}{\partial \rho}$$

$$= -\frac{n}{\rho} J_n(k\rho r) \sin n\phi e^{-jk_z z} \quad \text{--- (3)}$$

$$H_\phi = -\frac{\partial \psi}{\partial \rho}$$

$$= -J_n'(k\rho r) k\rho \cos n\phi e^{-jk_z z} \quad \text{--- (4)}$$

$$\alpha = \frac{P_d}{2P_f}$$

Now, H_ρ will have $\frac{1}{\rho} \frac{\partial \psi}{\partial \phi}$ and this will have $\frac{-n}{\rho} (j_n'(k\rho r)) \sin(n\phi) e^{-jk_z z}$. So, this is equation number 3.

Now, H_ϕ is minus of $-\frac{\partial \psi}{\partial \rho}$ that is equal to $-(j_n'(k\rho r))(k\rho) \cos(n\phi) e^{-jk_z z}$. So, this is equation number 4. Now, to find out the attenuation constant due to the conductor loss, we can write this as α given as $\frac{P_d}{2P_f}$ where, P_d is the dissipated power and P_f is the power flow.

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$$P_d = \mathcal{R} \int_0^a \rho d\rho \int_0^{2\pi} d\phi \left\{ (H_\rho)^2 + (H_\phi)^2 \right\}_{\substack{\rho=a \\ z=0}}$$

$$P_f = \int_0^a \rho d\rho \int_0^{2\pi} d\phi \left[E_\rho H_\phi^* - E_\phi H_\rho^* \right]$$

$$\alpha = \frac{P_d}{2P_f}$$

So, where P_d we can write as $P_d = \mathcal{R} \int_0^a \rho d\rho \int_0^{2\pi} d\phi \left\{ (H_\rho)^2 + (H_\phi)^2 \right\}_{\substack{\rho=a \\ z=0}}$. So, H_ϕ and H_ρ are known,

so we can just simply substitute and calculate P_d whereas power flow P_f is given by $\int_0^a \rho d\rho \int_0^{2\pi} d\phi \left\{ (E_\rho H_\phi^*) - (E_\phi H_\rho^*) \right\}$. So, again $E_\rho H_\phi$ and $E_\phi H_\rho$ are known, so we can again substitute and then calculating P_d and P_f we can get, the attenuation constant as P_d upon twice P_f , so next for the TE modes.

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TE modes

$$\psi^{TE} = J_n(k_\rho \rho) \cos n\phi e^{-j k_z z}$$

$$E_\rho = -\frac{1}{\rho} \frac{\partial \psi}{\partial \phi}$$

$$= \frac{1}{\rho} J_n'(k_\rho \rho) \sin n\phi (n) e^{-j k_z z} \quad \text{--- (5)}$$

$$E_\phi = \frac{\partial \psi}{\partial \rho}$$

$$= J_n'(k_\rho \rho) (k_\rho) \cos n\phi e^{-j k_z z} \quad \text{--- (6)}$$

$$E_{\phi} = \frac{\partial \psi}{\partial \phi}$$

$$= J_n'(k_{\rho} \rho) (k_{\rho}) \cos n\phi e^{-jk_z z} \quad \text{--- (6)}$$

$$H_{\rho} = \frac{1}{z} \frac{\partial^2 \psi}{\partial \rho \partial z}$$

$$= \frac{1}{z} \frac{\partial}{\partial \rho} [J_n(k_{\rho} \rho) \cos n\phi e^{-jk_z z} (-jk_z)]$$

$$= \frac{1}{z} [J_n'(k_{\rho} \rho) (k_{\rho}) \cos n\phi e^{-jk_z z} (-jk_z)] \quad \text{--- (7)}$$

So, for TE modes, we have ψ^{TE} as $j_n(k_{\rho} \rho) \cos(n\phi) e^{-jk_z z}$. So, E_{ρ} we will have $-\frac{1}{\rho} \frac{\partial \psi}{\partial \phi}$. So,

we can calculate this. So this will be $E_{\rho} = \frac{1}{\rho} (j_n'(k_{\rho} \rho)) (n) \sin(n\phi) e^{-jk_z z}$. So, we can give this as equation number 5.

So, after E_{ρ} we will have E_{ϕ} . So, E_{ϕ} , we have as $\frac{\partial \psi}{\partial \rho}$. So, this is $(j_n'(k_{\rho} \rho)) (k_{\rho}) \cos(n\phi) e^{-jk_z z}$.

So, this is equation number 6. Now, H_{ρ} . So, H_{ρ} we found out that H_{ρ} can be written $\frac{1}{z} \frac{\partial^2 \psi}{\partial \rho \partial z}$.

So, first we will differentiate with respect to z, and again with respect to Rho we will have $H_{\rho} = \frac{1}{z} (j_n'(k_{\rho} \rho)) (-jk_z k_{\rho}) \cos(n\phi) e^{-jk_z z}$. So this is equation number 7. So, H_{ρ} and then H_z , similarly we will calculate.

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Handwritten equations from the whiteboard:

$$= \frac{1}{2} [J_n'(k_\rho \rho) (k_\rho) \cos n\phi e^{-jk_z z} (-j\omega)] \quad (7)$$

$$H_z = \frac{1}{2} J_n(k_\rho \rho) k_\rho^2 \cos n\phi e^{-jk_z z}$$

$$P_d = \Re \int_0^a \rho d\rho \int_0^{2\pi} d\phi \left([H_\phi]^2 + [H_z]^2 \right) \Bigg|_{z=0}^{z=a}$$

$$P_f = \int_0^a \rho d\rho \int_0^{2\pi} d\phi [E_\rho H_\phi^* - E_\phi H_\rho^*]$$

$$H_z = \frac{1}{2} J_n(k_\rho \rho) k_\rho^2 \cos n\phi e^{-jk_z z}$$

$$P_d = \Re \int_0^a \rho d\rho \int_0^{2\pi} d\phi \left([H_\phi]^2 + [H_z]^2 \right) \Bigg|_{z=0}^{z=a}$$

$$P_f = \int_0^a \rho d\rho \int_0^{2\pi} d\phi [E_\rho H_\phi^* - E_\phi H_\rho^*]$$

$$\alpha = \frac{P_d}{2P_f}$$

We get H_z as $\frac{1}{2} (j_n(k_\rho \rho)) (k_\rho^2) \cos(n\phi) e^{-jk_z z}$. So, again to find out the attenuation constant we

need to find out P_d and P_f . So, power dissipated P_d can be written as $\Re \int_0^a \rho d\rho \int_0^{2\pi} d\phi \left\{ (H_z)^2 + (H_\phi)^2 \right\} \Bigg|_{z=0}^{z=a}$. And power flow P_f , we can

write $\int_0^a \rho d\rho \int_0^{2\pi} d\phi \left\{ (E_\rho H_\phi^*) - (E_\phi H_\rho^*) \right\}$. So, on substituting we will get attenuation constant as

the ratio of P_d upon $2P_f$, so this is all about the cylindrical wave functions. Thank you.