Advanced Microwave Guided-Structures and Analysis
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Lecture 65
Cylindrical Wave Functions Tutorials (cont.)

Hello everyone, so this is the second tutorial session on cylindrical wave functions.

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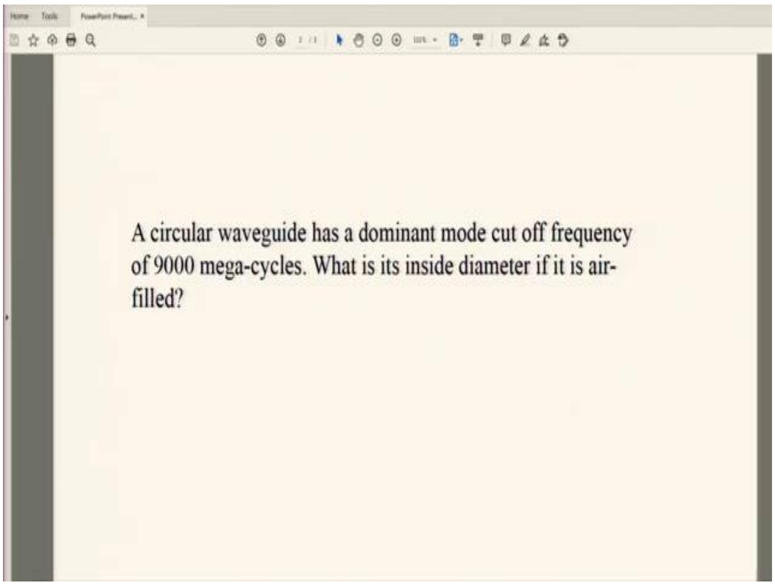

So, the first question says that we have a circular waveguide and the dominant mode cutoff frequency of 9000 mega cycles. So, what is its inside diameter if it is air filled?

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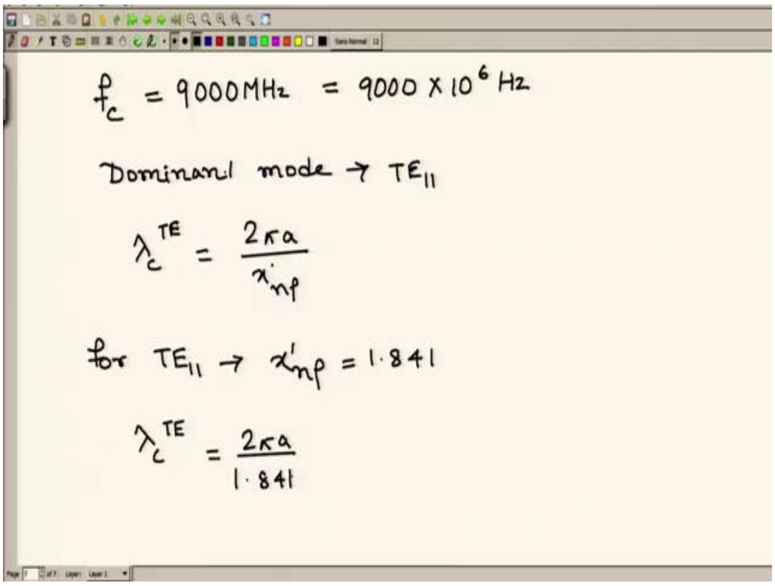

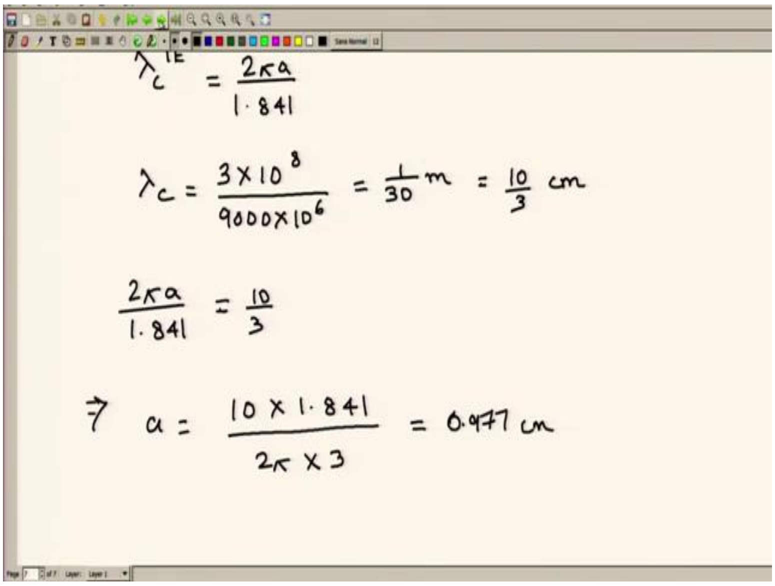

So, the cutoff frequency for the circular waveguide is given as 9000 Mega Hertz that is equal to 9000 into 10 to the power 6 Hertz. Now, in the question it is said that it is in the dominant mode operation. So for circular waveguide the dominant mode is, we know TE11. So, for dominant mode λ_c^{TE} , we can write it down like $\frac{2\pi a}{x'_{n\rho}}$, where a is the unknown that we need to find out. $x'_{n\rho}$ is given as four 1.841, so we will have then λ_c^{TE} as 2 Pi a upon 1.841.

Now, we can also write λ_c^{TE} as c by f. So f is 9000 into 10 to the power 6 that gives us 1 upon 30 meter. So, that is 10 upon 3 centimeter. Fine so, therefore we can write, 2 Pi a upon 1.841 as 10 upon 3, so then we will have a as 10 multiplied by 1.841 divided by 2 Pi multiplied by 3. So, this will give us 0.977 centimeter.

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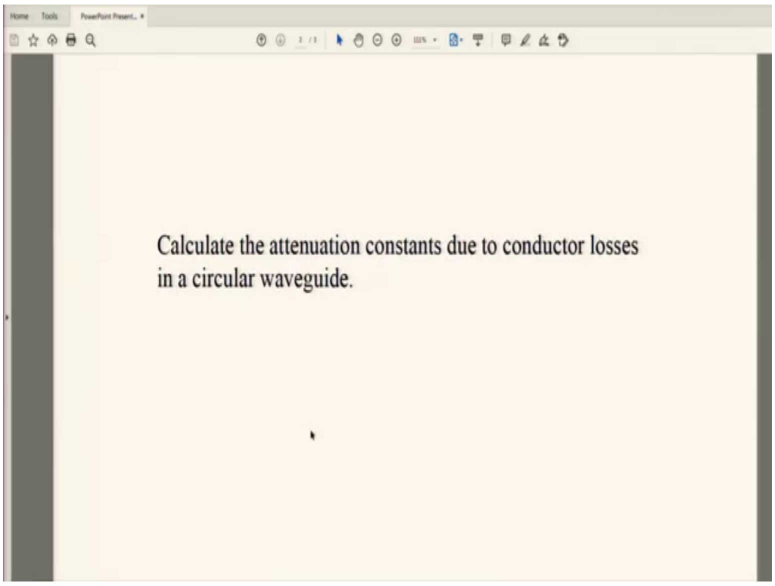

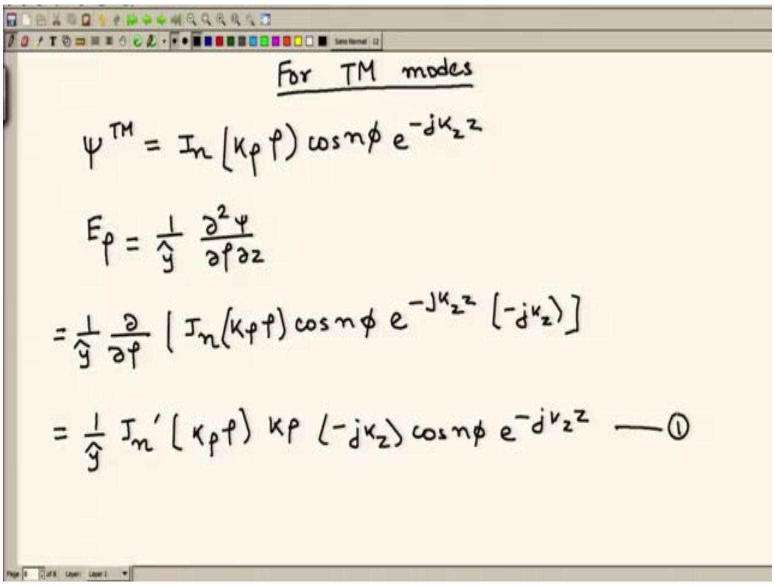

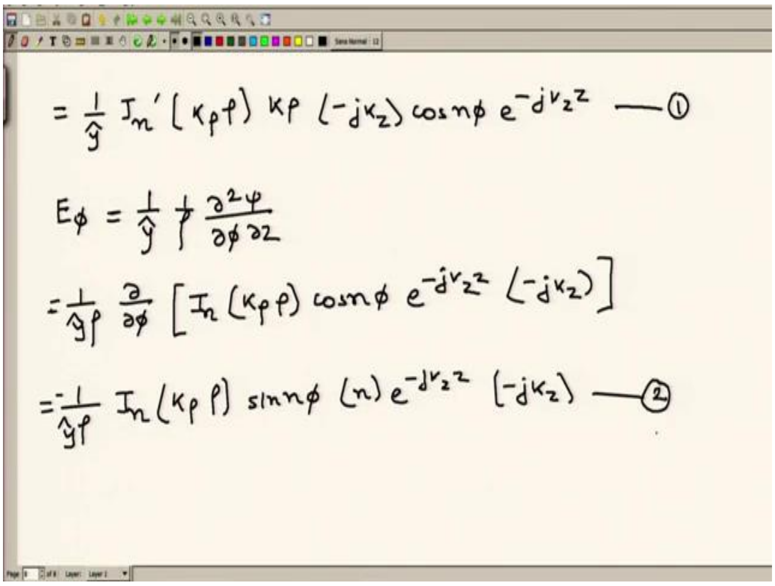

In the next question, we need to calculate the attenuation constants due to conductor losses in a circular waveguide. So, for finding out the attenuation constant, we will consider both the modes that is for TM modes and TE modes. So, to start with TM modes, we can write, Psi of TM as $j_n\left(k_\rho \rho\right)\cos\left(n\phi\right)e^{-jk_z z}$. Therefore, from the fields that we had previously found out that E_ρ is given by $\frac{1}{\hat{y}}\frac{\partial^2\psi}{\partial\rho\partial z}$.

So, this will be $\frac{1}{\hat{y}}\left(j_n'\left(k_\rho\rho\right)\right)\left(k_\rho\right)\left(-jk_z\right)\cos\left(n\phi\right)e^{-jk_z z}$ .So let us give this equation as equation number 1.

Now, again E_ϕ for TM modes, we have $\frac{1}{\hat{y}\rho}\frac{\partial^2\psi}{\partial\phi\partial z}$. So, this will give us $\frac{1}{\hat{y}\rho}\left(j_n\left(k_\rho\rho\right)\right)\left(n\right)\left(-jk_z\right)\sin\left(n\phi\right)e^{-jk_z z}$. This is equation number 2.

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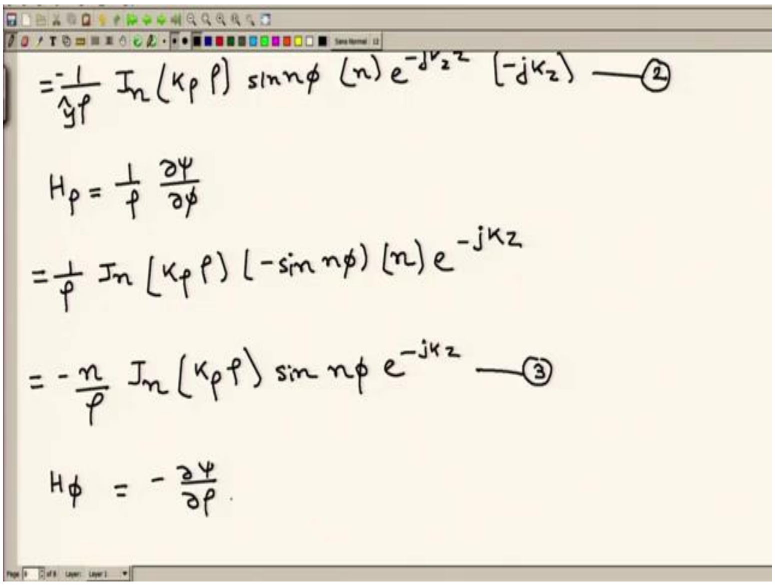

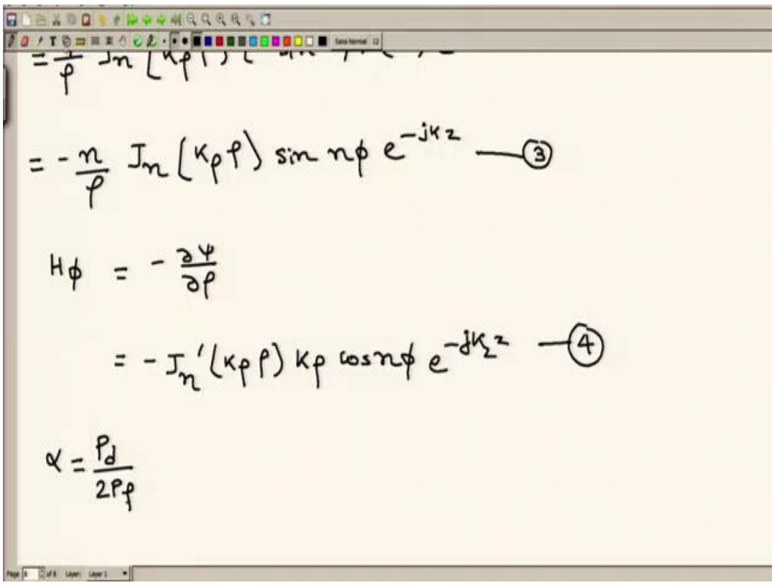

Now, H_ρ will have $\frac{1}{\rho}\frac{\partial\psi}{\partial\phi}$ and this will have $\frac{-n}{\rho}\left(j_n\left(k_\rho\rho\right)\right)\sin\left(n\phi\right)e^{-jk_z z}$. So, this is equation number 3.

Now, H_ϕ is minus of $-\frac{\partial\psi}{\partial\rho}$ that is equal to $-\left(j_n'\left(k_\rho\rho\right)\right)\left(k_\rho\right)\cos\left(n\phi\right)e^{-jk_z z}$. So, this is equation number 4. Now, to find out the attenuation constant due to the conductor loss, we can write this as α given as $\frac{P_d}{2P_f}$ where, P_d is the dissipated power and P_f is the power flow.

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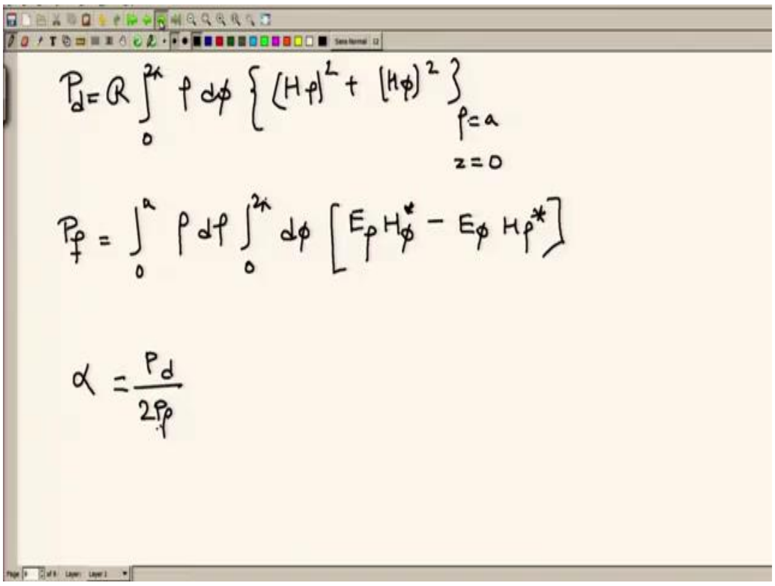

So, where Pd we can write as $P_d = \Re \int_0^{2\pi} \rho d\phi \left\{ \left(H_\rho\right)^2 + \left(H_\phi\right)^2 \right\}_{\substack{\rho=a \\ z=0}}$. So, H_ϕ and H_ρ are known, so we can just simply substitute and calculate Pd whereas power flow Pf is given by $\int_0^a \rho d\rho \int_0^{2\pi} d\phi \left\{ \left(E_\rho H_\phi^*\right) - \left(E_\phi H_\rho^*\right) \right\}$. So, again E_ρ H_ϕ and $E_\phi H_\rho$ are known, so we can again substitute and then calculating Pd and Pf we can get, the attenuation constant as Pd upon twice Pf, so next for the TE modes.

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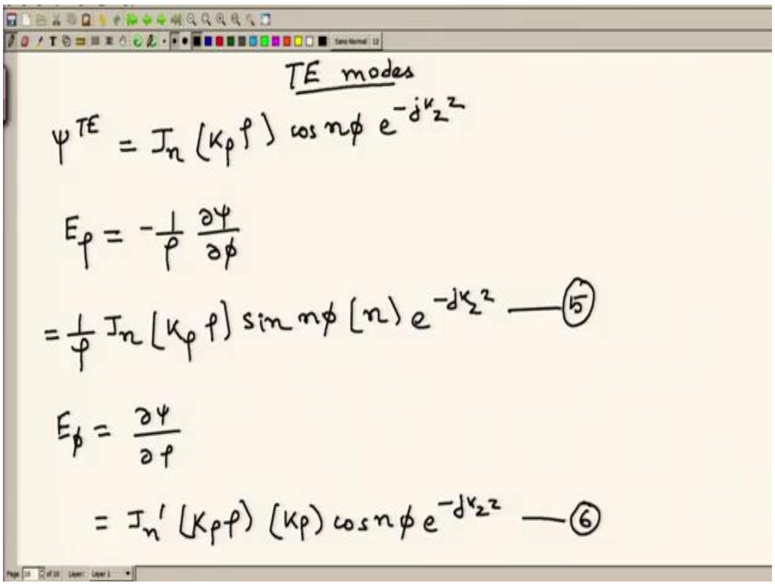

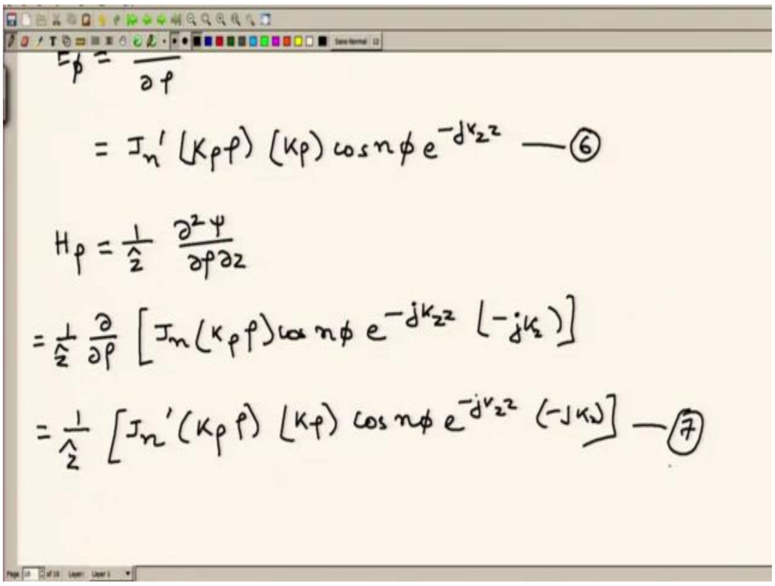

So, for TE modes, we have ψ^{TE} as $j_n\left(k_\rho \rho\right)\cos\left(n\phi\right)e^{-jk_z z}$. So, E_ρ we will have $-\frac{1}{\rho}\frac{\partial \psi}{\partial \phi}$. So, we can calculate this. So this will be $E_\rho = \frac{1}{\rho}\left(j_n\left(k_\rho \rho\right)\right)\left(n\right)\sin\left(n\phi\right)e^{-jk_z z}$. So, we can give this as equation number 5.

So, after E_ρ we will have E_ϕ. So, E_ϕ, we have as $\frac{\partial \psi}{\partial \rho}$. So, this is $\left(j_n'\left(k_\rho \rho\right)\right)\left(k_\rho\right)\cos\left(n\phi\right)e^{-jk_z z}$. So, this is equation number 6. Now, H_ρ. So, H_ρ we found out that H_ρ can be written $\frac{1}{\hat{z}}\frac{\partial^2 \psi}{\partial \rho \partial z}$.

So, first we will differentiate with respect to z, and again with respect to Rho we will have $H_\rho = \frac{1}{\hat{z}}\left(j_n'\left(k_\rho \rho\right)\right)\left(-jk_z k_\rho\right)\cos\left(n\phi\right)e^{-jk_z z}$. So this is equation number 7. So, H_ρ and then Hz, similarly we will calculate.

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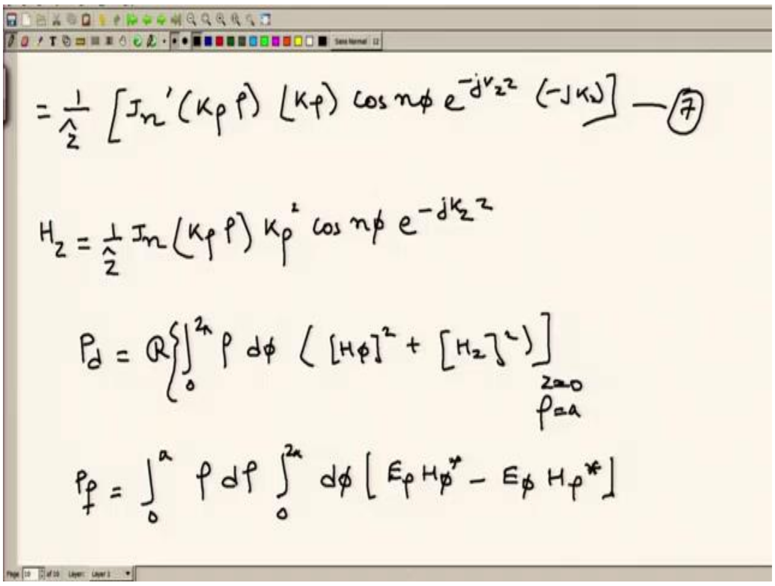

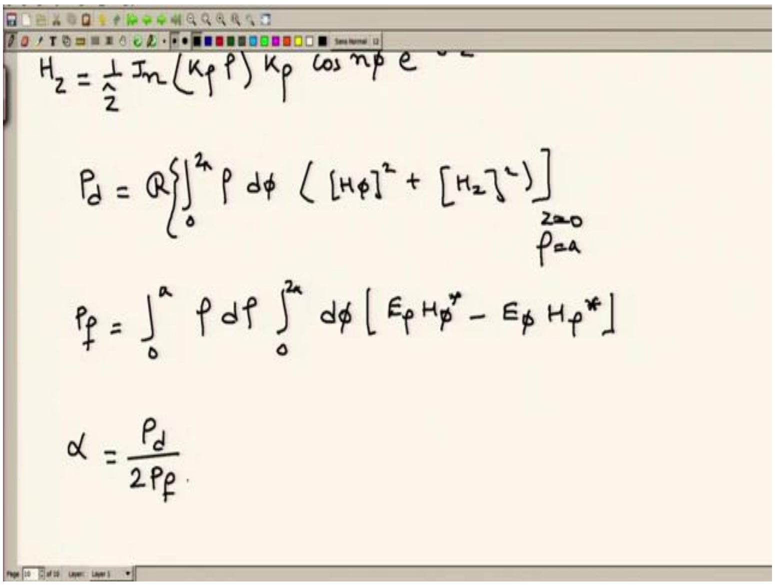

We get Hz as $\frac{1}{\hat{z}}\left(j_n\left(k_\rho \rho\right)\right)\left(k_\rho^2\right)\cos\left(n\phi\right)e^{-jk_z z}$. So, again to find out the attenuation constant we need to find out Pd and Pf. So, power dissipated Pd can be written as $\Re\int_0^{2\pi}\rho d\phi\left\{\left(H_z\right)^2+\left(H_\phi\right)^2\right\}_{\substack{\rho=a\\ z=0}}$. And power flow P_f, we can write $\int_0^a \rho d\rho \int_0^{2\pi} d\phi\left\{\left(E_\rho H_\phi^*\right)-\left(E_\phi H_\rho^*\right)\right\}$. So, on substituting we will get attenuation constant as the ratio of Pd upon 2 Pf, so this is all about the cylindrical wave functions. Thank you.