Advanced Microwave Guided-Structures and Analysis
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Department of Electronics & Electrical Communication Engineering
Lecture 69
Application to the Coupling Problem: Tutorial

Hello everyone. Today's session is on Application to the Coupling Problem.

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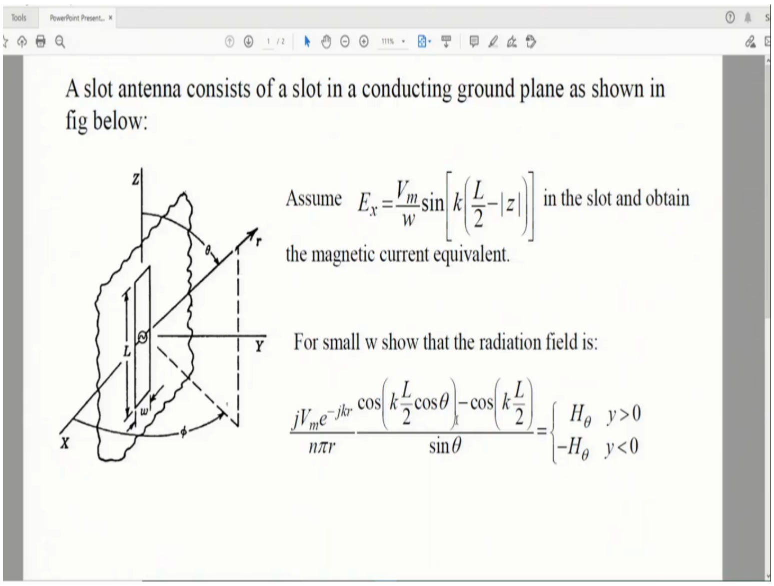

So, the first problem states that there is one slot antenna and it consists of a slot in a conducting ground plane as shown in the figure. So, there is a slot present. The length of the slot is given by L and the width is w and the length of the slot is along the z direction whereas the width of the slot that is w is along x- direction. It is assumed that Ex equals $\frac{v_m}{w}\sin\left[k\left(\frac{L}{2}-|z|\right)\right]$ in the slot and we need to obtain the magnetic current equivalent. Also, we need to show the radiation field for small w. So, we will start solving it.

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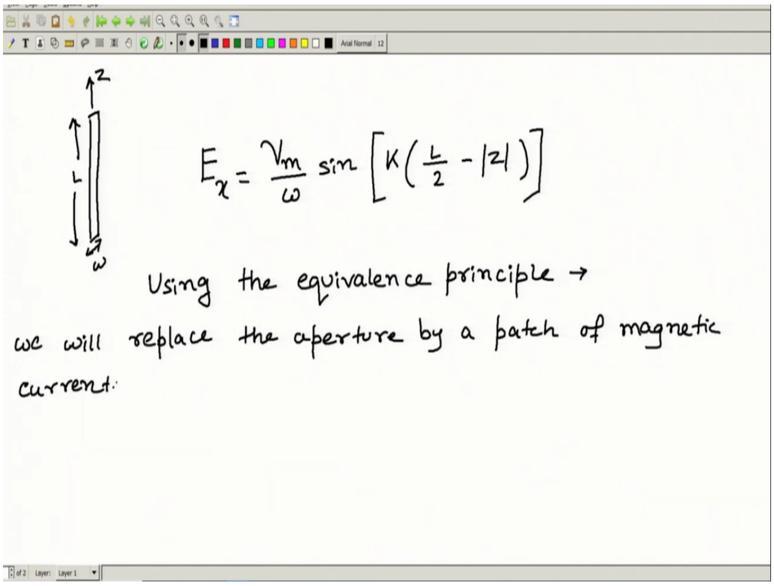

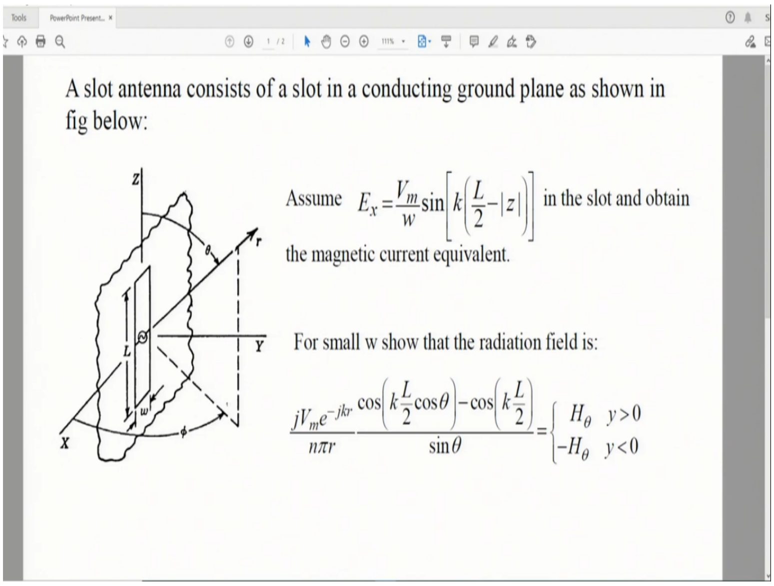

So, we are given a slot antenna. So, we will start with. So, basically, it is a dipole slot antenna when fed by a voltage that is impressed across the center of the slot. The slot and the ground plane, it can be viewed as a transmission line and the field in the slot will be essentially a harmonic function of k_z . So, basically, we were given a slot of length L and with w and this is z.

So in the question it has been asked that we need to assume Ex as, so Ex is $\frac{v_m}{w}\sin\left[k\left(\frac{L}{2} - |z|\right)\right]$. So, this is provided to us. So, what we will do now? We will take the help of equivalence principle and then solve the problem. So, basically, we can start with the equivalence principle. Using the equivalence principle, what we can do? We can replace the aperture by a patch of magnetic current. So, we can write that using the equivalence principle, we will replace, we will replace the aperture by a patch of magnetic current.

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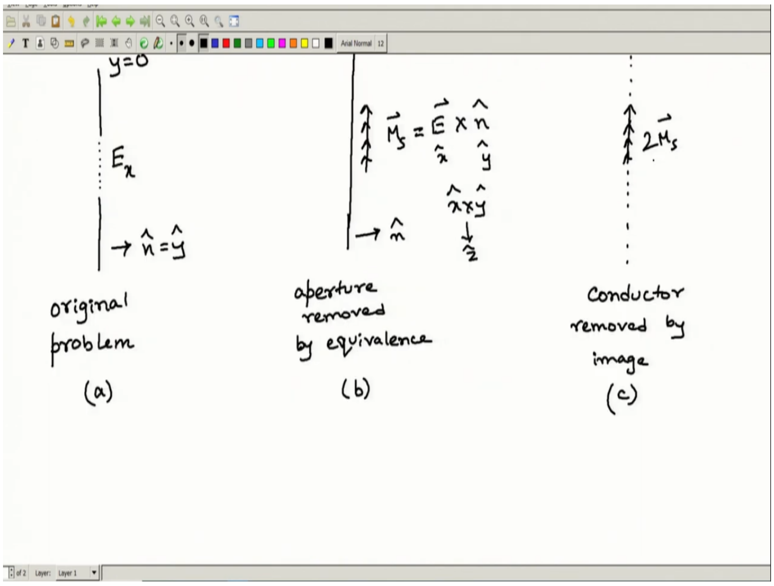

Now, then we can see like suppose, this was Ex, so this is the slot, this is Ex, so we can write this. So, this was our original problem. We have a slot on the conducting ground plane. So, this is our original problem, fine. So, here is the $\hat{n}$ is $\hat{y}$, normal. Now this original problem, what we will do? By using the equivalence principle, we will replace the aperture by a patch of magnetic current.

So, the next step if this is 'a'. This was an original problem. Now what we will do? We will replace the aperture by a patch of magnetic current. So, now it will look like, so we have replaced this aperture by magnetic current. So, **Ms** is the magnetic current. Then we can say in this we have applied the equivalence principle and let us write this as aperture removed by using equivalence theorem, by using equivalence. So, this is 'b'. Now this **Ms**, we know this is $\vec{E}\times\hat{n}$. Here we have $\hat{n}$ is $\hat{y}$, **E** is $\hat{x}$. So, x cross y, we will get as z. So, **Ms** is $\vec{E}\times\hat{n}$. So, this state is when we have removed the aperture by the equivalence and we have replaced the aperture by a patch of magnetic current.

Next, we will apply the image theory. For that this will be like this. So, here we can write. Now while we use image theory, so this will be, this we can write as conductor removed by image theory. Now see while we have removed the aperture and placed the magnetic current over there, so now what happens is that in the vicinity of the ground plane, there is magnetic source. In the vicinity of the ground plane, we have the magnetic source.

Now therefore, if we apply image theory in this state that is in figure c, if we apply image theory then what will have? Then after removing the ground plane, there will be the image and both of them, the image and the actual will be in so vicinity that it can be assumed as like this, twice of **Ms**, fine.

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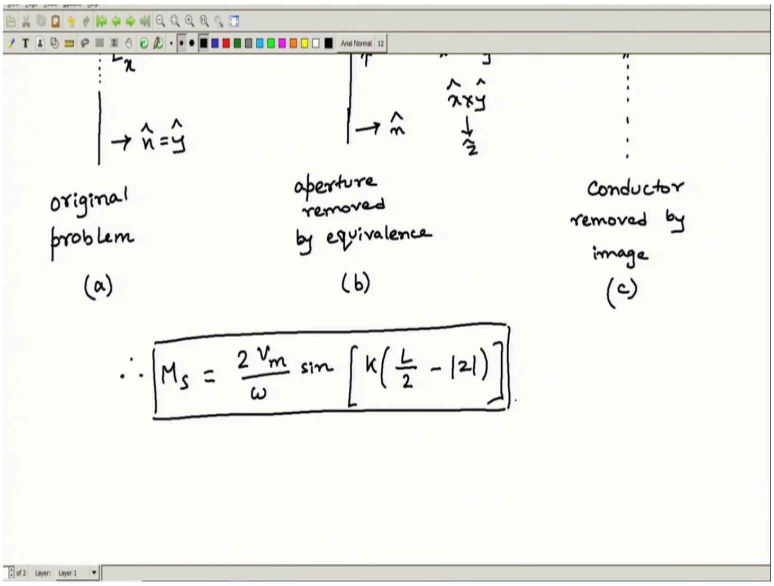

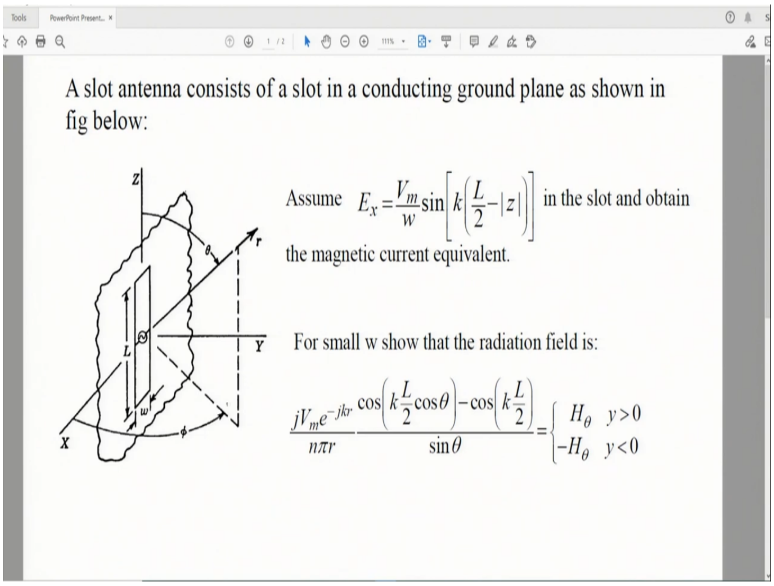

So therefore, we can then write **Ms** as $\frac{2v_m}{w}\sin\left[k\left(\frac{L}{2}-|z|\right)\right]$. So, this was the magnetic current equivalent. The first part is now obtained. Now what the second part says? It says that if we assume like w is very small then what will be the radiated fields?

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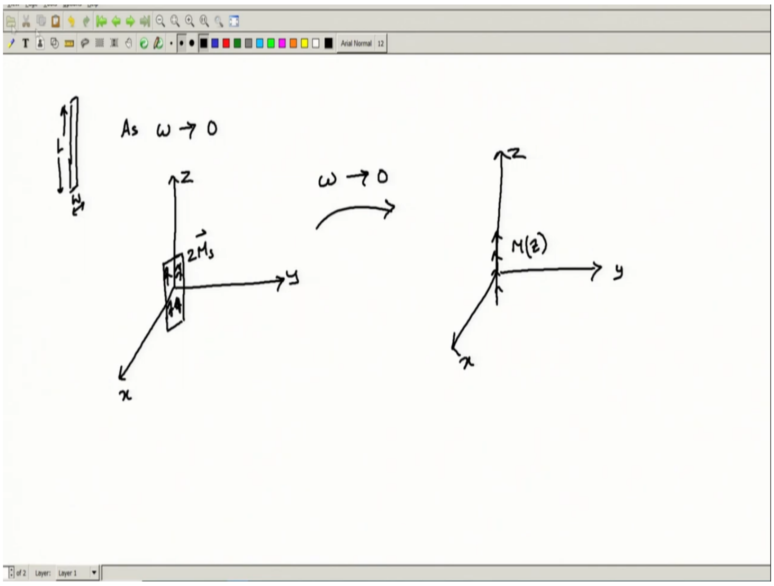

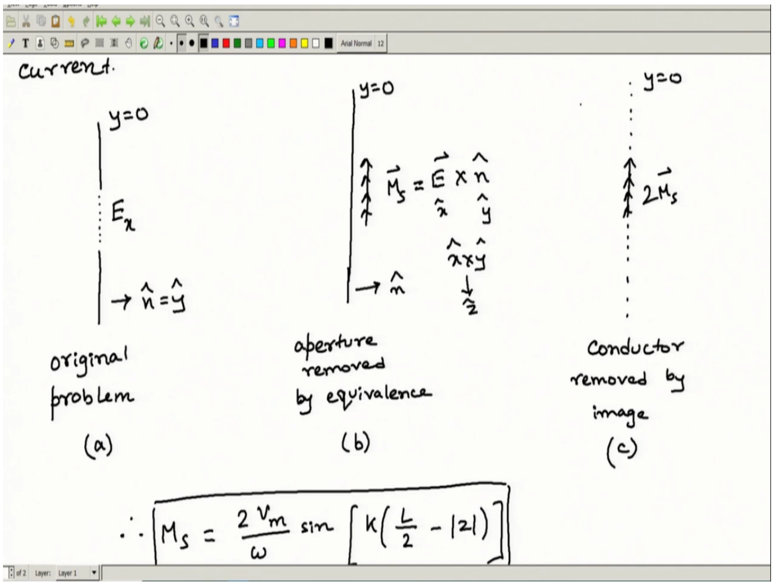

So now, previously the slot was like this. It was having a width of w and this was the length of the slot. Now it has been said that we need to assume w is very small that is w when tends to 0. Now when w tends to 0, we can draw like this. So, when we like change the original problem to this state, like when conductor is removed by the image theory and we have 2 **Ms**, so this state we can draw like this. So, basically, we were having this as z, this is y, this is x and this was 2 **Ms**.

Now, when w is made very small, what will happen? The equivalent magnetic current, this one, the equivalent magnetic current will then behave as a line filament. So, now we can draw this problem like this. So, this will be z, this is y, this is x and since w is tending to 0, so at origin, we will have this.

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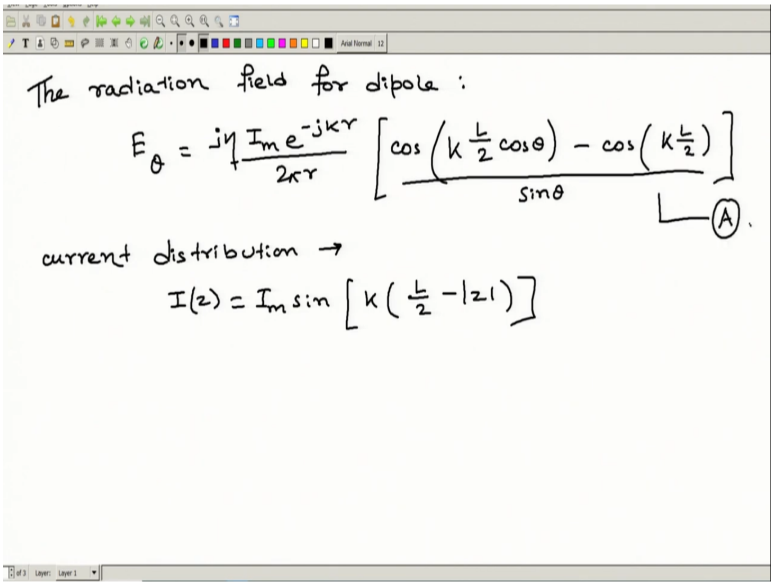

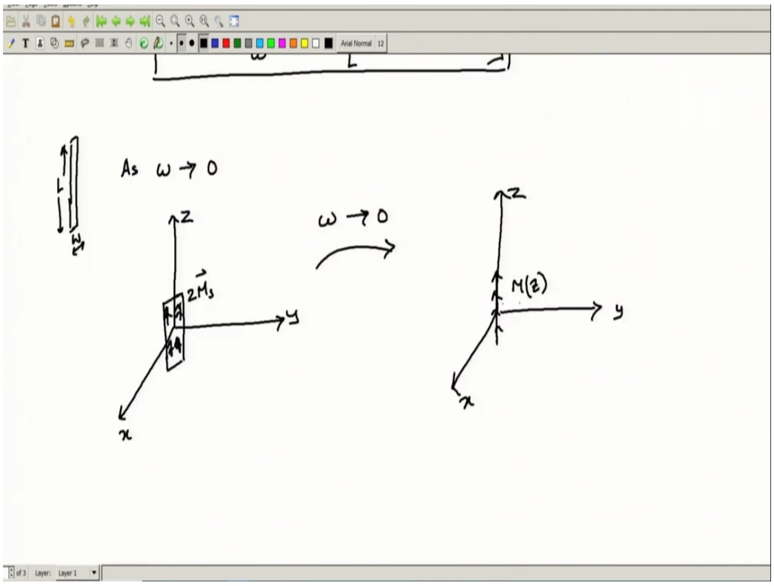

So, now we know that the radiation field for dipole, we can write, ok. For the current distribution of the radiation field can be written like $E_\theta = \frac{j\eta I_m e^{-jkr}}{2\pi r}\left[\frac{\cos\left(k\frac{L}{2}\cos\theta\right)-\cos\left(k\frac{L}{2}\right)}{\sin\theta}\right]$. So, this was the radiation field for dipole. Considering in this case, the current distribution used to be Iz is $I_m \sin\left[k\left(\frac{L}{2}-|z|\right)\right]$. So, basically, with this equation, if this is equation number (A), so now the radiated field for this figure for Mz, we can write down using the duality.

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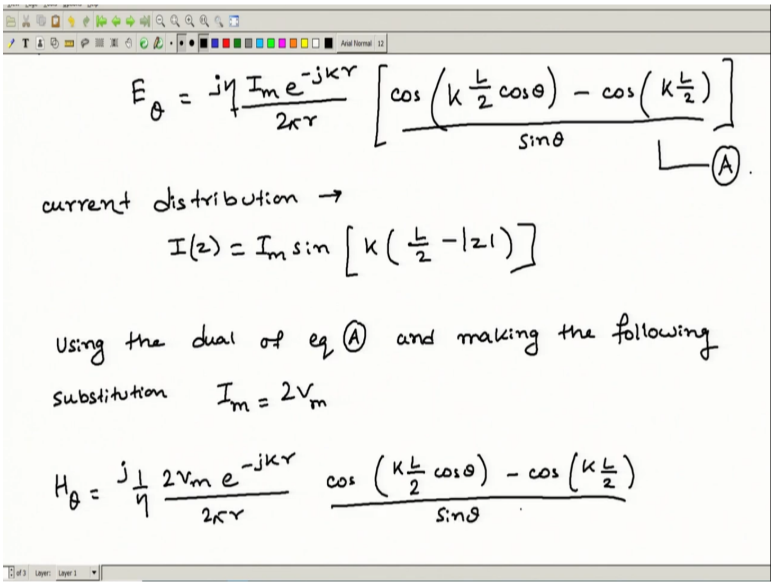

So, by using the dual of equation (A), and here we can write like and making the following substitution. So, we will use the dual of equation number (A) and we will substitute in place of Im. Now we have 2 Vm. So, now using duality from equation (A), we can write E_θ will be H_θ , since we are applying duality.

So, H_θ will be, in place of j we will have j, in place of η we will have $\frac{1}{\eta}$. Since we are applying duality and then in place of Im we have twice of

$$\frac{2v_m e^{-jkr}}{2\pi r}\left[\frac{\cos\left(k\frac{L}{2}\cos\theta\right)-\cos\left(k\frac{L}{2}\right)}{\sin\theta}\right].$$

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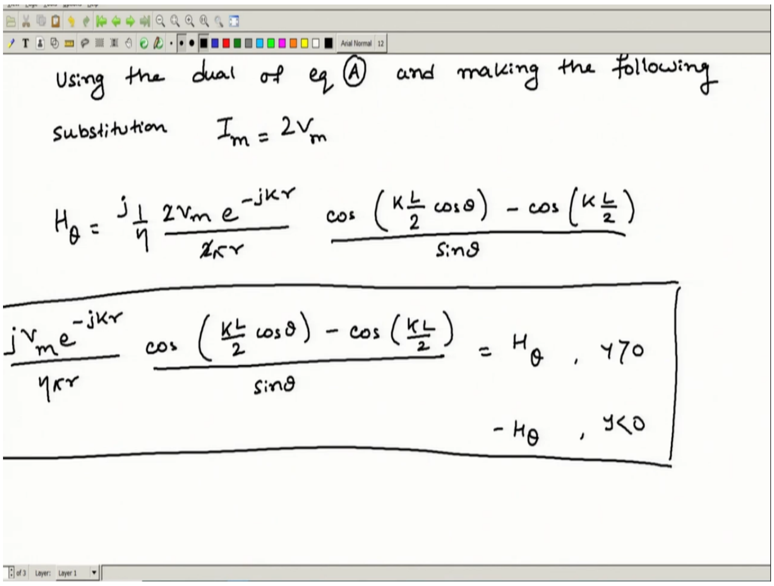

So thus, we will have. So, this 2 2 will cancel out. We can write down like $\frac{jv_m e^{-jkr}}{\pi r}\left[\frac{\cos\left(k\frac{L}{2}\cos\theta\right)-\cos\left(k\frac{L}{2}\right)}{\sin\theta}\right]$. So, this is equal to H_θ. So, this applies when we are considering y ≥ 0. And if we consider y ≤ 0, this will be $-H_\theta$. So, this is the radiated field.

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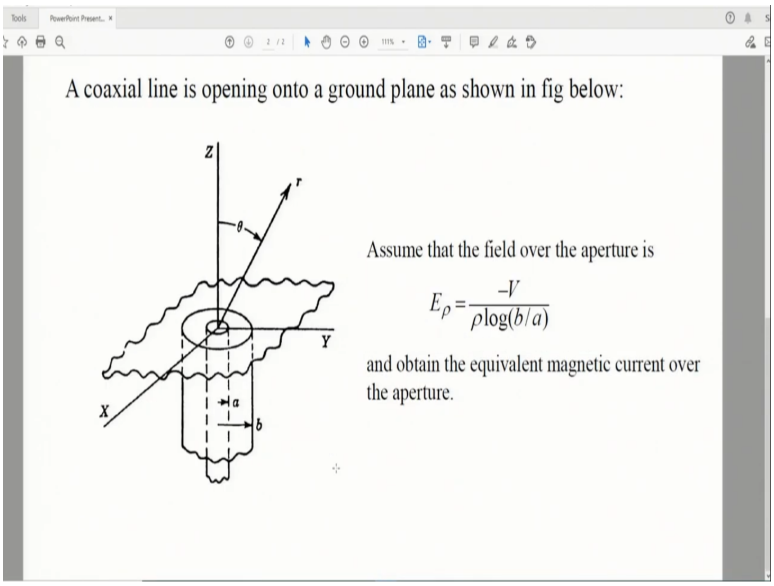

Now let us move to the next question. The next question says that there is one coaxial line up and it is opening onto a ground plane. So, here we have a coaxial line which is opening onto a ground plane. And again, it has been told that we need to assume that the field over the aperture is given as E_ρ equals to $\frac{-V}{\rho \log\left(\frac{b}{a}\right)}$, where 'b' is the outer and 'a' inner radius that we can see from the figure. And then we need to obtain the equivalent magnetic current over the aperture. So, we will solve this problem in a similar manner using the equivalence principle.

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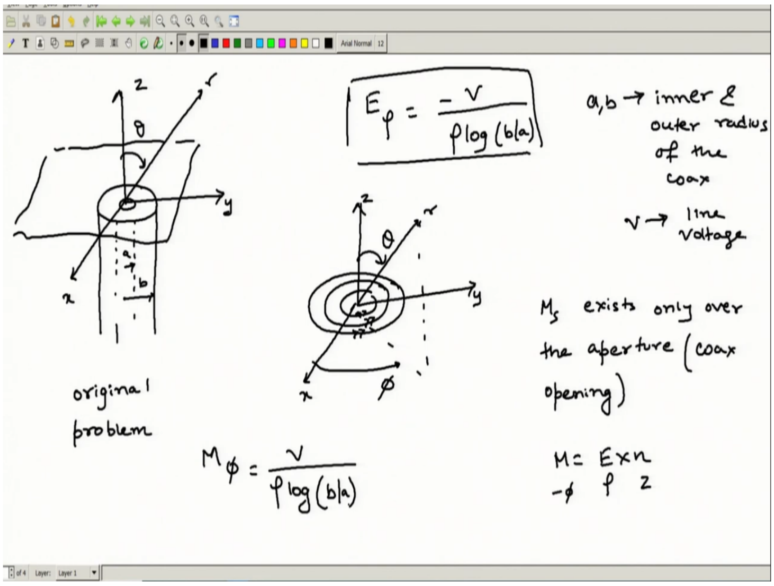

So, let us start solving this. So, it was given like we have a coaxial cable. This is the inner radius 'a' and this is the outer radius 'b'. This is z y x and we have a ground plane over here. This angle is theta. Now in the question it is asked, so basically this was our original problem given in the question. It was asked, it was told that the field over the aperture was given by, it was told in the question that is E rho is $\frac{-V}{\rho \log\left(\frac{b}{a}\right)}$. So, where 'a' and 'b' are the inner and the outer radius a coax and V is the line voltage.

Now so what we need to do? We need to again, using the equivalence principle; we will change the problem, original problem into the equivalent problem. So, basically, what we have? We have a coaxial transmission line that is being opened into a ground plane. So, it is to be noted like that in the question, it is assumed E_ρ is this one, $\frac{-V}{\rho \log\left(\frac{b}{a}\right)}$. Now we can change this original problem. We can draw like this. This is our, this is z, this is y, this is x, this aperture will be, replace this aperture by magnetic surface current.

So, we can write like this. **Ms**, ok. **Ms** exists. Where it exists? **Ms** exists only over the aperture. **Ms** exists only over the aperture that is the coax opening. So, **Ms** exists only over the aperture, for tangential **E** is 0 over the ground plane. So, we can write like this. So, we will have, we know **M** is $\vec{E}\times\hat{n}$. So, **E** is E_ρ, $\hat{n}$ is $\hat{z}$. So, $\hat{\rho}$ cross $\hat{z}$, we have $-\hat{\phi}$. So, M_ϕ will be like this.

So this is now M_ϕ. This is r, this angle is theta, and this will be phi. So, now we can write. So, this is the equivalent problem, so now we can write M_ϕ as $\frac{V}{\rho \log\left(\frac{b}{a}\right)}$. So, this is M_ϕ, so basically what it is? This is a loop of magnetic current which if b is very very less than lamda can act as an electric dipole.

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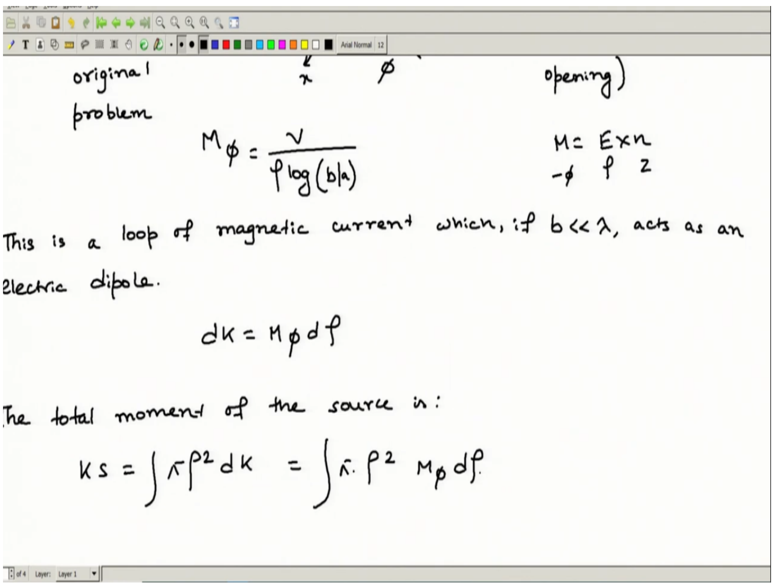

So, we can write this is a loop of magnetic current. This is a loop of magnetic current, which the condition is if b is very very less than lambda, acts as an electric dipole. Now this current is visualized like say, a continuous distribution of magnetic current filaments, in which the strength we can write like dk is $M_\phi d\rho$. So, then the total moment of this source, we can write the total moment of the source is, we can write it as k s is integration of $\int \pi\rho^2 dk$. So, that can be written as $\int \pi\rho^2 M_\phi d\rho$.

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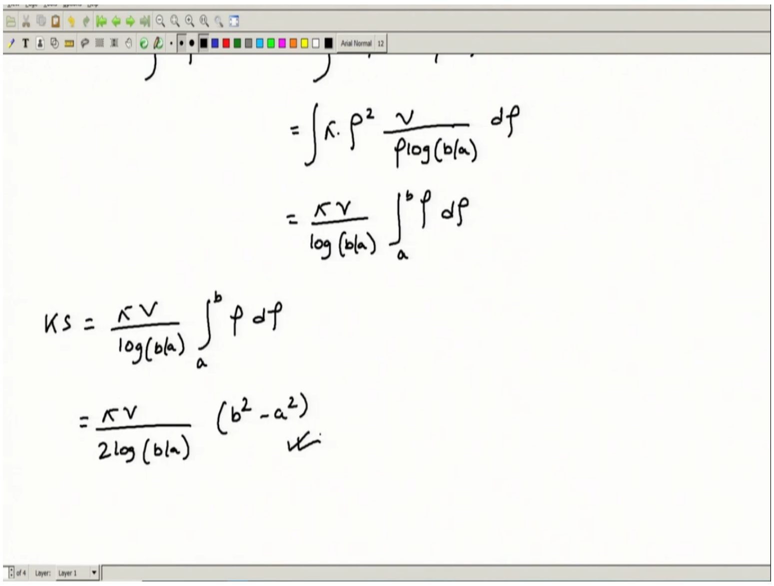

Which will give us $\pi\rho^2$, in place of M_ϕ; we have $\frac{V}{\rho \log\left(\frac{b}{a}\right)}$. So, basically, we will have $\frac{\pi v}{\log\left(\frac{b}{a}\right)}$. So, this integration will run from a to b rho. So, then we can have ks as $\frac{\pi v}{2\log\left(\frac{b}{a}\right)}\left(b^2 - a^2\right)$. Now this magnetic loop is an equivalent to electric dipole if we make certain substitution like IL equals to $-j\omega\varepsilon KS$. So, in place of ks then we can substitute this value and then we can eventually find out all the fields. So, this is all for today. Thank you.