Advanced Microwave Guided-Structure and Analysis
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Lecture No. 07
Scattering Matrix Concepts Tutorials (Contd.)

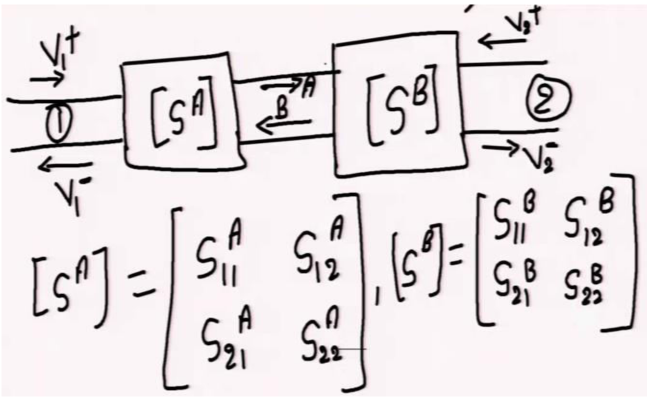
4. Consider two two-port networks with individual scattering matrices, $[S^A]$ and $[S^B]$. Show that the overall S_{21} parameter of the cascade of these networks is given by

$$S_{21} = \frac{S_{21}^A S_{21}^B}{1 - S_{22}^A S_{11}^B}$$

we have to find out the overall S_{21} parameter of the cascade of these networks is given by S_{21} equal to

$$S_{21} = \frac{S_{21}^A S_{21}^B}{1 - S_{22}^A S_{11}^B}$$

So, here it is a two-port network like this.



This is $[S^A]$ and $[S^B]$ is known.

$$\begin{bmatrix} V_1^- \\ A \end{bmatrix} = \begin{bmatrix} S^A \end{bmatrix} \begin{bmatrix} V_1^+ \\ B \end{bmatrix}, \quad \begin{bmatrix} B \\ V_2^- \end{bmatrix} = \begin{bmatrix} S^B \end{bmatrix} \begin{bmatrix} A \\ V_2^+ \end{bmatrix}$$

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = [S] \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

So, here we can see for the total overall system V_1^- is a reflection, first reflection and V_2^- is a second reflection. Similarly, V_1^+ is the first incoming and V_2^+ is the second incoming. So, it can be written like this. So, we have to find out the S_{21}.

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = [S] \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}, \quad [S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$$S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+ = 0}$$

So, V_2^+ equal to 0 that means, it is terminated with matched load. So, if $V_2^+ = 0$ so, in that case B will be B = S_{11}^B A.

$$A = S_{21}^A V_1^+ + S_{22}^A \cdot B$$

$$= S_{21}^A V_1^+ + \underbrace{S_{22}^A S_{11}^B}_{} A$$

$$V_2^- = S_{21}^B A = S_{21}^B \left[S_{21}^A V_1^+ + S_{22}^A S_{11}^B \cdot \frac{V_2^-}{S_{21}^B} \right]$$

After this we can take the common V_2^- and this will come

$$V_2^- [1 - S_{22}^A S_{11}^B] = S_{21}^B S_{21}^A V_1^+$$

From this we can write

$$\frac{V_2^-}{V_1^+} = \frac{S_{21}^A S_{21}^B}{1 - S_{22}^A S_{11}^B}$$

5. A four-port network has the scattering matrix shown below.
(a) Is this network lossless?
(b) Is this network reciprocal?
(c) What is the return loss at port 1 when all other ports are terminated with matched loads?
(d) What is the insertion loss and phase delay between ports 2 and 4, when all other ports are terminated with matched loads?
(e) What is the reflection coefficient seen at port 1 if a short circuit is placed at the terminal plane of port 3, and all other ports are terminated with matched loads?

$$[S]=\begin{bmatrix} 0.1\angle 90^\circ & 0.8\angle -45^\circ & 0.3\angle -45^\circ & 0 \\ 0.8\angle -45^\circ & 0 & 0 & 0.4\angle 45^\circ \\ 0.3\angle -45^\circ & 0 & 0 & 0.6\angle -45^\circ \\ 0 & 0.4\angle 45^\circ & 0.6\angle -45^\circ & 0 \end{bmatrix}$$

For the lossless we know it should satisfy the unitary condition. So, to satisfy that we have to add all this

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2$$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 0.1^2 + 0.8^2 + 0.3^2 = 0.74$$

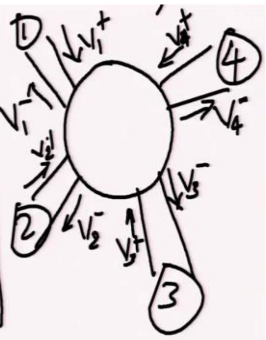
Since, this is not equal to 1. So we can say that this is not a lossless network.

Since network is symmetrical, then we can say that this is reciprocal network.

What is the return loss at port 1 when all other ports are terminated with matched load?

So, first we have to see that how this four-port networks are looking.

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \\ V_4^+ \end{bmatrix}$$



So, for the four-port network, we can write like some system is there and it has Port 1 like this, Port 1, Port 2, Port 3, Port 4. So, if it is Port 1, Port 2, Port 3, Port 4. So, Port 1 so, at Port 1 this will be V_1^+ V_1^+ and this is reflected from Port 1 V_1^-. It is incoming wave have at Port 2 V_2^+ reflected from Port 2 V_2^-. Incoming at Port 3 that will be V_3^+ and reflected from Port 3 that will be V_3^-. Incoming at Port 4 it is V_4^+ and reflected from Port 4 that it is V_4^-.

So, like this for the four-port network we can write the voltage that incoming and outgoing. So, in the, for the matrix term, we can write like this

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \\ V_4^+ \end{bmatrix}$$

For the four-port network it can be written as

$$V_1^- = S_{11}\ V_1^+ + S_{12}\, V_2^+ + S_{13}\, V_3^+ + S_{14}\, V_4^+$$

$$S_{11} = \frac{V_1^-}{V_1^+} = 0.1\angle 90^\circ = \Gamma$$

$$RL = -20\log|\Gamma| = -20\log(0.1) = 20\text{dB}$$

Next part is what is the insertion loss and phase delay between Port 2 and 4, when all other ports are terminated with matched load? So, between insertion loss between 2 and 4, when other ports are terminated with matched load, so, we have to find out S_{42}. S_{42} between port 2 and 4 is

$$S_{42} = \frac{V_4^-}{V_2^+} = 0.4\angle 45^\circ$$

$$IL = -20\log|S_{42}| = -20\log(0.4) = 8\,dB$$

Phase delay $= 45^\circ$

Insertion loss will be

$$\text{IL} = -20\,\log|S_{42}| = -20\,\log|0.4| = 8\text{ dB}$$

phase delay will be 45°

For the next part, means short circuit is placed at Port 3 and all other ports are terminated with matched load.

$$V_2^+ = V_4^+ = 0$$

$$V_3^+ = -V_3^-$$

$$V_1^- = S_{11} V_1^+ + S_{13} V_3^+$$

$$= S_{11} V_1^+ - S_{13} V_3^-$$

$$V_3^- = S_{31} V_1^+$$

$$V_1^- = S_{11} V_1^+ - S_{13} S_{31} V_1^+$$

$$\frac{V_1^-}{V_1^+} = S_{11} - S_{13} S_{31}$$

$$= 0.1j - (0.3\angle -45°)\,(0.3\angle -45°)$$

$$= 0.1\,j + 0.9\,j = 0.19\,j = 0.19 \angle 90°$$