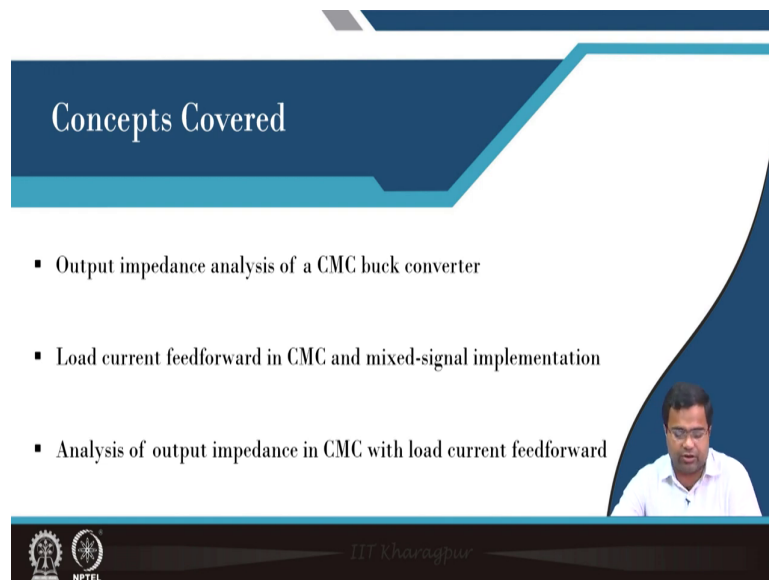


Digital Control in Switched Mode Power Converters and FPGA-based Prototyping
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Module - 11
Design and Validation Case Studies using Digital Voltage and Current Mode Control
Lecture - 108
Analysis of Output Impedance in Digital CMC with Load Current Feedforward

Welcome to this particular lecture, we are going to analyze the Output Impedance in Digital Current Mode Control With and Without Load Current Feedforward.

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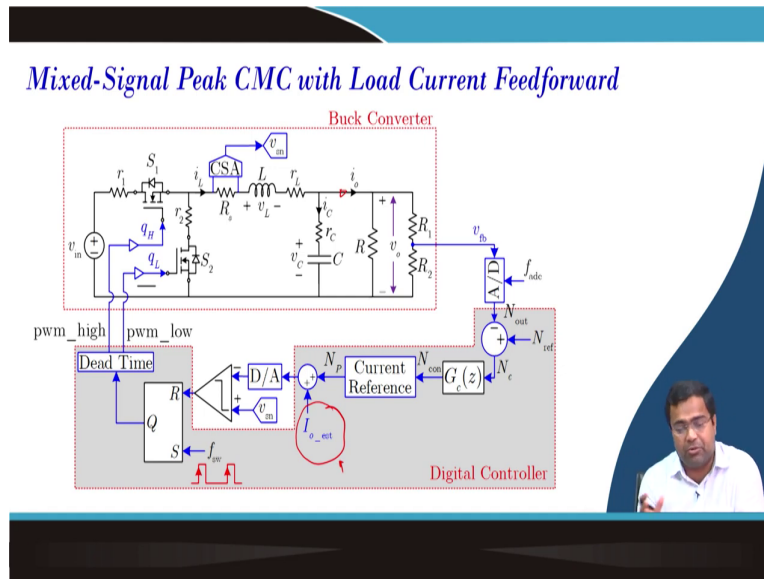
Concepts Covered

- Output impedance analysis of a CMC buck converter
- Load current feedforward in CMC and mixed-signal implementation
- Analysis of output impedance in CMC with load current feedforward

The slide features a dark blue header with the title 'Concepts Covered' in white. Below the header is a white area containing a bulleted list of three topics. In the bottom right corner of the slide, there is a small video inset showing a man in a white shirt speaking. At the bottom of the slide, there are logos for IIT Kharagpur and NPTEL.

So, here output impedance analysis under current mode control we will discuss. And then what will happen with the output impedance if the load current feedforward is added and what is the output impedance, with and without load current feedforward are we going to consider load current feedforward and how it is going to alter the output impedance?

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We know about this mixed signal current mode control we have discussed it multiple times. But here we have considered the load current feedforward inside the digital platform and it is like an estimated load current. Because actual load current we are not sensing and we are assuming this converter is driving a processor or it can be LED load and for LED load anyway the load current information can be indirectly obtained.

Because we know the nominal string current and we are turning on and off multiple strings. So, we can estimate the load kind pretty accurately. But in the case of the processor if we communicate then we can get the information of the load current list.

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Closed-loop Output Impedance under CMC

$$K_{loop} = \frac{k_f}{k_{dac} \times k_c} G_{ic} G_c z_o e^{-s\tau_d}$$

$$= \frac{k_f}{k_i} G_{ic} G_c z_o e^{-s\tau_d}$$

$$= \frac{k_f}{k_i} \times G_c \times \left(\frac{\frac{F_m V_{in}}{z_{in}}}{1 + \frac{F_m V_{in}}{z_{in}}} \right) \times z_o e^{-s\tau_d} = \frac{k_f}{k_i} \times G_c \times \frac{F_m V_{in} z_o}{F_m V_{in} + z_{in}} e^{-s\tau_d}$$

[For details, refer to [Lecture-38, NPTEL "Control and Tuning Methods ..."](#) course ([Link](#))]

So, if we incorporate load current then first of all the current mode control we have considered that delay can be added. Because we want to talk about the small signal design of current mode control and this small signal model of traditional continuous time current mode control and the digital current mode control will be more or less the same sorry mixed signal current mode control by incorporating a delay.

But since the delay is very small it has an insignificant effect I would say you can treat $e^{-s\tau_d}$ almost approximately equal to 1 because it has a negligible effect ok. But we want to analyze what will happen with and without load current feed-forward. So, this is the out loop transfer function, and what will be the output impedance?

Now in our condition on our case, we are taking this 1 by k_c . Because we have a current sensor gain that we have discussed because here actual inductor current we are not sensing and there is a current sense amplifier current sense resistance followed by the current sense amplifier and this is 0.01 volt per ampere ok.

Next, we have a feedback voltage gain which is like 0.27 and we are dropping some bit. So, it is resulting in a gain of 1 by 4 and this is clubbed together. So, now, your loop transfer function will look like this, and here we will also have $e^{-s\tau_d}$, but you can drop this term because this is almost 1.

Because the delay is negligible is small compared to the time period and this is the overall loop transfer function. We have discussed in lecture number 38 in the NPTEL course without delay; that means, what is the output impedance of current mode control like for continuous time current mode control or analog current mode control?

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Closed-loop Output Impedance under CMC

$$z_{oc} = \frac{z_o}{1 + K_{loop}}$$

$$z_o = \frac{r_e}{L} \times \frac{\left(1 + \frac{s}{\omega_o}\right) \left(1 + \frac{s}{\omega_o}\right)}{\left(1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}\right)}$$

Where

$$Q = \alpha \times \left[\frac{(r_c + r_e)}{Z_c} + \frac{Z_c}{R} \right]^{-1}$$

$$\omega_o = \sqrt{\frac{R + r_e}{R + r_c}} \cdot \frac{1}{\sqrt{LC}}$$

Now, the closed-loop output impedance will be open-loop output impedance divided by 1 plus loop transfer function this thing we already know. And for a typical buck converter, we know the output impedance is a function of ω_o here it is the ω_o L inductor that time constant, and then it has a pole. This thing we know and we have discussed in our earlier course.

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Output Impedance with Load Current Feed-Forward

$$\hat{v}_o = z_o (\hat{i}_L - \hat{i}_o)$$

$$= z_o \left[\frac{G_{vc} e^{-sT_d}}{k_i} (G_c k_f \hat{v}_c + K_n \hat{i}_o) - \hat{i}_o \right]$$

Where $K_n = \begin{cases} 1 & \text{Buck converter} \\ \frac{V_o}{V_{in}} & \text{Boost converter} \end{cases}$

$$\Rightarrow \hat{v}_o = z_o \left[\frac{G_{vc} e^{-sT_d}}{k_i} (G_c k_f (-\hat{v}_o) + K_n \hat{i}_o) - \hat{i}_o \right]$$

Handwritten notes in red:
 $K_n = 1$ buck
 $= \frac{V_o}{V_{in}}$ boost

Now, if we incorporate load current feed-forward. So, this is a load current feed-forward and this is a normalized gain, and this normalized gain equals 1 for the buck converter. For the boost converter, it will be V_o by V_{ref} for a boost for normalizing because we want to normalize this with respect to the inductor current. But here we are giving this and then as if there is a DAC gain there is a current sensor, so you have to scale this reference.

So, what we are doing? Although we are showing this for analysis purposes, in actual implementation we are using a, I_o estimate and the value we are setting according to this scaling factor. So, the average inductor current that scales inside the digital platform should be consistent with the load scaling.

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Output Impedance with Load Current Feed-Forward

$$\hat{v}_o = -\frac{G_{ic}e^{-sT_d}}{k_i}G_c k_f z_o \hat{v}_o + \left(\frac{G_{ic}e^{-sT_d}}{k_i}K_n - 1\right) z_o \hat{i}_o$$

$$= -K_{loop} \hat{v}_o + \left(\frac{G_{ic}e^{-sT_d}}{k_i}K_n - 1\right) z_o \hat{i}_o$$

$$\Rightarrow \hat{v}_o = -\frac{\left(1 - \frac{G_{ic}e^{-sT_d}}{k_i}K_n\right) z_o}{(1 + K_{loop})} \hat{i}_o \Rightarrow z_{oc} = -\frac{\hat{v}_o}{\hat{i}_o} = \frac{\left(1 - \frac{G_{ic}e^{-sT_d}}{k_i}K_n\right) z_o}{(1 + K_{loop})}$$

Now, if you analyze with the load feed forward v 0. So, if we write those steps one by one this is for buck and boost. Then v 0 if you write then what are you going to get if you further write this expression in terms of so this is our total loop transfer function. So, the loop transfer function is a product of this term this all this term product including this. So, all these products will give you a loop transfer function. So, v 0 can be written like this, and if we write output impedance. So, it will look like this.

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Output Impedance with Load Current Feed-Forward

$$z_{oc} = \begin{cases} \frac{\left(1 - \frac{G_{ic}e^{-sT_d}}{k_i}K_n\right) z_o}{(1 + K_{loop})} & \text{with feedforward} \\ \frac{z_o}{(1 + K_{loop})} & \text{without feedforward} \end{cases}$$

$\frac{z_o}{1 + K_{loop}}$ with feedforward ≈ 0
 $\frac{z_o}{1 + K_{loop}}$ without feedforward ≈ 1

Now, since we have taken this term to be approximately equal to 1 and we know from our earlier course that control tooth current because if we take the inductor current whether it is a buck or boost. So, this is our inductor current and if you take the small signal inductor current; that means, i_L tilde we have to consider in current mode control this can be replaced by a control current source in the small signal sense and this will be inductor so; that means if we take i_L perturb by i_C perturb.

So, this will be this transfer function is nothing, but our $G_i C$, but this will be approximately 1 with the first-order transfer function. That means, which is a very rudimentary model we saw and that was good enough to design current mode control without ram compensation. And if you make that then this $G_i C$ will be 1 and these two this will can be scaled to make it 1 and if you do that because the whole loop gain.

Then it can be shown that $1 - G_i C$ this into Z_0 divided by $1 + \text{loop transfer function}$ is my output impedance closed loop with load current feed-forward. Of course, there is a k_n term there k_n term is there. So, if k_n is equal to 0 then it is simply Z_0 by $1 + \text{loop}$; that means, there is no load feed forward.

If this equals 0 then there is no load feed forward then this will be the transfer function. But if k_n is equal to 1 this can be approximately equal to 0. Because this $G_i C$ is approximately equal to 1. So, which means your closed-loop output impedance will become nearly 0 with load current feed-forward and we will be taking an experimental case study showing that will have an insignificant impact on the undershoot and overshoot of the output voltage when there is a load step transient. Because this will make the output impedance almost 0. So, it will take somewhat closer to an ideal voltage source.

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Summary

- Output impedance analysis of a CMC buck converter
- Load current feedforward in CMC and mixed-signal implementation
- Analysis of output impedance in CMC with load current feedforward

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So, in summary, we have discussed output impedance analysis under current mode control and we have also considered load current feed-forward and their mixed-signal implementation aspect and we have analyzed the output impedance under current mode control with load current feed-forward.

So, in the subsequent lecture, we are going to consider an experimental case study using with and without output load current feed-forward and show what the impact on the load transient performance is for today.

Thank you very much.