Digital Control in Switched Mode Power Converters and FPGA-based Prototyping

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Module - 04
Modeling Techniques and Mode Validation using MATLAB
Lecture - 31
Continuous- Time Small-Signal Modeling under Digital Control

Welcome, today we are going to talk about Continuous Time Small Signal Modeling under Digital Control.

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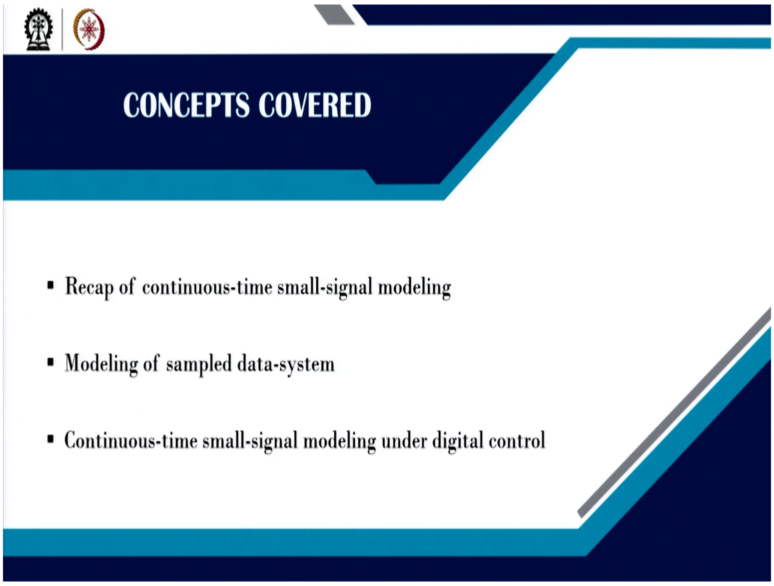

So, in this lecture we want to recapitulate our continuous time small-signal modeling for analog control, then we want to go for like modeling of sample data system for the digitally controlled converter and then continuous-time small signal modeling of switched mode power converter under digital control.

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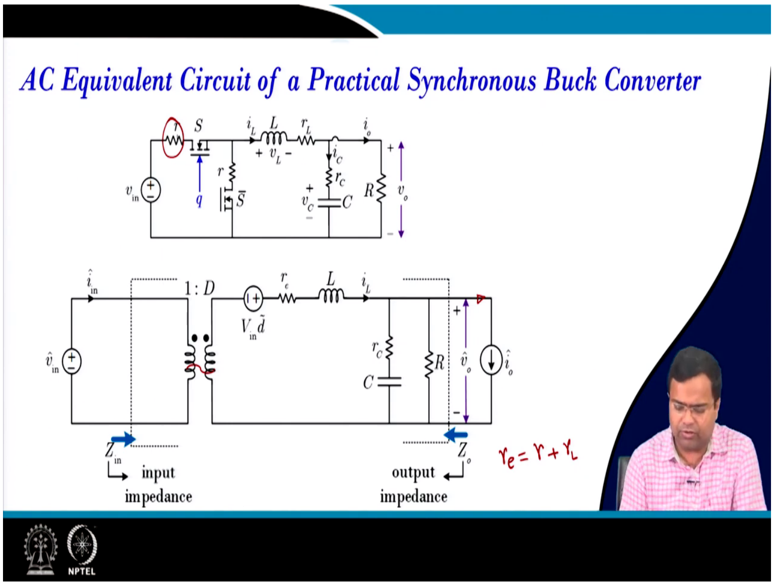

So, we will start with our know AC equivalent circuit of a practical synchronous buck converter, which we have discussed in our earlier course NPTEL course. And in this circuit, we start with our actual synchronous buck converter and this is the AC equivalent circuit. And here the r equivalent we know that this r equivalent is nothing, but RDS on that is r of this on state resistance which is here plus r L.

The resistance of the inductor is this resistance. So, this circuit shows also a current sink and that will be helpful to characterize the output impedance and input impedance. So, we will know about we know about this equivalent circuit.

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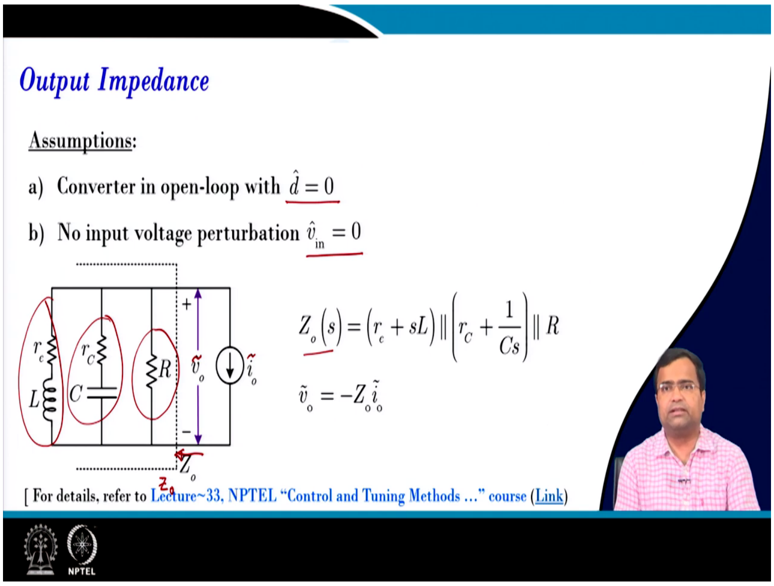

Now, if you want to find output impedance expression then under open loop conditions, we want to keep the duty ratio constant. So, the perturbation of the duty ratio is set to 0. We also want to keep the input voltage constant. So, the input voltage perturbation is 0 and then the earlier AC equivalent circuit will be transformed into this circuit. This is what you are looking at from this point and this is my output impedance.

Where we have a current-sinking load which is i 0. And because it is a sinking load. So, the output impedance expression will be a parallel combination of this branch and this branch as well as the resistance, ok.

And output voltage perturb model that is the v 0 tilde you can say or v 0 hat tilde is equal to minus Z 0 into i 0 tilde. Where since it is a sinking load that is why this negative sign will come and this is discussed in lecture number 33 in our earlier course.

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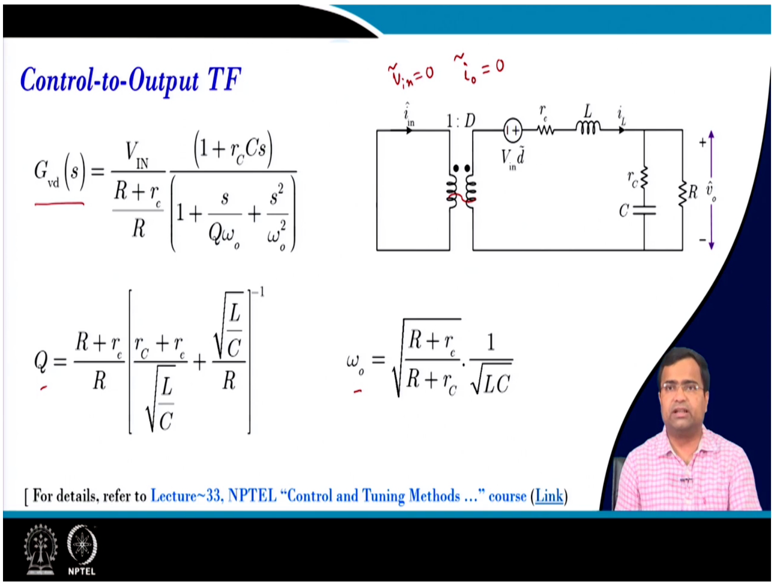

Now, we can also find out the control to output transfer function and that is under the open loop system. So, where we will set the input voltage perturbation to 0. As well as we will set load perturbation external load perturbation to 0.

So; that means, that is removed and then we can get control of the output transfer function using this expression. And we can find out all the Q factor undamped natural frequency and that is also discussed in lecture number 33.

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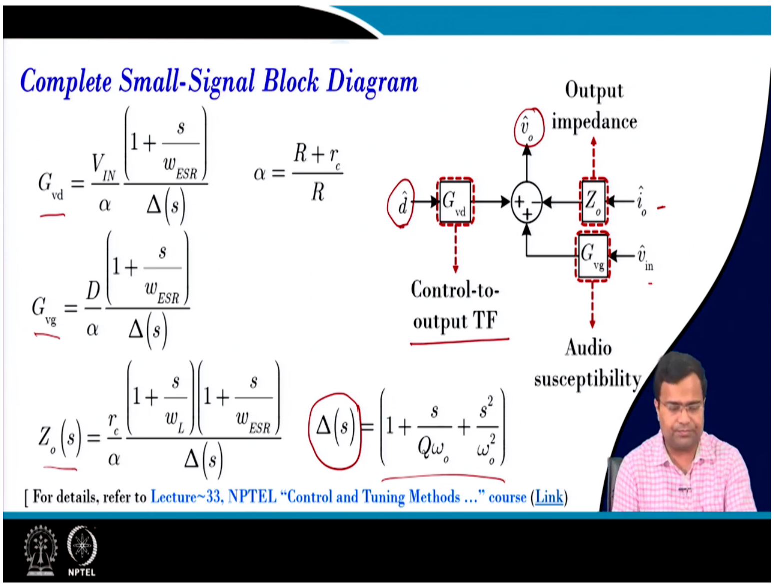

Now, with the complete small signal model, we know that the perturbation in the output voltage can be expressed in terms of the perturbation of the duty ratio the perturbation of the output current, and the perturbation of the supply voltage.

Output current perturbation and supply voltage perturbation are external and generally considered to be disturbance loads. And duty ratio perturbation is a control variable to regulate the output voltage for changes in load current or change in input voltage or if you want to maintain certain achieve certain transient performance. So, that G vg is known as audio susceptibility which shows what is the effect in the output voltage for a change in input voltage

And control to output transfer function is the one since it is a control input. So, how do you know to anticipate the effect of disturbance in the output voltage by adjusting this duty ratio? And another is the output impedance. Now we know that G vd is the control to output transfer function and G VG is the auto susceptibility and this is the output impedance expression.

And for all these expressions the denominator term is common and represents the poles of the system this is also discussed in lecture number 33 in our earlier course.

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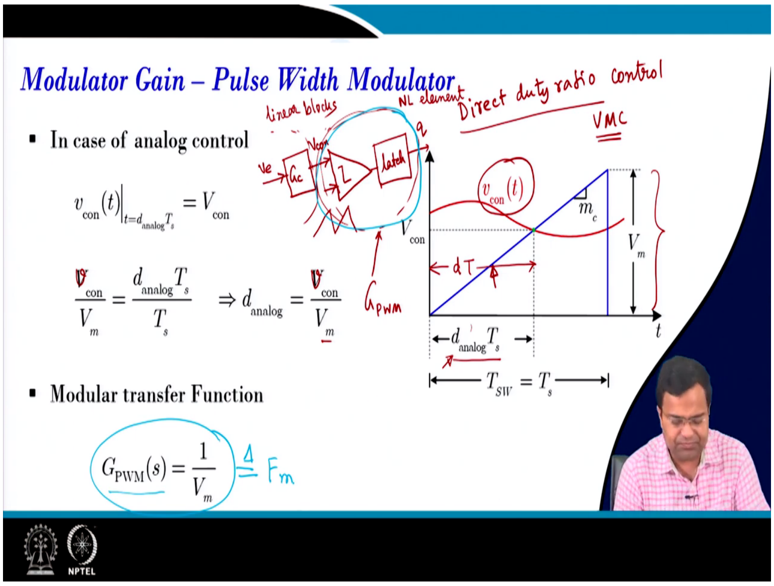

Now, what is our objective? First, we want to go for digital control. So, in this modeling of the digital control, we will start with the continuous time model and we want to start adding digitization from our continuous time block under analog control.

So, the first block will come under modulator gain. And if we talk about direct duty ratio control, direct duty ratio control and such control one such control is the voltage mode control. So, voltage mode control is an example of direct duty ratio control, where we are controlling the duty ratio directly using a feedback loop, ok.

So, if we go by this then, you know our control voltage typically is the output of the voltage controller that is a control voltage and that is coming from the outer loop. And this is compared with the sawtooth waveform. So, this is our sawtooth waveform if we compare it with the sawtooth waveform wherever they intersect that represents the duty ratio because it is d analog into T s.

So, T s is the total time period which is also the switching period, and d is analog as if we are talking about analog control. So, the duty ratio using analog control is represented by D into T which is the on time. And we know about this and V m is the maximum voltage for this sawtooth waveform and we know under voltage mode control V con by V m can be written as d analog and in that way, d analog can be written as Voltage control.

So, V control we like small because it is an instantaneous variable where V m is the constant variable that is the peak that maximum value of the sawtooth signal. So, why we are talking about this? Because if you take a voltage mode control, we will have an error voltage. Then this error voltage we pass through a controller and this controller output we call a V con and that will be compared with this sawtooth waveform. This is our sawtooth waveform and then this control output will go to a latch circuit.

So, there will be a latch circuit and this will generate the gate signal. Now, the gate signal in the gate signal, we will be controlling this on time and typically it is D into T. So, here the duty ratio is due to the analog control being d analog. So, we are controlling the duty ratio or we are adjusting the duty ratio using this control voltage.

So, if we want to get the small signal model, we know up to this portion we know it is a linear block all blocks are linear, linear blocks. But this portion is non-linear. So, this portion is non-linear you know these are the non-linear element and we want to relate the duty ratio of this q in terms of the control voltage using linear by a transfer function. So, we need to get the modulator gain.

So, we are trying to get the transfer function of this G PWM; that means, can we write a linear can we get a linear representation of this block which will ultimately generate the duty ratio? And, it turns out this is well known that this G PWM is nothing but the 1 by V m and that we also denote as the modular gain Fm and this is well known and that we have discussed earlier.

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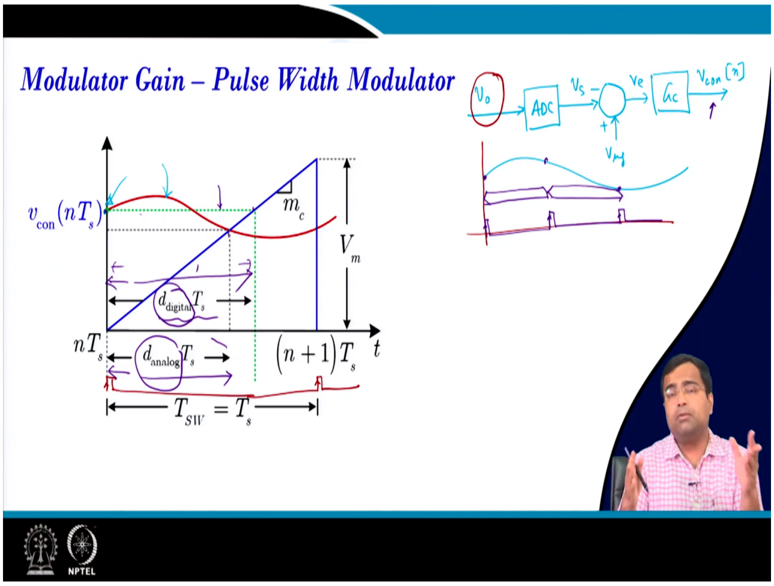

Now, we are going for digital control. So, if you go for digital control typically in analog control you know, we allow this red signal which is a control voltage to swing. Because this will be changing with respect to your know output voltage which is a time-varying signal.

And since there is no sampling, it is an analog signal. So, if output voltage changes because it has a ripple parameter. So, similarly, your control voltage will also change, it will also capture some aspect of ripple. And then that will be scaled by the compensator. But in the case of digital control when we talk about uniform sampling; that means, we are talking about one sample per cycle.

And if you do not consider any analog to digital converter delay any ADC or nothing if you do not consider any other delay then, as if we are capturing a sample here and output voltage and that output voltage; that means, we are talking about this is our output voltage it passes through an ADC and this is our sample voltage and this sample voltage is subtracted from the reference voltage and this error voltage is passed through our controller.

And this is now our V con and we call in terms of n these are digital. So, since output voltage can change, we are only considering once per cycle and that is since there is no delay, we are not considering any delay. So, this is the edge of this clock, when the switching will happen will start. It is the edge of the switching clock. And under trailing edge modulation the sawtooth waveform starts rising at the edge of the switching clock.

Now, at the same clock we are getting the sample; that means, the samples are getting updated; that means if we draw the output voltage waveform. So, then if we draw this kind of waveform what we are getting is the sample, this is the sample. So, these are the clock edge like this. So, we are getting the sample at these edges. So, once these samples are captured they pass through a controller which is a discrete-time controller.

And that will be an algebraic expression and this controller will be updated if there is no delay in the controller, we are not considering initially any delay. So, this will also get updated as if it will get updated here, it will remain constant, then again it will get updated here, then again it will remain constant like this. So, we are not considering any delay. So, it will get updated at this value. So, that means, this controller will get fixed at the beginning and it will remain constant for the whole cycle.

And for such digital control, the yellow one indicates the intersection of the. So, that is why we are calling the duty ratio under this the duty ratio under digital control which is from here to here. But if we take at the same time if we take the continuous time control analog control, then the actual signal of the control output is there is no sampling.

So, it will intersect the ramp here so; that means, this is my on time for the analog control and the corresponding duty ratio which is d analog and this is the on time for the digital control. And this is the digital duty ratio under digital control. So, you can see there is a difference between the duty ratio due to the analog and digital processes. And naturally, this is due to the sampling, and since we are sampling once per cycle the uniform sampling.

So, it is obvious that we are getting some difference in the duty ratio. Now we want to get the modulator gain for this digital block. So, we want to see how can we get the modulator gain for the digital PWM block under this sampling uniform sampling which will take somewhat similar to that of analog. But maybe something different, but can you get a linear approximation or the linear version of the transfer function or not?

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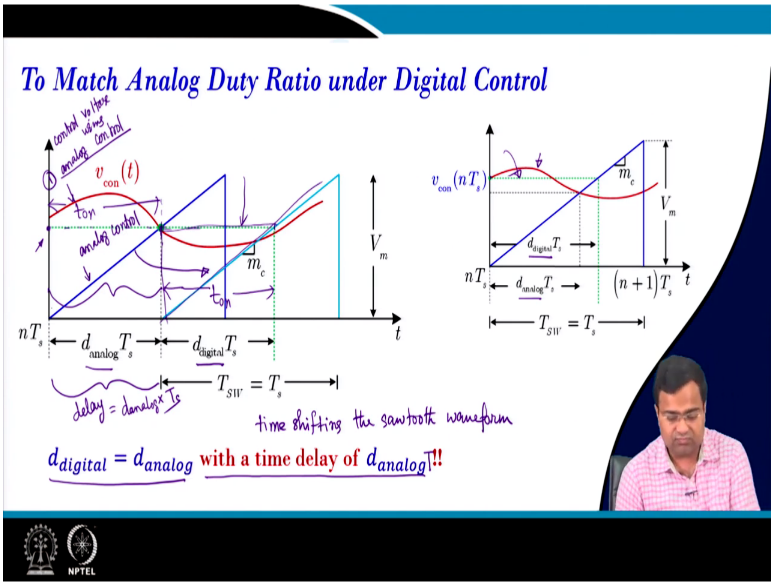

So, to answer this question the first thing, this is again the previous scenario where we have the sample control voltage and we have the actual control voltage this is the actual control voltage and the green one is a sample control voltage. And that two different waveforms result in two different duty ratios.

So, for our first job can we match the duty ratio of the digital control equivalent to the duty ratio of the analog control can we match can we try to do. So, to do that if we look carefully so, this is the analog control. So, this is the sawtooth waveform under analog control.

And since it is the analog control corresponding to analog control under analog control this is number 1 and it is the control voltage. So, we are talking about the control voltage. So, it is I would say it is the control voltage under analog control. So, I would say it is the control voltage using analog control.

So, this is case 1. So, it is resulting the duty ratio d analog, and; that means, if you see carefully this is the point of intersection and if we extend the line here so if we can start with this green line then this line will intersect with the sawtooth waveform at the exactly at the same point.

So, we want to match this on time which is the on time due to the analog control in such a way if we extend this point. So, this should be the sampling point, if we could start the like a sample value of the control voltage with this value initial value then we can perfectly match. So, that means, now what we can do is this blue waveform can be time-shifted right side delayed. But what amount?

As if this point of intersection will be as I said the initial value of the control voltage and that is a sample voltage and we want to hold this value. So, if we as if we take the sample here and if we start the digital control sawtooth waveform then this will intersect here and this on time.

That means, this on time will be identical to the on time of this, because their values are the same as the green waveform which is the extension of the point of intersection due to the analog control. And we are using the same value of the control voltage as a sample value to the starting point and then we can achieve exactly.

So, which means we can make the digital duty ratio and analog duty ratio identical by time shifting, by time shifting, by time shifting the sawtooth waveform, which means we have to consider a delay and what is the delay. It will be d analog into T. Because we do have not got duty ratio is just a dimensionless quantity. So, we have to d analog T so; which means, this will be a delayed amount.

So, this will be my delay and that will be d analog times T s. So, if we can delay this sawtooth waveform by this amount and if this becomes your sample quantity of the control voltage, then you will get the same duty ratio as analog control.

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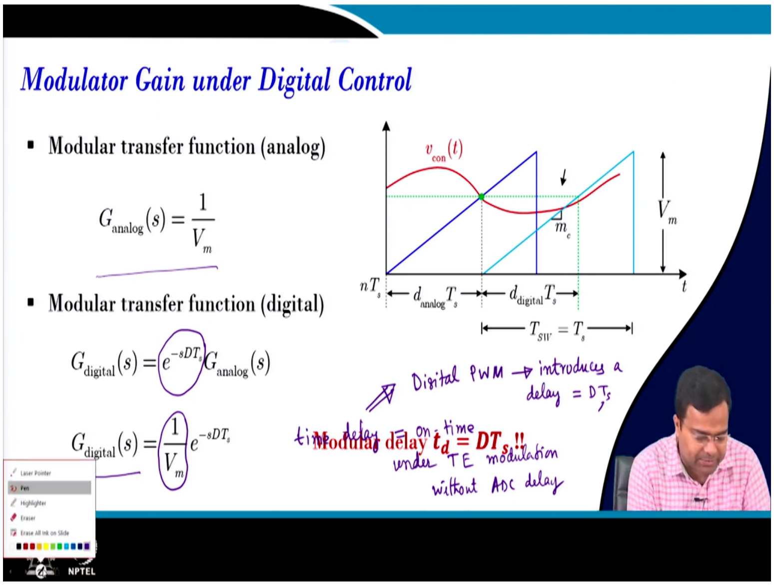

So, in summary, if you want to get the modulator gain this is the analog controller modulator gain 1 by V m. So, if you want to get the equivalent modulator gain of the digital block then it looks like you have to incorporate a delay amount of this.

And that delay amount is D into T s and. So, the digital modulator transfer function will be it is the same as the analog modulator transfer function multiplied by the delay. So, it will introduce a time delay, in this case, equal to the on time. So, this is under trailing edge modulation, trailing edge modulation without you know ADC delay; that means, the modulator itself is the digital modulator.

So, what is the summary? That means the digital pulse width modulator digital PWM block introduces a delay which is equal to D into T and T is the time period. So, in this case, we have taken T s as the time period and D as the duty ratio which is the capital duty ratio. So, modulated delay; means, it is written already here. So, it will be like this, ok. So, modulated delay.

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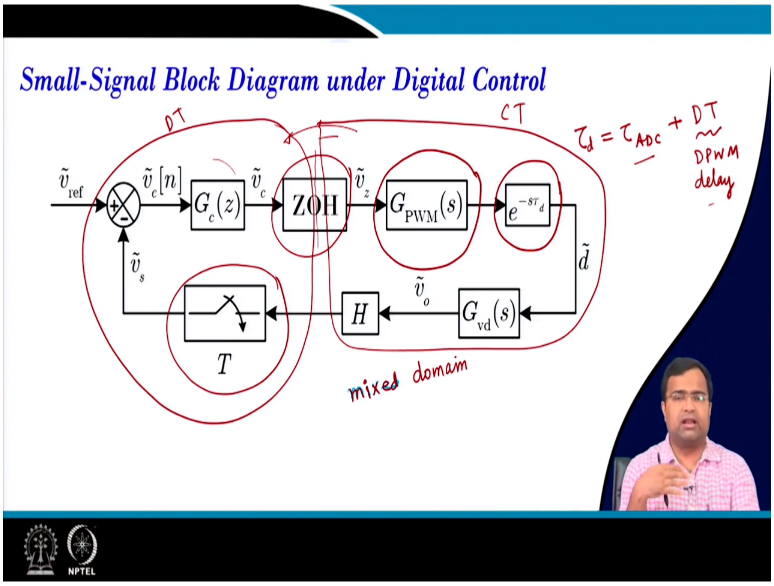

Now, small signal block diagram. So, we can write the small signal block diagram like this is the PWM transfer function for the analog control that is 1 by V m. This is the delay, now here the delay can be grouped into two, one delay we saw the DPWM delay another delay we have to consider due to the ADC sampling delay.

So, the total delay here turns out to be tau ADC plus you know D into T which is your DPWM delay. And this is again under trailing edge PWM with 0 trailing edge PWM under trailing edge PWM, ok.

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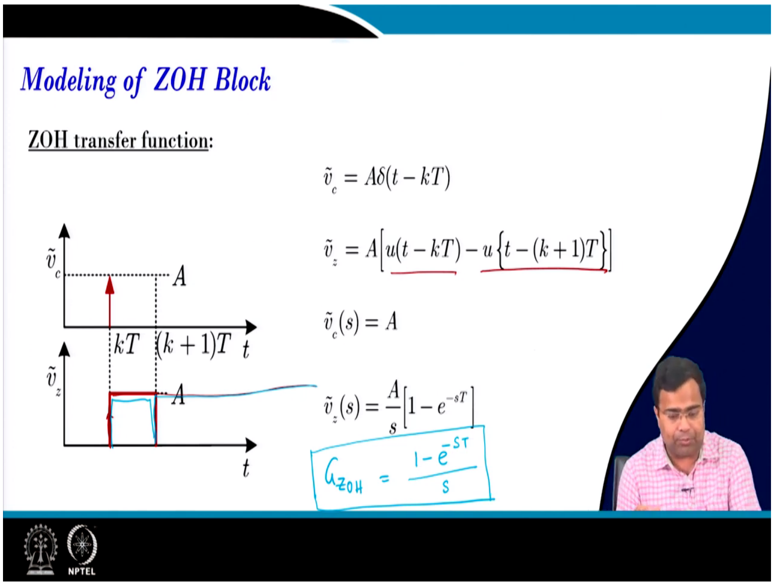

So, now if you want to model the zero-order hold block. So, because there will be if you take the sampling process will also have a zero-order hold effect here and this is the digital controller. So, for the zero-order hold block, we can get the transfer function of the zero-order hold block from this unit step response as well as the delay.

So, these are one unit step responses here minus delay; that means, once you apply one unit step here and you apply another unit step here and then you subtract. So, you will get this particular block and that is here and then if you take the Laplace transform. So, this will be; that means if you zero-order hold transfer function will be 1 minus e to the power minus s T by s.

And it turns out it can be shown the zero-order hold primarily affects the delay, ok.

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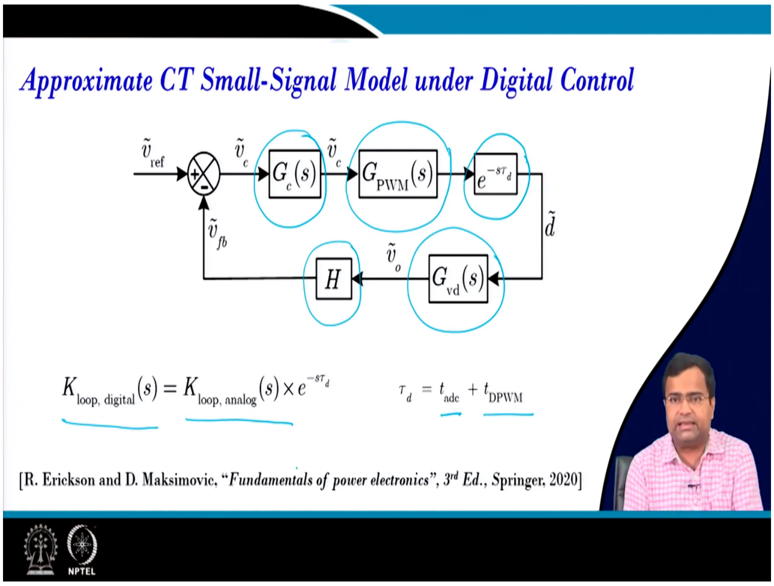

So, we can approximate continuous time small signal model under digital control can be approximated as if want to get. If we ignore the delay, I mean we are talking about the analog controller the I mean the equivalent analog controller the PWM modulator transfer function, this is the total delay loop delay then, and this is the control to output transfer function and this is the feedback gain.

And what is the total loop delay? So, the digital control loop transfer function will be the same as the analog control loop transfer function multiplied by the delay and the delay will be

ADC delay plus DPWM delay. And under trailing edge modulation t DPWM is nothing, but D into T.

And these approximate results will work reasonably fine and we will show when we will compare the continuous time you know transfer function using this approximate continuous time model and transfer function using the exact or accurate discrete time model that you want to compare. They will work reasonably fine and this preliminary model you know is also described in detail in this book.

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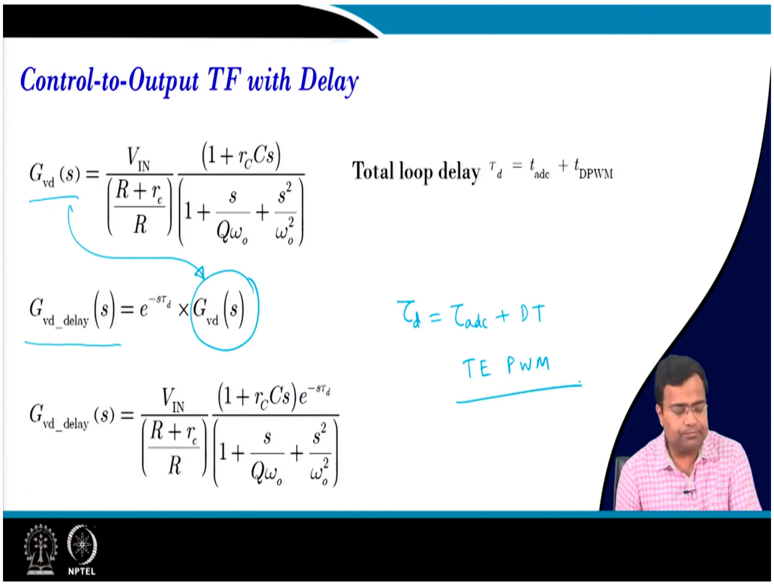

Now, the control to output transfer functions with delay. So, we know about the control-to-output transfer function of a buck converter synchronous buck. And if we consider the total loop delay then the control to output transfer function will have this one which is coming from here multiplied by the delay.

And we also know the what is the delay amount; that means, we know tau d equal to tau ADC plus D into T under trailing edge PWM, ok. Now in this approach, we converted all models. As you know if we go to the previous block, you will find this block is a mixed domain, it is a mixed domain which is the meaning of mixed domain.

So, it is a mixed domain; that means, this part from here to here these are continuous time part. And here to here is the discrete-time part. So, this is the discrete-time and this is a continuous time. So, we cannot design any controller or we cannot analyze in this mixed domain. Either we have to convert all the discrete equivalence of continuous domain and that we have approximated, ok.

We replaced the delay and then we continue with all controller transfer functions everything and in that delay with we considered all the delays possible delays. The other approach we want to convert this continuous time model into a discrete-time model by using zero-order hold equivalent.

Because there is an inherent zero-order hold action because it is one sample per cycle. So, you can approximately you can consider that there is a zero-order hold effect.

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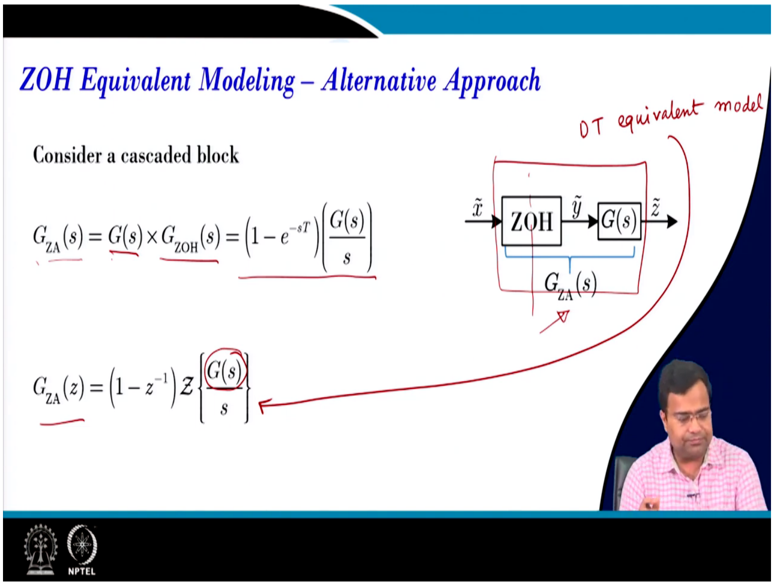

And that is why if you go for an alternative approach; that means, you go for zero-order hold equivalent, how to get that? And this is a standard textbook you can get the detail. If you have a loop transfer function consisting of one continuous time block another zero-order hold block.

Then you can get the equivalent; that means, if the loop has a zero-order hold element and there is another part of the loop, we are talking about if it is a part of a loop where we will club together all continuous time transfer functions and you know as if the left side there is a zero-order hold.

So, we will get this complete model to be a zero-order hold equivalent model and that can be expressed in terms of ZA s one minus s Z s by s. So, this is the continuous version of this, but if you want to get the z inverse because we want to get the discrete time equivalent model and this will be here; that means, it is the one minus z inverse Z then it is Z transform of G of s by s, and what is G of s?

It is you know the combined block which consists of all continuous-time transfer functions.

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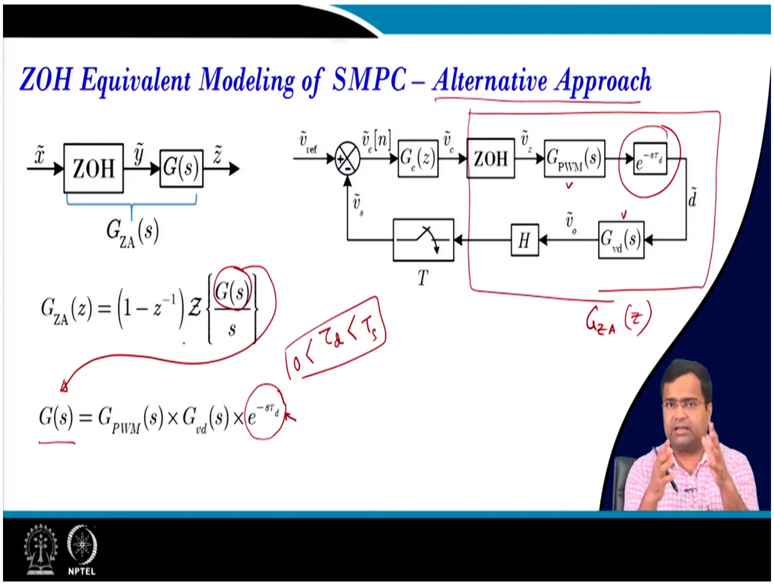

Now, how can we extend this approach to a switched-mode power converter? So, if you again consider the block consisting of mixed domain elements. So, we want to use this approach, where we want to get them from this part to this part we want to get G ZA Z. And what will be this?

This will be G of s in this form if you want to get G of s will be the product of G PWM s G vd s and this delay. And this if we can consider and we may consider an approximate version of this delay may be first order Pade approximation. Because if this delay this tau d equals to complete sampling time, then you know it is straightforward.

So, that delay can be accommodated here by z inverse, but since this delay maybe it is greater than 0, but less than this sampling time then we may approximate this delay may be a first-order pade approximation or something like that. Then you can get the total equivalent model.

So, in summary, one thing at the end because we want to compare this model in you know may be the latter part of this week. That is what will happen if we consider the continuous time model only; that means, all blocks are taken into the continuous domain, and adding that delay e to the power minus sT d then you will have everything in the continuous domain.

And you can design the control in the continuous domain which is standard and then you can get back once you design the controller, then you can get back in the z domain using you know a transformation like; that means, s to z transformation and that we will be discussing in the subsequent lecture.

And we want to first see if we use the continuous-time approximate model with a delay, what is the accuracy of the transfer function, with respect to what we learned about the accurate discrete time model?

So, if you compare the transfer function using an accurate discrete time model or if we compare it with a continuous time model with delay and maybe we can also consider the alternative approach the zero-order hold equivalent of this transfer function. So, combining all these what I mean is what are the accuracy and which one should be selected for the design purpose.

So, this modeling approach gives us some kind of information and when we will compare one with the other then it will help us to select. Because of this transfer function in continuous time, you know the small signal model is much easier to deal with because we know output impedance control to output transfer function all these pole-zero we know very easily. And we will see when we go to the z domain in pure discrete time model accurate discrete-time model.

It will be accurate, but it will not be so intuitive in terms of pole-zero location. So, we need to compare the accuracy as well as we need to find the mechanism of design in such a way it will also give some insight, in terms of impedance you know, or other parameters loop transfer function, pole-zero expression, and so on.

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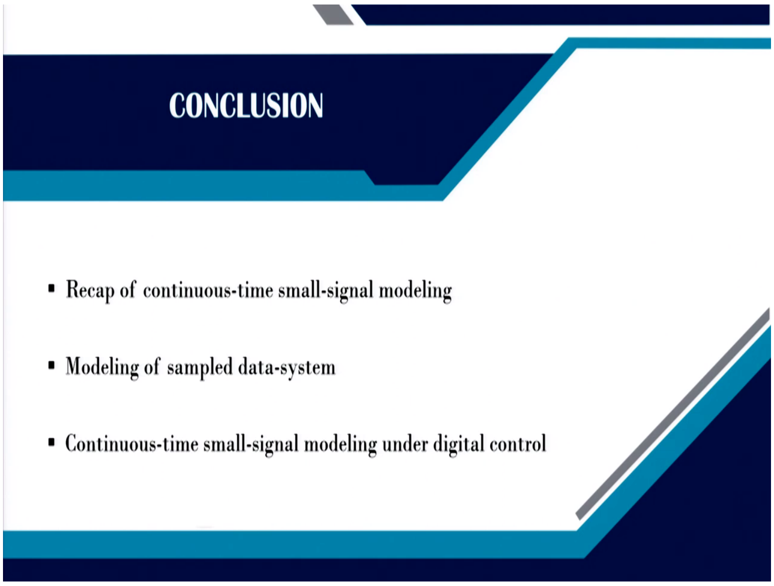

So, in summary, we have recapitulated the continuous time small-signal modeling, then you know of the AC equivalent circuit model, then we have discussed the sample data modeling perspective for this digitally controlled converter and finally, we have also discussed the continuous time small signal modeling under digital control. And we will be discussing the accuracy of this method with respect to the other method in the subsequent lecture. That is it for today.

Thank you very much.