Digital Control in Switched Mode Power Converters and FPGA-based Prototyping

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Module - 04
Modeling Techniques and Mode Validation using MATLAB
Lecture - 34
Derivation of Discrete-Time Large-Signal Models

Welcome. In this lecture, we are going to derive the complete Discrete Time model Large Signal Model.

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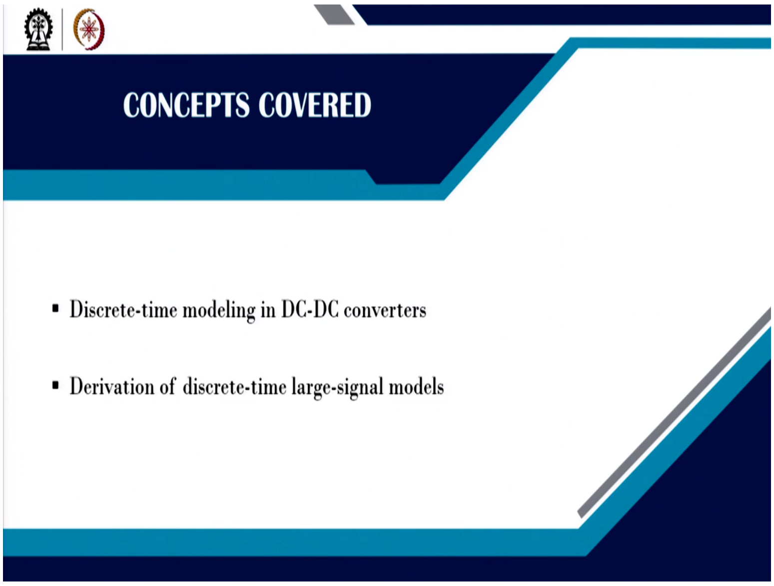

So, we will discuss the first discrete-time modeling in the DC-DC converter that we started in the previous lecture. And then we want to derive the complete discrete time large signal model.

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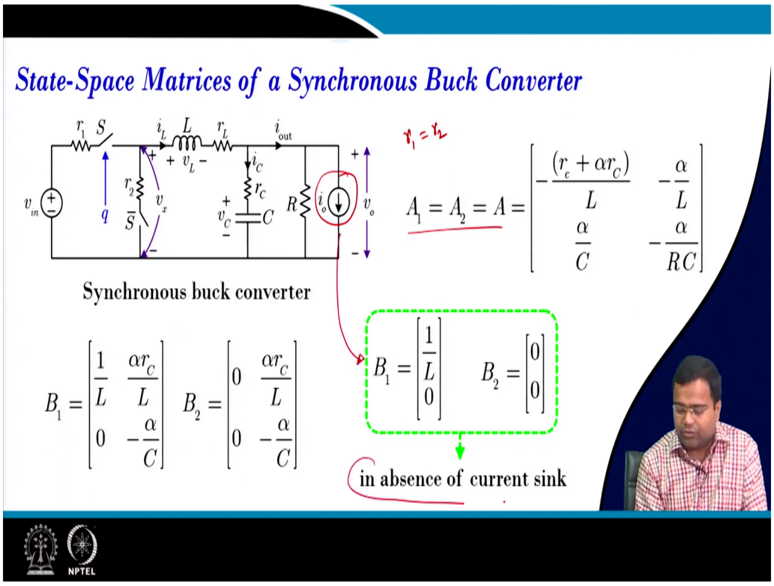

So, first, if we take a synchronous buck converter and this is the circuit diagram more or less practical synchronous buck converter, then we know how to get A 1 A 2 matrix and for r 1 is equal to r 2, then this A 1 A 2 matrix are same that we have discussed. Then we will get beyond the B 2 matrix and if we ignore the external current load if we ignore this part then you will get this matrix if it is ignored, ok.

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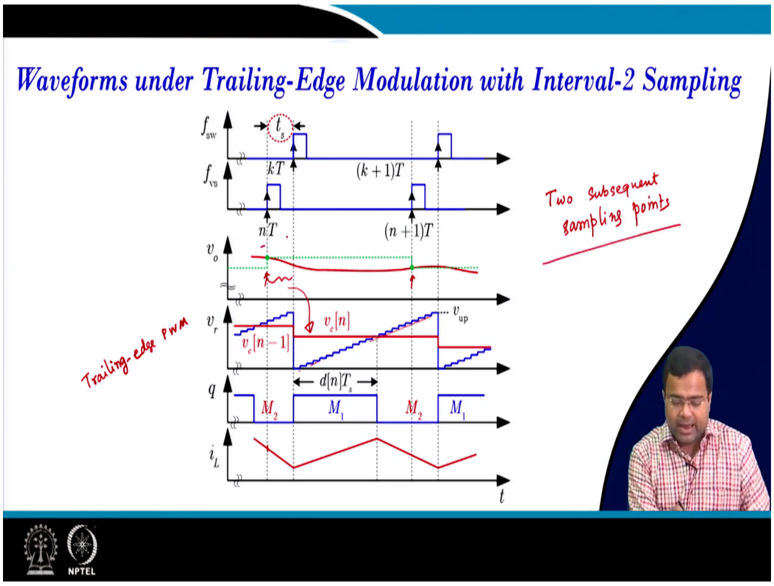

Now, so this is the diagram of this complete interval 2 sampling that we have discussed; how does it work? So, we will take the output voltage sample right here. So, we will take the sample, somewhat earlier than the actual switch turns on because we are talking about the trailing edge modulation. So, this is under trailing edge modulation trailing edge PWM.

So, under this modulation we want to take the sample earlier than the switch is turned on and that is why, it is called interval 2 because as if we are taking the sample during the switch-off time, which is mode 2. What does it do? So, we take the sample here and we will use this sample in the subsequent cycle, but this sample is used and it is this time is provided for ADC conversion time and the computational time and then your control output; that means, the output of the controller is ready after this time.

So, then your actual DPWM start and here we are taking a steer k approximation of the ramp signal. So, if we ignore the effect of quantization, then how do you derive the discrete-time model? So, you see we want to derive the discrete-time model between two subsequent sampling points, two subsequent sampling points. So, earlier we wanted to derive a point. So, earlier we want to derive between two switching points there was no delay, but if there is a delay in the sampling.

So, we will take the sampling clock as a reference clock to derive the discrete-time model. So, that is used for n T and n plus 1.

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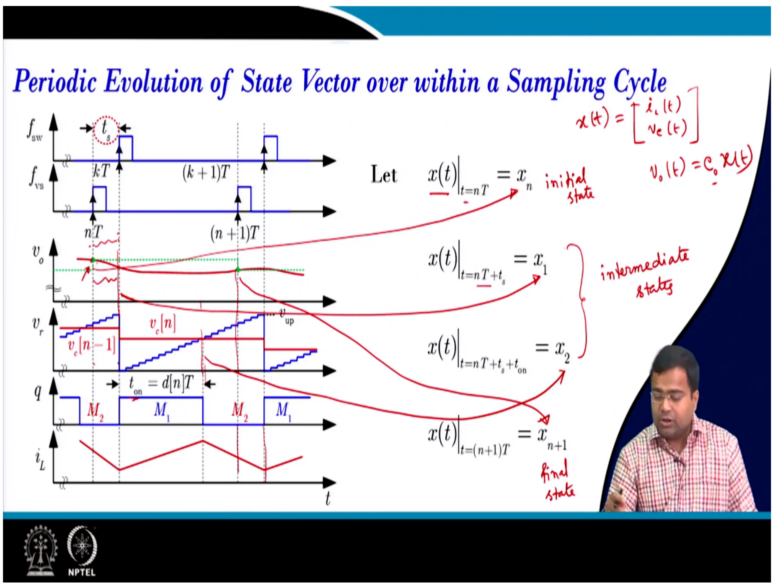

Now, how do we start? So, if you take this waveform, let x of t; that means, any state variable. So, what are the state variables? We know the state variable here we have taken, the inductor current, and the output sorry capacitor voltage. And the output voltage will be C matrix time x of t right, that we know. So, C we want to make C 0 because C is the output capacitor just to separate that notation, we are using C 0.

Now, if you take the sample here you will see that first, it will undergo mode 2, and at this point, we are calling the state variable we are calling x n. Then at this point what is the state variable? We will call here x 1. So, this is my x 1, then it will come to this point when the switch will change its state.

So, this point we will call x 2 and if you see the duration, the first point will come to T equal to n T, the second point will come to n T plus t s this is the duration t s and the third point will come n T plus t s plus d n t on, that is the on this is the on-time t on x 2 and the final point the point which will be here; that means, the sampling point; that means, this point we are taking this state here x 1 t.

So, your intermediate variables are the intermediate states, right? This is my initial state and this is my final state. We want to write the final state in terms of the initial state and we want to substitute the intermediate state. How to start with?

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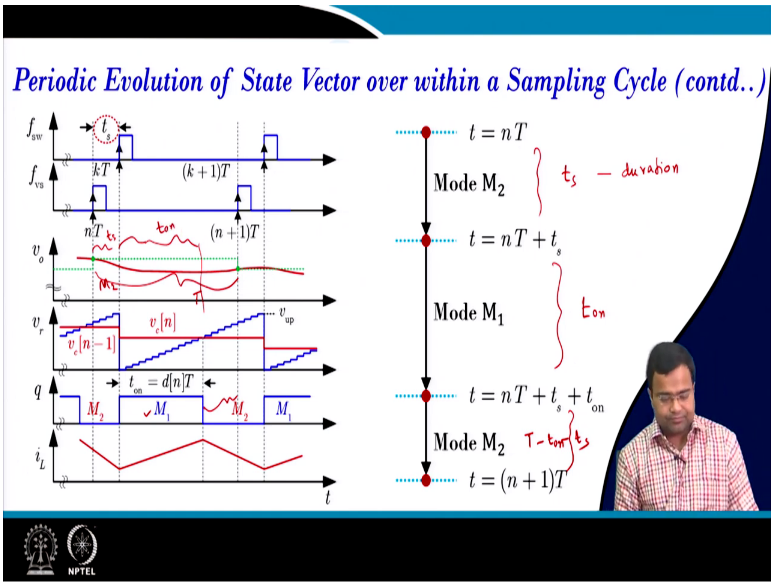

So, let us start t equal to n T, then it will undergo mode 2. So, this is my mode 2, then so the duration of mode 2 is t s, that is the duration. Then mode 1, the duration is what? It will be t on and this is under mode 1. So, you can see it is mode 1, then the rest duration which is this one; what is the duration? This duration if you different then it will be total time; that means, t minus t on minus t s right if you just subtract this; that means, from this point to this point is our total time, this point to this point in my time is t.

So, this point is t s and this point is t on. So, naturally, the rest of the point will be t minus t on minus t s ok.

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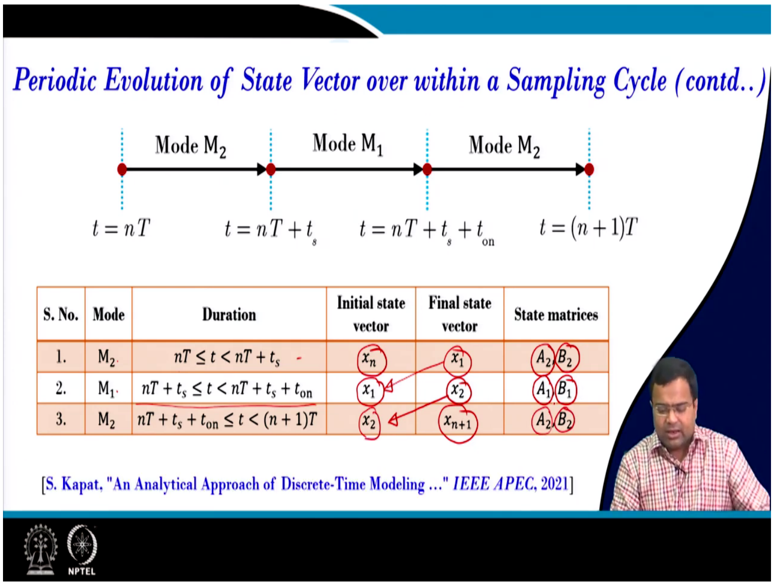

So; that means, this is the model diagram; that means, how the state propagates. So, the total duration; means, first serial M 2, then M 1 then M 2. So, it will be for the duration n T to n T plus t s, then it will be the duration between n T plus t s to n T plus t s plus t on and then n T plus t s plus t on till the n plus 1 T.

And for this mode x n is the initial state x 1 is the final state and the matrix corresponds to off-state matrix A 2 and B 2. Then this mode's initial state is x 1, this state will become an initial state and the final state is x 2 in this mode 1, it will be A 1 and B 1 matrix will be considered. Again this will be the initial state for the next cycle, the next configuration.

So, x 2 is the initial state and the final state is x n plus 1 and in this mode again it is mode 2. So, A 2 and B 2 matrix. So, in that way we will get; so we will get detail about this, there are different books also you can refer to in this paper.

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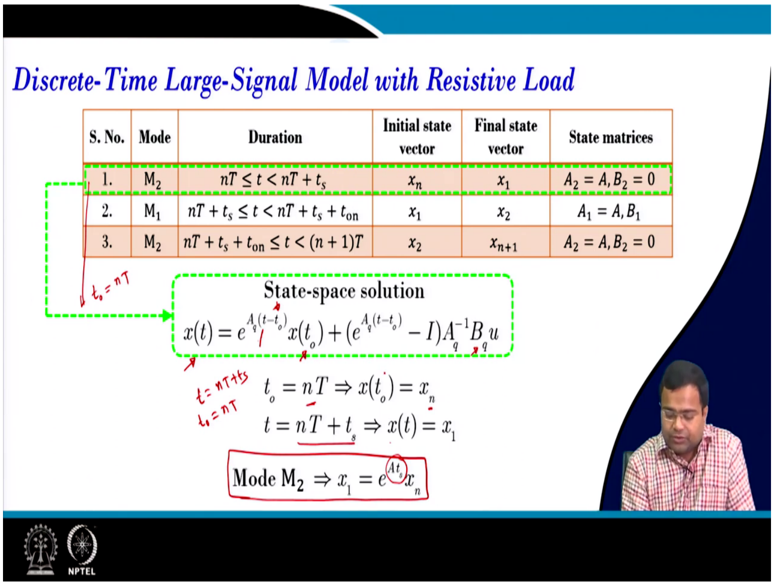

So, here we want to get the complete state. So, we know for each mode the state space solution. So, for this state, it starts with x 1. So, you substitute here, what is the t 01? So, t 01 is n T right, that is the initial time for this state, and t 0 is equal to x n, which is starting value of x n. Because if you take this mode the time starts t 0 from equal to n T its starting value is right. And what is the n value? n value of this state will be n T plus t s. So, x 1 is equal to x of t.

Then mode 2, is under mode 2. And what is the duration? If you take the difference it will be a t s and during mode 2 of a buck converter, there is no output voltage disconnected. So, your solution is simply A into t s because we know this solution. If you substitute total t, what is total t? t equal to n T plus t s and t 0 equal to n T. So, then the difference is t s, this one will be x n that we have defined and since B 2 is 0. So, it will simply be this solution.

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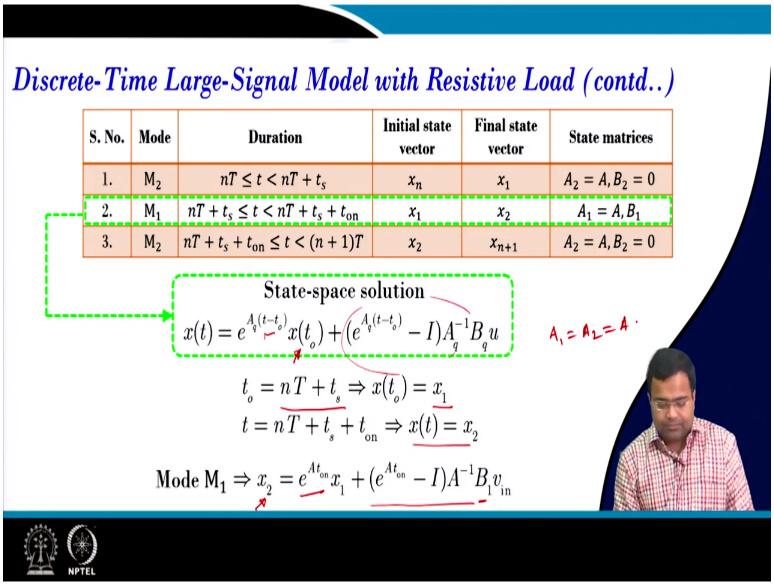

Next for mode two, again you substitute. Here, what is the initial state is my n T plus t s. So, n T plus t s and the state is x 1 and the final state n t plus t s plus t on and the state is x 2. So, what is this difference? This difference will be simply t on. So, it will be t on and the initial state here will be x 1 for this mode and the final state is x 2 and this state of a buck converter your now input voltage is connected, it is beyond the matrix.

So, you will write this and again the solution here this invertible A q; that means, A 1 is invertible and here for a buck we know that A 1 equal to A 2 equal to A we have taken. So, here we will get the solution.

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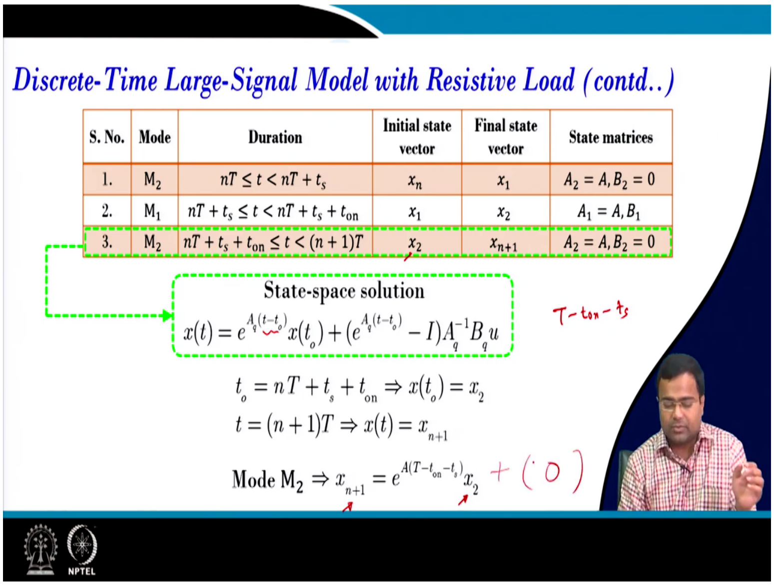

Next, for the third mode, we will substitute what is the initial value. It is x 2. What is the initial time? n T plus t s plus t on. What is the final time? n plus 1 T. So, what is the difference? It will be t; that means if you subtract it will be you know T minus t on minus t s. And during this state x 2 is the initial state, x n plus 1 is the final state for this mode and again it is the off state of a buck converter where the input voltage is disconnected. So, there will be no solution here. So, it is 0, right?

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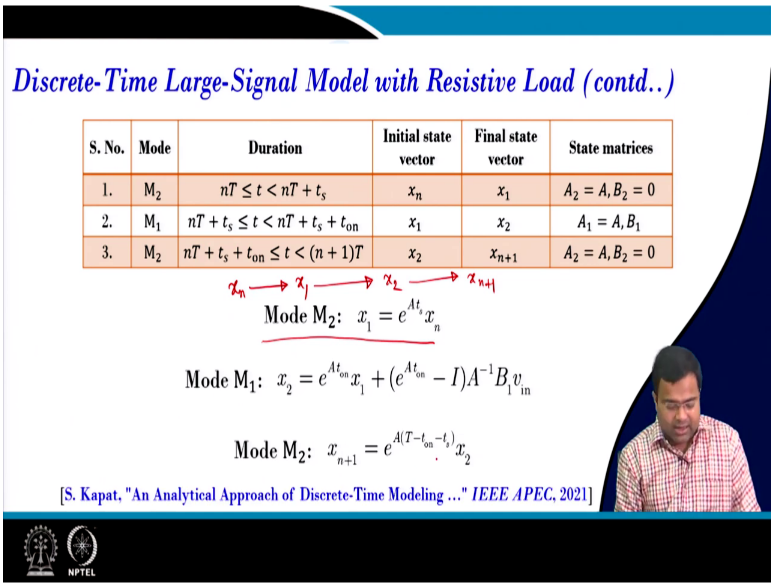

So, now if you combine together mode 2; that means, how it is propagated? So, your propagation is x n propagated to x 1 it is here, then mode 1, x 1 propagates to x 2 it is coming like this, then mode 2 x 2 propagates to x n plus 1. So, we will get x n plus 1 with detail and you can get the detail process in this paper.

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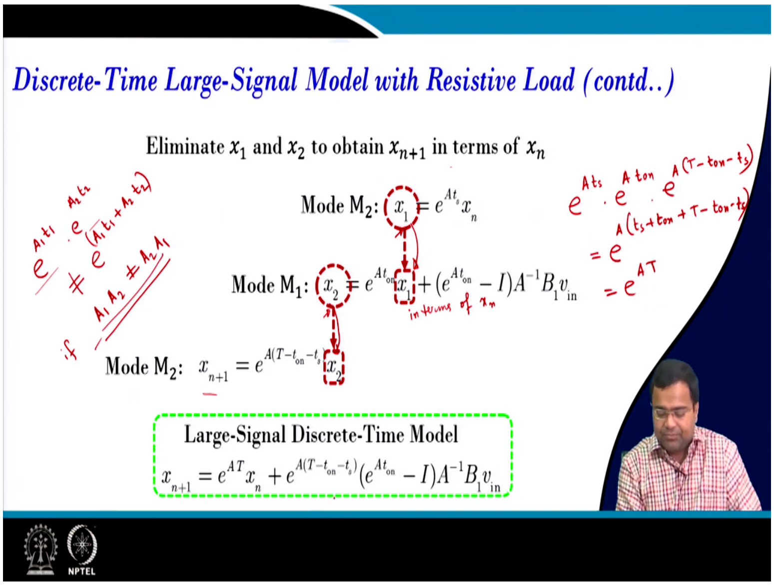

So, the complete solution x n plus 1, if you propagate; that means, you will backpropagate. So, first, you start with eliminate x 1 x 2 to obtain x n plus 1 in terms of x n. So, in mode 2, we have know x 1 and x 2 also we know. Now, you substitute; that means, this x 1 you substitute here, and then it will be in terms of x 2 and the x 2 term, x 2 you substitute here sorry. So, in this case, it will be in terms of x n and you substitute x 2 since x 1 is already replaced. So, the whole term will be in terms of x n.

So, you will get the complete discrete time model like this and you see for all these three cases it will be product; that means, e to the power A t s into e to the power A t on into e to the power A T minus t on minus t s. So, since all a matrix is identical. So, you can simply write e to the power; that means, e to the power t s plus t on plus T minus t on minus t s. So, it will be e to the power A T s that is it.

But, if the A 1 A 2 matrix is different; that means, can you always write e to the power A 1 t 1 into e to the power A t t 2 will not be equal to e to the power A 1 t 1 plus A 2 t 2 if A 1 A 2 is not equal to A 2 A 1. If we can show that means, commutative property if they are not equal then we cannot write.

But if the matrix is identical then there is no problem. So, x a square. So, here since this case is very straightforward for buck converter you can very easily get it, but you will get a different scenario for a boost converter when the A 1 A 2 matrix is different. Then this property cannot be satisfied. So, you cannot write like this. So, then you have to write in the product.

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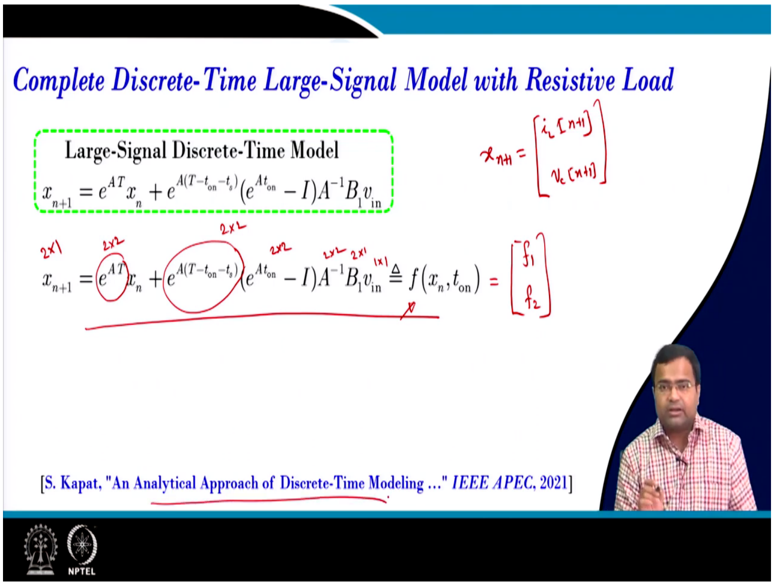

So, the complete discrete-time model for a buck converter can be derived by this method. As a complete discrete-time model and we will take this complete discrete time model as a vector function, because this x n plus 1, is consisting of i L n plus 1 and v c n plus 1. So, naturally, this one will be a vector consisting of f 1 and f 2. Why we are writing? Because we want to show this the large signal model can be used to derive the discrete-time small signal model using the Jacobian approach ok.

So, right now I am just representing this in the complete vector form. So, this will be A 2 cross 1 matrix, because there are only two states and you will find this matrix will be 2 cross 2, again this will be a 2 cross 2 matrix and B matrix is 2 cross 1, input is 1 cross 1 and this whole term again 2 cross 2, this will be also 2 cross 2. So, we will use this vector function to derive a discrete-time small signal model in the subsequent lecture.

So, we will get detail about this how this vector function can be used for deriving discretized small signal models, straight away mathematically. And that can be used for you know for buck-boost as well as it can be used for you know constant on-time modulation, constant off-time modulation, then trailing edge modulation, leading-edge modulation, and all the derivations of small signal models are summarized in this paper. So, for detail, you can refer to this paper.

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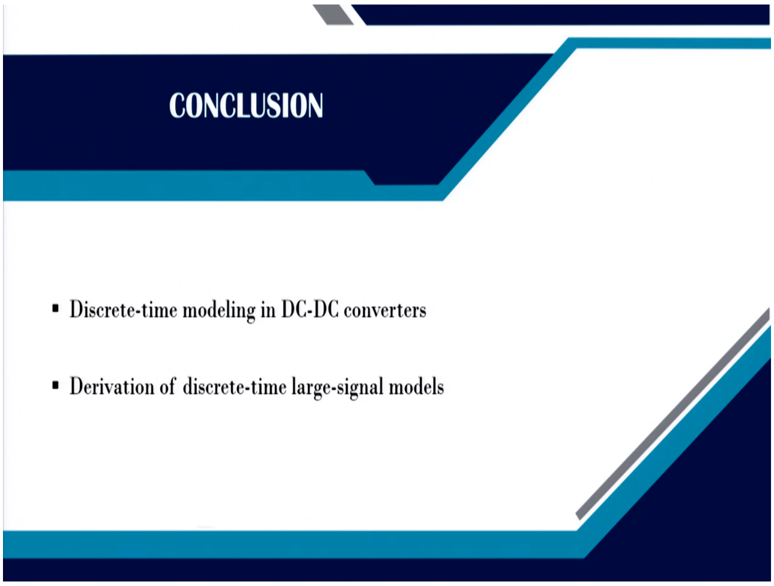

So, in summary, we have discussed discrete-time modeling in DC-DC converters, then we have discussed the derivation of discrete-time large-signal models. So, here we have shown only a buck converter, but as we move forward for stability analysis we can also do the for a boost converter because we already know the method. So, I think this is enough for today, we want to stop it here.

Thank you very much.