Digital Control in Switched Mode Power Converters and FPGA-based Prototyping
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Module - 04
Modeling Techniques and Mode Validation using MATLAB
Lecture - 39
Discrete-Time Transfer Functions and Closed Loop Block Diagrams

Welcome. So, in this lecture, we are going to talk about Discrete Time Transfer Function and Closed Loop Block Diagrams under-voltage mode and current mode control.

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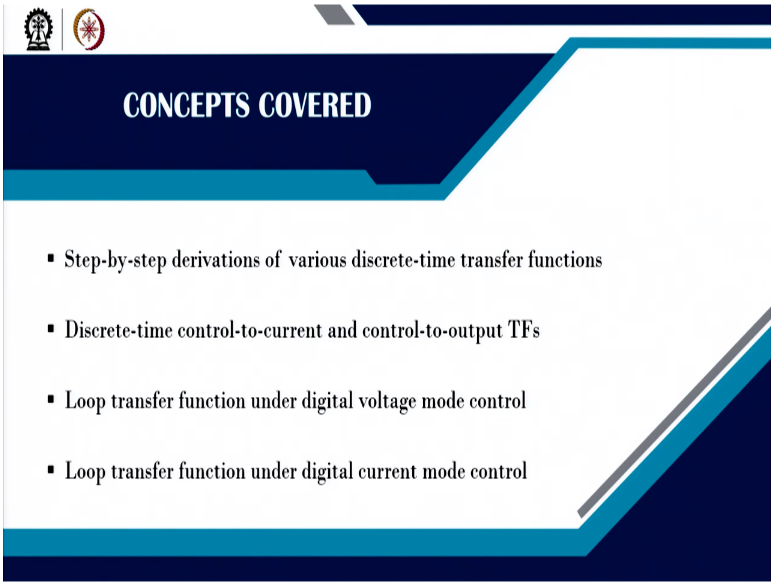

So, here we are first going to talk about the step-by-step derivation of various discrete-time transfer functions, then discrete time control to current as well as control to the output transfer function, then loop transfer function under digital voltage mode control, and the loop transfer function under digital current mode control.

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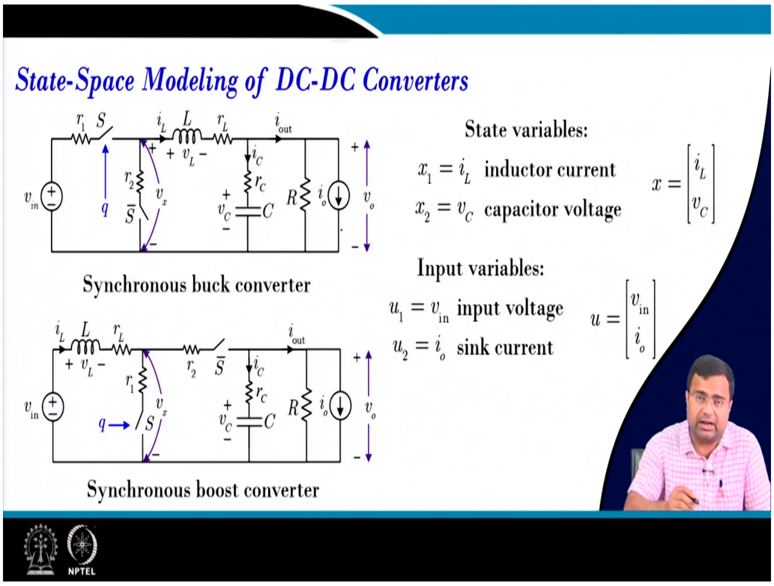

So, if you recall you know the state space modeling of a DC-DC converter if you take a synchronous buck converter with practical parasitic and we can also consider a synchronous boost converter. So, where we can consider the inductor current and the capacitor voltage as the state variable and we can also take the input variable as the input voltage and the load current if we consider an additional sinking load.

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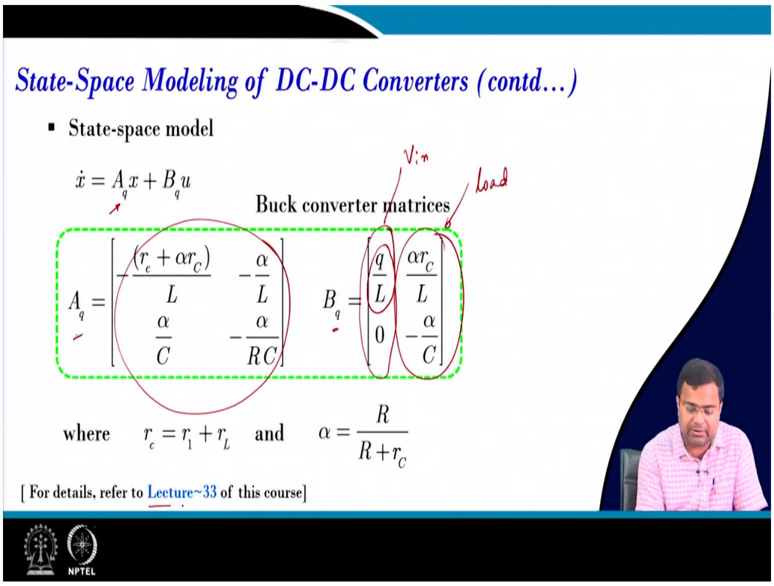

Next, if you want to write the state space model then based on the switch configuration whether the q is the gate signal whether q is 1 or 0, we will get accordingly A 0 or A 1. And here A q for a buck converter can be written A q in terms of the parasitic. Here, we are assuming that the two switches of the buck converter are identical, so, they're on state resistance are the same. Then the A matrix will be common you can see the A matrix is independent of q.

But if we take q B q, it is a function of q; that means, when the switch is on then 1 by l, and when the switch is off it is 0. And this particular term is associated with the input voltage, and this particular term is associated with the load current. So, load current. You can see the load current term there is no q dependency, but in the input voltage, there is A q dependence. So, whether the input voltage is connected or disconnected and we have discussed this derivation in detail in this lecture course in lecture number 33.

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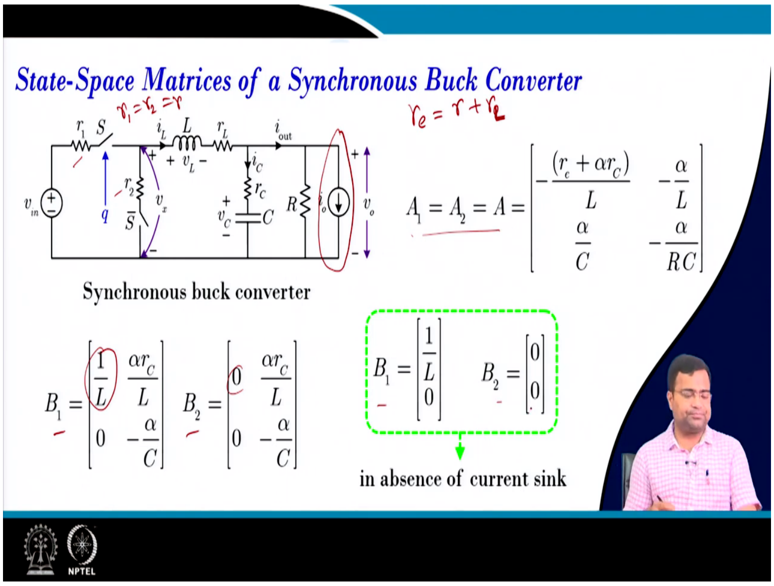

Now, if we take the synchronous buck converter and since we are considering this r 1 and r 2 are equal because we have taken r 1 is equal to r 2 equal to r and where r e we have taken r plus r on that is sorry r L that is the DCR of the inductor. Then, we can write all A matrices are common; B 1 matrix will be 1 by L term will be there for B 2, it will be 0. If the load sink is not there if we remove this path then B 1, and B 2 will be simple like this. So, B 2 is a null matrix.

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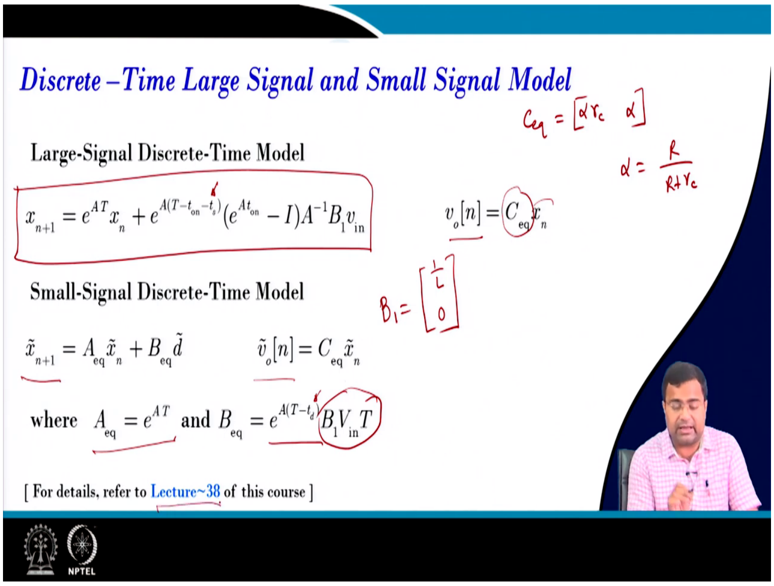

And, we have also discussed how to derive discrete time large-signal models. So; that means, we have discussed you know even we have discussed in lecture number 33, and 34 as well as we have discussed in lecture number 38 about the large signal model where for a buck converter under trailing edge modulation interval two samplings. This is the delay and this delay can accommodate the ADC conversion time, controller computation type as well as DPWM delay.

So, you can write the expression like this and where the output voltage is a function of state multiplied by the C matrix; that means, C into x. And, the C equivalent matrix also we have discussed with you know here, here can get the C matrix here. So, what is the C matrix? Ok, let me write down the C matrix C equivalent. C equivalent for a buck converter is simply alpha r c into alpha.

And what is alpha? It is r by r plus r c ok so; that means, the C equivalent does not depend on any q function. Now, the discrete-time small signal model that also we have discussed that if you take the x n plus 1 tilde which is equal to A equivalent into x n tilde plus B equivalent into d tilde. And similarly, the v 0 tilde will be equal to the C equivalent of the x n tilde.

Then for a buck converter, we have discussed A equivalent is AT and B equivalent is A e to the power A sorry A equivalent equal to e to the power AT and B equivalent equal to e to the power AT minus this is the delay and B 1 V in, B 1 is the input matrix. What was B 1? We know B 1 is equal to in this case 1 by L 0 V in into T. And this we have also discussed in lecture number 38 of this course.

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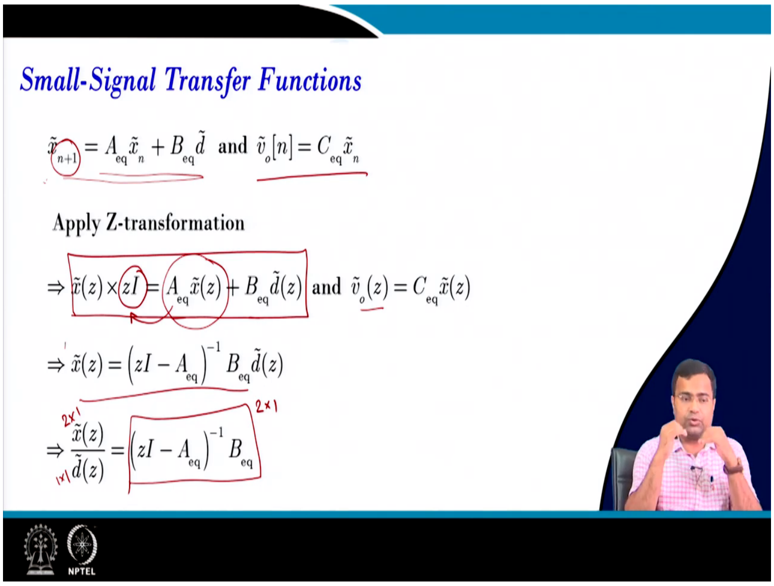

Now, for the discrete times, we have to derive the transfer function. So, we have one model part of the state space model and part of the output model all are in discrete time. Then, we apply Z transform. If we apply Z transform, then we will get zI because there is an n plus 1. And then all A equivalent and B equivalents are there, and v 0 z equal to C equivalent to x z. Now, x z from this equation if we write this equation if you take this term in the left side, you can take x z tilde common it will be zI minus A equivalent into inverse B equivalent into d z.

So; that means, we can obtain the x tilde in terms of the d tilde along with the matrix A equivalent, B equivalent, and z. Then, the x z tilde by d z d tilde will be this, but this is a 2 cross 1 and this is a scalar. So, we will get a transfer function matrix ok. Matrix will be 2 cross 1; that means, what are the 2 cross 1? So, you will get 2 cross 1, the first term will be the current-to-duty ratio and the second term will be the capacitor voltage-to-duty ratio.

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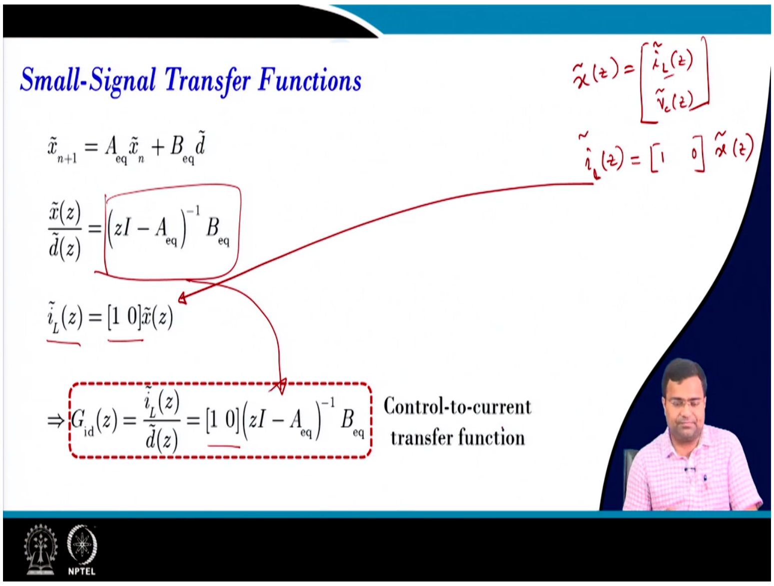

That means if we want to get the current too. So, how to extract current from the x z? You simply multiply with 1, 0 matrix, then it will give you the first term because we know what is my x z tilde. It is nothing but i L z tilde and this is the v c s z tilde where v c is the capacitor voltage. So, if we want to take the first element; that means, my i L z tilde is equal to I will simply multiply with this matrix into x z.

And this is exactly what we have done here. Then d i z will be simply i L z by d z. This is 1 0 term will be there and then whatever we have obtained here we should substitute here, and this is called control to the current transfer function.

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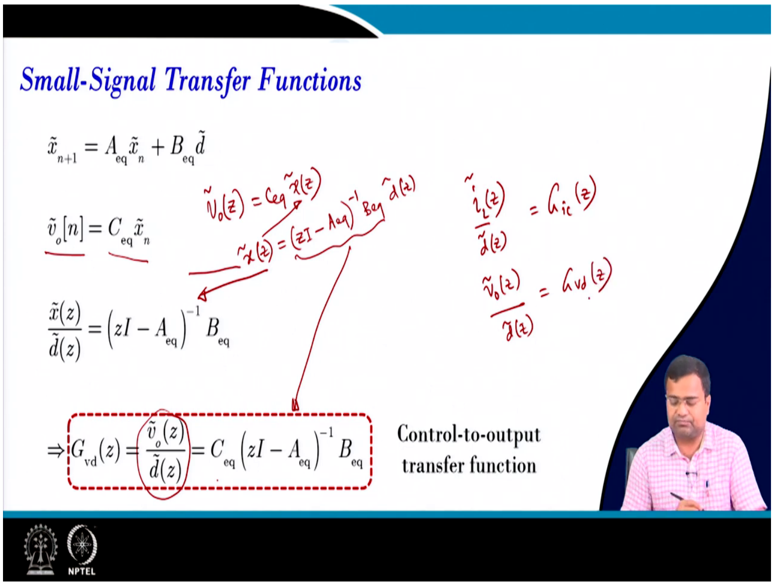

Next, we want to derive control of the output transfer function. So, v 0 n is already a function of x n; that means if we apply the Z transform; that means, we can get v 0 z as a function of C equivalent to x z. And what is x z? We know the x z tilde is equal to z I minus A equivalent inverse B equivalent into d hat z and this is exactly shown here. Then, if we substitute this into this equation we can get this equation.

So, it will be C equivalent when this term will come. And that is your v 0 z by d z and this is called the control to the output transfer function. Now, we have derived control to the current transfer function; that means, we have derived the i L to d transfer function in the z domain. And this we call G ic z control to current. We have also derived control to output transfer function and this we called G vd z right.

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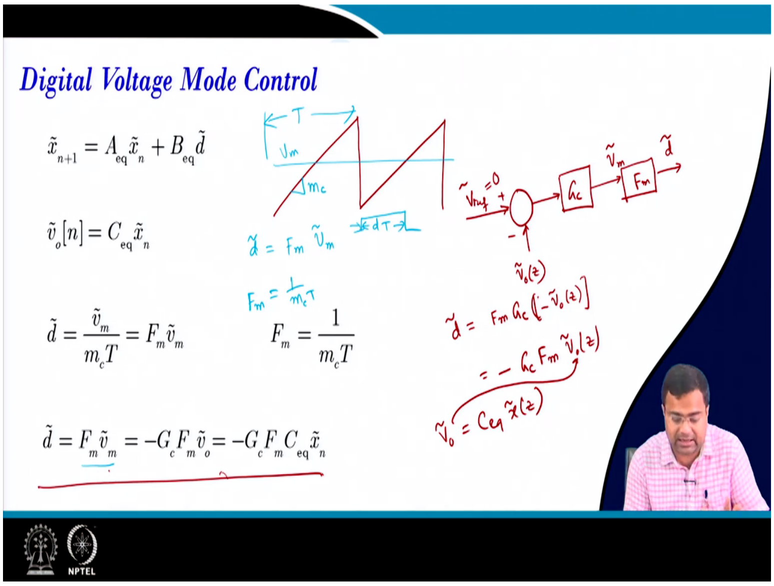

So, in digital voltage mode control if we consider it, what is the block diagram? That means, how do generate duty ratio in a digital voltage mode control? So; that means, again you take the sawtooth waveform like this and there is a modulator gain, and if this is, v m, and if this is m c then what will be our v m? And this will be our let us say our duty ratio d into T. So, then our duty ratio tilde will be F m into v m tilde where F m is nothing but 1 by m c T where T is the time period ok; this is written m c and we can express that d tilde F m this.

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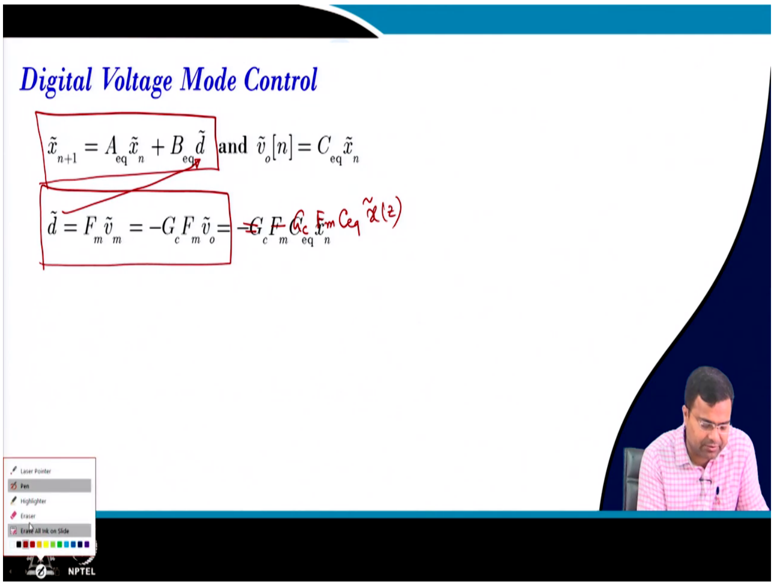

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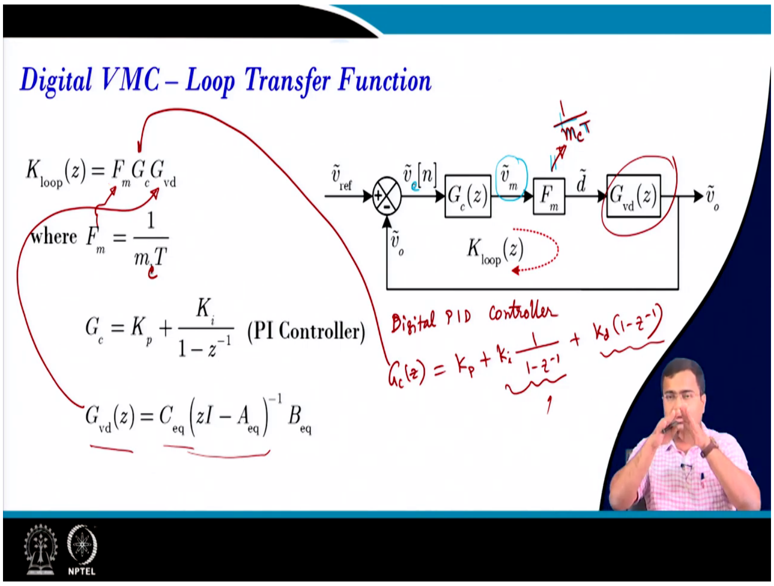

Now, we will show if we draw the block diagram. So, this is our error voltage, then this error voltage goes to the controller, the controller will generate the modulating signal, and then it goes to the duty ratio. So, here this F m is nothing but 1 by m c T ok. So, if you use a different color it is 1 by m c into T. This is our Fm. Then, we have already derived the v 0 to d transfer function in the z domain and that itself takes care of the delay and all.

And; that means if we go back to voltage mode control you know what we are discussing here; that means, what is my V m? So; that means, if you draw the loop this is our ref and this is our v 0 z and if we keep this thing to be 0 for a regulator purpose. Then, it goes to our controller and the controller gives us v m perturb. And then this goes to F m and this gives rise to our d perturb.

So, how can we write d perturb in terms of v 0? So, it is simply F m into G c into minus v 0 z which is nothing but minus you can write G c F m whatever G c F m v 0 z. And what is v 0 z? v 0 z is nothing but C equivalent to x z. So, you can replace this term here then you will get this expression. Why we are writing this expression? We want to get the closed loop characteristic equation; that means, we have the original equation like this.

And this is the original equation irrespective of what control technique we use. If we use a voltage mode control our duty ratio is generated from the outer loop and we have already discussed this expression as a function of v 0. So, we can substitute here and we further know this is nothing but minus G c F m into C equivalent into x z, and if you substitute ok. So, this is there; that means, we can erase this part.

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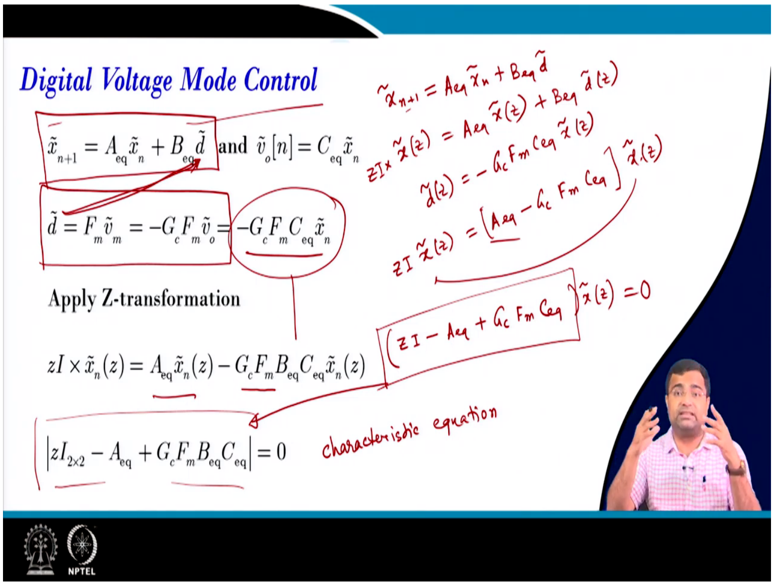

Then, if we apply the Z transform sorry it was originally it was in the time domain. So, we apply Z transform. So, here we have to substitute this d into this term; and all these expressions will be substituted here. So, we will get G c F m B equivalent; and this term is coming here. That means if you start with this expression let us start. Our original expression was x plus 1 is equal to A equivalent x m perturb plus B equivalent d perturb.

If we take the Z transform then what we will get? z I because n plus 1 is equal to A equivalent to x z plus B equivalent to d z. And what is our d z? We have discussed it is minus; that means, you can refer here to minus G c F m C equivalent into x z. If we replace it here, then what we will get? z I x z equal to A equivalent minus G c F m C equivalent, this whole thing into x say.

And if you take everything in this side; that means, this is the expression I am showing here then you get the characteristic equation z I minus of this which will be zero; that means, the final expression will be z I minus A equivalent plus G c F m C equivalent this whole thing into x z equal to 0. And, if we take this part and if you take the mod; that means if you take the determinant of this and this determinant is equal to 0 is the characteristic equation.

What is the role of this characteristic equation? So, the role of this characteristic equation, you have to ensure that all eigenvalues are should be within the unit circle otherwise it will be unstable. So, while we can derive the stability concept from the loop transfer function, we have a representation to make sure that you do not miss out on anything.

That means you can ensure that by suitable choice of the controller, you have to make sure that because it is directly in the digital domain. So, none of the eigenvalues should go outside the unit circle because this is under closed-loop voltage mode control. And when you will go to the design we will consider such you know equation; that means, we have to make sure that no eigenvalue.

So, if one of the goes eigenvalues goes outside the unit circle then it will lead to a different type of instability. So, if you take the voltage loop transfer function; that means, we have this loop which will be the product of all, then the modulator gain we know should be m c, m c T where m c is the compensating ramp; which means, sawtooth waveform slope and the controller we have taken if we take a PI controller, but typically for voltage mode control we consider a PID controller.

So, if you take a digital PID controller, the controller then we will get G c of z equal to K p plus K i 1 by 1 minus z inverse plus K d 1 minus z inverse. So, this is a discrete-time integral term and this is a discrete-time derivative term; and these are all coming from the backward difference formula. So, then we can get and we know that G vd is nothing but C equivalent multiplied by this inverse matrix into B equivalent.

So, you can easily get the loop transfer function by substituting here, by substituting the controller here, and also by substituting here. So, we can obtain and then we can draw the loop gain plot to make sure that the closed loop is stable.

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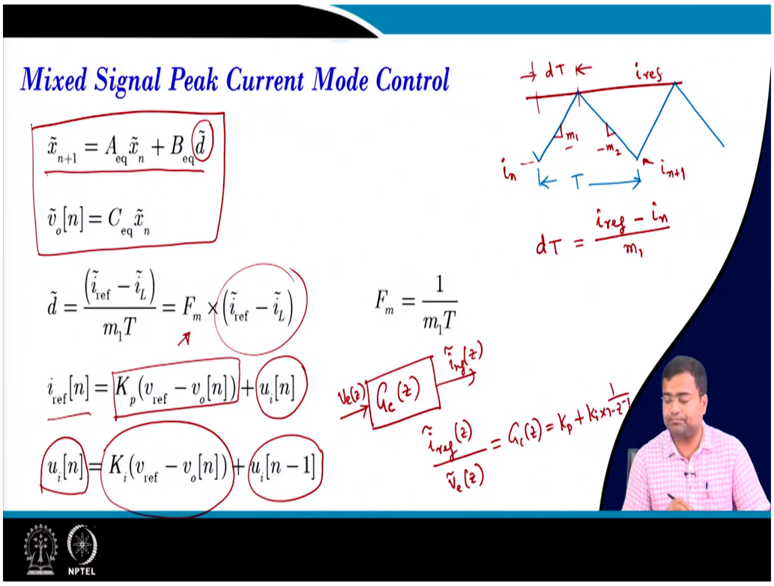

Similarly, if you take the mixed signal peak current mode control we can start with the very fundamental equation which is the perturbed model, but in current mode, the difference is that the generation of the duty ratio is different. In voltage mode, the duty ratio is generated from you know, how it is generated? If you take a controller the output of the controller is compared with the sawtooth waveform and that generates the duty ratio.

But, in the case of current mode control, it is not like that. So, in current mode control if you consider if this is your inductor current let us say. So, here to here, we have a time period that is fixed because we are talking about peak current mode fixed frequency control. Then, what we can consider here? This is our rising slope and this is our falling slope ok. Now, how do we generate the duty ratio? So, this is our DT and if this current is my i n beginning of this cycle and this is my i n plus 1.

And this is my peak current or here we can write the reference current. I will say it is the reference current. So; that means, my duty ratio can be derived from here my reference current minus i n divided by m 1 ok. So, we can substitute output. So, these are the two basic equations of the state space model. Then, we have discussed; that means, if we take from here the duty ratio perturbation can be a function of i ref tilde minus i L tilde where we can write in terms of modulator gain by this current error.

And what is the modulator gain? It is simply 1 by m 1 T under peak current mode control where m 1 is the rising slope of the inductor current. Then, we can write I, ref, n it is coming out of an as a PI controller because typically for current mode control we take a PI controller and this is a discrete-time PI controller. So, this part is the proportional part, proportional part and this is coming from an integral part; how is it written?

So, the integral controller can be written as the previous integral term plus the integral gain into the error voltage. And this is using an incremental algorithm or the discrete-time integral control; that means if you take the; means, if you take the transfer function of the controller; that means, the controller output, what is the input to the controller? The error voltage and output of the reference current. And if we want to get the transfer function of this; that means, v e z which is nothing but our G c z. So, it can be written as K p plus K i into 1 by 1 minus z inverse ok.

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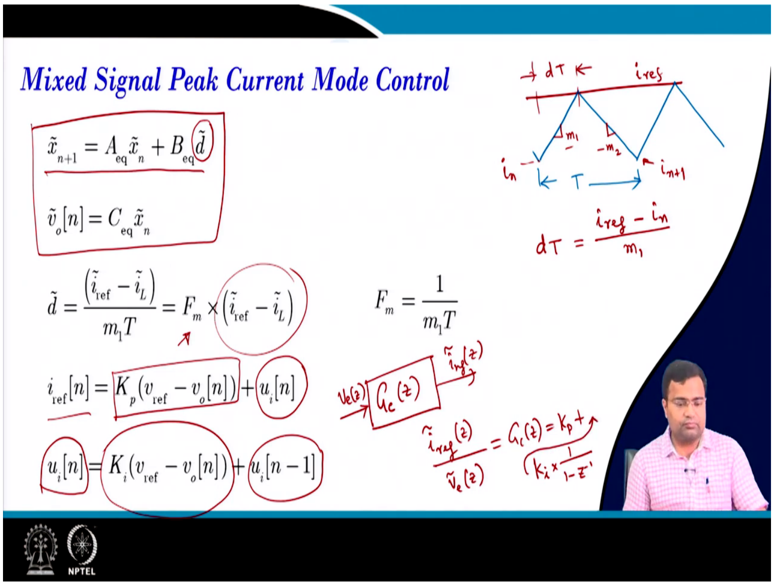

So, if you write below to make it clear it is K i into 1 by 1 minus z inverse. So, this term will go here ok.

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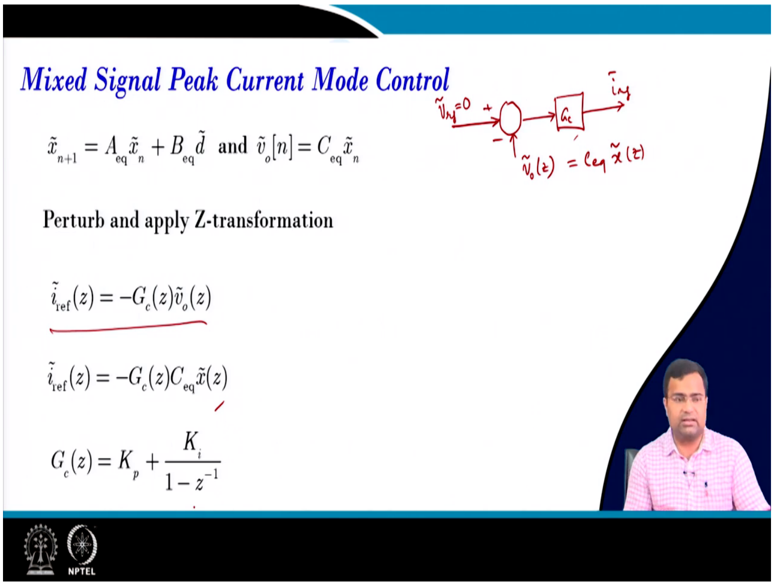

Next; that means if we apply Z transform I ref can be written in terms of the controller because if you take the loop transfer function in current mode control you have a reference voltage perturbation simply it is 0 for regulator purposes. Then, we have this v zero z perturb then we have this controller. And this controller generates our reference current so; that means, the reference current is simply a product of G c into v 0 z then v 0 z we know it is nothing but C equivalent into x z. And this is written here. Next controller for a PI controller you know it is K p plus K i by 1 minus z inverse.

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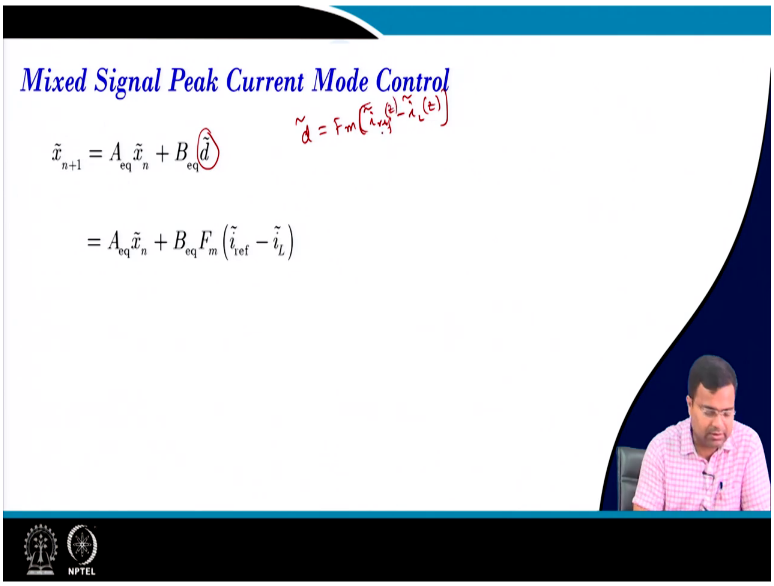

Now, we start with this. So, we replace the duty ratio, what is the duty ratio we know? The duty ratio is the modulator gain then I ref perturbation minus i L perturbation.

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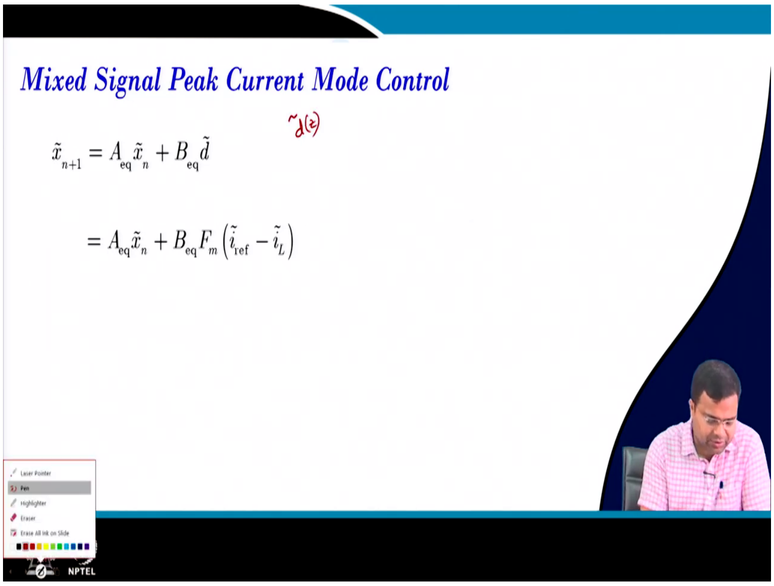

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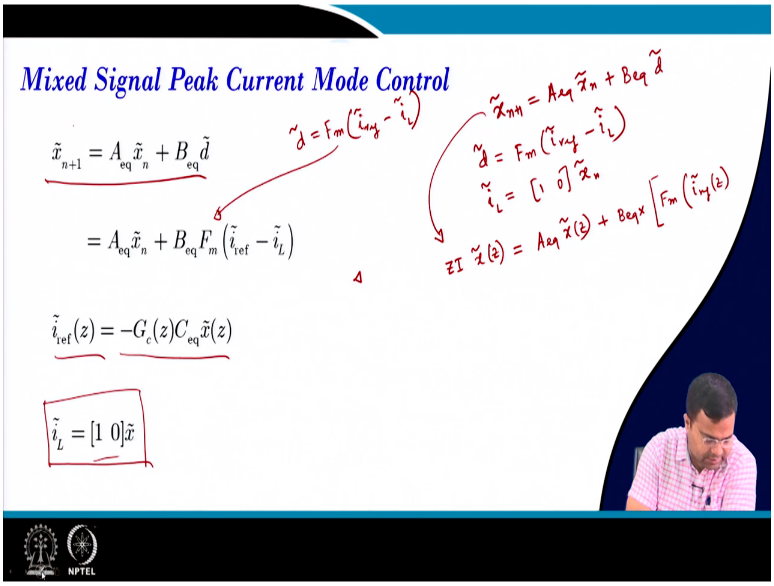

So, if you take in terms of z this will be, or if you take you to know. So, d if you write in terms of z; here it is in the time domain. So, it is just the perturbation we have to learn that d perturbation is F m into i ref perturbation minus i L perturbation and that we have substituted here. Then I ref we know it is minus of this. And what is i L? We know that i L is nothing but a 1, 0 matrix into x.

So, after Z transformation we have to write this; that means, we are writing this. So; that means, if we apply Z transformation to this equation, what we will get? Or let us write from this beginning. So; that means, we have x n plus 1 A equivalent into x n plus B equivalent into d z, then what is d z? This is nothing but F m into i ref z minus or i ref into i L. Then, what is i L? i L is nothing but 1, 0 into x n then.

So, if we apply; that means, we apply here Z transform; that means, it will be z I into x z left side will be A equivalent into x z plus B equivalent into what we will get. So, F m then i ref z.

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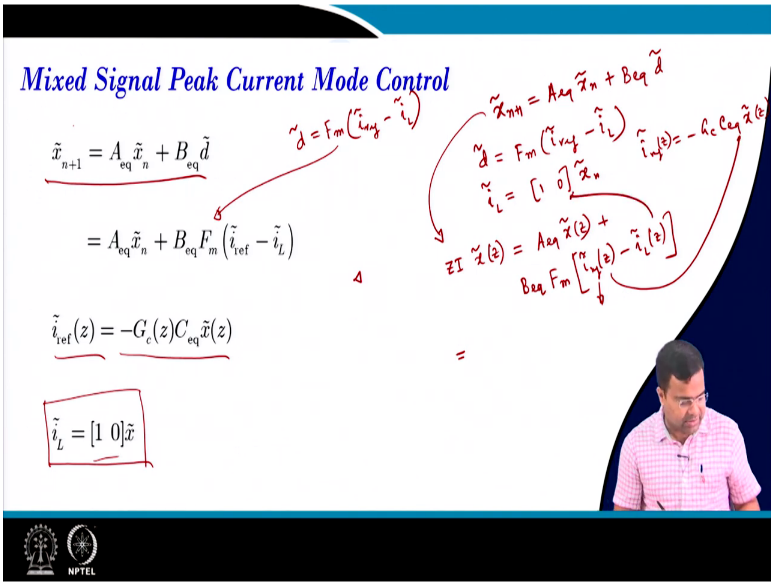

So, we can write this out. Let me write this term below. So, it is B equivalent to F m. Now, what is i ref z? i ref z is minus G c into C equivalent into x z. So, if you write Z domain. So, this is z; which means. So, here it will be what? i ref z minus i L z. So, what is i L z? That means, this you replace with this, and this you replace with this.

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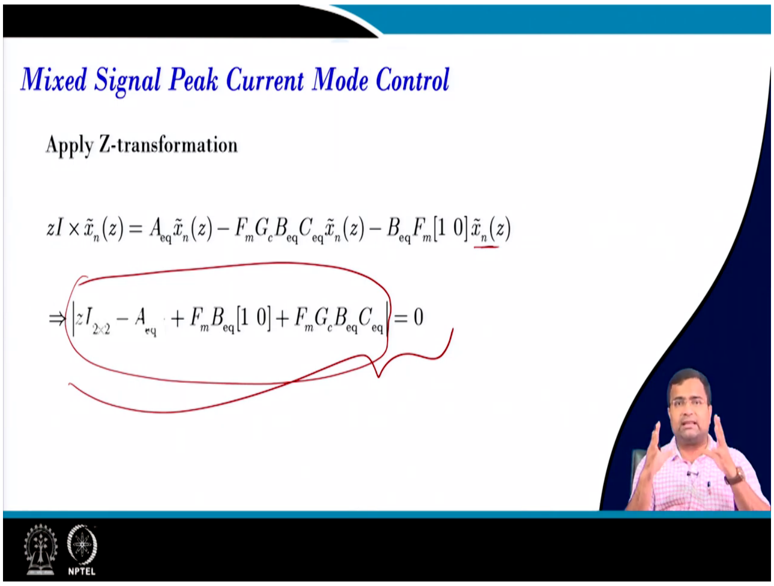

And after that, you will get the overall characteristic equation. And you can take everything on the left side because x z can be taken commonly. So, we will get the characteristic polynomial like this. And this equation is the characteristic equation. So, from here we can find out the eigenvalues to check whether all are inside the unit circle or not.

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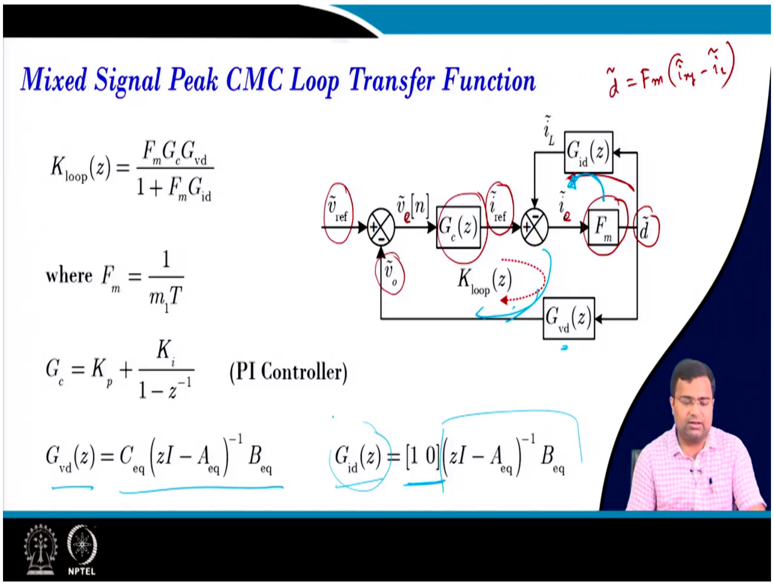

So; that means, this is the loop transfer function under peak current mode control. You see this is the reference voltage minus the current voltage output voltage. This is the error voltage that goes to the controller. It generates the reference current level, then this reference current level minus i L is nothing but your error current. And, if you multiply with the modulator gain it will generate the duty ratio because we know that the duty ratio is nothing but F m i ref minus i L.

Now, how i L and d are related? It is nothing but the control to current transfer function right. And then how output and duty ratios are related? It is nothing but the control of the output transfer function. So, you will get the complete loop transfer function. Now, if we want to write. So, you can see there are two loops one loop is the inner current loop, the inner current loop and this is the outer voltage loop, the outer voltage loop.

So, it is a two-loop control. So, we can get the loop transfer function by block diagram reduction the complete loop transfer function where the modulator gain is 1 by F m and the controller is the PI controller. So, we can obtain the loop transfer function of the current mode control. And this will be used when you want to design the controller in the subsequent week. And this is the G vd of this G vd that also we know.

So; that means, in digital current mode control you can accurately get all the small signal transformations very easily by just applying the Z transformation and control to output transfer function we know the current to control to the current transfer function. So, it is 1, 0 matrix has to be multiplied with this term ok. And this will be your scalar quantity.

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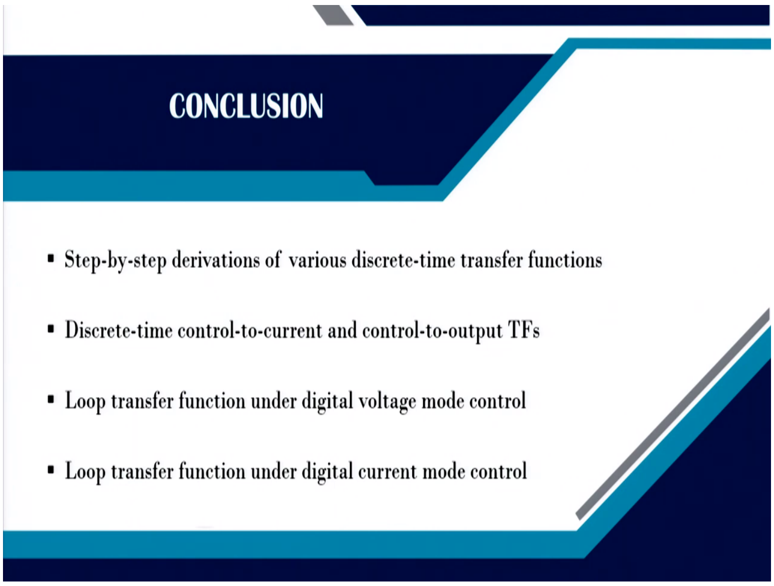

So, in summary, we have discussed step-by-step derivation of various transfer functions, we have talked about discrete time control to output control to current as well as control to the output transfer function, then we have derived loop transfer function under digital voltage mode control. And we have also derived the loop transfer function shown how to derive we have shown how to derive loop transfer function under digital current mode control.

So, this loop transfer function derivation and after getting the loop transfer function by using various transfer function control to output control to current, we will be using this transfer function for the design of both voltage mode digital voltage mode and current mode control. And subsequently, we will also implement that digital controller using Verilog HDL and we will prototype using FPGA. And we will also show a hardware case study for the buck and boost converter for voltage mode and current mode control. So, I want to finish it here.

Thank you very much.