Digital Control in Switched Mode Power Converters and FPGA-based Prototyping

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Module - 05
Frequency and Time Domain Digital Control Design Approaches
Lecture - 42
Recap of Frequency Domain Design of Analog VMC and CMC

Welcome to this lecture we are going to Recapitulate the Frequency Domain Design of Analog Voltage Mode and Current Mode Control.

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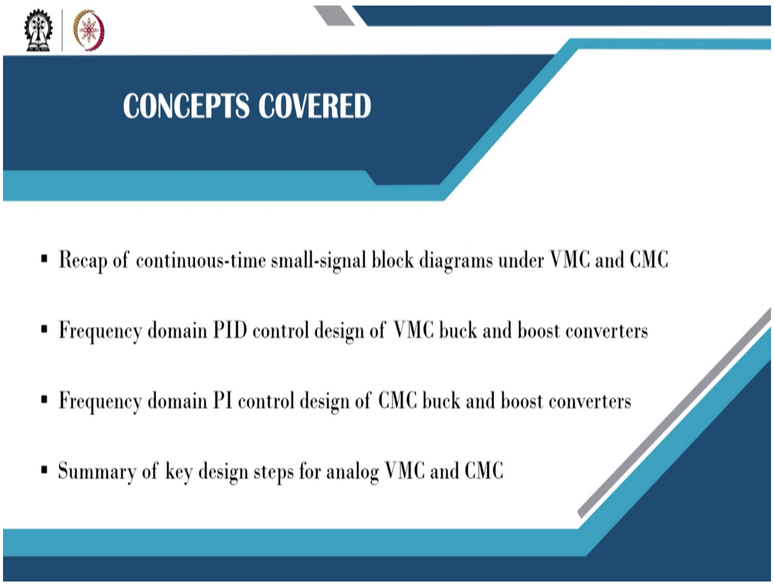

And this will you know provide a path for going for digital control design. So, in this course lecture, we are going to recapitulate a continuous-time small-signal diagram under voltage and current mode control, frequency domain PID controller designed for voltage mode control buck, and boost converter. The frequency domain PI controller design for current mode control buck and boost converter and summary of key design steps for analog voltage mode and current mode control.

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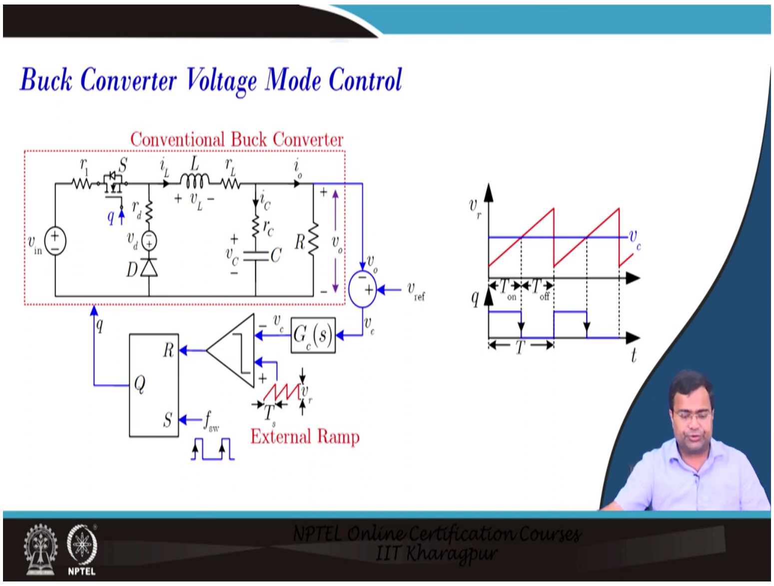

So, here we will first you know recall want to recall the voltage mode control buck converter we have discussed this you know diagram multiple times and you know this is a saw tooth waveform to generate the duty ratio.

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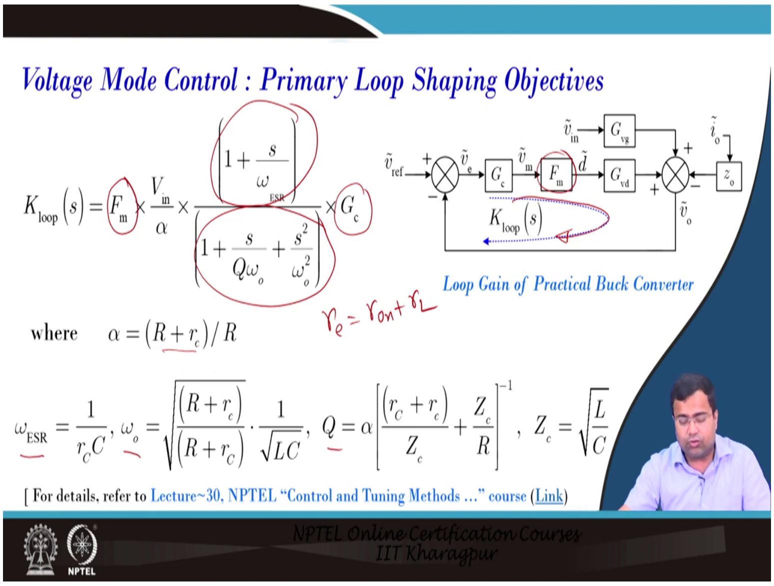

Now, if you go for digital voltage mode control what are our primary loop-shaping objectives? So, you can see this is our loop transfer function, loop yeah loop transfer function, then the loop transfer function is a product of the modulator gain and this modulator gain is coming for saw tooth waveform it is 1 by V m that is 1 by the peak voltage, then V in by alpha and there is an ESR 0 and then there is a double pole that is coming and this is a controller that we have to design.

Then, what is alpha? Alpha is equal to R plus r equivalent by R and r equivalent is what, it is nothing, but the r on state plus r L. So, throughout this lecture, we will be using r e. Now, we know the ESR 0, then omega 0, Q factor, and Z c. So, I am not going to repeat, how to design and you will get all this detail in Lecture number 30 in NPTEL the Controller and Tuning Method.

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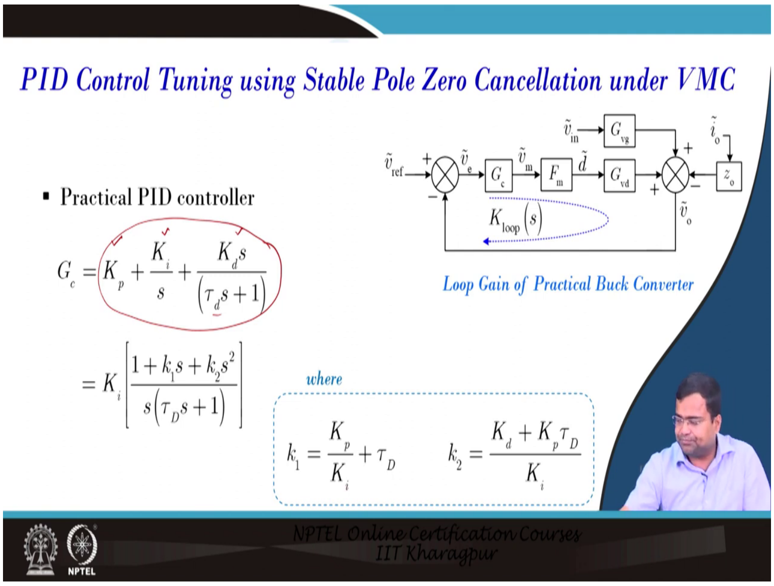

Then, we PID controller tuning for this voltage mode control we want to do for stable pole-zero cancellation. So, we can write a practical PID controller that we have discussed multiple times K p K I by s and K d this is a derivative filter time constant. Then, we can rewrite this equation in this form where k 1 and k t; that means if you simplify if you write the whole expression as a polynomial of numerator and denominator we can take K i out and these are the expression.

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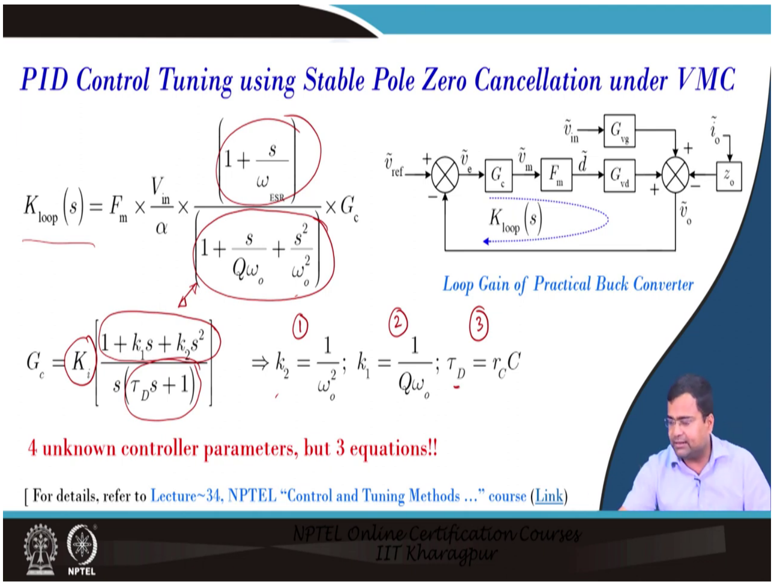

Next, in the loop compensation what is our objective, we need to set this numerator 0 in such a way as to cancel the denominator pole of the converter; that means, there is a pole-zero cancellation stable pole-zero cancellation, and we have discussed multiple time. Then next; that means, we will set k 2 equal to 1 by omega 0 square k 1 equal to 1 by Q omega 0 and we are also setting ESR 0 we are canceling using this derivative pole ok.

That is why you are setting tau D equal to it should be yeah tau D equal to r c into C ok. Next; that means, we have one equation here, another equation here, another equation for the controller we have 4 unknown one is k 1 2 k 1 k 2 which is again a function of PID controller tau D, another unknown is the K i so; that means, we have 3 equation 4 unknown, then we need another equation how to do that?

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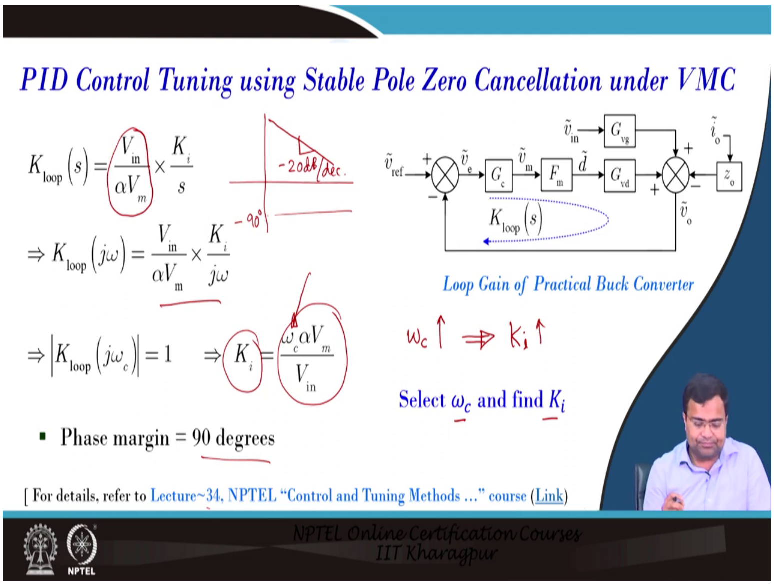

And this is discussed in Lecture number - 34. Now, if we do this pole-zero cancellation then our loop transfer function will be just like a gain by integrator; that means, it will be some gain V in alpha V m into K i. So, it will look like you know if you take the gain plot what will be this it will be like this minus 20 dB per decade, and the phase will be simply minus 90 degree. So, it will be minus 90 degree ok.

Next, if you write the polar; that means, the complex realization; that means, your frequency response then what will get, at crossover frequency the loop gain will be 1 and this will give us K i equal to omega c alpha V m by V in; that means, if you choose the crossover frequency then you will get the integral gain. And phase margin is by default 90 degree because the phase is minus 90 degree and the phase module is 180 degree minus 90 degree. So, this is 90 degree.

Now, if we increase k c omega c that is the crossover frequency then this will lead to; that means, we need a higher integral gain or it is vice versa; that means if you increase the integral gain your gain crossover frequency will increase. As we have discussed in lecture number; that means, select omega c and find K i ok and we have discussed in lecture 34 that this way of PID controller design has a problem with the model matching.

Because even the model was not matched much lower than one-tenth of the switching frequency because it is very difficult to get a first-order expression, particularly using the exact pole-zero cancellation that we have discussed.

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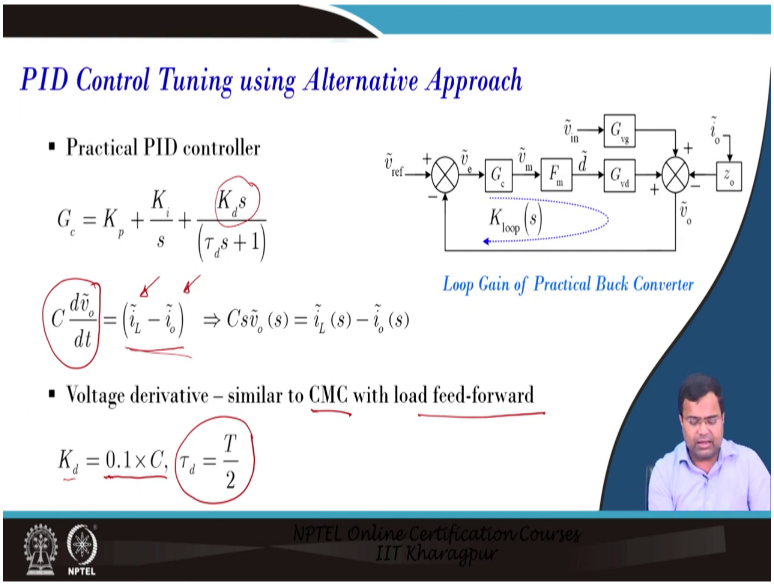

There can be an alternative approach to PID controller design because in digital control will mostly we are going to use digital PID control, which is why I am primarily focusing on PID controller, but in analog voltage mode control people use type - 3 compensators over PID controller.

Where we will have an additional pole apart from; that means, over the practical PID controller if you take an additional pole it will be a type - 3 compensators and which we have discussed in Lecture number - 35 in our earlier NPTEL course. Now the alternative approach is again the same PID controller, but in this case, if you look at the C DV dt in the small signal range. So, this is nothing, but the capacitor current for the buck converter and that will be I l minus i 0.

That means the derivative of the output voltage will carry the information of the inductor current as well as the load current; that means, it inherently provides similar to current mode control with load feed forward. Now you want to take the advantage of this so; that means, to take you can use a PID controller and you can view the derivative action similar to your inductor current with load feed forward.

And you can set K d to be some because here you can see C DV is equal to I C, if you set K d to be exactly C then it will represent a current mode control load feed forward. But the larger K d can have some effect because it can inject a lot of high-frequency noise. So, that is why we may take a smaller value of K d. Another thing is the time constant of the derivative filter you can choose T by 2 you can choose T by 10 ok. So; that means, this is somewhat flexible.

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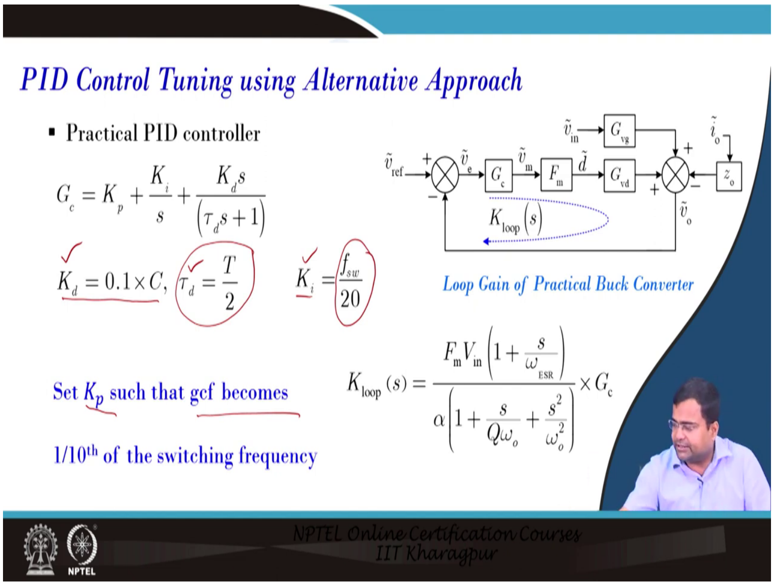

Then, what is the design step in the loop shaping? So, for this design you set K d to be let us say 0.1 times C, you can 0.4 or 0.5 times C no problem, you can set some tau d T by 2 you can take T by 10 also and K i let us say K i is an integral gain and dimension it is 1 by the integral time constant. So, it is the dimension it represents the frequency in the heart. So, here it is we have taken the switching frequency by 20 it is also a kind of a conservative approach.

Finally, that means, you know K d, you know tau d, you know K I, now you have to find K p. So, you can get K p such that you set. So, the gain cross-over frequency becomes one-tenth of the switching frequency, by that way you can get the proportional gain and this approach is far better in terms of shaping the loop. And it will also have a better model matching and we will be discussing the digital control case study by this approach indirect approach in the subsequent lecture with model matching.

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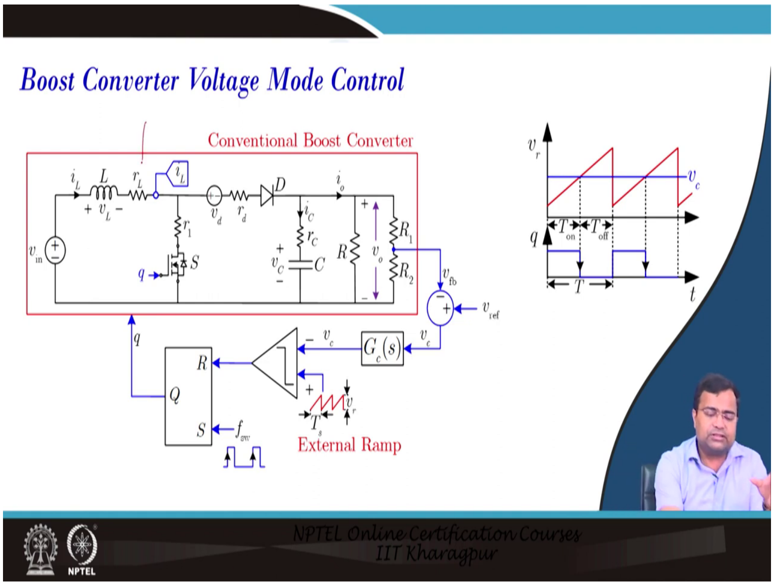

Now, we will be considering boost converter voltage mode control. So, again it is the same as the buck converter voltage mode control only the topology is different.

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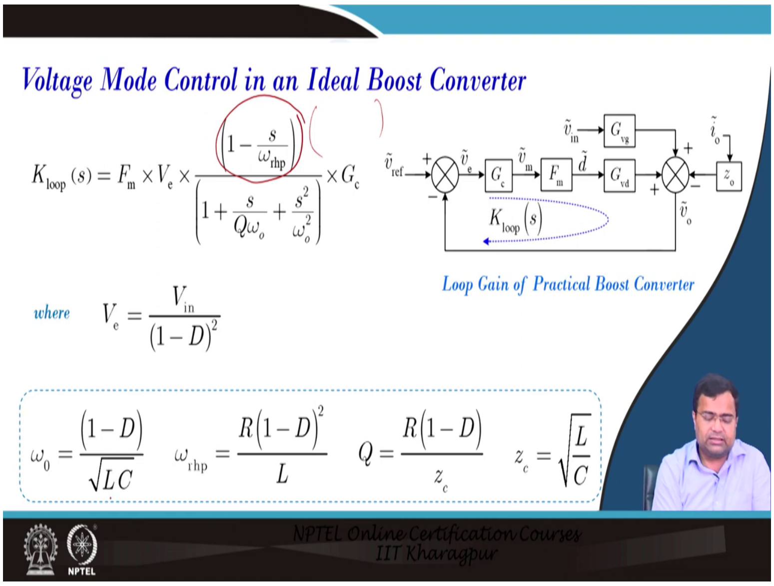

The same thing we can write the loop transfer function, the only here is that in the boost converter, we will have a right half plane 0. Here we are talking about the ideal boost converter in a practical boost converter we may also have an ESR 0. Since ESR 0 is far right-hand side so, it will be primarily dominated by the right-half plane 0. And what is V e?

It is V in by 1 minus D whole square and in for a boost converter, we know that omega 0 is 1 minus D square root of LC. So, all these expressions are well known.

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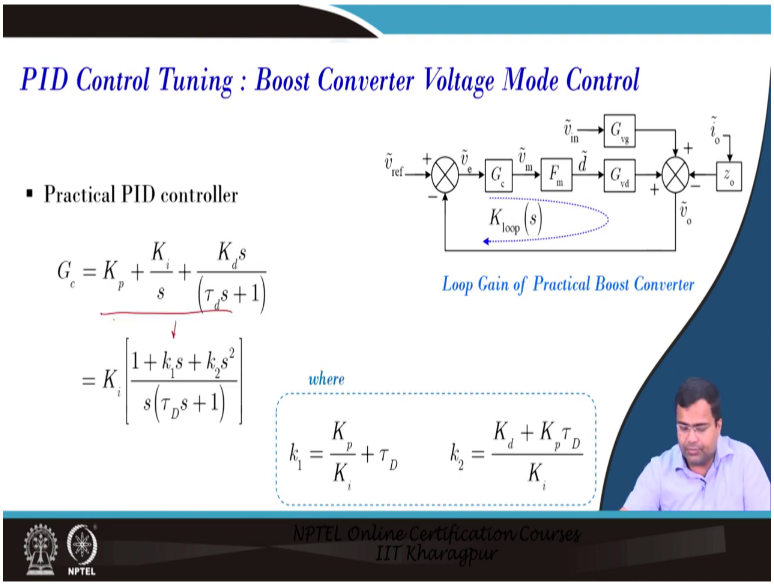

Now, if you take a PID controller in a boost converter how to design it? Again we write this PID, again you are initially going for exact stable pole-zero cancellation, again we are we can rearrange this PID into this form.

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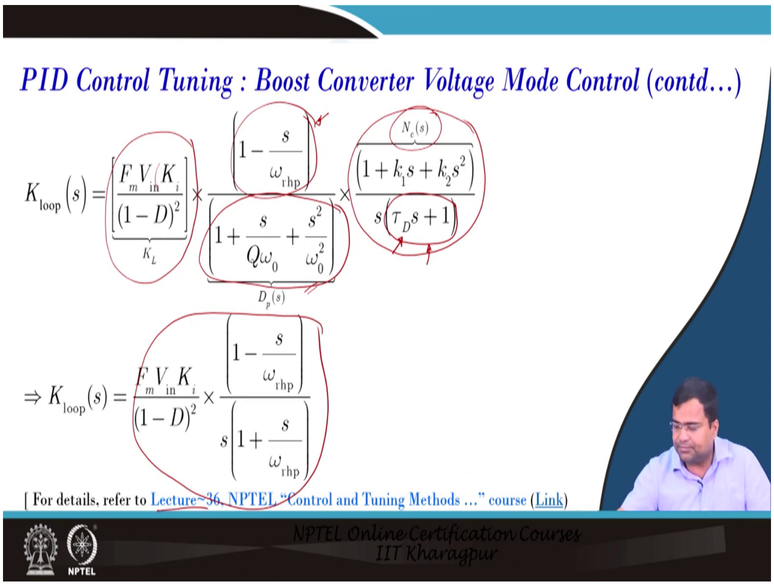

Now, going for the design so, you write the loop transfer function and we know the loop transfer function of the boost converter there is a right half plane zero, this is the pole of the converter and this is the expression of the controller. And we are taking the K i out here so; that means, we have clubbed all the gain here then all the pole zeros on the right-hand side. So, our initial objective we set the numerator stable zeros controller 0 as equal to the denominator of this; that means, the stable pole of the boost converter.

Then if you do that and then we will choose tau D in such a way that this 0 should coincide with the right half plane zero; that means, it is a stable pole of the controller that should be placed in coincidence with the right half plane zero of the boost converter. If you do that then the total loop transfer function will look like this ok. And next, it is discussed in lecture - 36 in the NPTEL in Control and Tuning Method.

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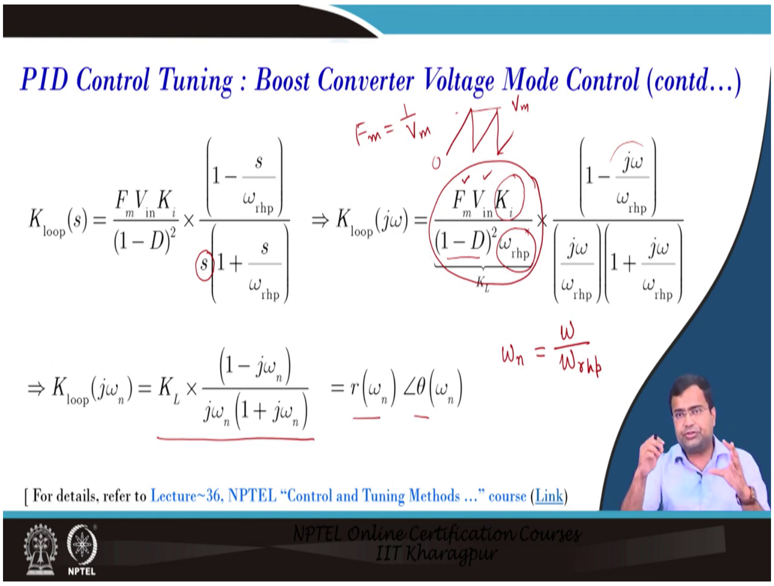

Then what will do? The loop transfer function is written like this. Now, we want to obtain the frequency response; that means, you replace s equal to j omega and you can get the expression. Now I am rearranging by take by modifying because here it will be s equal to j omega. So, I am dividing j omega by omega rhp and multiplying here. So, that all will look like omega by omega rhp, and then I can realize this loop transfer function by the normalized frequency where the normalized frequency is omega by omega rhp ok.

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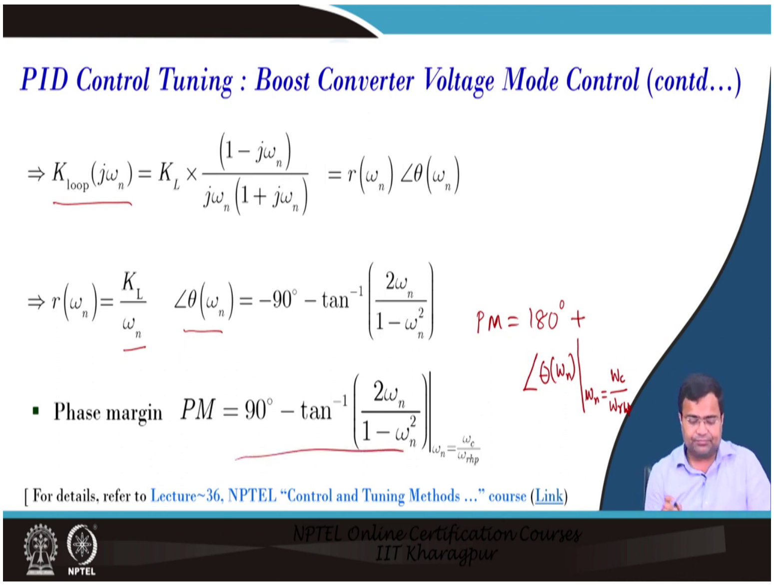

And this can be written in the polar form r and theta, which are this loop transform function, r is nothing, but K L by omega n. What is K L? If you write so, we have taken K L here this is the term we have taken K L; that means, K L is equal to F m which is the modulator gain of the voltage mode control; that means, the saw tooth. What is F m? We know F m is equal to 1 by V m. And what is V m? It is nothing, but the sawtooth voltage peak value is by V m peak value ok.

Then we know this is the input voltage K i that we need to find we have not yet designed and D is the duty ratio we know, the then omega rh is also known because it is a function of the resistance duty ratio and the inductor. Now once we write it then we have to design; that means, we know the real part of the loop transfer function, frequency response, and the imaginary that sorry the real part and the angle magnitude and the angle.

Now, what is phase margin? So, phase margin we can write; that means, the phase margin is generally 180 degrees plus theta of omega n, for this case where omega n will be should be equal to omega c by omega rhp; that means, let us write here bottom. So, I am just writing here it will be theta of omega m, where omega n is computed at crossover frequency by rhp 0 ok. So; that means if you write this is the expression.

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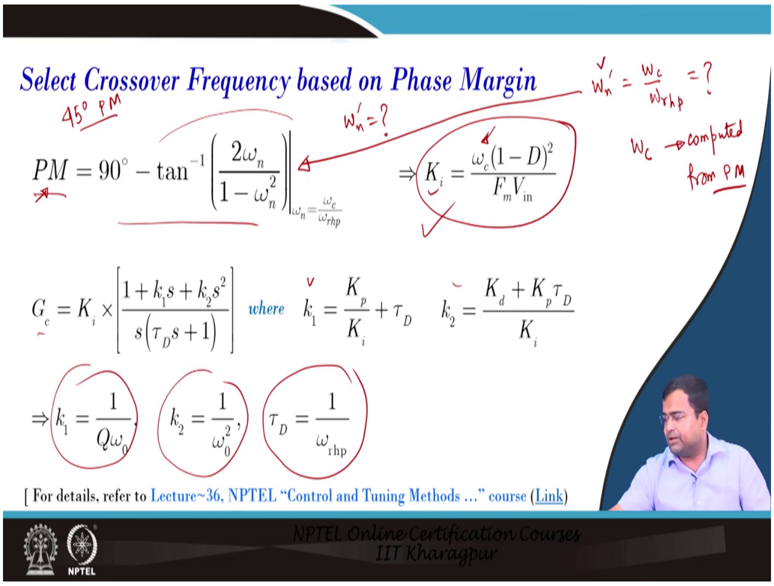

Now, how to design first you write the; that means, the select the crossover frequency based on the phase margin. So, if you set a desired phase margin to suppose I want to achieve let us say 45 degree is my desired phase value then you set this here to be 45 degree and solve this tan inverse, and there you will get what is by omega n value for the 45 degree phase margin.

Once you get that then you can; that means, that omega n dash that means what is omega n dash? It is nothing but omega c by omega rhp ok so; that means, you will get the omega n dash value. So, I am writing here dash because omega n is just a frequency omega n dash or it is already written sorry I am saying here. So, it is already written here because we have written omega n equal to omega rhp; that means, in this case, omega n dash is nothing, but omega c by omega rhp, and that you can find from this expression.

Once you find then you substitute then you will write what is my K i right because if you know what is my from here what we will get; that means, you can obtain the value from this equation; that means, it is known, omega rhp is known. So, omega c will be known, it is known from phase margin or it is sorry it is computed I will not say known it is computed from phase margin.

Once it is computed then you have to plug in here and find the integral gain so; that means, then in the controller, we already know k 1, k 2 you can find out that tau D was coming from the right half plane zero, that is a 0, these are all known and you also know the K i. So, you know all the parameters, by that way you can design.

Now, we are talking about the current mode control design for a buck converter.

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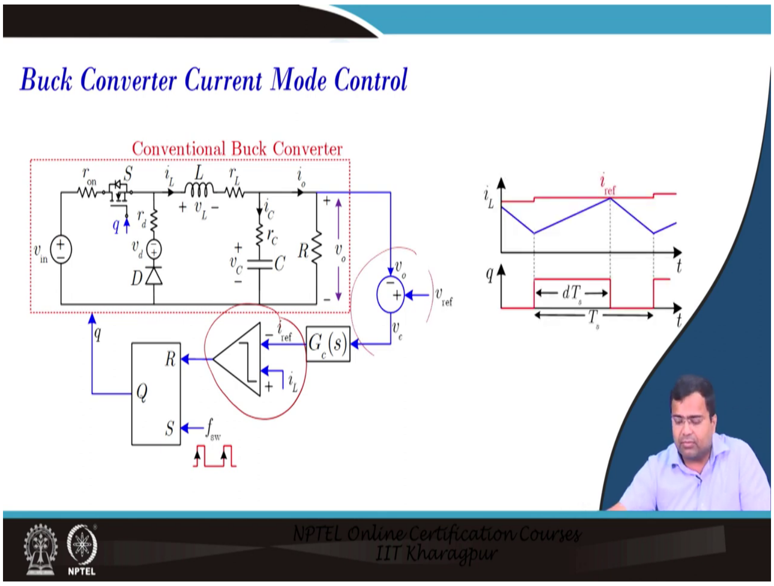

We know this is the implementation of current mode control, we have an inner current loop here and we have an outer voltage loop here and this is the peak current mode control.

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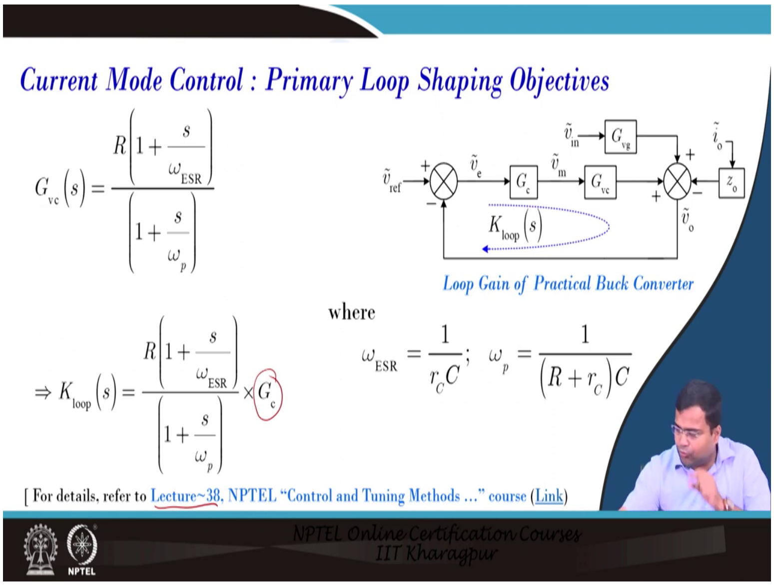

Then in the current mode control, we know the G vc in the current loop and if we take a first-order approximate model it will be like this G VC where these are the function and this thing we have discussed in our lecture -38 and this is we have to shape the loop.

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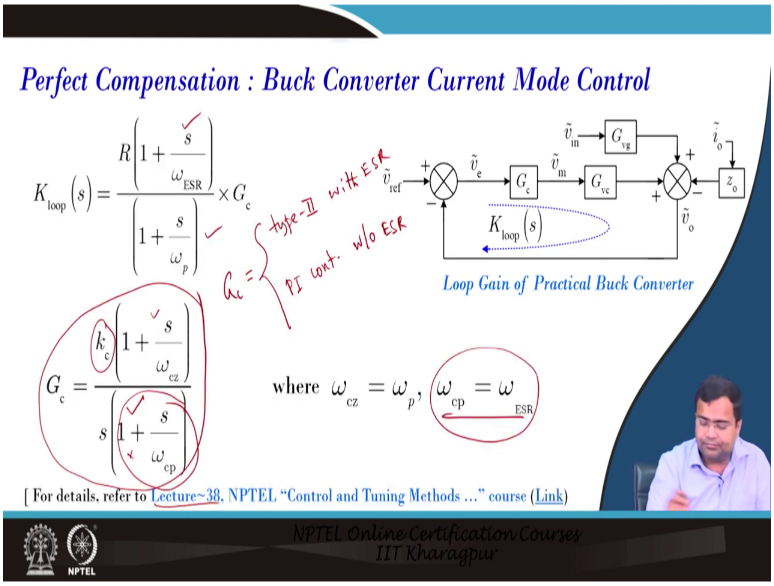

So, what will be the omega c? So, if you do not consider any ESR as 0 then it will be this representation is a type - 2 compensator, but if ESR 0 is ignored; that means, we do not need to consider this; that means, omega c will be type - 3 sorry type- 2 for with ESR and it will be a pi controller without ESR; that means if you ignore this; that means, you do not need a pole controller pole and if this is not required then it will be simply 1 0 by s which is the structure of a pi controller and this thing we have discussed in Lecture - 38.

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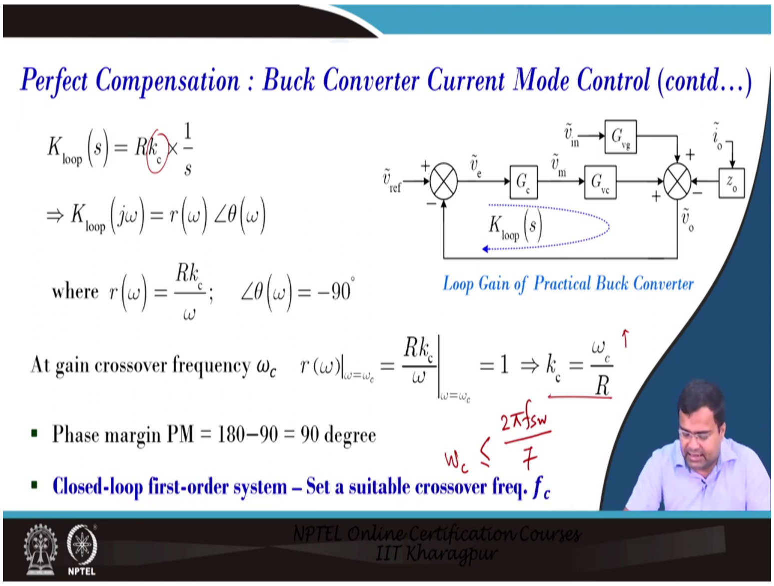

Next, once you do loop compensation because we are using this cancel with the ESR and this zero with the pole. So, it will look like a first-order system and this is Rk c by s where kc is the controller gain that is coming from here. Then if you do that you will always get a phase margin of ninety degrees and we have to find out the gain crossover frequency from here which means the phase margin is always 90 degrees.

Now, if we increase the omega c; that means, if you increase the crossover frequency gain crossover frequency then your controller gain will increase, but we know that we cannot increase significantly because there is a model validity problem and we know that model for current mode control will be valid, you know nearly you can say it should be less than equal to 2 pi fsw by maybe 7 or 8 one-tenth is the very like a very good number, but in current mode control you can slightly increase and it can go up to let us say one-seventh of the switching frequency.

So, then you can set the crossover frequency accordingly.

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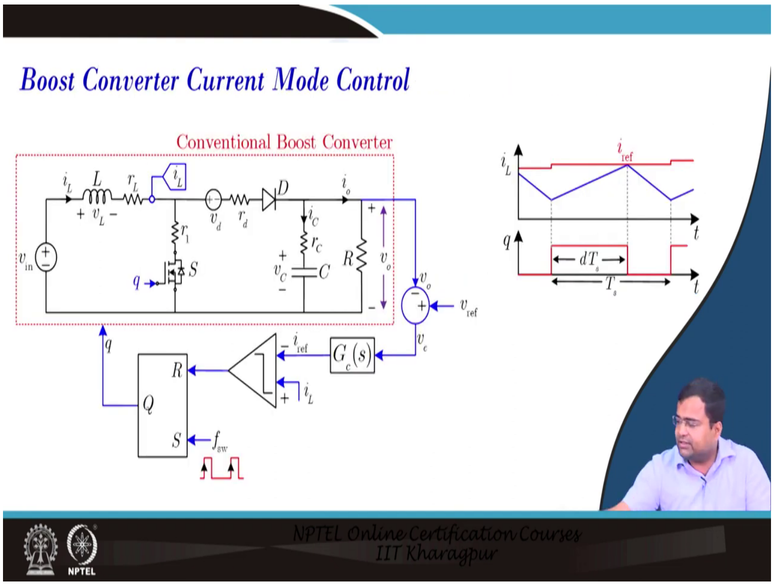

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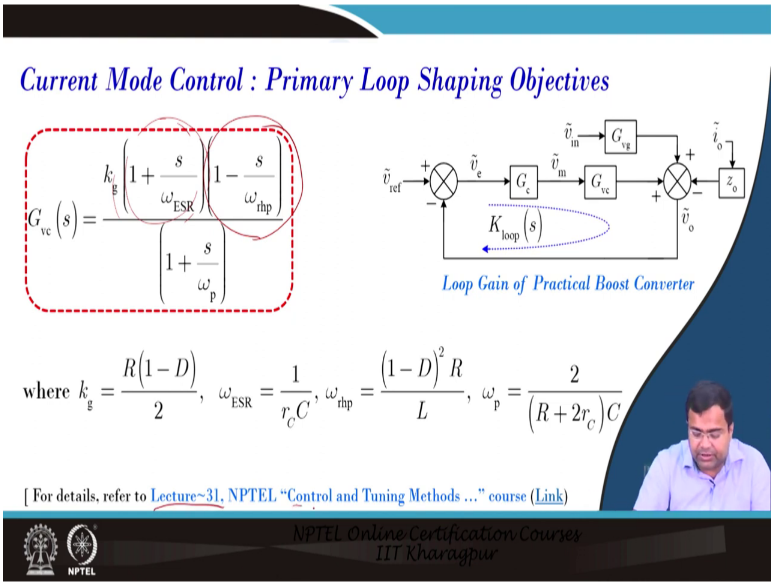

Now, if you take a boost converter, with the same current mode control only the topology got changed then we can write the loop transfer function for a boost converter if you write the control to output transfer function under current mode control we will get a right half plane zero and if we take an ideal boost converter we can drop this term and these are the expression it is given in Lecture 31 in our earlier NPTEL course.

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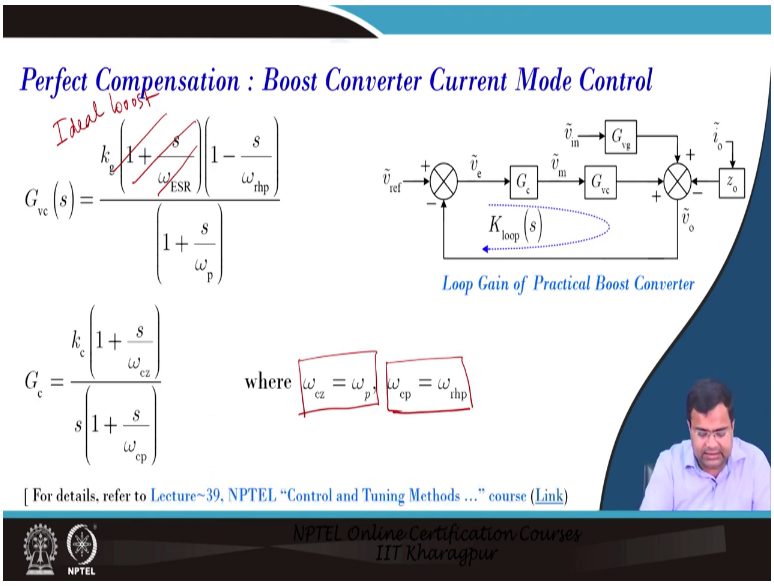

And if you take a current mode control compensation again you can ignore this because we are talking about an ideal boost converter if you are taking the ideal boost converter ideal boost. So, you can drop the ESR term and if you drop it then you need to do compensation. So, for the boost converter, we need a type-2 compensator where the controller pole will be placed coincident with the rhp 0 and the controller 0 will be placed in the stable pole of the converter.

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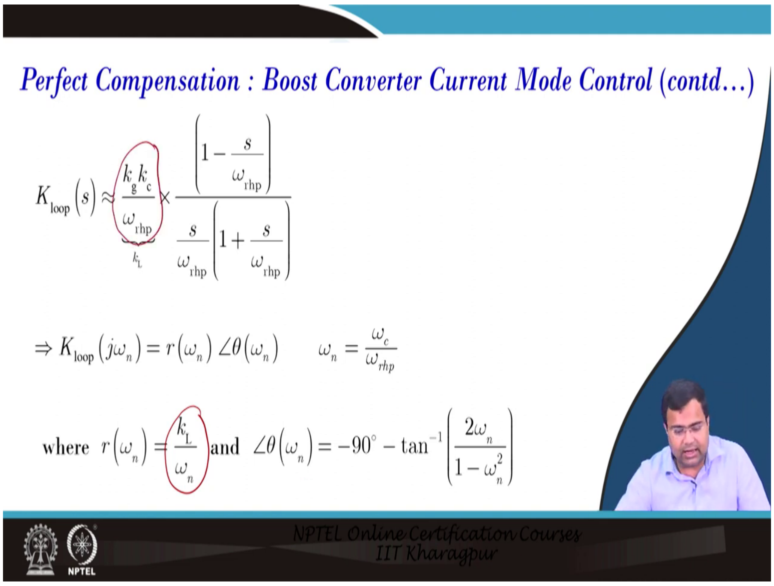

And then if you do this and this is discussed in Lecture number 39, you will get a loop transfer function which in the polar form it will be written like this and r omega n k L by omega n, where this is your k L ok and then theta omega like this.

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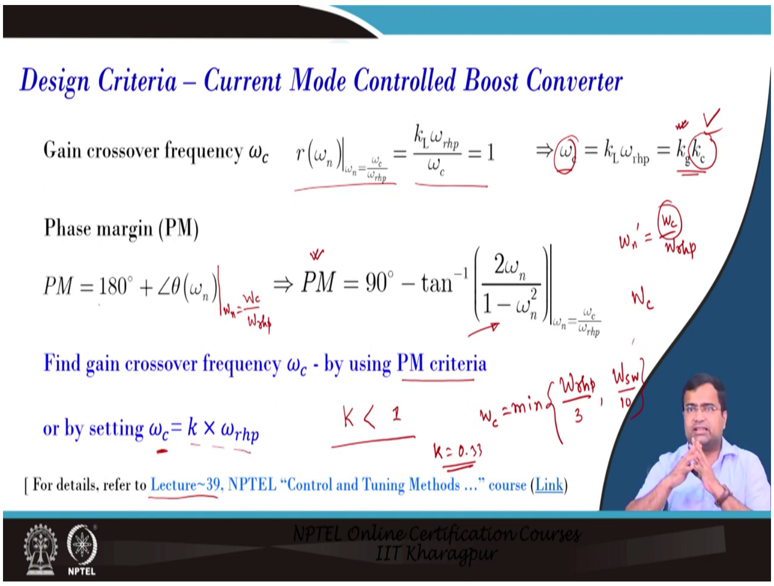

Then how to design? This is your r omega n and the gain crossover frequency r will be 1. So, you can get from here and this will give you crossover frequency is nothing, but k g into k c this is the system parameter. And once you; that means, this k c will be the controller here; that means, if you set omega c then you can find k c because this is unknown and this is known perfectly known.

The next phase margin is 180 degree plus phase computed at omega n equal to crossover frequency by rhp 0 right, and then it will be 90 degree minus tan inverse this. So, for a given phase margin find the gain crossover frequency either by using phase margin criteria; that means, you set a desired phase margin, then you can find out what my omega n dash which is nothing, but you have to find omega c by omega rhp; that means, you can find omega c from the phase margin and if you know omega c then you can find out the controller gain from here ok.

Another way we can set omega c to be k times rhp 0 and it is well known we have discussed in our NPTEL Lecture number 39, K must be smaller than unity if K is equal to 1 the phase margin will be 0 degree. So, it will be unstable. So, K must be smaller than 1 if K is small typically for current mode control we take you to know rhp 0 by 3 less than equal to this, but this is for high load conditions, under light load conditions we have to take the minimum of; that means, you know I would say you should take a minimum of omega rhp by 3 comma omega switching frequency by 10.

These two are the constant otherwise for the light load if you go for high bandwidth then your model will be invalid ok and those things we have discussed. So, if you set a K let us say K you can choose 0.33 like this then also you can find out the gain crossover frequency right from here and you can compute the phase margin directly and plug-in here. So; that means, and it is typically one-third is taken where you will get nearly close to a 6-degree phase margin.

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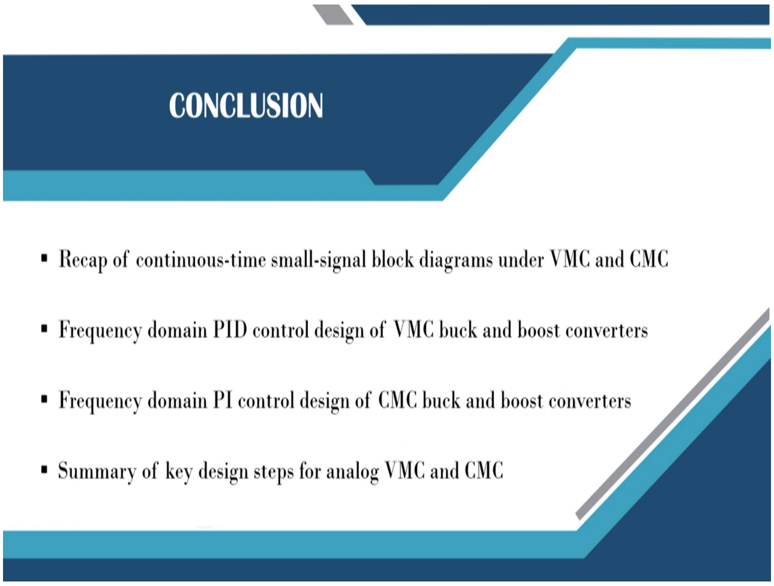

So, in summary, we have discussed we have recapitulated our continuous- time small- signal block diagram, and we have discussed the frequency domain PID controller design for voltage mode control and current mode control boost converter. We have discussed frequency domain pi controller design for the current mode control buck as well as the boost converter and we have also discussed some design key design steps for analog voltage mode and current mode control.

And these are the primary step before going to design digital control because we will be using this frequency domain approach and we will incorporate additional delay terms which will be coming due to the DPWM as well as you know the ADC delay and then we can modify our design step for design the digital voltage mode and current mode control and that we will be discussing in the subsequent lecture that is it for today.

Thank you very much.