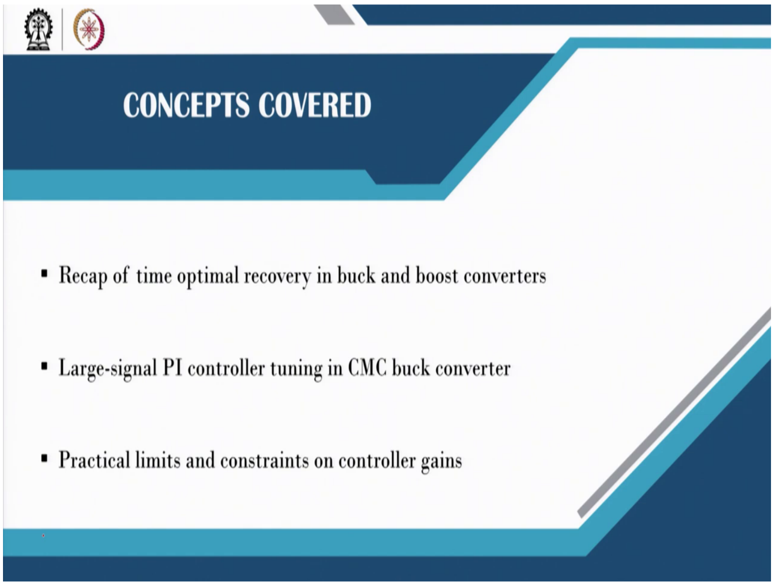
Digital Control in Switched Mode Power Converters and FPGA-based Prototyping
Prof. Santanu Kapat
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Module - 05
Frequency and Time Domain Digital Control Design Approaches
Lecture - 48
Trajectory-based CMC Design for Proximate Time Optimal Recovery

Welcome. So, in this lecture, we are going to talk about Trajectory-based Current Mode Control Design for Proximate Time Optimal Recovery.

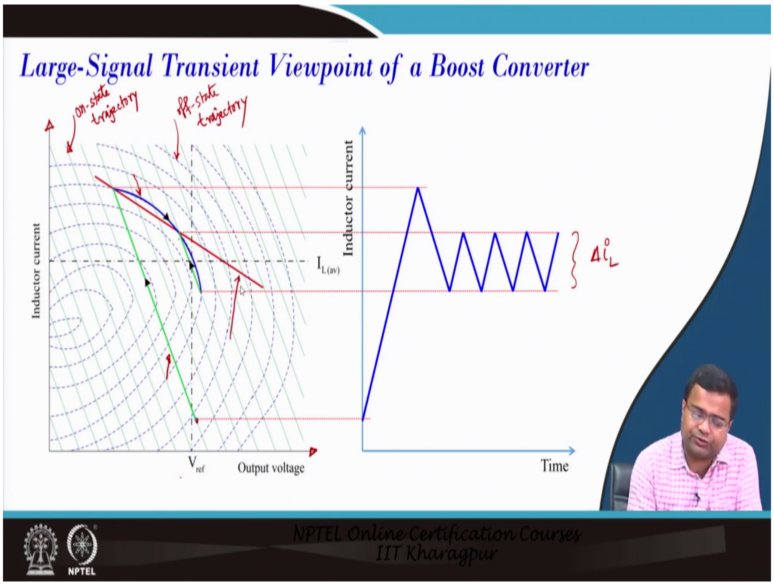
(Refer Slide Time: 00:33)



So, this lecture if you recap our previous lecture we talked about time optimal recovery in a buck converter, and in this lecture, we want to see how the current mode control can be designed by using like a using a time domain approach so, that we can proximately or nearly achieve the time optimal recovery using a closed loop control that is one of the objectives.

So, in this context, we will also talk about large signal PI controller design tuning in the current mode control buck converter, and finally, we will try to identify what are the practical limits and constraints on the controller gain selection.

(Refer Slide Time: 01:14)



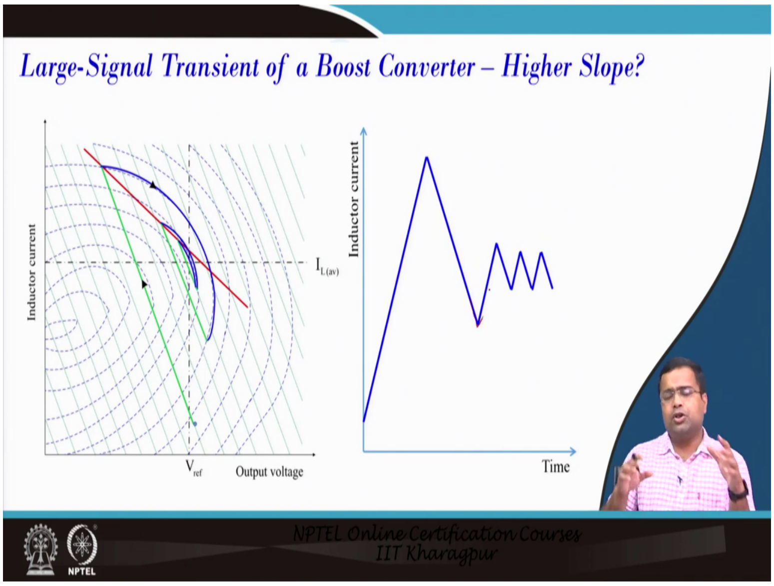
So, now we are going to start with the trajectory behavior of a boost converter in which here we are drawing the x-axis as the output voltage and the y-axis as the inductor current.

And here these particular lines indicate these are the trajectory of the family of curves this is the on-state trajectory on state trajectory on state trajectory and these are the off-state trajectory off state trajectory. So, this is the initial condition that we will start with. So, there can be an arbitrary initial condition.

So, here the green line is the on-state trajectory for the given initial condition, and the switch is turned on when it is hitting this red line, then the switch is turned off and the blue line is the offset trajectory and then it comes to the steady state which there is an on-off operation and if you draw the time domain waveform it will look like this in one cycle it recovers and it comes to steady state and this is the inductor current ripple.

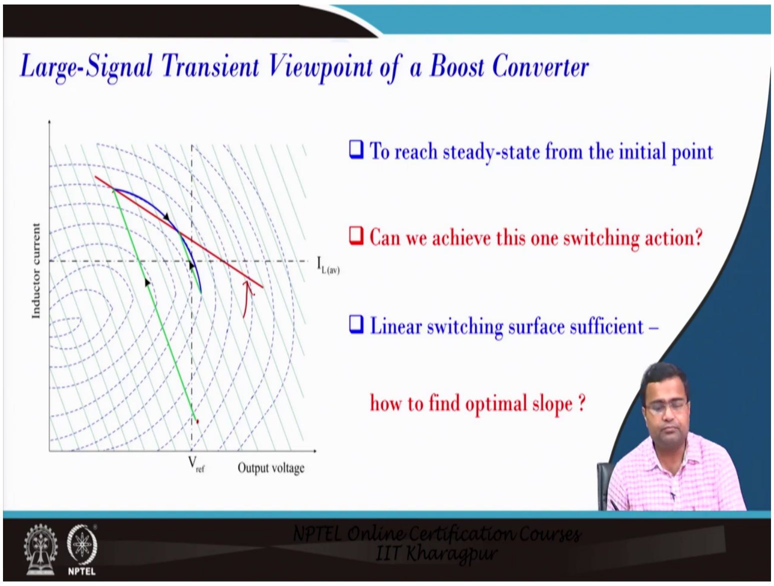
Now, what is this line we will discuss this is a first-order switching surface and it has a certain slope now what happens if we increase the slope of this line?

(Refer Slide Time: 02:40)



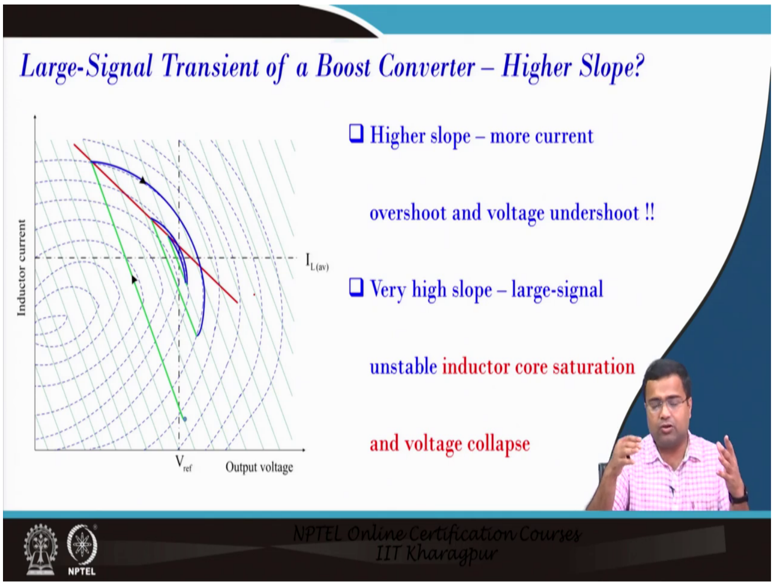
That means this was the earlier case now we will increase the line slope. If you increase the line slope you will find the equivalent current waveform the current overshoot has increased and also you will see additional transient even after reaching here; that means, there is an additional undershoot. So, this is because we have increased the slope

(Refer Slide Time: 03:03)



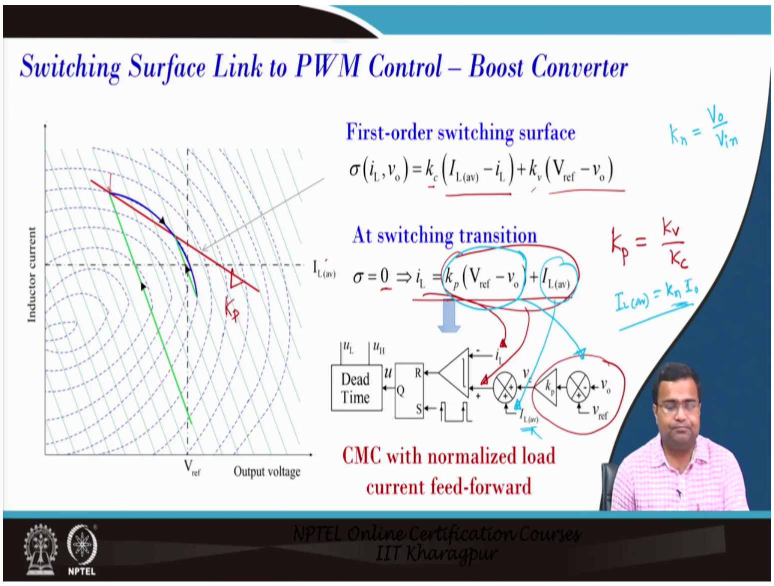
So; that means, to reach a steady state from the initial point here can we achieve in one switching action? That means, after one switching can we reach there and then whether this surface is it a linear surface straight line, then if it is a straight line then what is the optimal slope?

(Refer Slide Time: 03:24)



So, if we increase the slope then what happens? The higher slope more current overshoot voltage undershoot and very high slope there can be inductor core saturation or voltage might collapse.

(Refer Slide Time: 03:40)



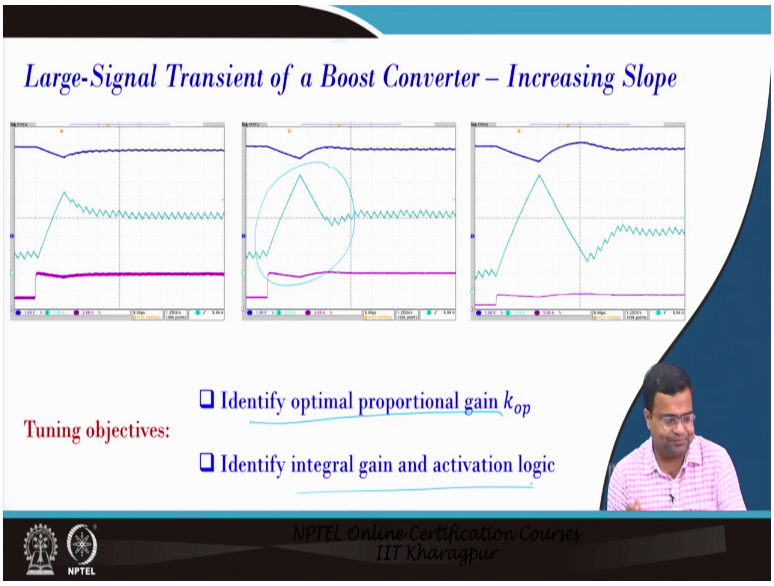
Now, we want to link how this switching surface is linked with our controller. So, one of the links is that this switching surface can be written as because you can see the y-axis is the output voltage axis and the x-axis is the inductor current axis. So, this straight line can be written as some gain into current loop error current where this is the average current some gain into voltage error.

And if you normalize k c; that means, you divide k c because they are equal at the point of transition then this k p is nothing, but. So, this k p is nothing, but k v by kc. So, we have one degree of freedom and this k p is my slope this is the slope of this line. So, this equation indicates what? This equation indicates that this inductor current is compared with the rest of the quantity, it is here, and at the point of switching they are equal and that is becoming 0 which is a switching law.

And what is this you can see? There is a proportional control into error voltage which is contributed by this term. So, if you use a different color. So, this is the term which is here, this particular block and this is a term which is here, and what is the I L average for boost converter? The average inductor current is a normalized gain into load current and what is that fact?

So, this normalized gain; means, if we want to normalize the load current in terms of the average current it will be kn will be equal to v 0 by v in, and if you multiply then you will get will normalize the load current so; that means, this term should carry some sort of load feed-forward with normalized. So, this is normalized; that means, we have represented this.

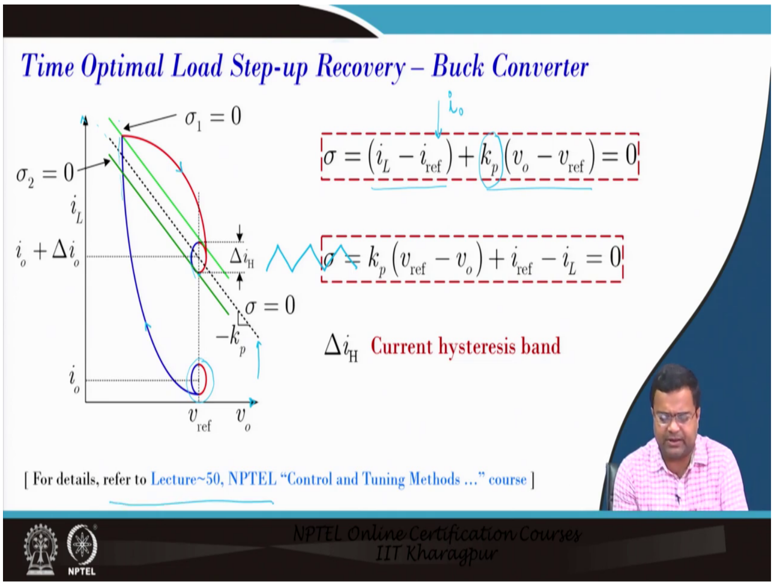
(Refer Slide Time: 05:45)



That means if we increase the slope. So, this is a test result if we use a smaller slope then it is not recovering in one switching cycle, if you increase the slope it is more or less recovered in one switching cycle. So, these are test results, and if you further increase there will be a further current overshoot which might cause more like an inductor saturation or it can increase you know the current overshoot, and also it will cause some voltage to undershoot and overshoot. So, as a result, it may take a longer time.

So, what are the tuning objectives? Identify the optimal gain which will achieve something similar to this and then how to incorporate integral gain so, that you can also because only proportional gain is not enough we need to incorporate the integral gain so, that you can ensure 0 steady-state error.

(Refer Slide Time: 06:36)



So, now we want to summarize what is the process for linking buck converter current mode control with phase plane trajectory and how can we achieve those gains mathematically, and what are the steps.

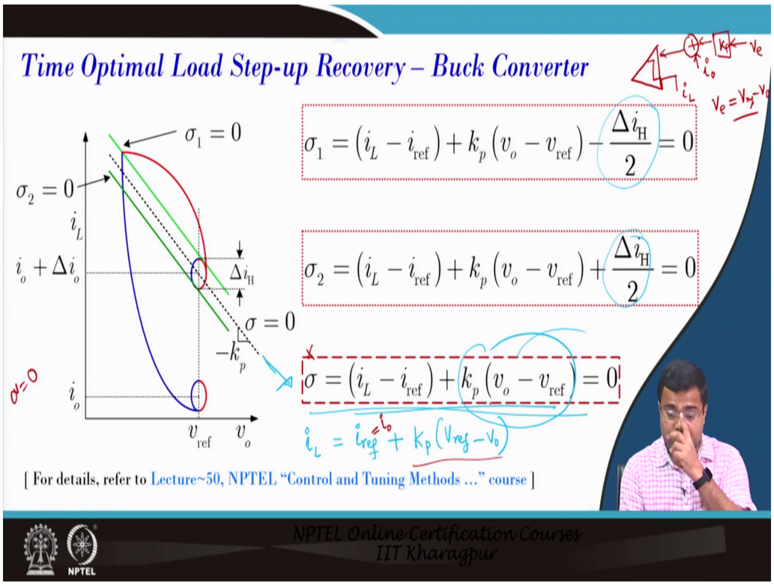
So, if you look at the back converter again this is a phase plane output voltage in the x-axis inductor current in the y-axis. So, initially, it was running under one load condition which is i 0 and now the i 0 has changed to i 0 plus delta i 0 and this blue line is the on-state trajectory and the red one is the off-state trajectory and here earlier we saw just one line for the intersection, but now we have introduced a hysteresis band of delta i H.

So, our current ripple and the band will be more or less the same, and now once it hit this limit then it turns off when it hit the lower limit it turns on, by that way, this periodic behavior will be retained and this is under steady state it will achieve something like this kind of current ripple.

So, what is the equation of this dotted line sigma equal to 0? So, it is again we have discussed in a normalized sense it is i L minus i ref plus k p into v 0 minus v ref where k p is the proportional gain and i ref is in the buck convertor it is the average inductor time which is nothing, but the load current.

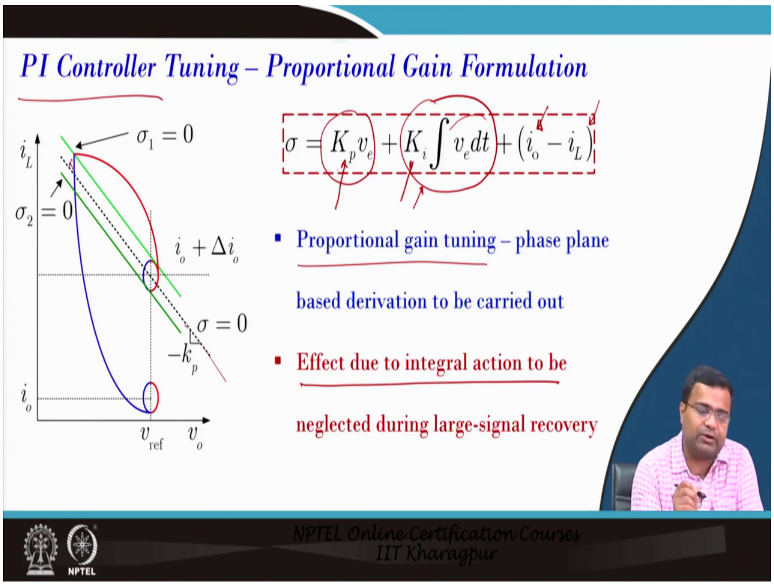
And if we incorporate this hysteresis band then we can figure out how can we get the sigma 1 and sigma 2 equations which are nothing, but the same they have the same slope of the sigma parallel line, but with the separate there we have introduced a hysteresis and these things are discussed in detail in lecture number 50 in our earlier NPTEL course which is a controller tuning method.

(Refer Slide Time: 08:31)



Now, this is the equation of this particular dotted line and if we take sigma 1 there will be a hysteresis band, sigma 2 there is another hysteresis band again this is discussed.

(Refer Slide Time: 08:46)



So, now you see what the surface looks like here if you remember our earlier at the point of switching if I take this term right side. So, I will get i L equal to i ref plus k p v ref minus v 0. So, what is I, ref? If this is a load current feed-forward. So, if you recall us earlier for the boost converter we have shown that if you put a comparator. So, the comparator will compare or inductor current and this is a comparator and then we have the load feed forward; that means, this is our load current.

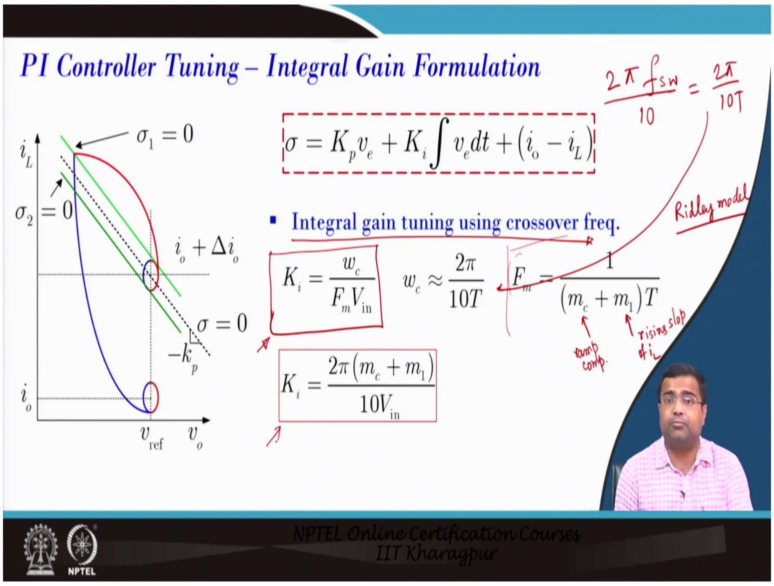
So, here it is equal to load current plus. So, this is an additional term plus we have a proportional control and then we have the error voltage, and what is the error voltage? It is v ref minus v s that is it. So, it is a current mode control with load current feedforward, but only proportional control may not ensure 0 steady-state error.

So, that is why we are modifying the sigma of the k p into the v error that we have discussed and if we take this term out; that means if we rewrite this equation what was of sigma if you rewrite? Everything you take; that means, it is i L minus i 0 and it is equal to 0. So, you can multiply a negative sign; that means, the sigma is equal to 0 if we even multiply with a negative sign sigma dot.

So, it will be just rearranging this is now i 0 which was the reference current this is the current feedback, this is the integral gain we have added an extra term and this was there because this is like this k p into this error voltage term. So, it will look like a proportional P I controller in current mode control with load feed forward now we have to tune we have to obtain this proportional gain and the integral gain that is the tuning objective.

So, now you will see the proportional controller will decide the slope of the surface and which is the first action that will drive the trajectory to reach more or less one switching cycle whereas, the integral action is designed to achieve zero steady-state error. So, it is a very slow process the objective is just to eliminate the error or any offset. So, the effect due to integral action is to be neglected during the large signal recovery and the proportional control is primarily driving the one-switching action recovery. So, that is the objective.

(Refer Slide Time: 11:42)



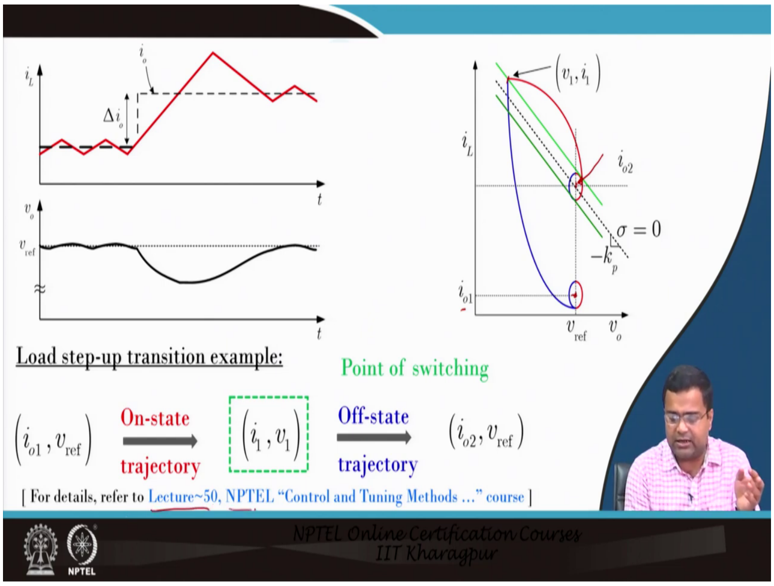
So, the next integral; means, integral gain since it is a slow process, it is neglected during large signal transient, but it has an actual effect when it comes close to the steady state we want to eliminate that steady-state error. So, that is why we are designing the integral gain using the small signal model.

So, in the small signal model if you recall you know the lecture number; that means, we are talking about PI controller design and you know if you go to go back to the current mode controller design we typically choose the integral gain to be crossover frequency by F m into V in and crossover frequency we can take you to know one-tenth of the switching frequency; that means, f s w 2 PI that by 10 which is nothing, but 2 PI by 10 T and this is here.

And what is F m? Here if you recall that we talked about the modulator gain in current mode control with a ram compensation it is 1 by mc plus m 1 into T where m 1 is the rising slope of the inductor current rising slope of i L and m c is the ramp compensation. See if you recall in our earlier NPTEL course Ridely model we discussed where the FM the modulator gain can be shown as 1 by m c plus m 1 into T and this is if you go Ridely model; that means, if you go to the Ridely model you will get the modulator gain 1 by m c plus m 1 into T.

Now, so, K i can be written as if you substitute 2 PI m c plus m 1 by 10 V in if you substitute all this and then so, that we can get the integral gain from the small signal model.

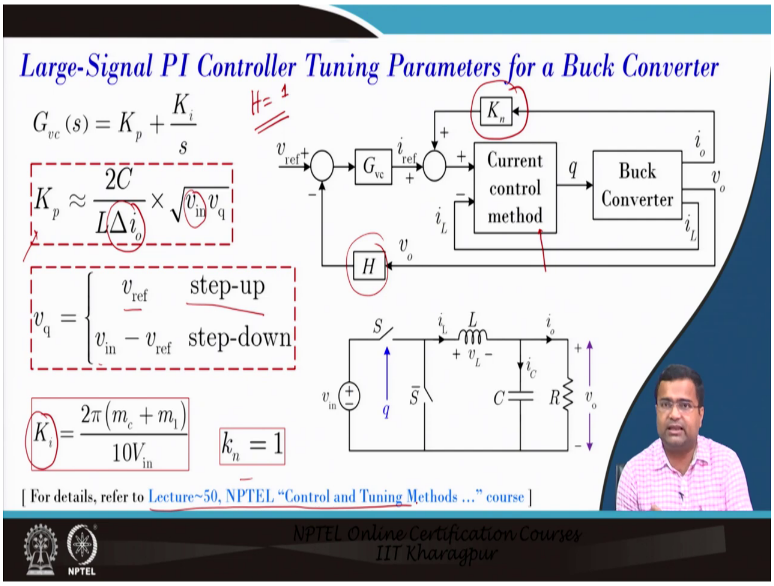
(Refer Slide Time: 13:50)



How do we get proportional gain? So, the proportional gain can be derived by using the phase plane geometry where we need to achieve the transient switching action.

Here we will consider the load will start from the initial load current i 0 1 with v ref. So, we are neglecting the ff d 2 ripple in this derivation as if it is starting from this point then it hit this point where the current is i 1 and v 1 current and voltage and then it turns off the off state trajectory it takes i 0, 2 this is this point is my i 0, 2 and v ref; that means, the current is i 0 2 voltage is v ref. So, this method of derivation is discussed in lecture number 15 in our earlier NPTEL control and tuning method.

(Refer Slide Time: 14:43)

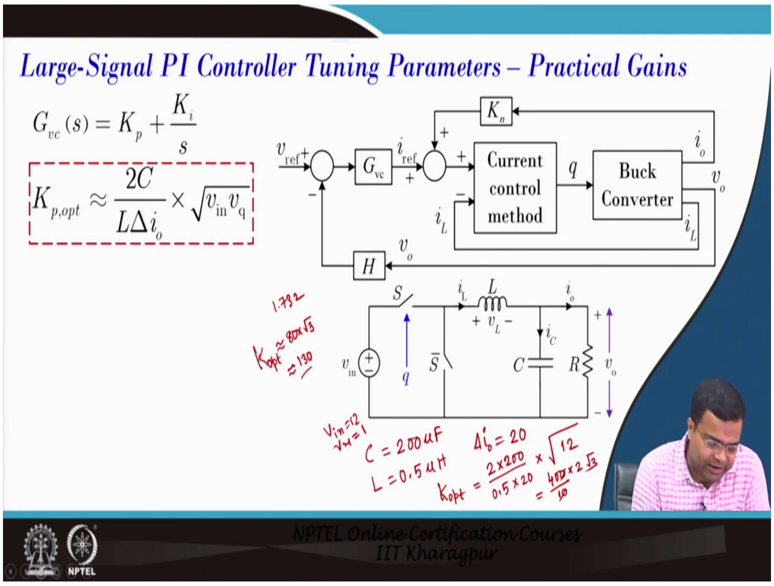


So, I am not discussing detail I am just summarizing the result. So, if you take a current mode control buck converter current mode control implementation we are using a load current feed with a normalized gain of K n feedback gain h. So, here G VC is the voltage controller. So, here we are assuming H to be 1; that means, we are not putting any voltage feedback gain, but if you use it; that means, any step-down factor then we have to accordingly change the large signal derivation we have to scale it.

So, here H is equal to 1, we are taking just a PI controller using the earlier derivation in lecture number 15 we can find the proportional gain is twice C by L delta i 0 what is delta i 0? It is a load step size v in is the input voltage, L is the inductor c is the capacitor what is v q? v q equal to v ref during the step-up transient v in minus v ref during the step-down transient and when the input duty ratio is near 50 percent they are identical.

And we have already discussed how to derive the integral gain. So, we have derived proportional gain integral gain using you know the large signal tuning and normalized gain here is 1 and this is also discussed in lecture number 50 in our earlier course.

(Refer Slide Time: 16:08)



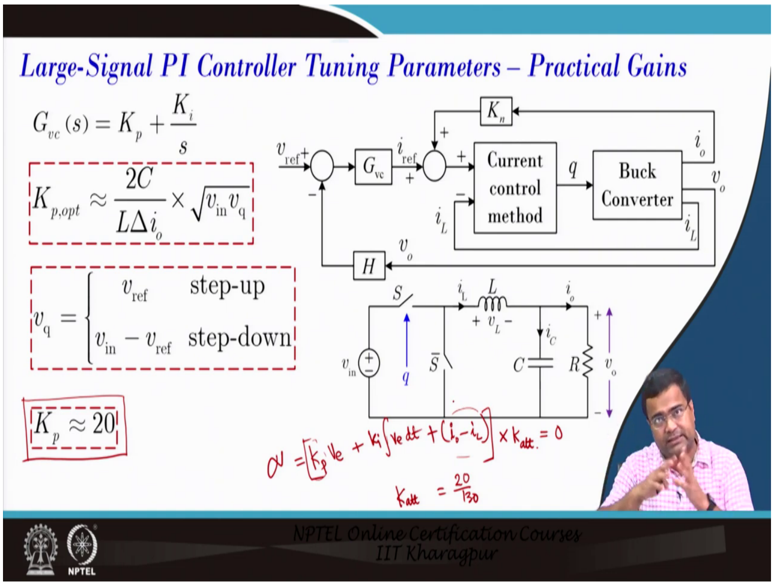
Now, the same method is the problem here if you find out; that means, you know if you calculate for example, in our case. So, we are talking about this derivation.

Now, suppose for our case our capacitor is let us say 200 microfarad inductor is 0.5 microhenry, and let us say we have applied a load step size of 20 amperes then what is my k optimal gain proportional? It will be twice 200 divided by 0.5 into 20 and it is v in into v ref. So, let us say our v is equal to 12 volts and v ref is equal to 1 volt. So, it will be 12.

So; that means if you calculate what you will get? You will get 400 divided by 10. So, if you just it will be 10 square root of 12; that means, it will be 2 root 3 and what will be the value you will get K optimal to be roughly around how much? 40; that means, 80 into root 3 and root 3 is 1.7 what is the root 3 value? So, it will be around 1.732 right what is that? 1.732 and you multiply by 80.

So, how much you will get? So, around 130 to 140 yeah. So, it is around 130 roughly around 130 like that. So, it is a very large gain and we will see this large gain may lead to instability; that means, what we want to mean the derivation of this large signal model gives us a large value that the system may not tolerate and it can be there can be large overshoot undershoot or it may be unstable.

(Refer Slide Time: 18:35)

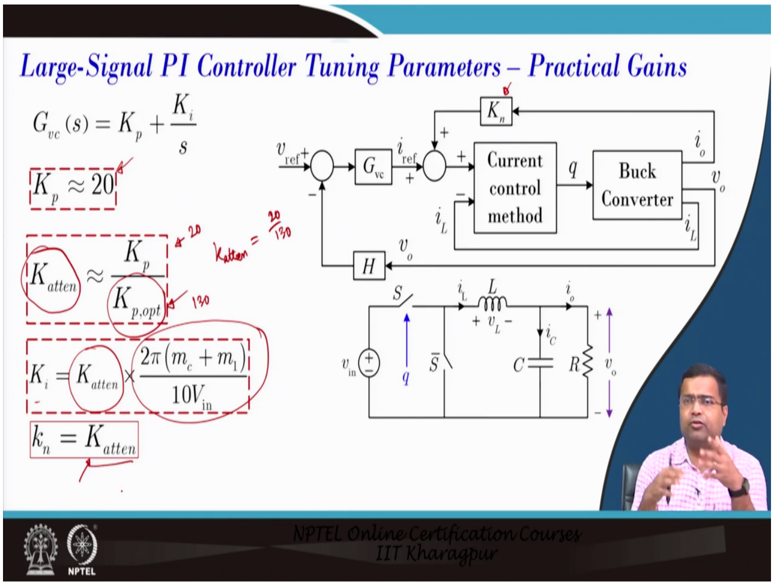


Then the next step. So, we have discussed this suppose we have a gain limit because we want to implement this controller using analog, but our proportional gain should not exceed 20 and here we got around 130. So, it is not acceptable then what to do? We remember the sigma what was sigma? It was if you remember correctly it was k p what was sigma? It was up into v error k i into error dt plus what is that? i 0 minus i L and this will be 0 at the point of switching.

Now, at the point of switching, we can always normalize. So, suppose the whole term i multiplied by; that means, what I am saying is i multiplied by some you know some fraction k f let us say I set k f to be 20 by 130 which means whatever k p we got now this k p or this is a factor of attenuation I want to attenuate this gain. So, if I want to attenuate this gain as a factor of attenuation then what is this attenuation factor?

So, this attenuation factor will be 20 by 30. So, all values will be attenuated; that means, that means my if you multiply then this KP will be attenuated ki will be attenuated and this current loop feedback gain also should be attenuated. So, everything will be attenuated then it will resemble the same thing.

(Refer Slide Time: 20:40)

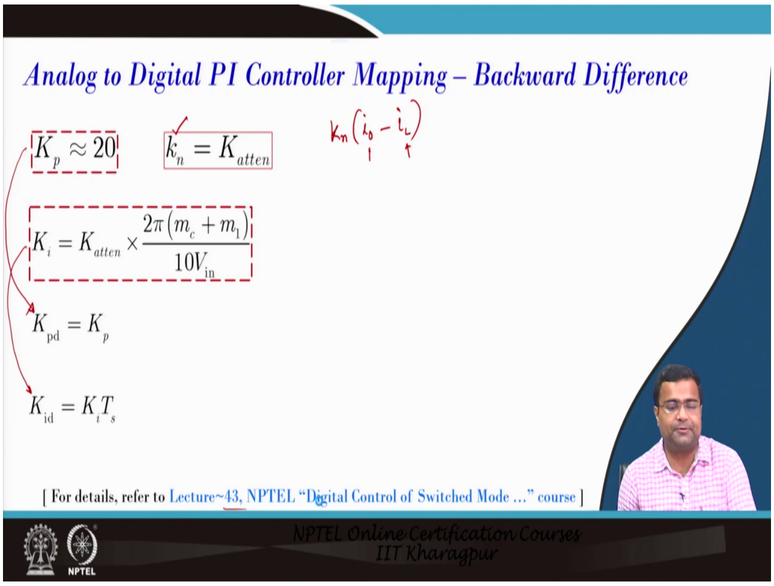


So, this is exactly what we are doing. So, suppose I have a gain upper range is 20. So, I want to find out what my calculated optimal gain from the analytical derivation I will take I will find out the attenuation ratio by K p by this actual.

So, which was around 130 and this we want to get 20 for example. So, my attenuation factor will be something like you know this attenuation factor will be 20 by 130 this K i now our present K i will be attenuated multiplied by this attenuated factor with the original K gain and similarly, this factor will be just the attenuation factor if you do that then we can implement this logic with a realistic value of K p and we should get.

But you may find that this gain may be too small because if this is too small the effect of the current loop I mean that we are stepping down the current loop by a large factor and which may inject noise. So, we have to be careful and we will see some practical constants particularly when you go to digital control.

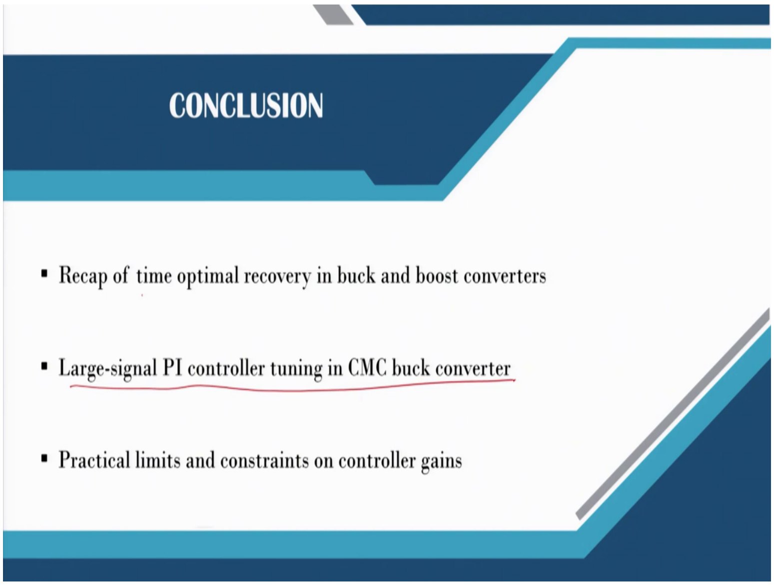
(Refer Slide Time: 21:55)



So; that means, if we take K p we have to find out the attenuation factor and we know how to find the attenuation factor and this k n will be our the feed-forward gain that the current loop which includes the load feed forward; that means, the k n will be i 0 minus i L.

So, this is the actual inductor current these are load current, but the actual inductor current load current will be the same scale then once you get K p you have to get discrete-time K p which is the same as continuous time and the discrete-time integral gain will be a continuous time integral gain into sampling time. So, this is discussed this conversion method in lecture number 43.

(Refer Slide Time: 22:42)



So, in summary. So, in the next lecture, we are going to take MATLAB case studies and we want to see what are the practical constants and what are their limit. So, in this lecture we have discussed we have recapitulated the time optimal recovery in a buck and also how does it look like in a boost converter.

We have discussed large signal PI controller tuning in a current mode control buck converter and we have identified some practical limits and some constants on the controller gain. So, in the next lecture, we want to show how does it look like when you want to simulate this using digital current mode control that is it for today.

Thank you very much.