Analog Integrated Circuits
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Lecture - 01
Introduction to MOSFETs

In today's class, we will start by looking at an Introduction to MOSFETs, their operation and their IV characteristics. So, let us start off with the IV characteristics because for from a circuit's point of view; that is the most basic thing that we require from the device.

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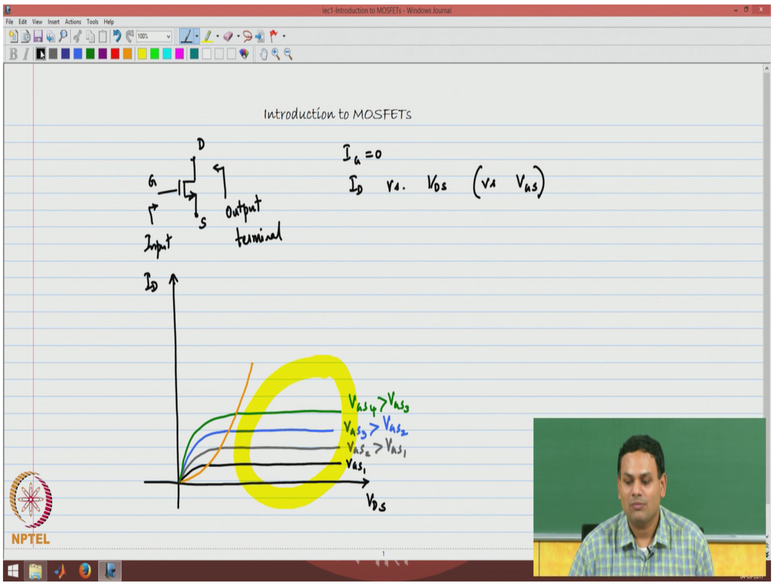

So, first of all the MOSFET happens to be a 3 terminal device, it has a gate terminal, it has a drain terminal and a source terminal. And by normal convention for analogue circuits this happens to be the input terminal and this happens to be the output terminal.

Now, it so happens that the MOSFET has a gate current that is an input port current that is 0; especially at low frequencies. And therefore, there are no input characteristics for the MOSFET. In other words, I G versus V GS need not be plotted because it is the gate current is just 0, we will plot I D versus V DS, but now remember that the drain current maybe a function of both the drain source voltage and the gate source voltage. So, we will plot in this form, I will put this in brackets because we are the basic plot is going to be the drain current versus the drain source voltage.

But we will plot; show different curves for different values of the gate source voltage. So, this is your basic plot and just a reminder that for good amplification to create a good amplifier using this MOSFET; you want the drain current to be a function of the gate source voltage only and it should not change with drain source voltage. And therefore, what you desire is straight lines parallel to the x axis; I will show them with different colors to show that there are different values of V GSs.

So, this is some V GS one this is V GS 2 which is greater than V GS one and. So, on now what is the MOSFET do closer for small V DSs it so happens that the MOSFET being passive device has to the IV characteristics have to transition into the third quadrant especially because I G is 0. So, I will show the border of this region in this manner using this, but what we are interested in is what happens here and you will see the MOSFET behaving in this manner and eventually it will transition into the third quadrant, but we do not have to worry about that for the purposes of these plots.

So, this shows the IV characteristics of the MOSFET and we will note that to use it as an amplifier, we will always use it in this region. In other words we will use it in the region where the IV characteristics that the drain current is not a function of the drain source voltage it is a function of the gate source voltage only. Now, we can go ahead and start writing the current equations of the MOSFET.

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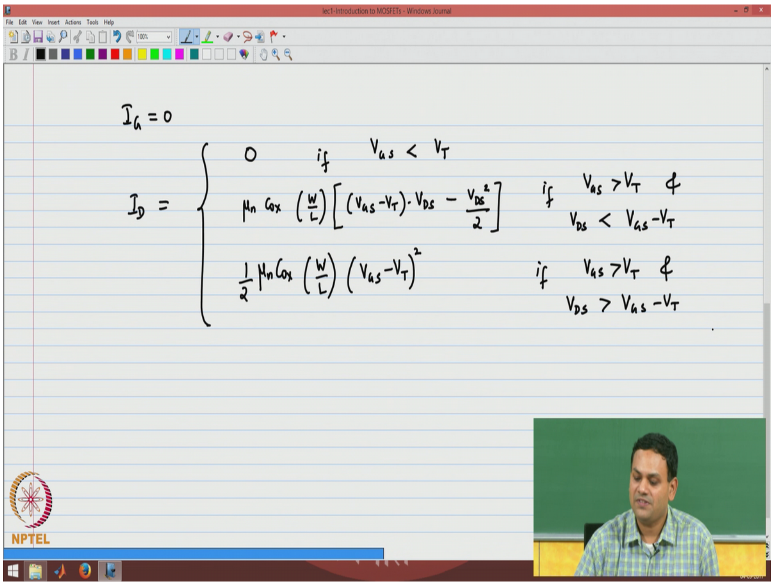

$$I_G = 0$$

$$I_D = \begin{cases} 0 & \text{if} \quad V_{GS} < V_T \\ \mu_n C_{ox} \left(\frac{W}{L}\right)\left[(V_{GS}-V_T)\cdot V_{DS} - \frac{V_{DS}^2}{2}\right] & \text{if} \quad V_{GS} > V_T \ \& \ V_{DS} < V_{GS}-V_T \\ \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)^2 & \text{if} \quad V_{GS} > V_T \ \& \ V_{DS} > V_{GS}-V_T \end{cases}$$

So, primarily there are 3 separate equations for the current of course, the gate current is always 0 for the MOSFET. So, there is no problem with that now if you look at the drain current there are 3 major region. So, I will show them like this maybe I will move this here first of all the MOSFET does not conduct if the gate source voltage is smaller than a specific value called the threshold voltage we will denote that by VT; V subscript T. So, if the gate source voltage is smaller than the threshold voltage the device does not conduct.

And now if the gate source voltage is larger the device starts to conduct and now it has to distinct regions. So, the current is given by this expression. So, if the gate source voltage is larger than VT and the drain source voltage is smaller than V GS minus VT, then the drain current has this expression it is mu n C ox; W over L times V GS minus VT times V DS minus V DS square by 2.

So, clearly in this region the drain current depends on both the gate source voltage and the drain source voltage and this represents this region of the device characteristic the third region is given by the following expression. So, it is half mu n C ox W over L into V GS minus VT the whole squared if V GS is greater than VT and V DS is larger than V GS minus VT. So, clearly if the drain source voltage is larger than the difference between the gate source voltage and the threshold voltage the drain current is a function of the gate source voltage only and v are going to use the MOSFET as an amplifier in the third region.

This region is called the saturation region of the MOSFET. So, I will show that here.

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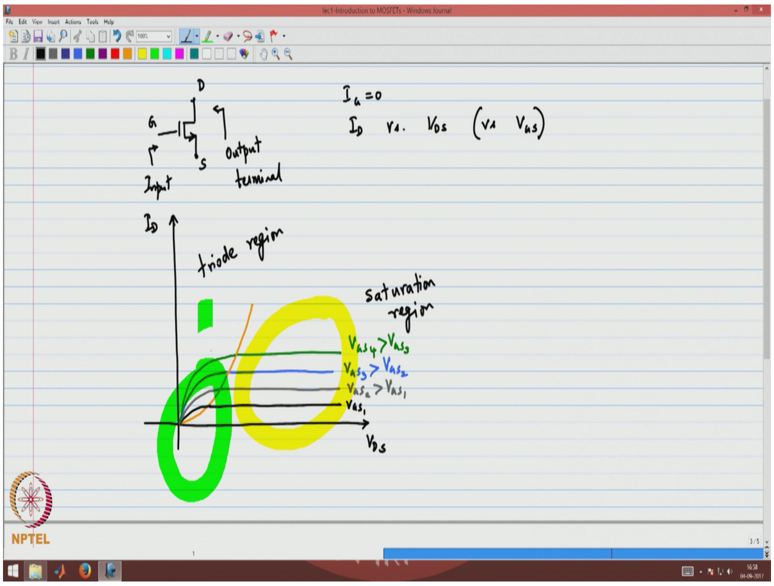

It is called the saturation region of operation and this region is called the triode region of operation; and as you can see MOSFET transitions between the triode region and the saturation region in depending on the value of the drain source voltage and the gate source voltage.

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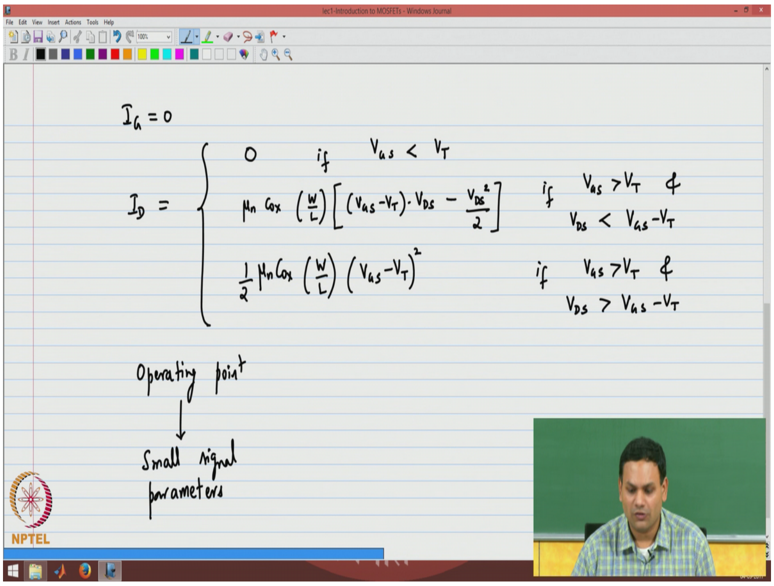

$I_G = 0$

$$I_D = \begin{cases} 0 & \text{if } V_{GS} < V_T \\ \mu_n C_{ox} \left(\frac{W}{L}\right)\left[(V_{GS}-V_T)\cdot V_{DS} - \frac{V_{DS}^2}{2}\right] & \text{if } V_{GS} > V_T \ \& \ V_{DS} < V_{GS}-V_T \\ \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)^2 & \text{if } V_{GS} > V_T \ \& \ V_{DS} > V_{GS}-V_T \end{cases}$$

Operating point

$\downarrow$

Small signal parameters

We have seen the behavior of the currents and voltages we have written down the expressions. So, this non-linear equation represents the dependence of the drain current on the gate source voltage in the saturation region and will decide the operating point of the MOSFET.

And remember the operating point of the MOSFET is used to calculate the small signal parameters of the MOSFET this is something you have learnt in the previous course the more basic analogue circuit's course. So, from the operating point you will derive the small signal or incremental parameters and the small signal model of the MOSFET. So, now, let us look at the small signal model.

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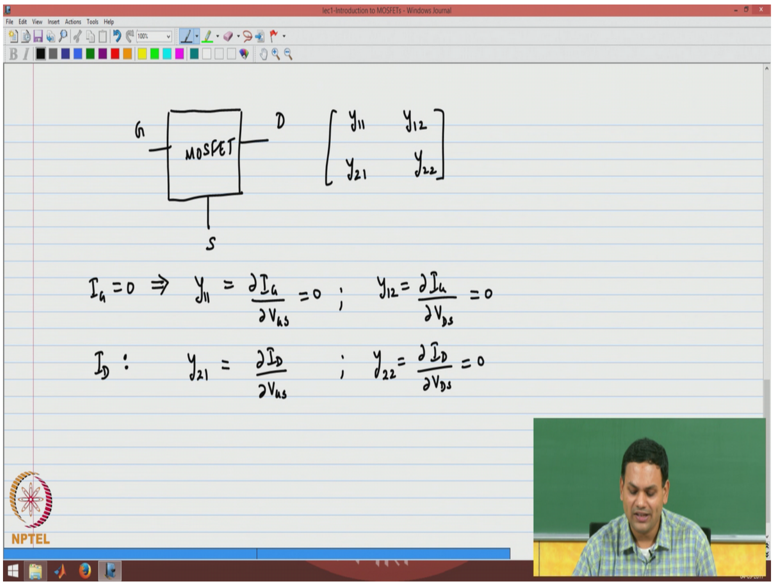

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$$I_G = 0 \Rightarrow y_{11} = \frac{\partial I_G}{\partial V_{GS}} = 0 \; ; \quad y_{12} = \frac{\partial I_G}{\partial V_{DS}} = 0$$

$$I_D : \quad y_{21} = \frac{\partial I_D}{\partial V_{GS}} \; ; \quad y_{22} = \frac{\partial I_D}{\partial V_{DS}} = 0$$

So, if you look at the MOSFET simply as a 2 port it has 3 terminals which is the gate the drain and the source the gate is the input terminal the drain is the output terminal and the source is the common terminal between input and output.

And therefore, this can be represented by the small signal y parameters and note that in the case of the MOSFET the gate current is 0 which implies that y 11 which is dou I G by dou V GS is 0 and y 12 which is dou I G by dou V DS is also 0 because the gate current is just 0. Similarly from the drain current expressions, we can derive y 21 as dou I D by dou V GS. Now y 21 has 3 expressions we will go ahead and calculate this, but I just wanted to point out that y 22 is dou I D by dou V DS for an ideal MOSFET this should be 0. And therefore, in the small signal model it looks like y 21 is the only significant small signal parameter of interest.

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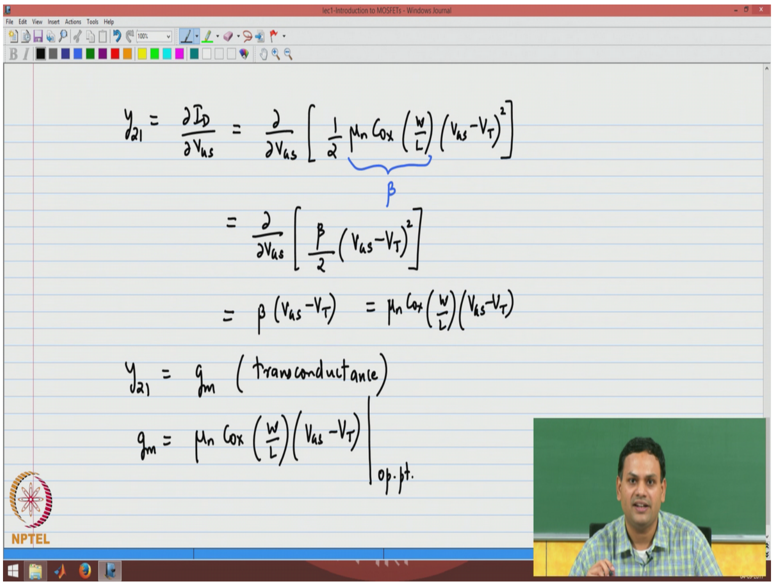

$$y_{21} = \frac{\partial I_D}{\partial V_{GS}} = \frac{\partial}{\partial V_{GS}}\left[\frac{1}{2}\underbrace{\mu_n C_{ox}\left(\frac{W}{L}\right)}_{\beta}\left(V_{GS}-V_T\right)^2\right]$$

$$= \frac{\partial}{\partial V_{GS}}\left[\frac{\beta}{2}\left(V_{GS}-V_T\right)^2\right]$$

$$= \beta\left(V_{GS}-V_T\right) = \mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{GS}-V_T\right)$$

$$y_{21} = g_m \ (\text{transconductance})$$

$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{GS}-V_T\right)\Big|_{op.pt.}$$

So, let us quickly calculate that y 21 is dou I D by dou V GS in the saturation region. So, let me make a small simplification here to is the flow of writing I am going to call this parameter as some beta just makes things easier. So, since these are device parameters and geometric parameters I can write this as beta over 2 times V GS minus VT D whole square. And therefore, this is nothing, but beta into V GS minus VT which is of course, mu n C ox W over L into V GS minus VT.

So, this is the first expression for y 21 that we are interested in y 21 is also called the trans-conductance or g m. Please note that- a normal conductance is a single port parameters. So, you talk of resistance and conductance for a 2 terminal one port element in this case you are talking about conductance referred the controlling voltage is the input port, but the conductance is seen at the output port therefore, it is called a trans-conductance.

So, the first expression for g m is mu n C ox W over L into V GS minus VT now from the basic current this is of course, evaluated at the operating point. But, now from the basic current equation you know the relationship between the drain current and the gate source voltage.

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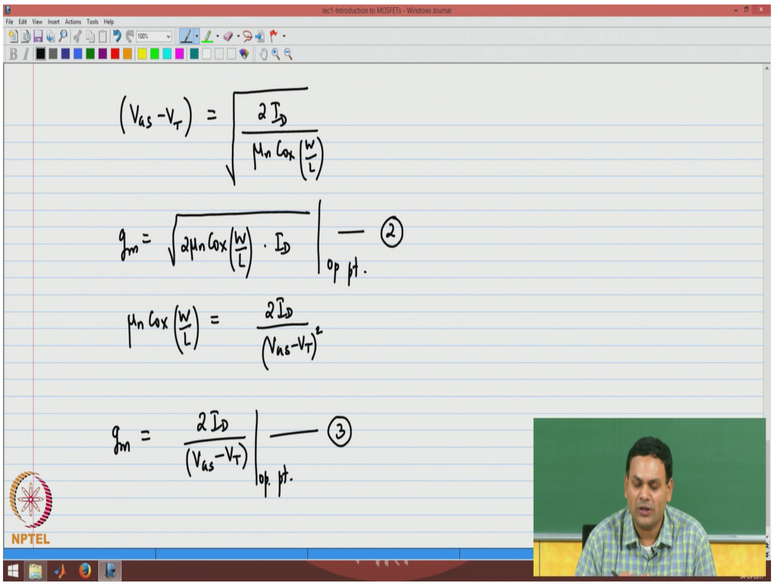

$$(V_{GS} - V_T) = \sqrt{\frac{2 I_D}{\mu_n C_{ox} \left(\frac{W}{L}\right)}}$$

$$g_m = \sqrt{2 \mu_n C_{ox} \left(\frac{W}{L}\right) \cdot I_D}\,\Bigg|_{op\ pt.} \quad — ②$$

$$\mu_n C_{ox} \left(\frac{W}{L}\right) = \frac{2 I_D}{(V_{GS} - V_T)^2}$$

$$g_m = \frac{2 I_D}{(V_{GS} - V_T)}\Bigg|_{op.\ pt.} \quad — ③$$

So, V GS minus VT from the current equation of the MOSFET in saturation is nothing, but 2 I D over W over L placed under the square root sign. And therefore, I will call this expression number one. And so, now from this you can replace the expression for V GS minus VT insight the g m equation.

So, you get the second expression for g m into I D this is I will call this expression number 2 and now I can also replace the device parameters mu n C ox W over L in terms of the current and voltage of the MOSFET. So, in other words, I can say that mu n C ox W over L is 2 I D over V GS minus VT the whole squared. And therefore, I get the third expression for g m which is 2 I D over V GS minus VT.

This is the third expression for the trans-conductance of the MOSFET and of course, all of these are evaluated at the operating point now they are all identical they are all equivalent it is just that in some cases you may know the device parameters and the V GS and the voltages. So, that you can calculate the value of g m in other cases, you may know the drain current directly in other cases you may not know the device parameters, but you are may know only the drain current and the gate source voltage. And therefore, you should be able to calculate the trans-conductance of the device.

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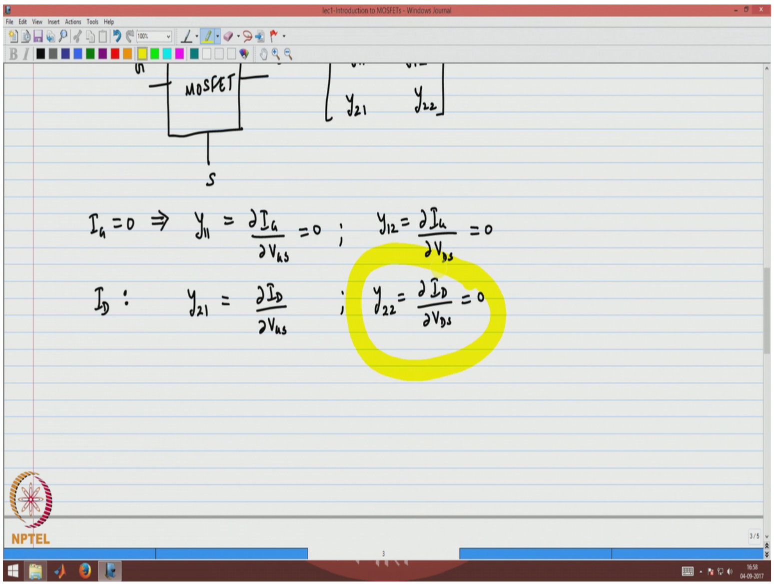

$$\begin{bmatrix} & \\ y_{21} & y_{22} \end{bmatrix}$$

$$I_G = 0 \Rightarrow y_{11} = \frac{\partial I_G}{\partial V_{GS}} = 0 \; ; \quad y_{12} = \frac{\partial I_G}{\partial V_{DS}} = 0$$

$$I_D : \quad y_{21} = \frac{\partial I_D}{\partial V_{GS}} \; ; \quad y_{22} = \frac{\partial I_D}{\partial V_{DS}} = 0$$

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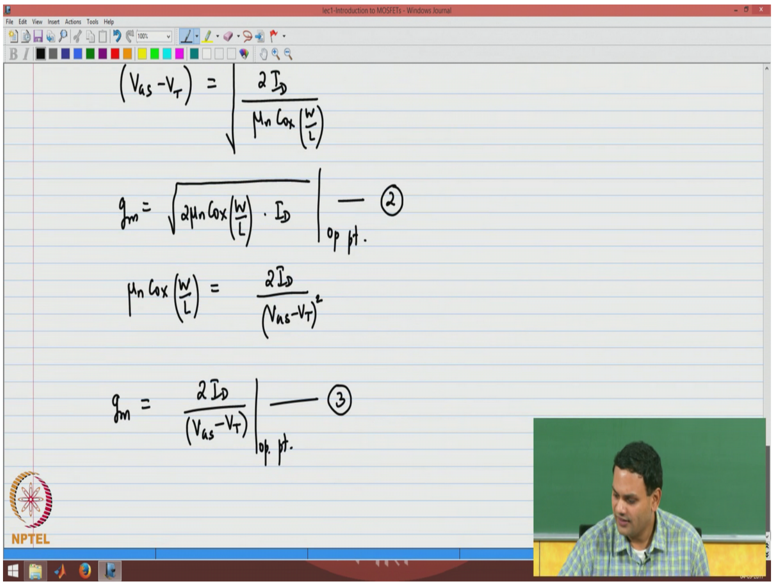

$$(V_{GS} - V_T) = \sqrt{\frac{2 I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}$$

$$g_m = \sqrt{2 \mu_n C_{ox}\left(\frac{W}{L}\right) \cdot I_D}\;\Bigg|_{op\ pt.} \quad — ②$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right) = \frac{2 I_D}{(V_{GS} - V_T)^2}$$

$$g_m = \frac{2 I_D}{(V_{GS} - V_T)}\Bigg|_{op.\ pt.} \quad — ③$$

Now, having said this we have to go back and I would like to point out that even though we said that for an ideal MOSFET y 22 could be 0 it turns out that for a real MOSFET it is not exactly 0 and it turns out that if you to measure the IV characteristics of any MOSFET even though I will show it for one V GS even though you expect the drain current to be independent of the drain source voltage the actual drain source voltage, we will follow the characteristic shown on blue.

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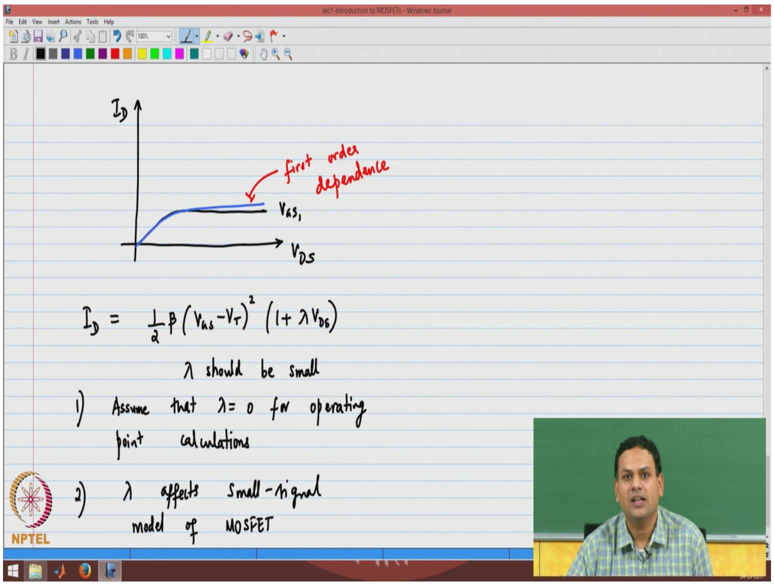

$$I_D = \frac{1}{2}\beta\left(V_{GS}-V_T\right)^2\left(1+\lambda V_{DS}\right)$$

λ should be small

1) Assume that $\lambda = 0$ for operating point calculations

2) λ affects small-signal model of MOSFET

That is they will show a slight dependence of drain current on drain source voltage and for engineering purposes we will represent this as a first order dependence. So, we will say that this is a first order dependence and we will modify the expression for the drain current in the following way this is only in the saturation region.

So, this was the original expression we will now add a small dependence on the drain source voltage please note that ideally lambda should be 0, but in general lambda should be very small. So, that the dependence of the drain current on the drain source voltage is small now what does this mean for us. So, first of all at the device level this is something due to it is due to something called channel length modulation. So, that is something you could learn more about in the devices course in the semiconductor devices course.

But for the purposes of this analogue ICs course we will assume that it does have a dependence and it can be modelled as a first order dependence and we want to find out what implication this has for IC design for analogue IC design. So, therefore, we should see what you know what effect this has on the operating point as well as on the small signal equivalent circuit. So, the first thing we will say to make things much easier for us since lambda is very small we will assume that lambda equal is equal to 0 only for the operating point calculations.

In other words if you want to find out the drain current for a given gate source voltage we will assume that lambda is 0 because the lambda is assumed to be quite small it should not affect your operating point significantly; in that case why even use a lambda the reason is even though it may not affect the operating point calculations it will still effect is small signal model.

So, please note that even though lambda may be assumed to be 0 for the operating point it can still affect the small signal model of the MOSFET and let us just see quickly see in what way it effects us for the small signal model.

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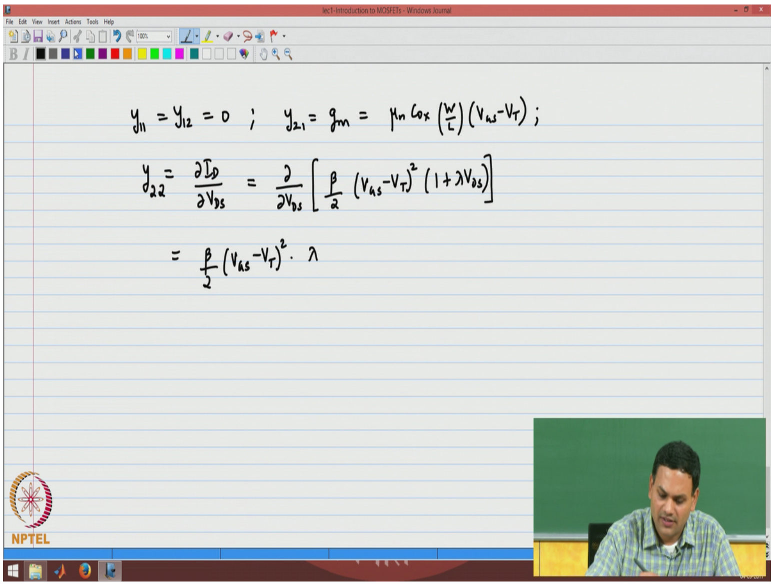

$y_{11} = y_{12} = 0$; $y_{21} = g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)(V_{GS}-V_T)$;

$$y_{22} = \frac{\partial I_D}{\partial V_{DS}} = \frac{\partial}{\partial V_{DS}}\left[\frac{\beta}{2}(V_{GS}-V_T)^2(1+\lambda V_{DS})\right]$$

$$= \frac{\beta}{2}(V_{GS}-V_T)^2 \cdot \lambda$$

So, we have seen. So, far that y 11 and y 12 are 0 for the MOSFET and we have seen that y 21 is equal to g m and for calculating g m, we will assume that lambda 0. So, I will write down the first expression that we wrote down which is mu n C ox W over L into V GS minus VT.

So, the value of g m does not change, but we still have the fourth parameter which is y 12 which was originally dou I D by dou V DS we had assumed that that was 0, but turns out that is not 0 anymore. So, let us go back and plug in the expression for ID. So, it is beta by 2 into V GS minus VT the whole squared into one plus lambda V DS.

This is beta by 2 into V GS minus VT the whole square into lambda I am sorry they should be y 22 into lambda. So, now, we will make one more approximation remember that we said lambda is 0 for the operating point.

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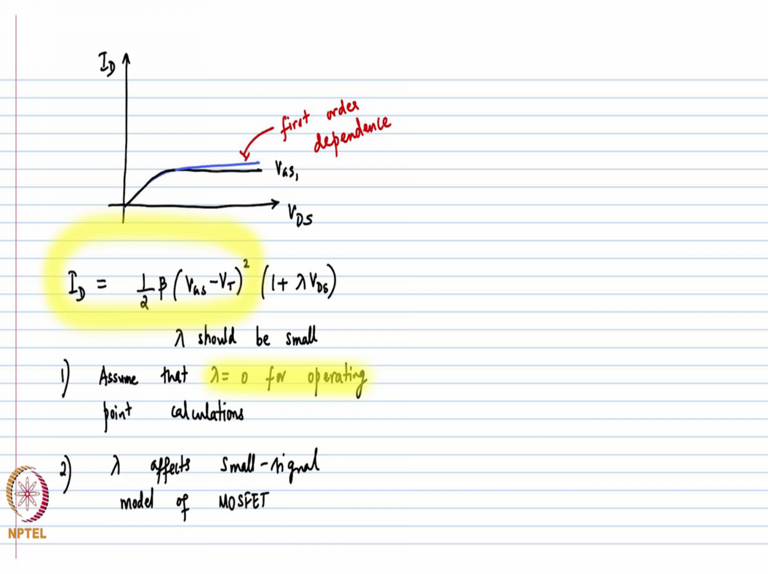

$$I_D = \frac{1}{2}\beta\left(V_{GS}-V_T\right)^2\left(1+\lambda V_{DS}\right)$$

λ should be small

1) Assume that $\lambda = 0$ for operating point calculations

2) λ affects small-signal model of MOSFET

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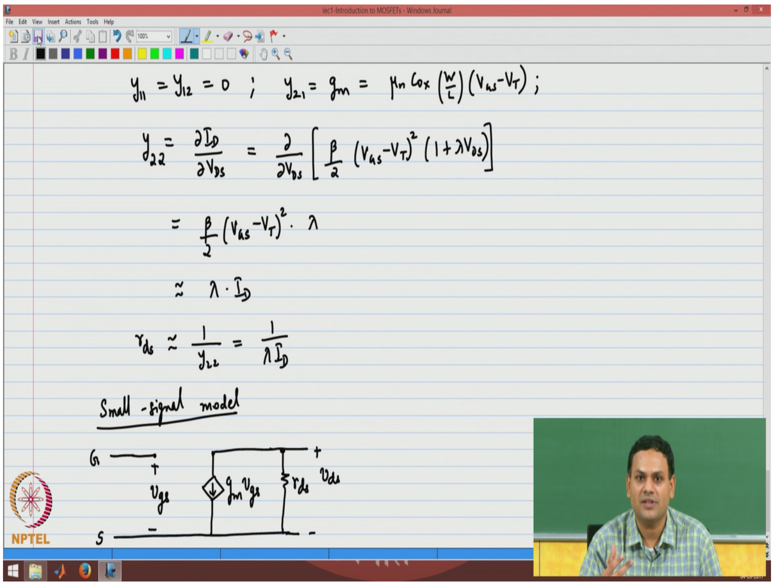

$y_{11} = y_{12} = 0$; $y_{21} = g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{GS}-V_T\right)$;

$$y_{22} = \frac{\partial I_D}{\partial V_{DS}} = \frac{\partial}{\partial V_{DS}}\left[\frac{\beta}{2}\left(V_{GS}-V_T\right)^2\left(1+\lambda V_{DS}\right)\right]$$

$$= \frac{\beta}{2}\left(V_{GS}-V_T\right)^2 \cdot \lambda$$

$$\approx \lambda \cdot I_D$$

$$r_{ds} \approx \frac{1}{y_{22}} = \frac{1}{\lambda I_D}$$

Small-signal model

And therefore, we will say that this is approximately equal to lambda times ID. So, the approximation we are making is that the expression for lambda. So, small that I D can be approximated to this expression alone.

So, now this is the conductance between the output ports and now if you were to find out the resistance which I will called r d s which is the drain source resistance of the MOSFET this is one over y 22 which is one over lambda I D. In other words if you know the lambda for the device and if you know the drain current at the operating point you can calculate the drain source voltage.

So, now we are in a position to draw the small signal model of the MOSFET. So, clearly I am going to take the source terminal as the reference terminal between the input and output ports at the gate the device is an open circuit and on the drain side you have 2 parameters of interest one of them is g m times V GS note that the voltages and currents we are talking about are now the small signal currents and not the total current.

So, far in the device equations we were writing they were the dc currents or the total currents now we are talking about the small signal currents alone. So, this voltage is V GS apart from this you also have the drain source resistance r d s and this voltage is the small signal drain source voltage of the MOSFET at this point we need to move ahead and look at some how the MOSFET is MOSFET is constructed in modern ICs.

So, we will stop here for this session and continue start from the MOSFET device geometry from the next class.