Analog Integrated Circuits
Prof. S. Aniruddhan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 11
Dominant Pole Compensation

In this lecture we will again continue our study into the stability of multiple feedback systems. So we have so far been looking at 3 pole systems, so in other words your forward block A of s has 3 poles so we will represent it as A naught by 1 plus s by omega p the whole cubed and as before we will assume that the that the reverse path the feedback path is frequency independent.

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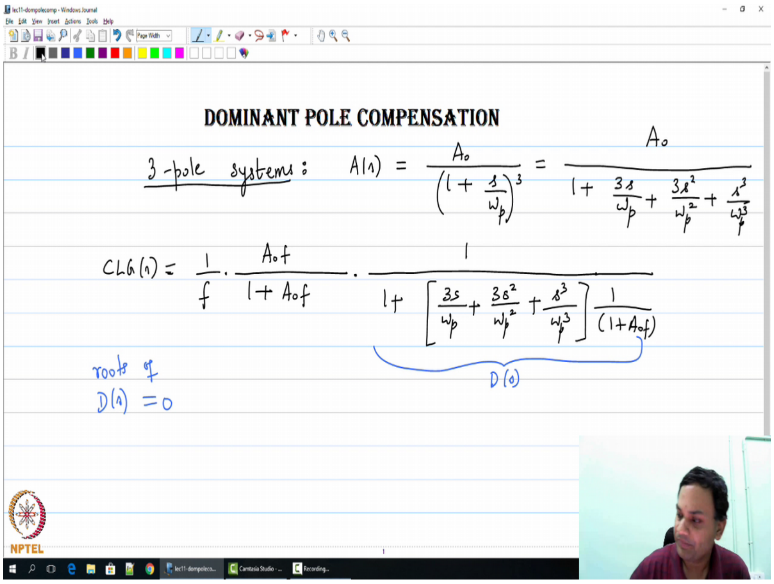

Now, if you write down the expression for the closed loop gain of the system, in terms of A of s we will then use that to study the stability of the system, so let us expand out the denominator because we are going to need to do this 1 plus 3 s by omega p plus 3 s squared by omega p squared plus omega p cubed.

Now if you look at the expression for the closed loop gain of the system as a function of frequency, so you will of course have the ideal gain of the feedback system which is 1 over f times the low frequency loop gain which is a naught f by 1 plus a naught f times the frequency response, so for the frequency response you will see 1 over 1 plus 3 s by omega p plus 3 s squared by omega p squared plus I am sorry I made a small mistake here they should be s cubed by omega p cubed, plus s cubed by omega p cubed into 1 by 1 plus A naught f, so this is the expression for the closed loop gain of the feedback system. So, now I am going to represent this portion as sum D of s the denominator polynomial and what I need to do I need to find out the roots of D of s equal to 0 if I do this will give me the expression for the roots of the closed loop system and I know that if the closed loop system has roots in the right half plane especially complex conjugate roots I know that the system is going to be unstable. So, let us now take this denominator polynomial and find out it is roots.

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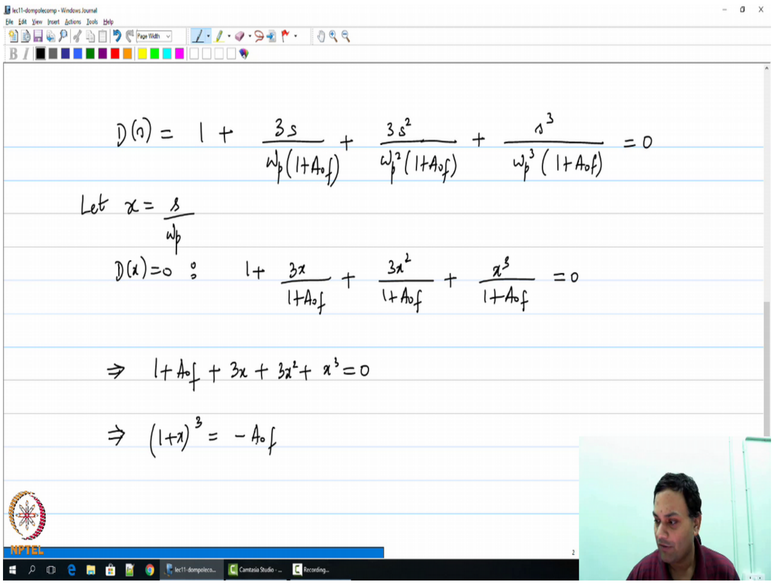

So, now the expanded expression for D of s is 1 plus 3 s by omega p into 1 plus A naught f plus 3 s squared by omega p squared into 1 plus A naught f plus s cubed by omega p cubed into 1 plus A naught f and I am going to set that to 0 to find out it is roots. Now this is of course the same as finding out the roots of. So, now I am going to make a small substitution to make things easier I am going to say some variable x is s by omega p, so I find out the roots of D of x and then I substitute x equals s by omega p.

So, the roots of D of x equal to 0 is the same as finding it is the same as finding the roots of 1 plus sorry 3 x by 1 plus A naught f plus 3 x squared by 1 plus A naught f plus x cubed by 1 plus A naught f equal to 0 and now I can multiply throughout by 1 plus A naught f, so 1 plus A naught f plus 3 x plus 3 x squared plus x cubed is equal to 0.

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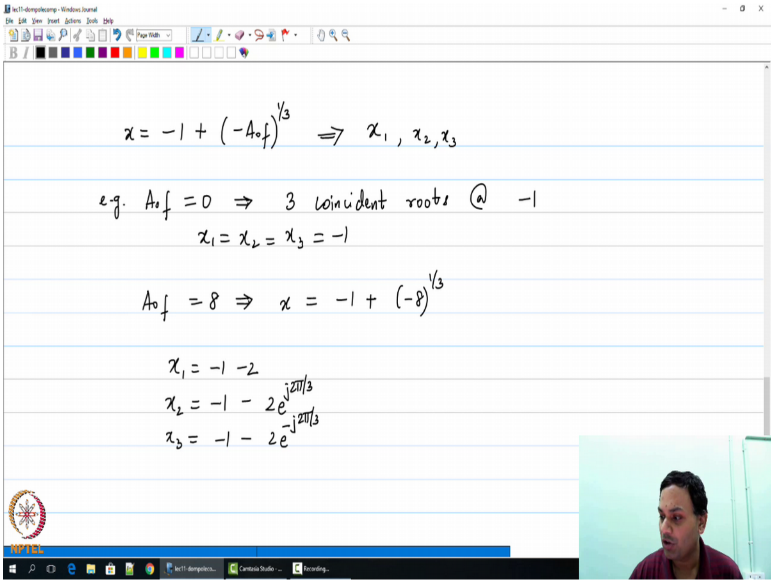

This again can be written in this form 1 plus x the whole cubed is equal to minus A naught f, now it is clear that we want to find out the solutions of this particular cubic equation. And of course the solution is clearly given by x is equal to minus 1 minus 1 plus the cube root of minus A naught f and of course the cube root of minus A naught of f has three solutions and each of these solutions will give you three roots x 1, x 2 and x 3.

For example, so let A naught f be equal to 0 right, so I have no loop gain in the system so this means there are three coincident roots at minus 1 that is x 1 is equal to x 2 is equal to x 3 which is equal to minus 1, so what happens if I try out a different value let us try out A naught f equals 8, if the total loop gain in the system is equal to 8 then the system has three roots x 1, x 2, x 3 so I need to find out the roots of minus one plus the cubeth root of minus 8.

So, this gives me three solutions x 1 is minus 1 minus 2, x 2 is minus 1 minus 2 e power j 2 pi by 3 and the third solution x 3 is minus 1 minus 2 a power minus j 2 pi by 3, so these are basically three solutions one of them happens to be real the other two solutions are complex conjugates of each other.

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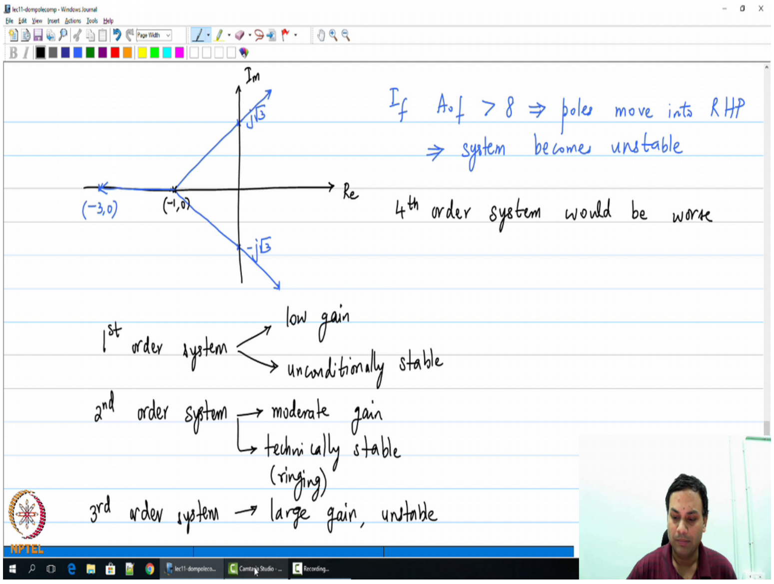

So, now I am going to show the progression of the roots in the in an x y plot, so this is the real axis and this is the imaginary axis and remember when A naught f was 0 you had three coincident poles at minus 1 comma 0 and when the loop gain increases to 8 the poles of course when the loop gain increases on 0 one pole always moves along the x axis, the other two poles split apart into a pair of complex conjugate poles.

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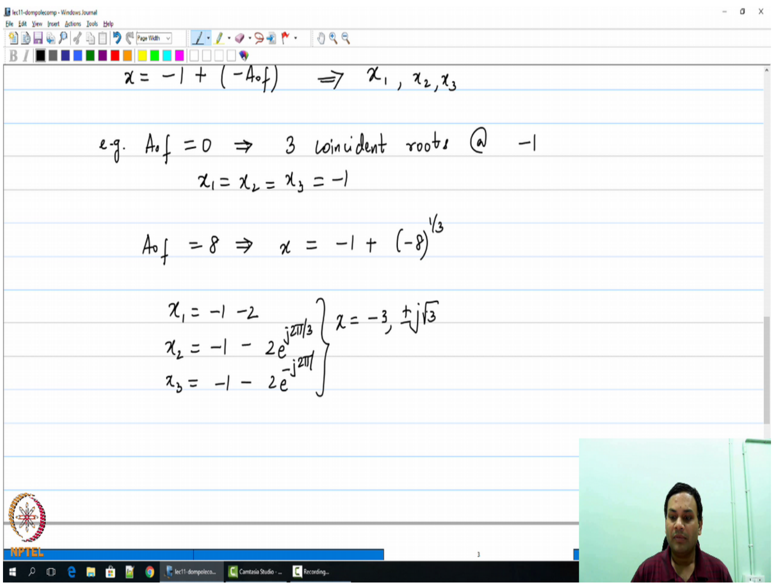

Now let us calculate the value of the poles at minus 2 for a at a naught f equal to 8, if you look at this particular value minus 1 minus j 2 pi by 3 so this gives you a value of x 1 equals minus 3, x 2 equals ah in fact, so this gives you three values x equals minus 3 and plus or minus j root 3. And therefore you find that at a value of A naught f these three poles now are such that the first pole is at minus 3 comma 0 so this is at A naught equal to f so they move in this way and the other two poles have move to plus or minus j root 3, and it should be clear now clear to you now that if the value of A naught f increases more than 8, then they the poles move into the right half plane and therefore the system will become unstable, the closed loop feedback system becomes unstable.

So, now you may notice that we had actually seen the first order system is unconditionally stable, the second order system is stable but when you try to achieve large loop gains the system has very high quality factor or very low damping factor and the closed loop response even though unconditionally stable can have a lot of ringing, what we find is for a third order system even for a value of only 8 the system becomes is barely stable and if A naught f increases if the loop gain increases beyond 8 the system definitely becomes unstable.

So, now this completely goes against what we have been trying to do so far, so we need to find a way to fix this, so obviously we can see that the third order system is already unstable at A naught f equal to 8 of a fourth order system would be worse. Now let us now regroup again so to kind of give a recap of what we have been studying so far, so we want to increase loop gain so that the study state error of the system is extremely small, so that it is as close to 1 over f is possible which is the ideal gain.

Now, it turns that the first order system is unconditionally stable so I will kind of summarize it here. So, the first order system is unconditionally stable but low gain, so it has low gain and which is an undesirable quantity and it is unconditionally stable, which is the desirable quantity that we will like. If you look at a second order system so it has moderate gain and it is technically stable but because the phase reaches minus 180 degrees only at infinite frequency but you can have ringing, and the third order system can have large gain but unstable for that large gain, so our strategy is going to be to build a higher order system which is stable.

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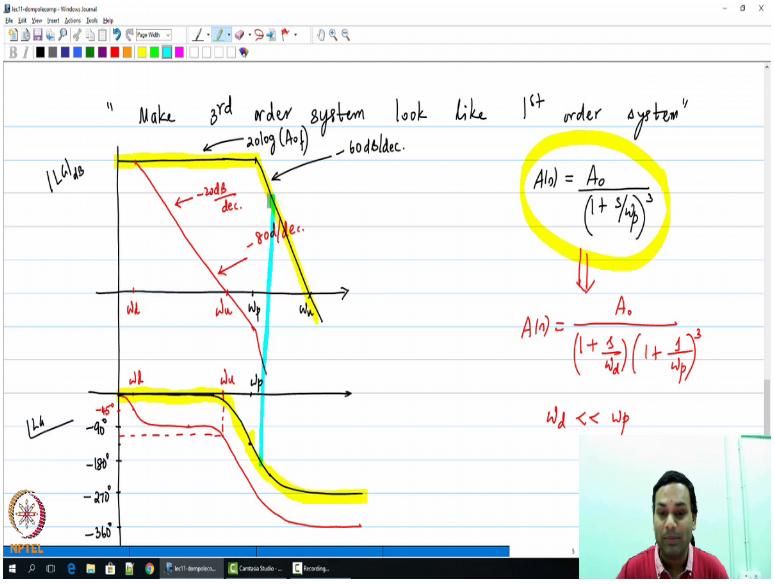

Now which the system that we know that is unconditionally stable is the first order system, so our strategy is to make the third order system which is the system with largest gain so look like first order system, so our strategy is going to be to make the third order system look like a first order system so this pretty much summarizes our strategy, so what I have done here is I have drawn out what happens when you take a third order system and make it look like a first order system.

So, let us look quickly look at the details of this, so your original system is completely shown in black so it has a forward block whose transfer function is A of s equals A naught by 1 plus s by omega p the whole cubed, so therefore the if you draw the bode plots of the system the magnitude stays at 20 log A naught f till it hits the three poles that omega p.

Once it does so, the gain starts dropping linearly in the bode plot at the rate of minus 60 dB per decade and the unity gain frequency of the system can be determined for this particular transfer forward block transfer function, if you look at the phase similarly the phase stays at 0 for a very long time now at approximately one tenth the pole frequency omega p the phase of the loop gain starts changing and at omega p the actual value of the loop gain will be three times minus 45 degrees or minus 135 degrees.

If you go to very large frequencies you will get minus 90 degrees from each of the omega p poles and the final phase would be minus 270 degrees. Now what can we say about the stability of the system clearly as I have drawn it the system has a gain I am going to show this in maybe light blue, the system has a gain which is larger than 1 when the phase hits minus 180 degrees. So, clearly when the phase hits one eighty degrees the system has a gain which is much larger than 1 and of course the system has a strong potential for instability. Now, let us see how you would make this look like a first order system as for a stability is concerned, so now I am going to take the same third order system.

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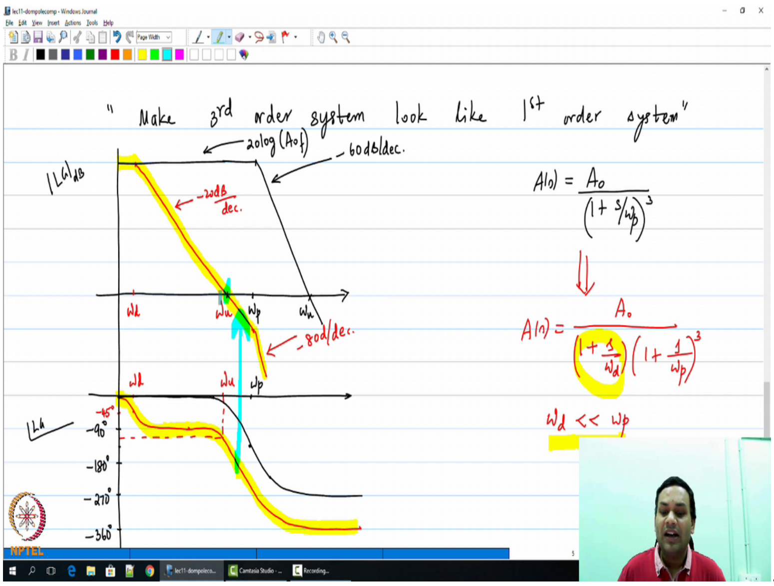

I am going to add a 4th pole omega d with the condition that omega d is much smaller than omega p and so the transfer function looks like this, now for this system I am going to draw the bode plot in red and as you can see the loop gain magnitudes stays at 20 log A naught f till it hits the first pole omega d, what happens subsequently at that point the gain starts dropping with a single pole response or at minus 20 dB per decade and for a well define system it should cross omega u such that it is still it still looks like a first order system. If you look at the phase response of course once it hits all the other three omega p the phase goes that minus 80 dB per decade so let me show that the phase response here the phase goes at minus 80 dB per decade at that at that point.

Once it hits the other three poles, now what happens to the phase response the phase again starts at 0 but at around one tenth of omega d the phase starts reducing at omega d you will get the a phase of minus 45 degrees from the omega d pole and once you go far enough beyond omega d the phase will asymptotically approach minus 90 degrees and it will stay at ninety degrees for a wide range of frequencies. Now at a certain point very close to one tenth omega p the phase of the three poles at omega p will start effecting the overall phase of the new system and eventually the at very high frequencies much larger than omega p, the total phase would be equal to that of a four pole system and therefore minus 360 degrees.

Now, most importantly I am going to show the when the phase crosses 180 degrees, so that happens at this point and as you can see when the phase crosses 180 degrees the gain the loop gain the magnitude of loop gain has already become smaller than 1, alternatively you can see that when the when the loop gain has actually hit 0 dB or a value of 1 the phase has not hit minus 180 degrees, so clearly the system is technically stable as I have drawn it, so now obviously if you increase the value of omega d the point where the phase is 180 degrees could occur when the magnitude of the loop gain is less than 1 as the designer it is our job to actually choose omega d appropriately. Now, having said this let us quickly look at two definitions of stability before we do the look at the analytical behavior.

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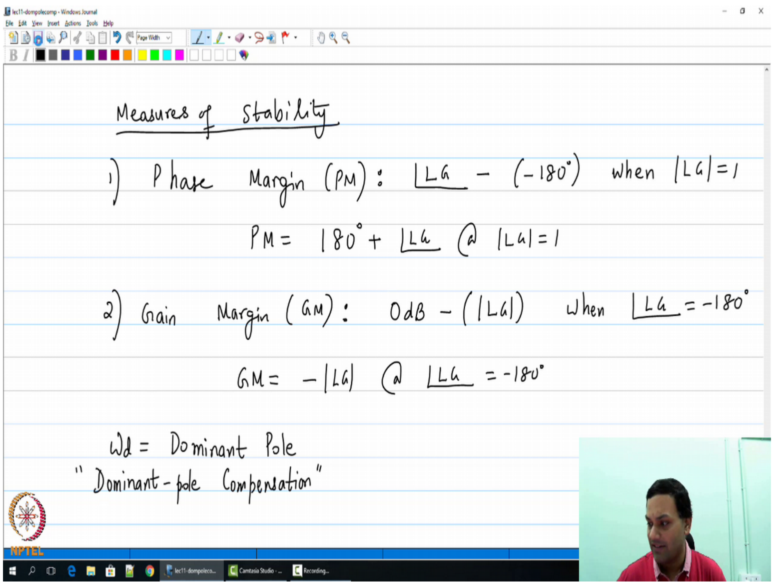

Now the analytical behavior of the system can be represented as a measure of sorry the measure of stability of the system can be represented in two ways or rather I will call it measures of stability, the first measure of stability that we will learn is what is called phase margin of an abbreviated as P M and this is defined as the difference between the actual angle of the loop gain and the point at which the system becomes unstable which is minus 180 degrees when the magnitude of the loop gain is equal to 1, so in other words you find out the frequency at which the magnitude of loop gain becomes equal to 1 and ideally your phase should not have already reached minus 180 degrees and the difference between the actual phase and this minus 180 degrees is the phase margin, in other words it can be written in this form 180 degrees plus the actual angle, for example if the actual angle were minus 135 degrees the if the system is said to have a phase margin of 45 degrees.

Now, the other measure of stability is what is called gain margin I am going to abbreviate it using G M, so this is defined as the difference between the actual between the point where between the point where the loop gain goes to 1 which is 0 dB minus the actual magnitude of the loop gain which is evaluated when the angle of the loop gain is minus 180 degrees, in other words ideally the loop gain should be less than 1 when the angle is 180 degrees so that the system is stable. So, this is clearly the negative of the magnitude of the loop gain when the angle is minus 180 degrees and similarly this is calculated when the loop gain is 1. So, these are two measures common commonly used measures of stability of the system and you will find that the phase margin is a much more commonly used measure of stability for various reasons.

Now let us go back to this particular system that we have seen we have seen now that addition of this pole omega d makes the system highly stable and this omega d is called the dominant pole of the system. Omega d is called the dominant pole because for a large range of frequencies it makes the system look like a first order system and this particular way of making the system stable is called frequency compensation and more specifically this particular way of compensating is called dominant pole compensation this is called dominant pole compensation this is one of the most commonly used ways to stabilize the feedback system. Now you will also find that you need to understand the system analytically so let us do that now let us find out how you would figure out where to place the dominant pole.

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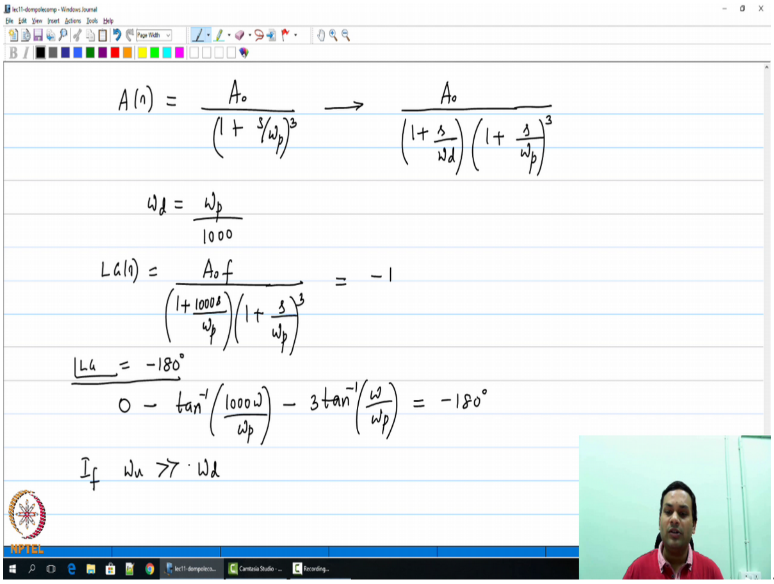

So, I am going to start off with the same third order system that I have started off with which is sum A naught by 1 plus s by omega p the whole cubed and what I am going to do I am going to assume that there is some feedback factor f so that the low frequency loop gain is A naught f as before I now need to find out where to place omega d the way I am going to do it now as an example is I am going to place omega d at a frequency at some frequency much smaller than omega p and figure out what the how large A naught f can be so that the system is stable.

So, let us quickly look at this, so now I am going to transform this into a dominant pole compensated system so into 1 plus s by omega d into 1 plus s by omega p the whole cubed, when we get to the details of the circuits we will understand how to make how to put this dominant pole into the system for now at the system level we need to understand what is happening. And what I am going to say is let us try placing omega d much smaller than omega p, so let us say omega d is going to be 1000th of omega p.

Let us say I am going to add a dominant pole which is at 1000 the frequency of the original 3 poles. So, now what is going to happen I can now write the expression for the loop gain of the system which is simply A naught f by 1 plus 1000 s by omega p which is the dominant pole and into 1 plus s by omega p the whole cubed and I am going to make this equal to minus 1 so in other words I need to apply a condition on the magnitude and a condition on the phase.

So, let us do both of those and of course I am first going to start off putting a condition on the phase because then I can find out omega as a function of omega p in other words I can find out the frequency at which the phase hits minus 180 degrees as a function of omega p so let us do that first. So, the first condition I am applying is that the angle of the phase response is minus 180 degrees, so this is the first constraint.

So this constraint tells me that I need to calculate the phase of the loop gain so of course the numerator gives me 0 phase and now I have to subtract the phase of the denominator from this, so the first dominant pole gives me a phase which is tan inverse 1000 omega by omega p minus now the three poles at omega p give me a phase of minus 3 tan inverse omega by omega p and I am going to equate this to minus 180 degrees.

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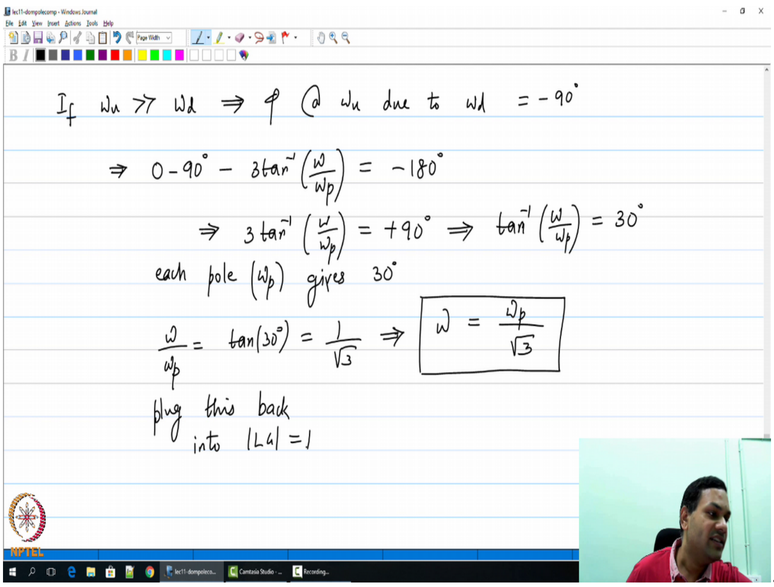

Now we can make some certain common sense approximations obviously the if the if the unity gain frequency were much larger than omega p or rather omega d what is the scenario then in this case if under this condition the phase of omega d, the phase due to omega d at omega u so I will So phi at omega u due to omega d would be minus 90 degrees, because at frequencies much larger than the pole frequency the pole offers a phase of minus 90 degrees please remember you can correspond this to the plot that we have drawn earlier ah if omega u is much larger than omega d then the phase due to omega d would be minus 90 degrees. So which means you are equation now becomes 0 minus 90 degrees minus 3 tan inverse omega by omega p is equal to minus 180 degrees or 3 tan inverse omega by omega p is equal to minus 90 degrees sorry plus 90 degrees or tan inverse omega by omega p should be equal to 30 degrees, now this is now clear what we are saying is that the each of the poles at omega p has to give you a phase of 30 degrees so that the overall minus 30 degrees so that the overall phases minus ninety degrees from the 3 poles the other minus ninety degrees comes from omega d as pointed out here, so that the total phases 180 degrees. Now this means that the contribution of each pole gives you 30 degrees right so what is the value of omega by omega p which is clearly tan 30 degrees, so we now have found out the frequency at which the phase hits minus 180 degrees this happens at omega by omega p by root 3.

What do we do next, we go back and plug this in plug in this relationship in to the magnitude condition and find out the maximum allowable value of A naught f which will give you the limit of stability when rather the upper limit of the loop gain when the system is marginally stable so we are going to find that out so let us go plug this back in into this equation and this gives you.

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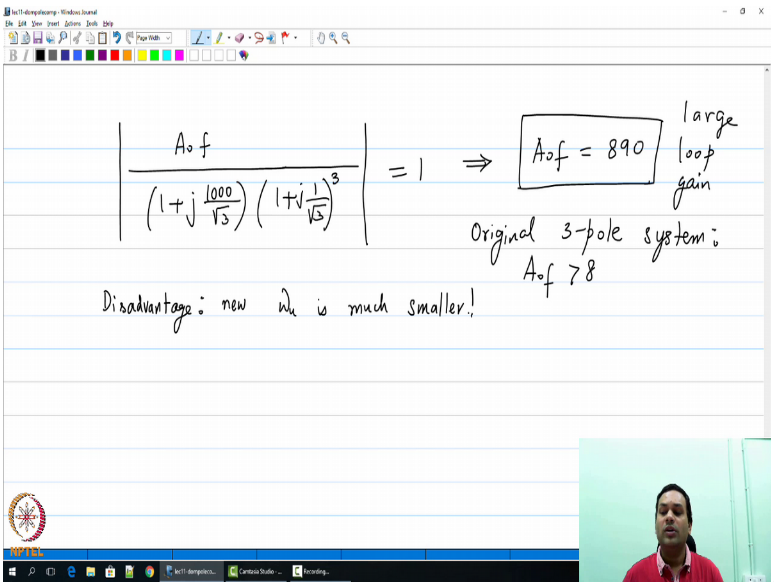

So, the magnitude of A naught f by 1 plus j 1000 by root 3 into 1 plus 1 by sorry 1 plus j by root 3 the whole cubed this should be equal to 1, and this of course tells you that this gives you the condition that A naught f equals approximately 890 and now you can see the power of this dominant pole compensation, if you had taken your original 3 pole system the upper limit of A naught f barely gave you two complex conjugate poles on the on the y axis, if you try to increase a naught f beyond 8 the system would have become unstable right.

Now, by adding the dominant pole the value of loop gain can now be much larger so you can see it is almost two orders of magnitude larger than the original value. So, this is a large loop gain compared to the original case, now what is this mean a let us try to get a little bit more insight if you tried to decrease the value of A naught f you can may push omega d a little bit further out. Similarly if you want more loop gain if you want to increase A naught f further you have to pull in omega d even lower than this particular value of 1 over 1000 of omega p.

What is the disadvantage of doing this, the disadvantage is you are constraining the bandwidth of the system, now disadvantage of course I should said I said disadvantage but very often you have no choice, so the nu omega u is much smaller but now when you this is the open loop omega u of the loop gain if once you place it in feedback the bandwidth can be much higher where you know depending on the actual gain that you need from the system.