Probability Foundations for Electrical Engineers
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Lecture - 01
Experiments and sample Space

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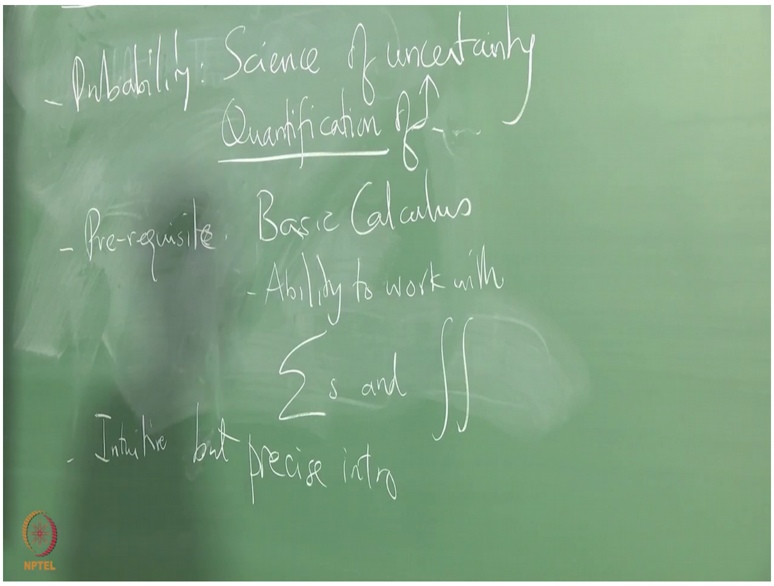

Probability is the signs of uncertainty. So, we are going to approach this business of uncertainty from a very right scientific perspective. In the sense that, yes we know that we cannot predict exactly what is going to happen, but in an average sense can we say something right, can we make predictions which are useful? These are general sentences which are cannot make, I mean be more specific right now, but as we go long hopefully you will right, you will understand what exactly, you know how we can quantify uncertainty right, whole business of probability is a quantification right of uncertainty. You can even use that. All these English is by the way very important, because without English all the maths right. Let me say it right in the beginning, English is more important as a maths to describe a scientific result right. So, the words finally, are the ones that used to conclude a paper right, or introduce a paper a technical idea and so on right. The maths is only the in the intermediary part.

So, that way I hope that this definition, that its a quantification as I did actually it captures right, they sense what probability theory is all about right. And this course is a first course as we write, as the name suggests it is probability foundations. The electrical engineering bit is added just to say that; yes most of the course is not going to be what something which is specific only to E E, but here and there we will take examples which are very relevant to E E right, which may not have been done in the maths courses. I will mention these examples as we go along right. The prerequisite for this course is basic calculus. First year calculus, both single and multi variable calculus right. Multi variable calculus is being closed for single variable; obviously, right, ability to work with summations and integrals right.

So, let me put double integral, because we will very quickly get to that right, single integral calculus alone, is not going to be sufficient right. At least integral in a plane right, which involves 2 variables right, you should be able to do. And you know which includes changing variables and. So, on right all those things will come to life in a big way out here right. These this is the basic tools that we will be using. And the goal of this course is a precise right, it is a intuitive, but precise introduction.

Why do I say intuitive, but precise, because it is just make an intuitive I mean throwing out right regarding position out the window. No right, it has to be both; otherwise its not worthy of being taught to, at a college level, upper under graduate level, third year and fourth year students, and even P G students benefit from something like this right, but at the same time it has to be intuitive also can if it is just precise and it just goes through the mathematics, without giving intuition that also does not help engineers right. So, it has to be combination of both intuition, strengthening intuition and being precise and as rigorous as possible.

Now in this context I must say that right, off late this. In advanced course in probability theory require, even more mathematics than what we will right be seeing here. We are not going to get into that right, like topics like measure theory and so on which are needed at the next level, will not be part of this right, because this is a U G course or entering P Gs right that level and. So, we will limit the mathematics to that right and the mathematical ideas to basic calculus ok.

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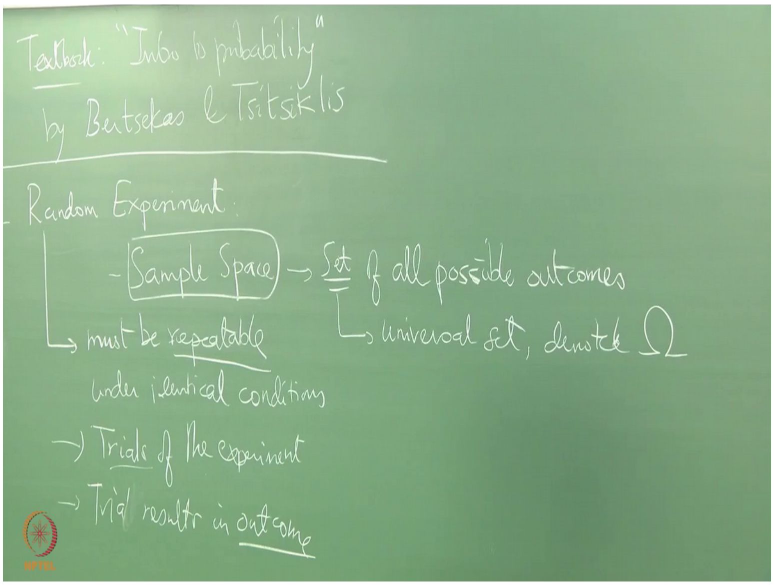

So, the text book which gives a very, this introduction to probability. This is a complicated name: I think I spelled it correct. Yes I think; that is what it is right. The second name is somewhat unfamiliar to us Indians out here anyway, but I think it is spelled correctly right. So, those of you who have not seen this book before, you can take a look at it, search for it online, and if it is possible, you can take it off the shelf right, and use it to whatever extent to you, I know that works for you right. We are not going to be covering everything in this book right, but the tone of this course is going to be very similar to, the level of the material in the book right.

So, what is this? What is this starting point right. It turns out that, this virtually all probability courses they start with a idea of random experiment right and a sample space right. So, then you have a sample space. What is the sample space, why do we need it.

All that will become clear when we define what. The random experiment basically right, is an experiment that you cannot. You can perform, but you cannot exactly control in the. I mean certainly you cannot control the outcome right, or it is something that nature throws at you, or some other persons do right, which you cannot control right. And the experiment itself must be repeatable, under nearly identical conditions; otherwise the probabilities of limited use to describe it right.

So, the probability can really come to your help, only if the experiment is fundamentally if its repeatable, under nearly identical conditions right. The same experiment should happen in the future; otherwise any analysis you do, of what is happening now, is not going to be helpful in predicting what is going to happen tomorrow right.

So, that is the strength of probability or in a way it is also the weakness right. It relies on the experiment being repeatable. So, the experiment must be repeatable. So, each run of experiment must happen with the same set of the same conditions. Like in other words right, this, the environment for running the experiment must be the same right. And different runs of the experiment are called trials right. So, we have what are called trails of experiment, the word.

So, each time you perform the experiment, you get an outcome, a result. So, the result of the experiment is called an outcome right. The trail results in, or it ends with an outcome. An outcome is the most basic observation that you have right, or the basic, the result that happens when you do the experiment. Like for example, the simplest random experiment the non trivial random experiment is tossing coins right, which all of you have seen since high school right. The classical theory goes well, a coin can come up only with either heads or tails right.

At this point we are not considered about any probability. We just identifying outcomes right. So, what is then coming back to sample space? What is this sample space? Basically it is a set of all possible outcomes, and that enumeration of all possible outcomes, is actually in some cases is the harder than you think it is right. So, the sample space is the enumeration of, or collection of all possible outcomes. And; obviously, then right straight brings in one of the fundamental prerequisites to deal with probability theory, which is sets, an idea of sets right, because I have used that word set right in the beginning right.

So, the sample space is therefore the universal set right, for that particular experiment right; so this right. So, let me not. So, please note that right. Just write universal set here, if I keep, I will write minimally on the board, but enough I hope to right make the point. And the notation we are going to use for this, which is typical in set theory, is this capital Omega right. Now there is again for electrical engineers you all know that this symbol is a more commonly used version for what for.

Student: Resistors.

For resistors right Ohm, but in this we will not have the need to look at resistors and capacitors at all right. So, we can go back to the original. I think the usage of omega is, for the sample space, or even older than, its usage of resistive element, but anyway. So, we will use right this notation rather than letter like s or something, because we can save the s for some, you know more mundane quantity right. So, we will exclusively use this omega right which is a important quantity for us, because it tells us all the possible outcomes, without right saying which is more likely or less like, nothing is a sort it just simple list right, it is only a list. So, the elements are listed without any particular ordering right. You can use whatever order you want. Remember in set theory right, a set consists of some collections of objects without regard to order right ok.

So, that fundamental idea will be very important here right. So, now, the question is, of course, I said earlier in some cases, in some real world experiment right, you will not even show exactly, you know what do you mean by this set of all possible outcomes. So, not every experiment is going to be as simple as, you know rolling coin or rolling a die or whatever, or some conisation of them or rolling 2 coins or coin twice and so on right not every experiment is going to be like that, by you know ahead of time yes this is only possible right. And the theory does not say anything about right, how. I mean the theory does not concern itself about the actual physics of an experiment or what, any experiment is going to come up with.

So, there are two distinct parts, which I think we should be aware of right from the beginning; one is the mathematical angle right, the other is a physical part of it right, and you have to keep them separate. The physical part is, which is specific to an experiment is. As I said listing, you know doing the experiment number of times, listing the outcomes and then some rare outcomes may show up once in a blue moon, but right, unless they happen. I mean you do not know if they are there or not right. I mean I let me not get into examples right now, but they are certainly very you know, lots of experiments with rare outcomes which happen once in a while, you know, and some even more rarer than others right.

So, this listing of outcome is actually right not as trivial as you think it is ok, but mathematics right does not care about how difficult it is obtain any data right. It only cares about, you give me an omega I will work with it; that is right what mathematics can do for you right. So, in this course we are mostly concerned with only mathematical manipulations right. Not the collection of data right. So, we will not concern ourselves is a good point, to say the limitation of this particular course right like most other courses around the world right. In this course we are not going to worry about, how we actually got a particular mathematical model. It is there we will work with it right. This is the mathematical model, 99 percent of are, I might even go as far as saying 100 percent of the examples here are going to be right of that nature. This is the mathematical model what can we do with it right.

So, where this, mean what is a actual physical situation is really well modelled by this model; that is not a question here, I can answer by enlarge. Occasionally we might say well this is used in this physical case that is used there and so on, but that is not the focus of this course right, just like other courses at this level right. So, the reason why I am emphasising that, is because this is a good time to say right. So, mathematics deals with omega, and it subsets right.

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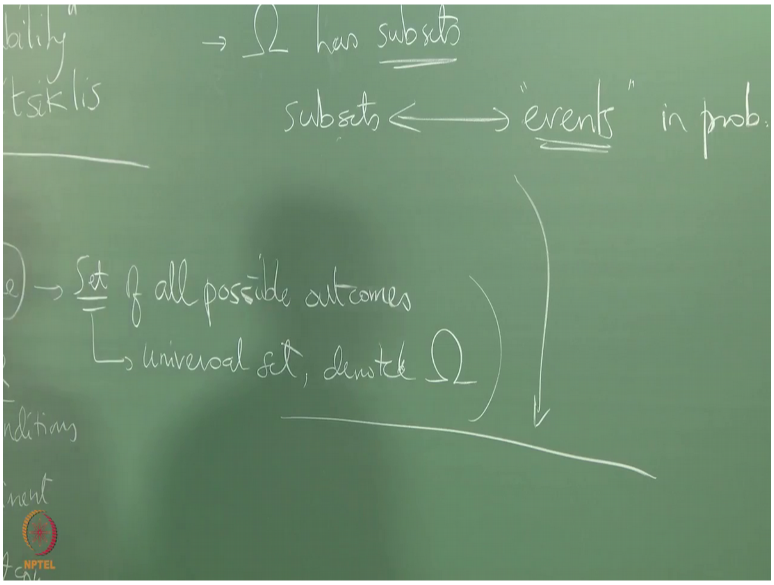

So, once you have a universal set, you can look at subsets of omega right. What are these subsets of omega? There is a very important thing about any sample space right. A smaller connection of outcomes; omega is a collection of all possible outcomes, somehow you listed them fine, but what of what is of interest in many cases is, what are the subsets you can form right, from a given sample space right. What do these subsets correspond to in practice right. They correspond to what we call events. Now the word event is used. Also just for clarification I am just going to, when I say use the word prob I will mean. I mean probability not problem, or any other expansion of prob right.

In probability theory, events is the term used for subsets. It is a specific use of the word event right. I guess when the theory was being developed, they needed a word to express ideas like right. For example, if you role a die right, the event that an even number shows up right, which is subset of the sample space right. If you say the sample space is the numbers 1 to 6, which is all the numbers that can show up in the die right, getting even numbers, the set of even numbers is a subset right and so on and so forth right.

So, you can appreciate the bigger the sample space is, the more points is has the more subsets it will have. So, our fundamental starting mathematical calculation is this right. Given that I have right, given, supposing I have, I am going to here. From next time onwards I will try to be more organised on this board right, but please bare with me right.

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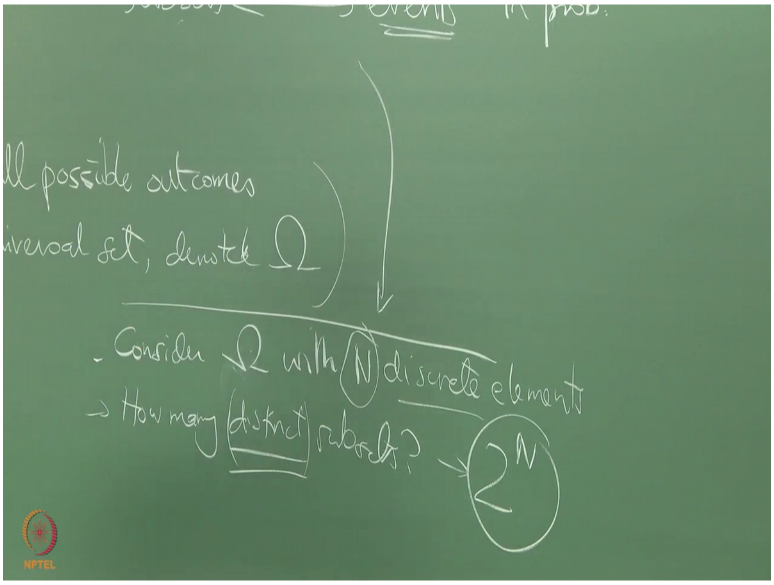

So, lest say consider omega with n discrete elements right, the most basic examples of in probability, or even in set theory for that matter right, are discrete sample spaces where you can count. If you have enough fingers in your hands feet whatever right, you can actually count n 100 200 whatever. Those are examples where you can actually count. And this is also extension to countable infinity right, which is corresponds to set of integers; 1 2 3 infinity right; that is also considered countable. So, countable is right a synonym for discrete right. So, in this case, I am saying well it has a finite number of elements right, countably infinite; obviously, is where that n goes to infinity ok.

How many distinct subsets can you form? What are distinct subsets first of all; a distinct subsets are subsets which differ at least in one element right. If you look at 1 2 3 4 5 6, if you look at 1 3 5 and 1 2 5. They are distinct, because 2 is right not the same as 3. And again in defining a subset you do not care which order you wrote them right. 1 3 5 is a same as a 5 3 1. Ordering of the elements in the set has makes no difference right. So, you also have by the way, the concept of equality and difference of 2 sets, two subsets are equal, if they consists of exactly the same elements written whichever order right. So, now, I am asking the question right. If there are n discrete elements in the sample space right, how many subsets can you form; distinct subsets. The key word is a. I mean we will not use this as, you automatically subsets are going to become useful subsets are going to be distinct, but for now I write that word downright how many.

Student: 2 power n.

2 power n, everyone knows this right. How do you get, what is simplest way of in getting to that number 2 power n. This is very important. I hope this is viewable right. I think it should be, there should not be any problem, unfortunately it came to the bottom right there, but I am assuming that. Why did this 2 power n come from? Each element is either in a subset or not in a subset right. So, if you form strings 1 0 0 0 1 1 0 0. So, for every element you put 1 or 0. How many strings of length n can you form?

Student: 2 power n.

2 power n with n bits right, it is as simple as that. You also include in this count of 2 power n, two very important quantities; one is the all zeros, where nothing is included. What is that set called? This 2 power n includes the empty set, is also includes the all ones which is the.

Student: Sample space.

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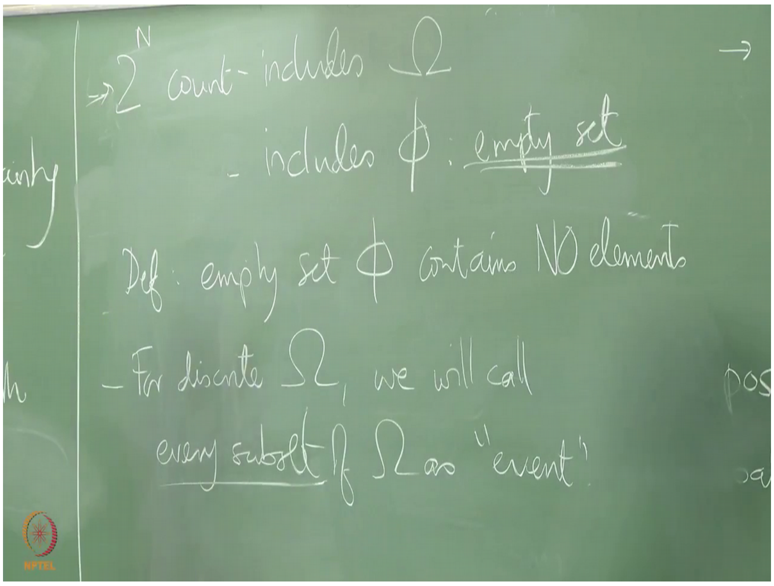

Sample space itself. So, let me talk about empty set. The count includes, and it also includes. So, for every sample space we have, every non trivial sample space which contains at least one element right, you have an empty set. Of course, the sample set with one element is not a very interesting sample space, because that is that is only thing is going to happen. There is no point in constructing a priority area around it right. So, in a way the right, the most non trivial: I mean the basic non trivial sample space is two points; head tail or 0 1 in different context right or whatever 2 or 2 distinct outcomes right. So, empty set, is a set that contains no element I have to write this down, because again it is a very important concept. So, definition empty set. I just write it as phi right. This is again a standard notation. So, you can be used in every experiment right, regardless of what sample space you have, you can always right formulate a set, which has no elements right. So, phi is in a sense, it is common to all sample space ok.

So, we have this very important calculation 2 power n right. And each of these 2 power n, including phi and omega itself right, can be considered an event right. In at least discrete case, we are going to consider. There is this fine line, we have to draw, will come, I will draw a talk about what about this fine line, about actually what sets can be right. What events can be included in our list, special list that we are going to talk about or not included right. There is a distinction there, but for the discrete sample space right. We will include all possible subsets right, we will, every subset of omega as event. Whether or not right, it makes any physical. Whether or not, this makes any physical sense to us or not right.

For example, if you role a die the number 1 4 and 5 may not mean anything physically, or 1 and 5 or whatever. You might say what is the point why do we want to include it, but for the for mathematical completeness, it completeness it turns out that right, it is important to, precisely define what is an event and what is not an event right, and right upfront I want to say right. For discrete sample spaces, we will basically allow every subset of that sample space to be called an event and so. What are they going to say, I am getting. So, if you have n points, now coming back to that example of n points, how many sets have that one particular element right? Before that let me write the notation right.

So, now I think it is important to introduce notation for the points in the sample space.

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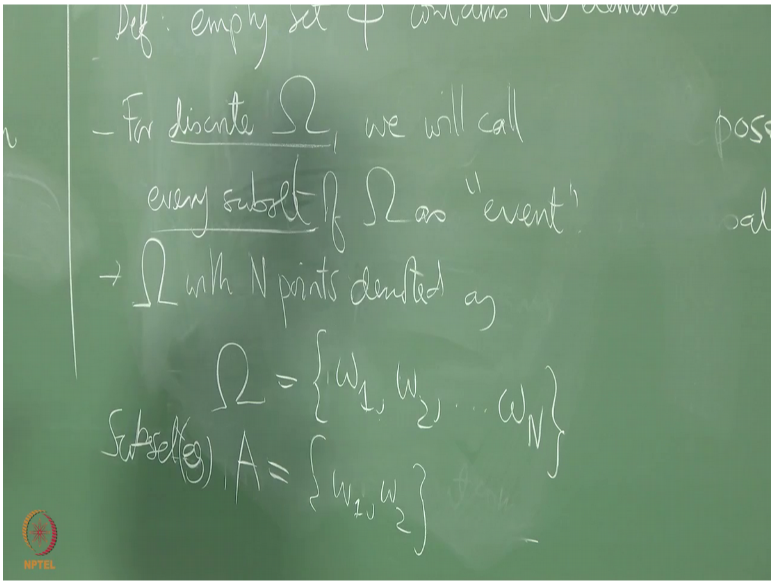

So, omega with n points, you are going to, you will notate it like this omega 1. This is a standard way in which the set is written with curly braces right. The same notation we can use right for the subsets also. We just use only right the smaller set of elements right, which are included in that; the elements of omega which included in the smaller subsets as that. So, each subset e g, some subset say a which can be omega 1 comma omega 2 whatever just an example right.

So, this notation I am. The reason I am doing this, is to emphasise that these curly braces will occur a lot in this course right. So, continuing with some more terminology right. So, basically we are saying now that outcomes of the experiment right, are the elements of omega, are outcomes of the experiment; that is basically have said. So, far in different words right, are identical to the elementary outcomes right.

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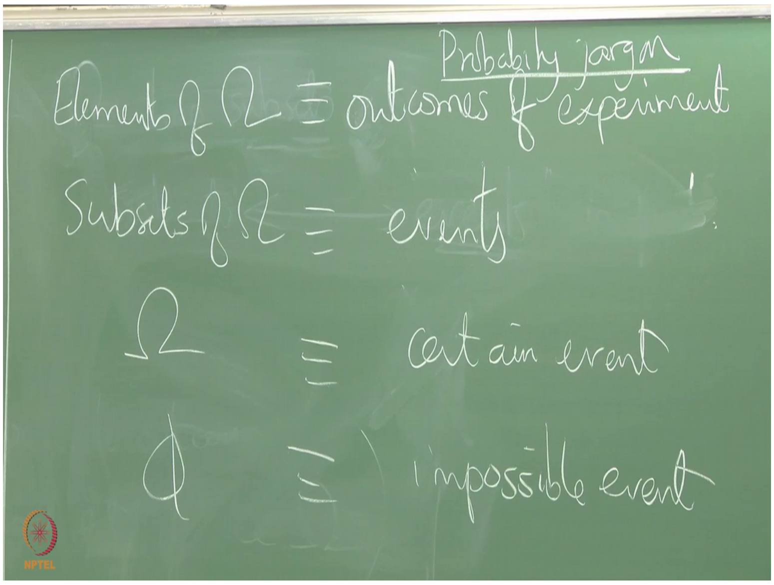

Whenever you are talking about outcome, remember right it is the basic observation you are making which will include in omega right.

So; obviously, elements of omega must be, or are the same as a elements outcomes of the experiment right and. So, this is like a summary. So, this elements then subsets of omega are events right. So, this is like a compilation of some terms right. So, omega itself, this is in the probability right jargon, as you might call it right in the jargon of probability theory. These on the right hand side you will list the few things right. What is omega? What is the probabilistic jargon for omega. It is a certain event, this is going to happen, no matter what right and phi. Of course, is impossible event which can never happen, why is the, why is phi called impossible, because.

Student: It has to be.

Some outcome has to happen right. So, you cannot have any trial of the experiment, nothing happens. One last thing before I conclude for today right, how do I say that a particular event has happened on an experiment right. If an experiment has these n outcomes in the sample space right, and I define my subset. See these subsets have to be defined and named, and all you knew if want to say associate name or a title, you should beforehand right. You say even number on a roll of die, whatever or a square whatever. So, you identify that event, that subset, earlier then; if the outcome of the experiment lies in this subset, if it right the outcome.

Remember on every trail of the experiment you cannot get both omega 1 and omega 2. You can get either omega 1 or omega 2. It is not possible to get multiple outcomes right; that is a fundamental theory. So, if either omega 1 or omega 2 happens, if it shows up in the experiment, then you say the event has occurred right.

So, since I am running out of time for today’s class, I will stop here.