Probability Foundations for Electrical Engineers
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Lecture - 05
Part A

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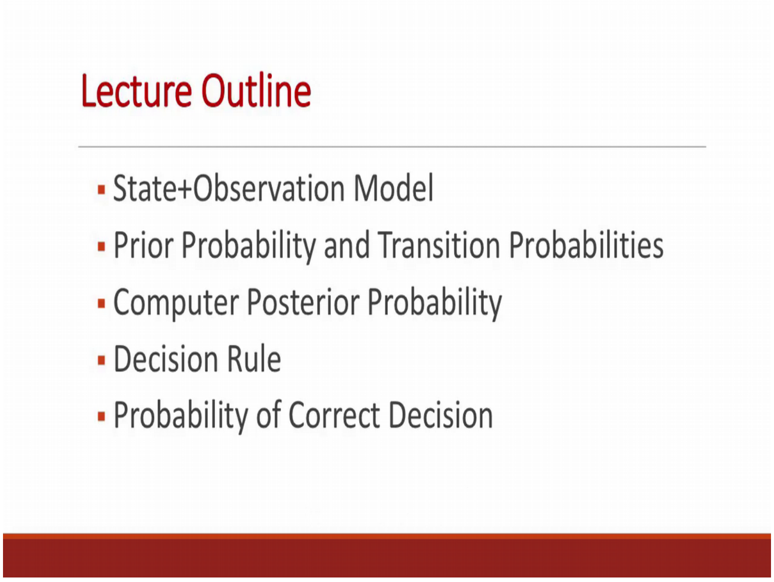

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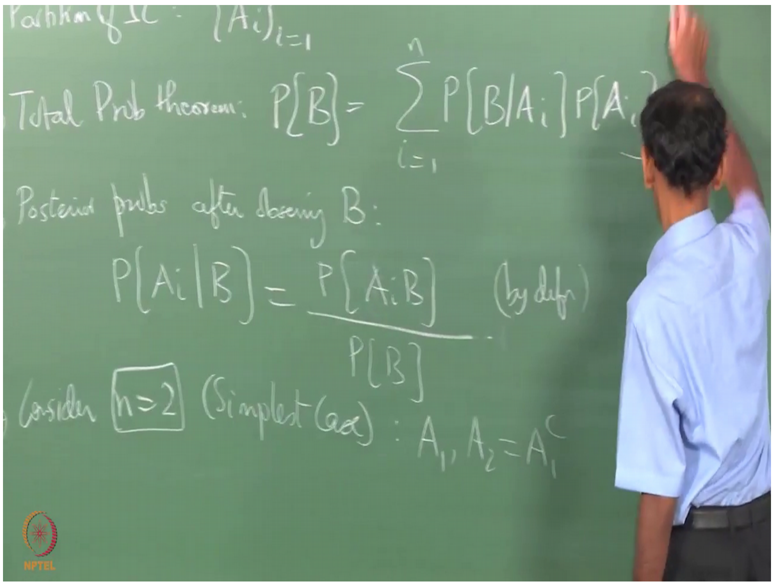

This formulation is a little on the abstract side we need to look at it some con concrete examples which I will do in the course of today's class. I went back and looked at these examples and it is a good point to bring them in actually. So, you know why the general n is a little complicated to look at. So, let us look at the simplest case which is what n equal to 2. For a useful partition of omega you need at least 2 events A 1 and A 2. So, let me just start off here and then continue on the simplest case.

So that means, you have only A 1 and A 2. Actually it is a very important case also as we will see. So, here we have A 1, A 2 with where such that A 2 is the complement of A 1 actually because you want them to be disjoint or mutually exclusive and add up to omega so obviously, A 2 must be the complement of A 1.

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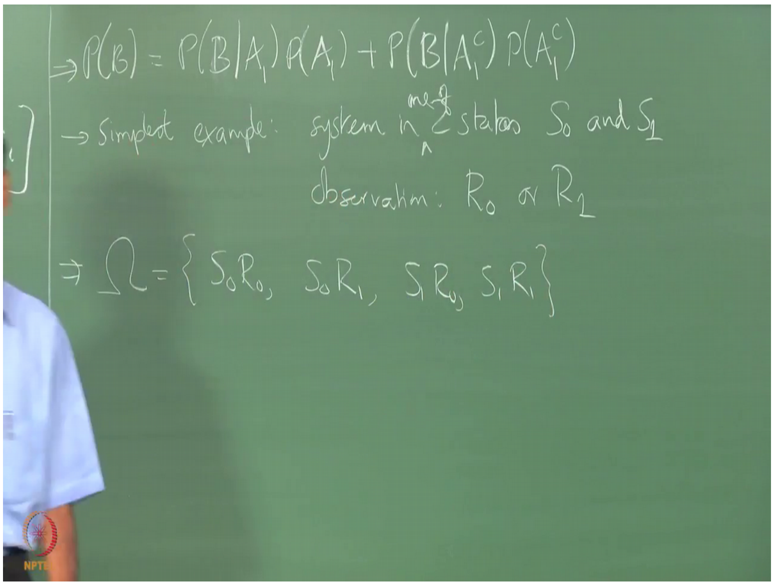

So, what is a probability of B now, any event B will be P of A sorry P of A B given A, P A plus P of A given B given A complement P of A compliment this is very commonly used formula which I am sure you have seen before. Let us see how far we can go with this. And of course, you can talk of the posterior probabilities will write in terms of both A given B as well as A complement given B both of them will be the posterior probabilities in this particular case. So, let me I define A 1 and A 2. So, let me write A 1 here with the proviso of course, that A 2 is A 1 compliment. So, you do you do not have to write A 2 if you do not want to give.

So, the most common manifestation of this situation happens where, this is not I would not say most common the simplest manifestation. Supposing you have a system in which can be in one of two states as we will call them S naught and S 1. So, the simplest; some system in one of 2 states not just in 2 states right, I will make sure that I write this 1 of 2 states with prior probabilities P of S naught and P of S 1. And then you make an observation which is one again 1 of 2 possibilities I just write it as R naught R or R 1 actually this is also an odd see there any S naught or the system is either in S naught or S 1. So, in a given run a trial of the experiment the system takes one of the 2 states S naught or S 1 and then you make an observation which is R naught or R 1.

So, it is essentially your omega at the sample space is what just has four points. This experiment is not complete like until and unless you make an observation. So, S naught S 1 by itself does not make up the sample space right. So, the sample space consists of one of these possible states and you know one of the observations. Therefore, obviously it is S naught R naught S naught R 1 S 1 R naught and S 1 R 1. So, you have four possibilities for omega. Now rather than writing this sample space like this is more the intuitive to write it in the form of, they state on the left and the observation on the as we will see just very in intuitive to do it that way or you represent it like this S naught S 1 R naught and R 1.

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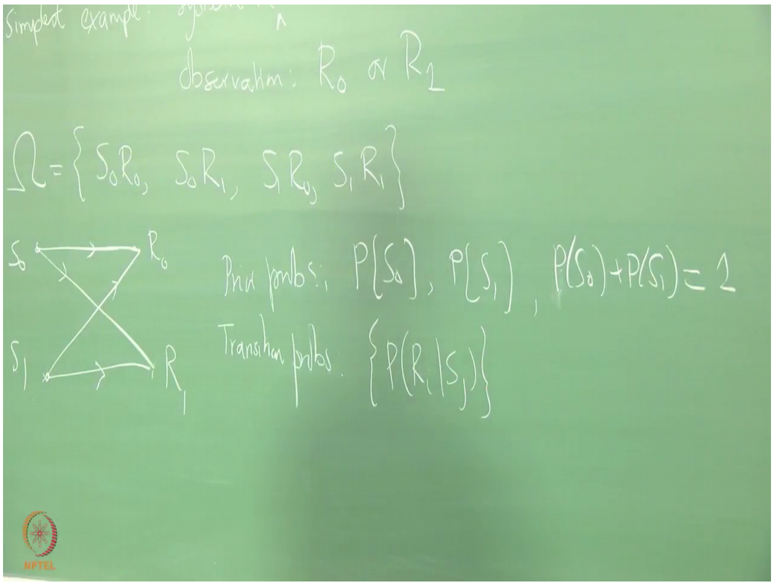

So, here on the you have you put down what you are what the observation is in and on the left you have the state which most cases is hidden you do not know what state the system is in. Then you have these connections, what are these connections? These are all the possibilities that you can have, you can have the system in S naught and the observation being R 1 and etcetera etcetera. All 4 connections are all 4 this, these 4 points are represented by connections.

So, where is this situation for example, effect communication engineers a lot it is the most basic digital communication system there is S naught S 1 is the hidden bit that is to be transmitted and R naught an R 1 let say you have your a fixed system is given which spits out again a single bit is observation R naught or R 1.

And let us say that you are at this point in the lectures we will assume that you receive or see R naught your only you are going to say S naught or well let us say you make you have already decided that if you see R naught your you are going to say that S naught is a hidden state. That means, actually if you get R naught S naught was sent and so on, and for a R 1 let us say use you say that S 1 is sent. So, there is one possible way in which this thing happens. Another very different setting, but the same mathematics is when and what, S naught and S 1 represents 2 medical states of an individual drawn at random from population sick or not sick or has a disease does not have a disease and what is R naught in R 1 for that.

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Exactly you run a test which can come up either positive or negative and can come up positive also for an individual who is healthy otherwise. So, that sometimes called a false positive. So, and there is something other applications of this particular model, so we will spend some time on it to understand exactly how the calculations go. So, what are the prior probabilities the prior probabilities are clearly P of S naught and P of S 1 which have to add up to 1 let me write it as. That is exactly, they going to part here well I just have to a partition. So, turns out that omega has partitioned in two ways by S naught and S 1 exactly. So, then you have to complete the probabilistic description you need the so called transition probabilities.

This, if the state is in S naught what is the proper given that the state is in S naught what is the probability that you going to make observation R 1 or R naught or R 1. Now the reason for might be choosing this notation 0 1 0 1 is again motivated from going in digital communication where or binary communication if you will not just you know more generally digital by binary you are sending 1s and 0s you are receiving 1s and 0s that is why I am writing it as indexing it 1 and 0 so called transition probabilities which are what they are in the context of communications this will be a collection of four of them which I am going to write like this R i given S j where i can assume 0 or 1 and j can also assume 0 or 1. Remember this i and j you should use 2 different letters here because they can be independently chosen to be 0 or 1 or separately chosen to be 0 or 1.

So, here what are the rules in the transition probabilities have to follow. So, given S naught you have you must have we in fact, we say said this did we did we maybe I did not make this point last class let me make it now. So, given S naught you can you can get only R naught or R 1. So, therefore, probability of let me expand it out and write it. So that, you have R naught given S naught plus P of R 1 given S naught this must be 1 and also the other one, the other the equivalent statement with R 1 sorry not R 1 S 1 R naught given S 1 plus P of R 1 given S 1 this also must be 1.

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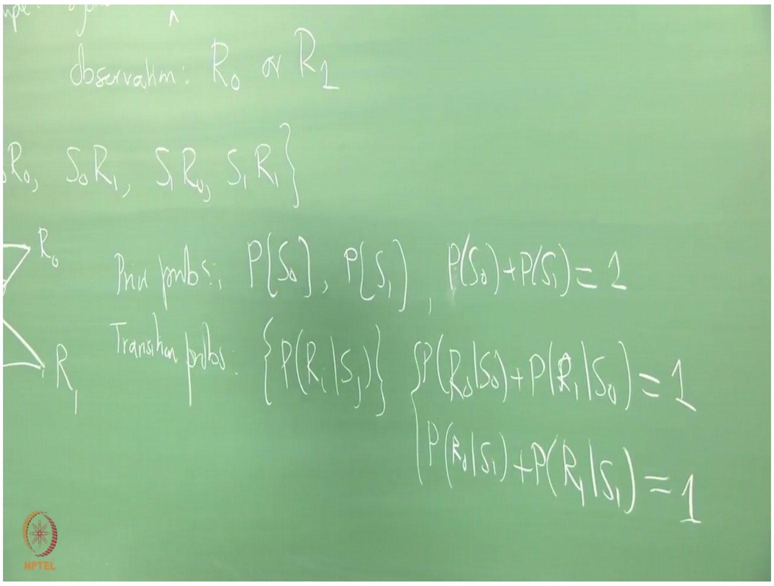

So, this collection of probabilities is the input to analysis. So, you assume that how many parameters have I introduced here I have introduced 6 parameters the 2 prior probabilities and the four transition probabilities. Actually they are not independently 6 of them there is only independently only 3 of them, if you know P of S naught you know P of S 1. So obviously, you do not count them as 2 independent parameters similarly. Therefore, essentially we have let say this and then for symmetry sake we can pick P of R naught given x naught and R 1 given S 1 or something as 3 starting numbers.

So, how we get these numbers that is it different issue as I said that is a complicated situation and needs a lot of statistical observation and testing and so on before you can say especially in the case of sickness and diagnosis and all that it is not at all its trivial thing to do. But we are not going to concern ourselves with how we get these numbers again. So, we get some numbers which are internally consistent in that definitely we require this and this and I mean these 3 equations to hold.

So, what can you now say about for example, what would be this B, this B would be let us say the event that R naught is observed. So, what would be the total probability of B here it would be, probability that R naught is observed remember I said R naught is an observation.

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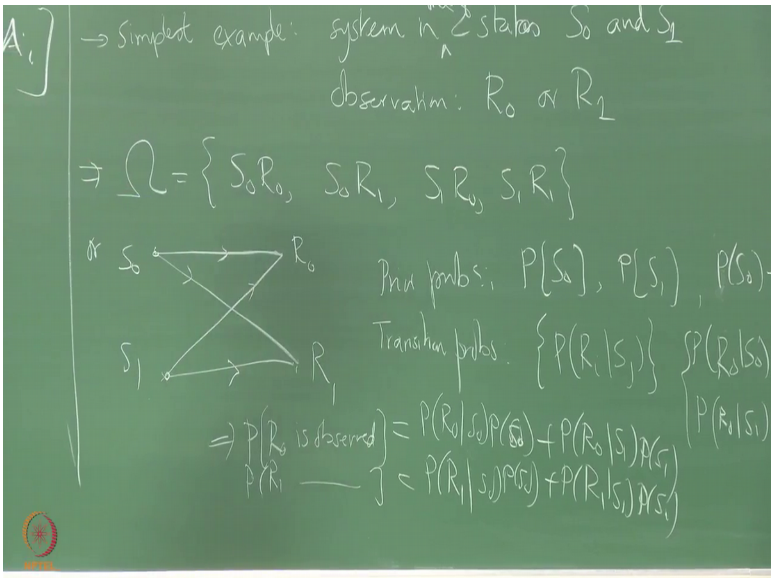

So, this is an event R the observation of R naught so obviously, this would be using total probability what would it be you condition on S naught and S 1, R naught given S naught given S naught plus P of R naught given S 1 P of S 1. And if you write a similar equation P for P of R 1 you would find that R naught and R 1 are also the probability is observing them at will add up to 1. It should be very clear. P of R 1 observed is P of R 1 given S naught P of S naught plus P of R 1 given S 1 P of S 1. Now, please add up these two and see for yourselves that that these they will have to add up to 1 because of these conditions we have written put on earlier.

Now what would be the posterior probabilities which are now going to I think is assumedly write down you can write them down yourself. Given that you observe R naught what is the probability that S naught was the actual hidden state as P of S naught given R naught been obviously, that can be written with you know with all the stuff that we have already said it is not a big deal.

Remember that, this joint probabilities what you should use in numerator not any conditional probability by itself. You can of course, express this joint probability as P of B given A i times P of A i the equivalently here this joint probability that S naught was actually the hidden state and R 1 was and R 1 was observed. So, that is a joint probability, it is not a conditional probability. So, the wording is very important let me just maybe write one of them, P of let say S naught given R naught.

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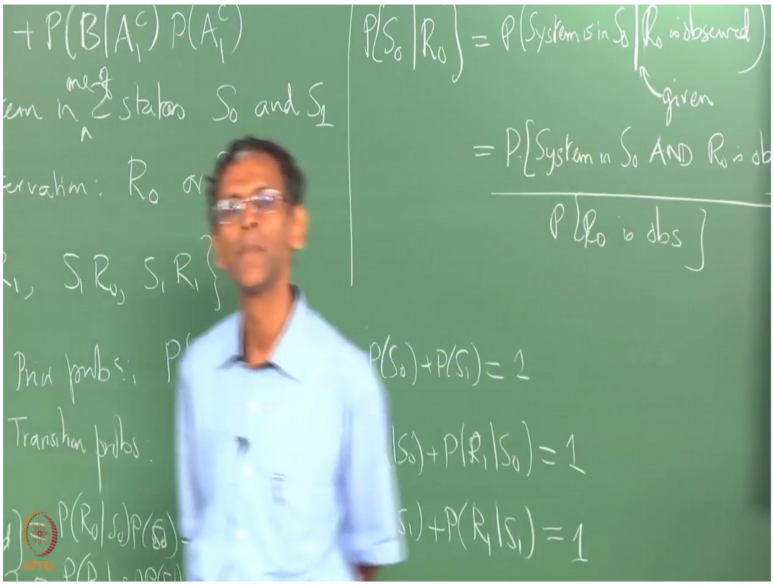

I am not going to write the expression, let me first, let me this is the probability that system is in S naught given that R naught is observed given that vertical bar is their conditioning statement. And again by definition or whatever based or whatever you want to call it this is the probability that system is in S naught and R naught is observed divided by what, the probability that R naught is observed. So, note the difference between this vertical line and this end right.

So, that would clearly be this joint probabilities is the probability of this point and always this is going to be smaller than this. So, let us use this simple formulation to figure out for example, the probability of error assuming that if you see R naught you are going to decide that S naught was they did not state. Let us say we have a decision rule like that we will come we will revisit this decision making business maybe a little later, but we will revisit delay later. But for now we will assume that the decision rule is fixed; that if you see R naught you are going to assume S naught if you see R naught you are going to assume S naught if you see R 1 you going to assume S 1. Let us assume; let us go with that decision rule and see what where it takes us fine.

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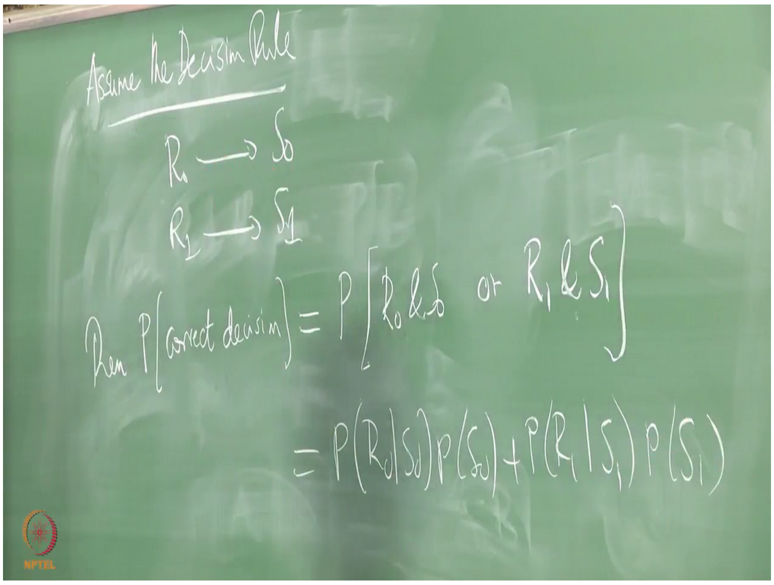

So, assume that it to then you are interested in the making correct decision, the probability of correct decision for this particular decision rule. Now, this c d correct decision I am going to write even later on abbreviated using c d right. So, again we will use conditional probability to write out this probability of correct decision condition on S naught or S 1. So, this is an event when do you get a correct decision supposing it actually was if S naught was ended actually you saw R naught you made a correct decision. Supposing the system was not S naught and you saw R 1 then you made a wrong decision.

So, what is the probability of correct decision? This making a correct decision is; obviously, also an event. So, how do you write this out? So, this is exactly the probability of getting this and getting this. So, in terms of S naught R naught, I will just write to abbreviate this, I will just say R naught and S naught which is the same as the R naught S naught R naught or R 1 and S 1 that is this point. So, clearly this can be written in terms of conditional probability.

So, these two again are obviously, mutually exclusive and so therefore, this is the probability of R naught S naught plus the probability of R 1 S 1. So, this is the probability of R naught given S naught probability of S naught plus probability of R 1 given S 1 times P of S 1. So, for the fix system this is the probability of correct decision. Alternatively you can also look at the probability of error and you can clearly check it out for yourselves; probability of error must be 1 minus probability of correct decision because there is no other third possibility, either you are or you are wrong right.

So, let me not give any numbers at this point I just want you know, we will be coming back to this example later on. So, I just keep this around. So, of course, in our goal in designing these communication systems to make them work reliably, so nobody would write would I mean this a very basic level of the discussion where you given a the whole thing just making measurements out of it you even told the decision rule.

So, the next thing would be is it in fact, the best thing to do that is if you see R naught should you or should you just close your eyes and always decide S naught not necessary you could decide the other way also, when does it make sense to decide a different way, so that is what we will look at slightly later. But for now I think I just wanted to you know introduce this concept of condition of how the condition probability comes in into this as analysis tool.