Probability Foundations for Electrical Engineers
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Lecture - 10
Part 2

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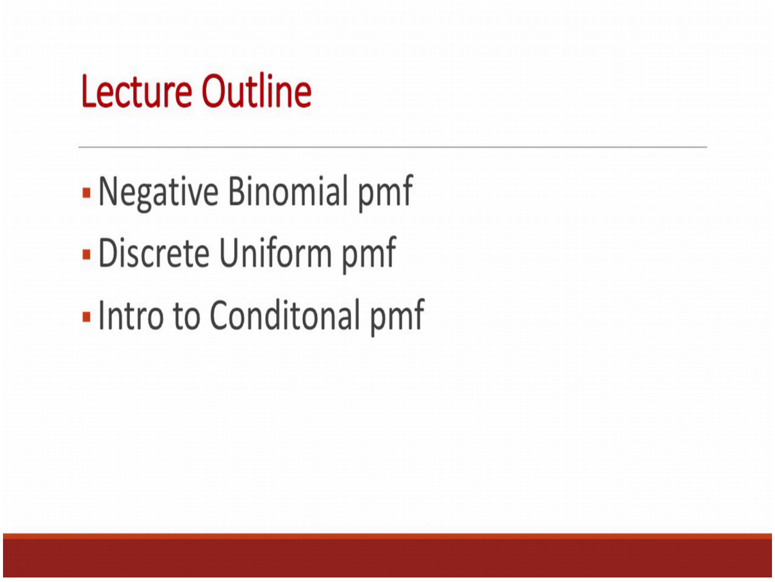

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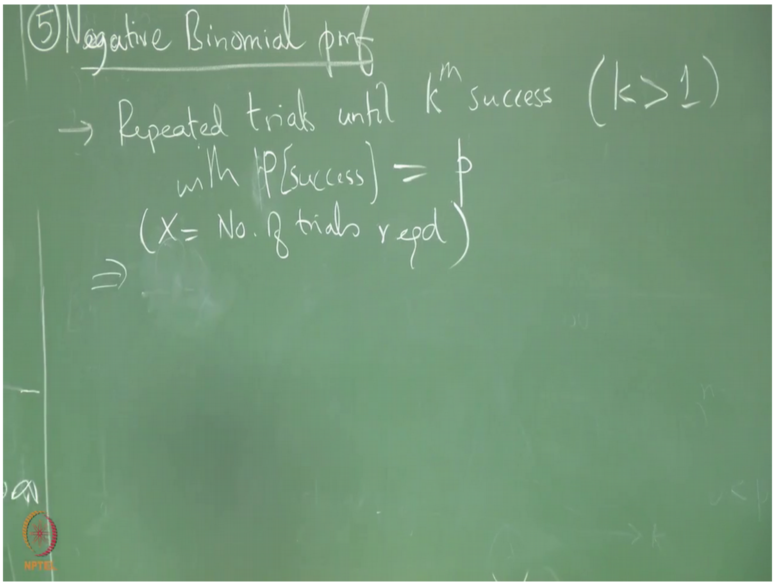

The next one, somewhat non trivial and I get the negative binomial pmf, which number should I use here, 5? For this I am going to make sure that I say correctly. So, what is this negative binomial pmf? Have you seen this? Is the number correct 5?

Student: Yes, sir.

Or is it should I put something else? I think it is ok. This says repeated trials until k-th success, where k is typically larger than 1; not, if you said k equal to 1 then you are back to geometric. Now, I am looking for repeated trials until the second success or the third success or whatever. Again the probability and each success is P, a small p. So, what happens to omega x? The range space, so, what will be, what is x first of all? X is number of trials required. That is a random variable, x. So, what is omega x?

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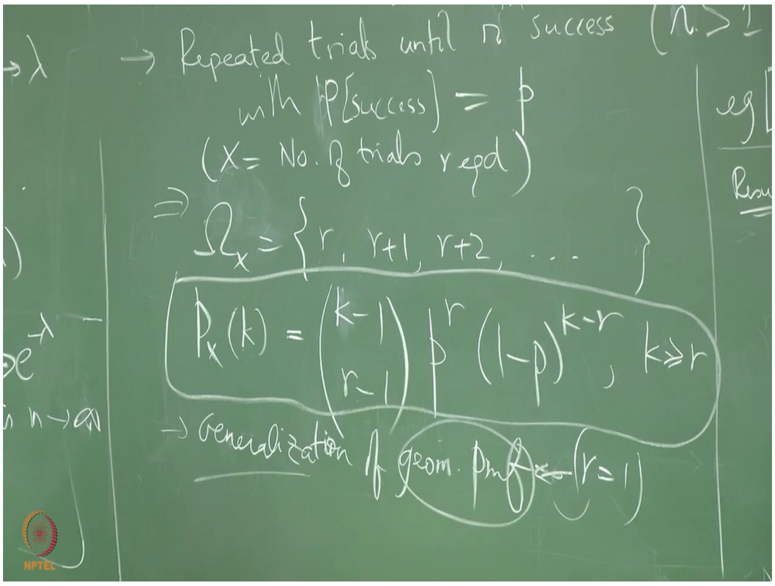

You need at least k trials for to get k success. So, it has to start from k, can I, cannot maybe should I use k or not. Let me not, again I may need k. k now becomes a fixed parameter; it is no longer the free variable of the pmf. So, maybe we will make a small change, so, that I will be uniform here. So, this k is not the same as this. So, I do not let me use r-th here, I am sorry about this, but I need to change this. Let me put r-th success. r is now an integer. Please make this small change. So, omega x in terms of r is what? r starts from r onwards, but you can get r, you can get r plus one, r plus 2 like that. Any of these are possible.

So, what is the probability that you need exactly k success, where k is one of these integers. How do you write p x of k? I am talking of r successes in k trials, where k is at least r obviously, but in general I want an expression. Again combinatorial terms should be needed here. So, what we are saying is, if you want r successes last one has to be a success, because you are going to stop counting at a success. So, how many successes do you need before that? You need k minus 1, sorry, r minus 1 success you need before the r-th success and how many spots do you have? You have k minus 1 trial for the r minus one success. Therefore, this there is a combinatorial term here which is basically r minus 1 choose or sorry it is k minus 1 choose r minus 1 and you have a sequence of successes and failures. How many successes you have? How many failures you have exact? You have exactly the r successes and you have k minus r failures.

So, what will be out come here? How many success do you? You have we have exactly r successes because that is what we want. You have exactly r successes and therefore, k minus r failures. So, what again you should this by this by itself is not complete you should say k greater than or equal to r and it is understood the k is an integer, which means to say k equal to r plus 1 and so on. So, this is the generalization of geometric pmf and to show that in general this adds to unity is not that simple you can go try it at home. I will just you know hint at what you have to do for k equal to 2, how do you go at it for k what is the next for one what we see for one sorry for r equal to 2, for r equal to 1 you have geometric. What happens when you said r equal to 1, you have to get back. So, what you have to get the geometric pmf has what r equal to 1.

What happens if you set r equal to 1 out here? This is always 1, k minus 1 choose 0 is always 1. No matter what k you take and you will get exactly p to the power of 1 minus p to the power of k minus 1, which is exactly what we wrote for the geometric case, isn't it. So, this is a generalization. What do you get for k equal to 2; for example, you can obviously, or sorry r equal to 2, you can always you know this will be k minus 1, choose 1, so, note that this fact, this is the same for all well I am sorry. No, it is not the same for all k. So, it is k what is k minus 1 choose 1? It is basically.

Student: K minus 1.

K minus 1 itself; so let me look at it here.

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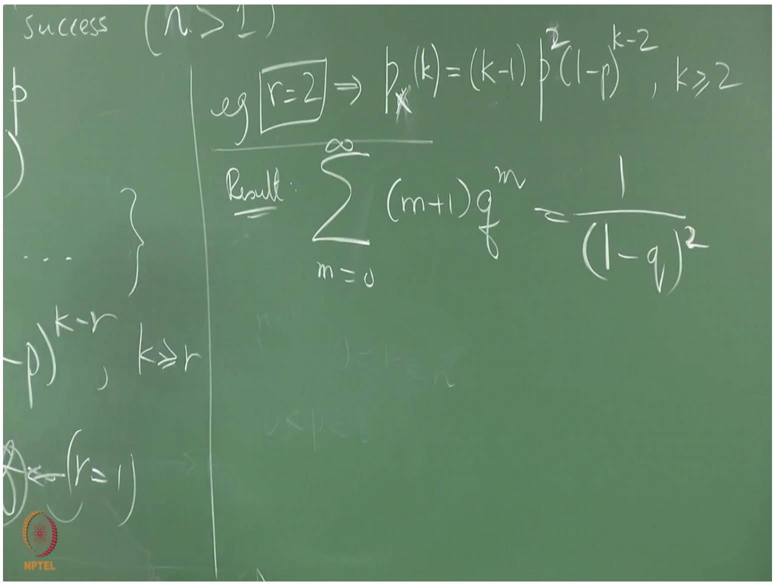

r equal to 2 gives you p x of k is what k? k minus 1 choose 1 is exactly?

Student: K minus 1.

k minus 1 and you have p power r, 1 minus p; so p square, 1 minus p to the power of k minus 2. Now this is somewhat easy to add up, because this p square does not depend on k. So, what tools would you would you use to add up this sequence? What is the tool you would use, itself? It is geometry, it is related to you I hope you have seen this kind of a sequence at least in your dhp classes the latest time you have seen it. You have the terms of the sequence being a product of an arithmetic and geometric progression. It is a very again very important type of sequence which arises a lot in analytical study.

So, the result is basically this, sigma let me write it down here, so I do not mess it up. This m plus 1, you remember this m plus 1, some q power m, from m equal to 0 to infinity. What is this is 1 by 1 minus q whole square. This is another result which I expect you to remember. That is why I am writing it. Of course it is q is between 0 and 1 that I do not have to say that. So, this q is basically there is 1 minus p and you can manipulate this summation to come in this form. Even though you are going to be summing only from r equal to 2 to infinity or sorry k equal to 2 to infinity, you can always manipulate it to get it in that form and use that you can show that easily for k equal to 2 and then maybe you can use induction if you want to show it for any arbitrary.

Student: (Refer Time: 8:54).

Any arbitrary r after that. So, I think I do not want to get any more into this then I will leave it at this, but this formula again is, I consider it fairly basic and I think it is fair game to expect you to know it, even if you can not exactly remember it you should be able to reason it out. Everybody in this class should be able to reason out this.

Once again, you have exactly r successes in k trails, you have this p power r, k minus r, 1 minus p power k minus r, the last one is a success you have r minus 1 successes in k minus 1 trial. So, you have this you can choose these locations in this many ways. So, that is just like the binomial. You get a binomial type coefficient out here. Without this there it will not work clearly, because this r minus 1 successes can come anywhere in the first k minus 1 trials.

You can get them all in the beginning and then you can get a string of failures, then you can get the k-th success sorry r-th success. So, I think we will let us move on again this is interesting pmf in many ways. So, we should keep this also in our list and so next. And let me just conclude this list of pmfs by looking at in a way something which could have been said earlier, but I am saying it now; the discrete uniform pmf, which we have sort of used implicitly in our dye throwing experiments, dye tossing.

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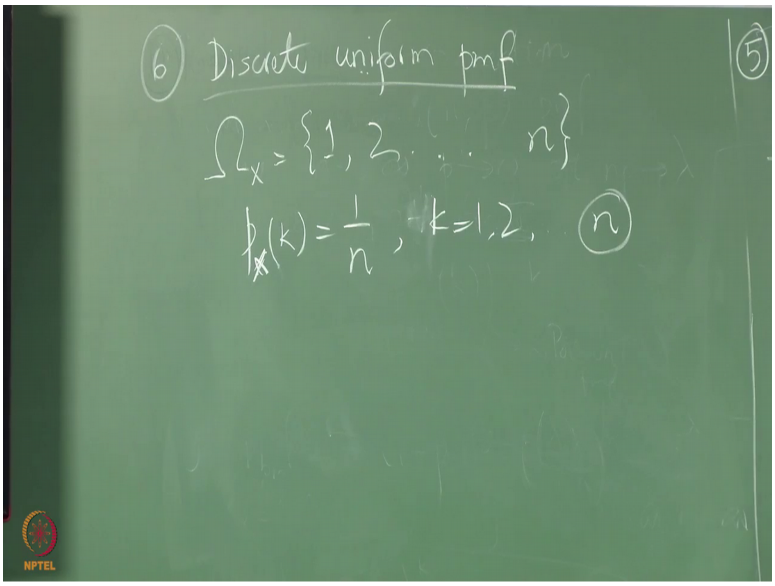

So, end this series on a very simple note. So, here omega x, it is very simple, it is just 1 to n. n is finite now, n cannot be infinity in this case. What is p x of k? A uniform, the term uniform tells you that p x of k is exactly 1 by n for all k. Let me say k equal to 1, 2, up to n; n has to be finite. This model cannot be applied to any situation where n rows without bound.

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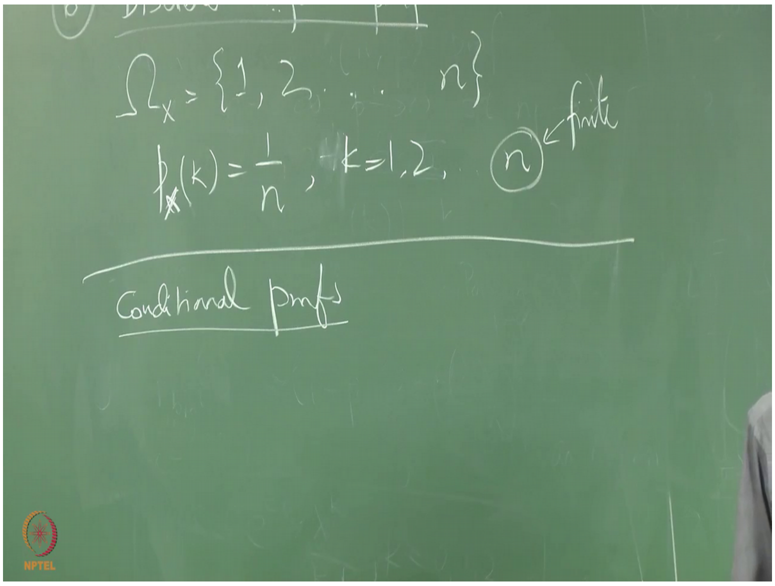

So, this I said just now is a generalization of dye tossing to any situation, where you have a bunch of equally likely outcomes and you can always saying is putting numbers on those outcomes from 1 to n.

So, we have equally likely outcomes you can be immediately map it to a discrete uniform pmf. This should not be confused with the continuous uniform pmf that we will study later on. The 2 are distinctly and totally different. They have some commonalities, especially the title is the same, but that is why the term discrete is very important here.

This is where our knowledge of conditional probability is going to be very helpful. So, there is no reason why you cannot ask for given that, for example; you did not see a success in the first n trials, what is they are going to be the pmf of number of trials needed for success. There is a memory less property as far as geometric pmfs concerned, but in general, supposing you know that the random variable x, something about it, you know some information about it, like it is at least 5 or at least some number m and r is between 1 and 10 or something, what can you say about the distribution of x, given that x lies in some interval. That is the most common form in which we see this conditional pmf. So, maybe I will take it up in a more detailed fashion tomorrow, I just want to introduce a point now.

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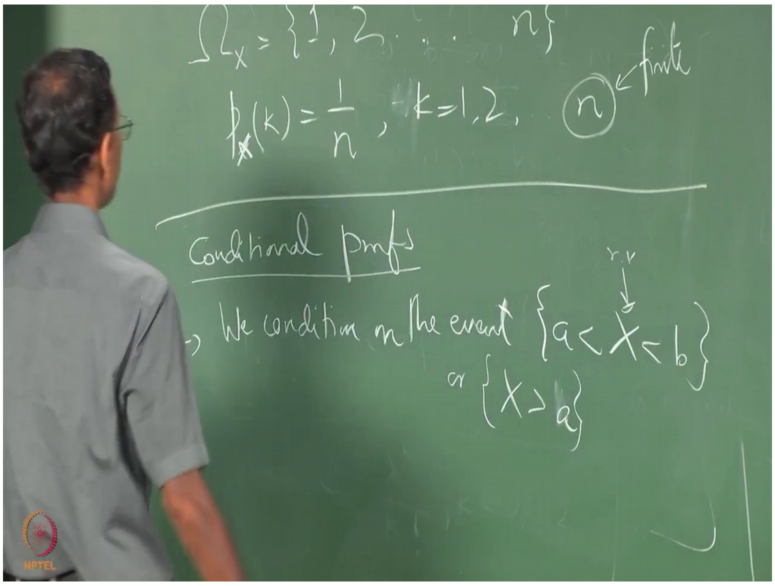

So, let us say, we condition on the event a less than x, less than b; so, this x is our random variable and which this includes for example, a less than x less than b or I mean could even be equal or for example, x greater than c or x well let me just say x greater than b, it is something like this. It obviously, the conditioning even has to be very clearly let me put a here not b, these 2 things are pretty much the same, if you say b is unbounded. So, I should use a only to make these 2 sort of look the same.

So, now, this is an event hopefully. You have chosen the numbers a and b such that that even has nonzero probability it has positive probability, you can say given this that the fact right that this is happened or this is where x lies in a particular experiment. What can we say about the probability is that x is some number k or some value x k not just k. We have to look at x k.

So, I think instead of looking at the general case, what we will do tomorrow is to start this discussion again. We will stress it ourselves to the case at omega x is some subset of integers rather than looking at some arbitrary x i and then we will give some put some flesh on this point. The most important case is the geometry case, where you know for example, what is the meaning of x greater than m, for example, the geometry case, it means that there is been no success in the first m trials. You had a string of m failures mfs, what is the probability that the m plus 1, is going to be a success, that kind of or for the same thing with the Poisson.

So, these kinds of questions are what we are going to answer, we are going to look at tomorrow. So, we will stop here today.