Probability Foundations for Electrical Engineers
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Lecture – 49
Important Continuous Distributions

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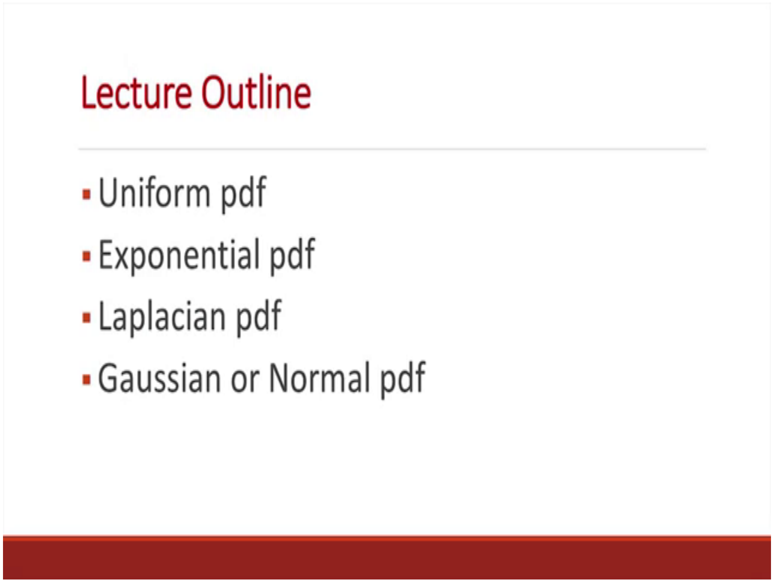

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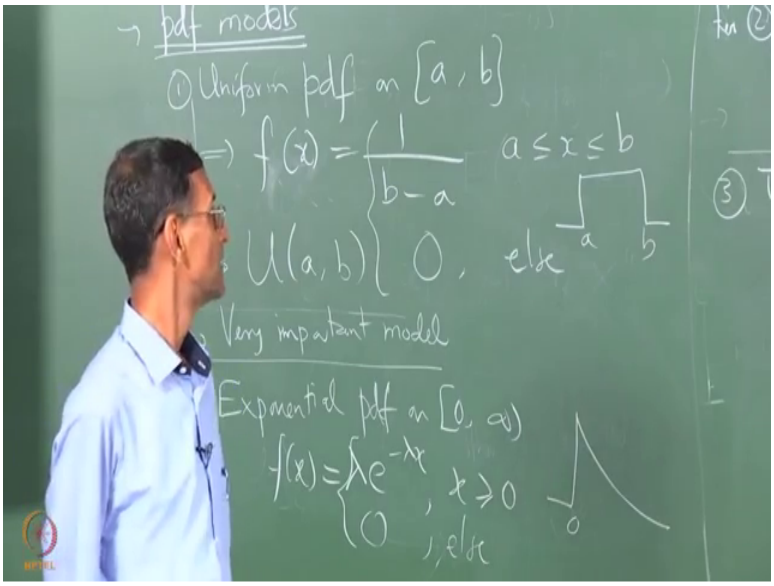

So, we look at important pdf models right. So, this is obviously, much a nicer territory for all of us right, we are not going to talk in abstract terms you are going to talk only in very very concrete examples. So, we can always talk of omega equal to r x of omega equal to omega being boom we have a pdf. I mean up we have the situation we can do this. So, we will look, I will do some numbering the uniform pdf right.

On some closed interval a b actually does not matter the interval is closed or open we will just assume close, but it is the same there is no difference whether you include or exclude a or b all right. So, what is this uniform pdf? Again I am, since I am not I do not need a random variable so I am not going to put subscripts all only if I define an x do I need to put f x. What is f of x? 1 by b minus a for a less than x less than b and I said it because of the nature of intervals and so on it does not matter whether you include or exclude, but for the sake of completeness I am taking it like this, that is all right.

So, this is one of the most importance continuous situations that you see in practice as I have we already seen this in the spinning pointer case. More importantly there is 0 1 that is even more in a sense more basic than 0 to 2 pi because that is the basis of all continuous random number simulation that you do with the computer. Every software package will definitely whether it is of the type of python every scientific computing package always includes a random number generator in 0 1 all right.

So, we will write that you know we need some notation like for example, we wrote binomial, Poisson and all that. So, uniform is a little on the big side, so we are as the big. So, it is say you know the u uniform is often written as U a b with some script u instead of writing uniform it is abbreviated to U. So, and of course, by domain extension which I forgot to say I have to say that. So, this is the domain extension here. So, a and b can be any two real numbers remember, a can be minus 1 and b can be ten does not it makes no difference. The only thing you want is you cannot allow either a or b to become infinitely large then of course, this reciprocal does not make any sense right. So, the idea is that you take some to any two finite numbers a and b and you can define a uniform pdf and that over that interval between them.

So, this is a very let me say this is a very important model because it says that what is the most important thing the probability depends only on the width of the interval and not where it is. Now, to go to some non uniform models all the other models of course, are going to be non uniform, the next one we will just list is this exponential pdf all this. This again is very important because a lot of life times and so on are observed to fall observed to follow this model. So, here there is a single parameter here lambda, non negative parameter lambda, all right.

So, note that both of these f of x is more, the more of these pdfs have this has a jump at a and b, this has a jump at 0. So, effects that means, x less than 0, here of course, it means x less than a or x greater than b that this more very convenient to write it like this rather than. So, this if you think of this is this going to be a b like this, this is going to be this is 0. So, this random vary any random variable with this pdf clearly has 0 probability of being of taking on an valuation and negative interval right. So, along with the CDF the pdf, sorry pdf you can also write the CDF. The CDF is always going to in this case be what be a ramp between a and b. And here what is it going to be? If this is the discharging of a capacitor.

Student: (Refer Time: 05:38).

The CDF will be a charging of a.

Student: Capacitor.

Capacitor right.

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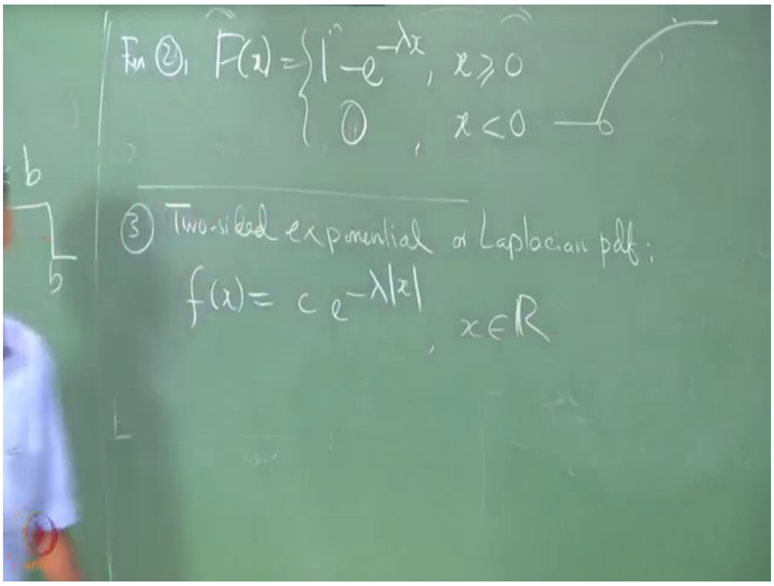

So, this gives part 2, I will just write only for 2, the F of x is what 1 minus e power lambda x for x greater than 0 and a 0 for x less than 0 I am writing it, let me write it explicitly like this. Please check that this must be the derivative of this for x greater than 0, is not it.

So, this is a single sided exponential, you can all obviously, also talk about two sided exponential, this is also is the its encountered in practice sometimes it is called Laplacian. Here, if I write the c lambda mod x where x is any real number. So, this is now defined over the entire the pdf itself is as nonzero values over the entire real line is only asymptotically decays 0. So, what would be the value of c? This is standard question right.

Student: (Refer Time: 07:28) example.

You have to do find the value of c by integrating this and setting it equal to unity. So, you can clearly or easily I hope verify that c must be lambda by.

Student: 2.

Right. So, basically it this lambda here it becomes lambda by 2 when you consider the same type of function over the whole real line. So, the CDF sorry the pdf looks like this. Remember the e power minus lambda mod x.

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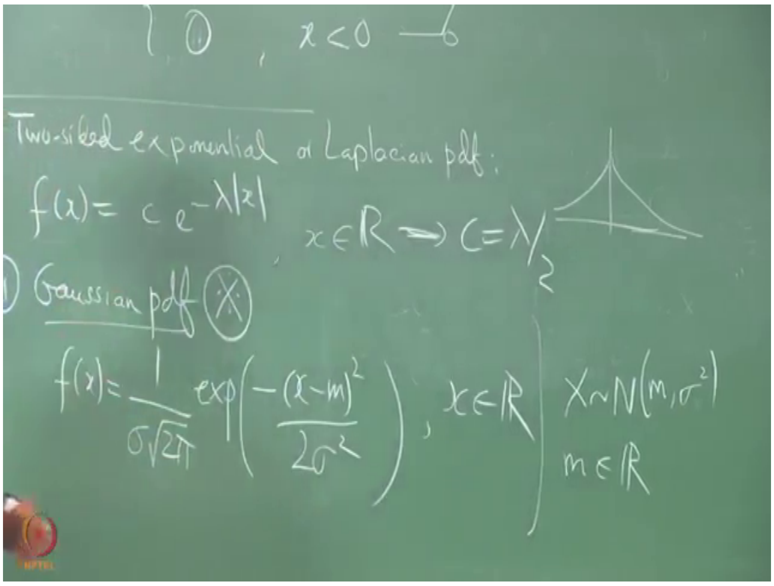

Now, we will encounter many complicated arguments with the exponential function. So, if once we get a more complicated argument than just simply lambda x or lambda mod x we will use exp. So, I hope that is with everybody here.

Just to do the formatting on the same line, we do not want to start a superscript which is all squishy and especially in (Refer Time: 08:28), please use a backslash exp all to typeset that this portion. Here it is ok, but as we will see when we look at Gaussian and so on. Let me look at Gaussian next some very important one right. So, I have write this, I have to do this you know for a Gaussian pc, you are the famous sign that we used to into to tell. This is very important, we can do here this is also extremely important right. So, here our f of x is going to be what? Does anyone remember? So, you have x for first of all you have this. What do you have?

Student: Minus x minus m (Refer Time: 09:17).

Minus x minus m some number m by.

Student: 2 sigma square.

2 sigma square. We will use the standard notation. What is the constant?

Student: (Refer Time: 09:30) input pressure of the pointer.

Ah?

Student: (Refer Time: 09:34).

Already, think of it like this. What values of x? All real.

Student: All real.

Right. So, this is such an important pdf because of the way in which it appears in such variety of applications that again the not. So, for Laplacian or the exponential Laplacian we are not giving it a name this is going to be called the normal Gaussian or normal and you say that appear random variable with this pdf is written N m comma sigma squared all right. So, the value of sigma squared or if you want you can think of this as somebody said is root 2 pi sigma squared, so this value of sigma squared you give us a parameter here. So obviously, that sigma squared is obviously, is a non negative, is a positive parameter, m can be any real number there is no restriction on m. Turns out that m is a point of symmetry of this pdf, so we have to look at you know what it looks like where we see right.

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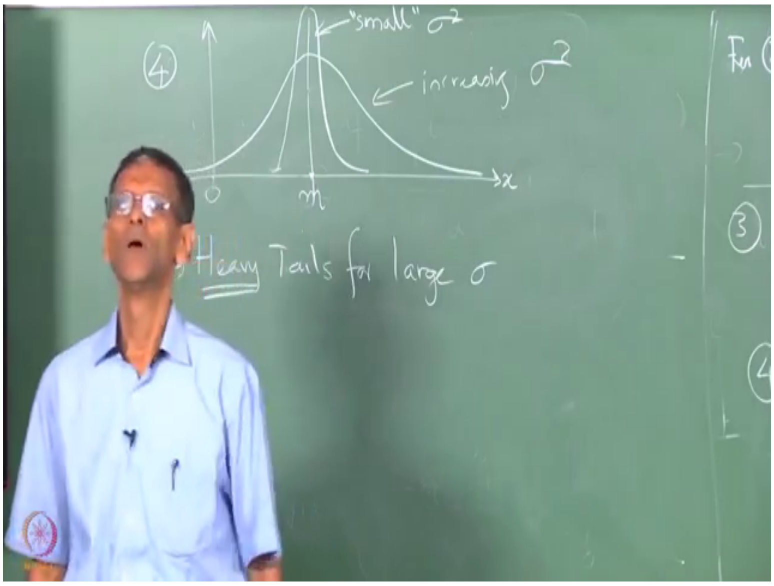

So, for what does it look like? The famous bell shaped curve, I cannot do justice to it this is the best I can draw. I think this is luckily we do not drew too much of this in this course, occasionally we do draw pictures. This point of symmetry is m. So, a 0 can be anywhere, it does not matter. This point, this can be 0 here. So, the point is symmetry can therefore, occur at any point on the real axis, can be negative or positive there is no restriction on that. But, what about sigma? What is role sigma in this?

Student: As just how (Refer Time: 11:57) random variable (Refer Time: 12:00)

That is the English, if fluctuates the bodies mean. I am interested in small as usual like in the Poisson case we want a very sigma from small to large right. So, what is it going to look like for small sigma and what is it going to look like for large sigma?

Student: (Refer Time: 12:17).

Hm.

Student: (Refer Time: 12:19).

For small sigma you get a nice high you know sharp high curve concentration around m.

Student: m.

Remember. So, this not you get for last small sigma. So, I sigma increases the spread increases always remember this. Sigma or sigma square does not matter its preferable to look at sigma squared the sigma does not matter really all right. Since I have written it as m, m comma sigma square I am focusing more on sigma squared than sigma itself, but it is a sigma is obviously, the positive square root of sigma squared so the two are basically inseparable. So, as you increase the sigma squared you spread out that the distribution has heavier tails for largest sigma. Similarly the exponential which I erased here for large lambda it is concentrated around 0, for smaller lambda the d k is smaller I have erased it here, but go back to your notes in may and make their point right. The same kind of f every pdf which depends on parameters you should understand the effect of those parameters on the shape of the pdf all right.

So, in the limit it turns out that you get a degenerate situation as sigma goes to 0. Similarly in the exponential switch case you get a degenerate situation as lambda goes to infinity it becomes very large. In other words the e power minus lambda x which is a visible to me here collapses almost to the single point x equal to 0, similarly as sigma goes to 0 here this distribution collapses to the point m. So, you get, you know a degenerate case where all the probability is concentrated in a small interval around m for very large sorry very small sigma. So, you get heavy tails heavier tails the use of this word heavy tail is very this comes up a lot. I just leave it a sigma.

So, I think, I will close here.