Introduction to Photonics

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Photon Interaction - 1

Okay welcome to introduction to photonics, we are, we have been talking about the particle nature of light in the past few lectures, so we introduce what a photon is and what are the properties of photons and then towards the end of last week you had talked about interaction of light with matter, so we started talking about absorption of light, we looked at the specific example of light getting absorbed in rhodamine 6G and that sort of absorption process essentially required us to tune into the photon picture and say that if the photon energy corresponds to the energy difference in this particular material than it is likely to get a absorb.

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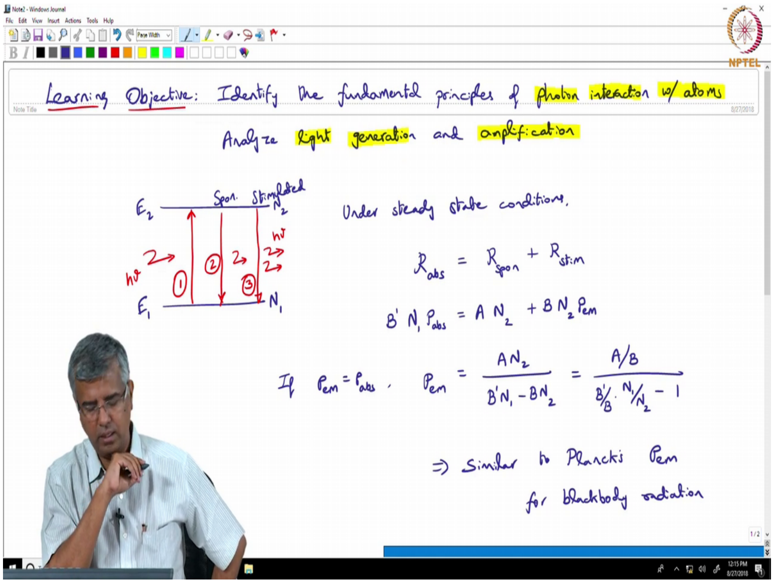

Learning Objective: Identify the fundamental principles of photon interaction w/ atoms

Analyze light generation and amplification

Under steady state conditions,

$$R_{abs} = R_{spon} + R_{stim}$$

$$B' N_1 \rho_{abs} = A N_2 + B N_2 \rho_{em}$$

If $\rho_{em} = \rho_{abs}$, $\rho_{em} = \frac{A N_2}{B' N_1 - B N_2} = \frac{A/B}{\frac{B'}{B} \cdot N_1/N_2 - 1}$

$\Rightarrow$ Similar to Planck's ρ_{em} for blackbody radiation

So we will continue to build on that picture, so what we will do for the next few weeks is to identify the fundamental principles of photon interaction with atoms, we just started doing that will go into that in little more detail and the specific motivation for considering that is we want to understand light generation and amplification okay, because so far we have been looking at ray optics, wave optics, electromagnetic optics, photon optics and an all of this we just figure there is some source out there which is giving us all this light.

Now that source how does that work right, so what goes into explaining light generation and amplification and in certain ways once we understand this then we will be in a better position to understand why certain sources are not so temporary coherent, why other sources are temporary coherent, why certain sources are specially coherent or not, you know all those things you know will be able to explain once we understand this.

Okay to understand this let just consider an atom, once again just like we did in known of the previous lectures we have an atom with energy levels let us say E1 and E2, E2 being greater than E1 and let us see what all are possible between E1 and E2 okay, so we already said one process that is possible is this light absorption through which if you come in with H new, such that H new corresponds to the energy difference between these two levels that is H2 equals to E2 minus E1.

Then you have an absorption process happening right, now that energy does not stay, cannot stay in E2 forever because it is one of the higher orbitals, it is what you call as metastable state, so it has to give up its energy okay, it can potentially you that energy by emitting a photon okay, what call as spontaneous manner okay, so it emit a photon under certain conditions enough spontaneous manner we want to, actual figure out what are those conditions as we move on but let us just assume that is one possibility or it could also have this case while this atom is held at E2, a photon with energy H new comes by and it stimulates this transition.

Okay it encourages this atom to give up its energy in which case it emits two photons, you come in with one photon you go out with two photons with the same energy H new and us that the other photon that is emitted is absolutely a cloned of the incoming photon okay, so what do we mean by that terms of light quantities, it has the same direction, it has the same phase, it has the same frequency as the incoming photon okay.

So it essentially generates a clone and this is something that we can build up towards amplification of photons, potentially as we move on okay, so that is process number three which we will call as stimulated emission, now spontaneous emission is sort of a misnomer because spontaneous means like the instant this atom goes to the higher energy level, it is spontaneously emits that right and it turns out that is not the case as you might have found out through the absorption experiments we did last week, it spends a finite time in that higher energy level before it gets emitted.

So as long as you understand that, that part is fine but stimulated emission on the other hand is something that happens, moment that photon comes by immediately if those other energy considerations are satisfied, it immediately emits photons, so stimulated emission in some ways is more spontaneous than spontaneous emission okay, if you can handle that.

Well, yes, the question is why is the frequency is the same at this point we are just considering two energy levels okay, we will go onto the picture where we are considered a brand of energy levels, the exerted state and of energy levels in the ground state and all that will come to that okay at this point it is, of course you may also I give that if you have a certain probability of emitting a photon between these two energy levels the probability of absorbing the photon is the same right, if what comes down can also go up and vice versa in this particular case.

But let just consider this case first and then began right what is call a rate question okay, what is a rate equation? We are tracking a rate at which all this three processes is happen okay, so we will write under steady-state conditions what do I mean by steady-state conditions? This is happening continuously okay, so the rates do not change, it is not like you just flipped a switch and you are immediately after that tracking this, you are letting things come to some sort of equality and then you are observing this, that is what the steady-state condition mean, if you are looking at immediately after you switch on something what is that called transient conditions right.

So we are not looking at the transient condition, we are looking at the steady-state condition, so under steady-state condition you can say that the rate at which the absorption process happens assuming that the energy does not get lost in any other way, then the rate at which the absorption happens to be equal to the rate or spontaneous emission plus stimulated emission right, spontaneous emission is this one, stimulated is this particular person process right.

So you absorb button photons right and go to the higher energy level from that energy level you either spontaneously emit or if you have an incoming photon at that time you can actually stimulated emission also happening okay, now if you want to quantify this as I mentioned earlier these are all probably sticky ones so that is got to be probability density associated with all of these things and so we can basically say for the absorption process happen, you need to have a certain number of atoms in the ground state right at energy level 1, if there are no atoms in that energy level 1 and that absorption and cannot happen right.

So let just also track number density of atoms in this level versus this level okay, so you are saying that this is got to depend on number density of atoms in that lower energy level, multiplied by some probability density which is called that P absorption right that is the probability density and they maybe some constant associated with this, let us call that constant B prime, we will define what this constant means a few minutes okay.

And similarly as far as spontaneous emission is concerned, spontaneous emission happens at a particular rate, it certainly depends on N2 right, only if the atom is in that exerted state is going to have spontaneous emission otherwise you cannot have emission right, so it certainly depends on N2 and let say that rate at which this emission happens is proportionality constant we call that as A and then when we consider stimulated emission that also depends on the number of atoms that are in the exerted state, it emits with a certain probability density that we can call as PM right.

And that also happens at a rate, let say it is proportional to B okay, now so what is essentially different between the spontaneous and stimulated process is that you know PEM the probability density of emission which actually depends on the probability that photon comes by okay, so that actually you know determines what that PM is okay, now in this sort of a system, simple system with two energy levels you can say that the probability density of emission and the probability density of absorption is roughly equal right.

So if we assume that, if this is equal than you can get an expression for this probability density of emission and so that just a matter of substituting instead of PABS you substitute PEM and then rearranging terms and when you do that is what you get is AN2 divided by B prime N1 minus BN2 which you can rewrite, let us say you divide the numerator and denominator by a factor of BN2, so you can write this as A over B divided by B prime over B multiplied by N1 over N2 minus 1 okay.

So row EM we have some expression like this and this is exactly what Einstein was doing you know 115 years ago right, so he wrote all this equations and then he came up with this and he was looking at this, 2 is credited he recognized that this looks very similar to what Plank had absorbed for a blackbody radiation okay, so he said this is similar, so in that case also Plank had derived an expression for this, emission probability density and so Einstein basically said okay this is similar to planks row EM for blackbody radiation okay.

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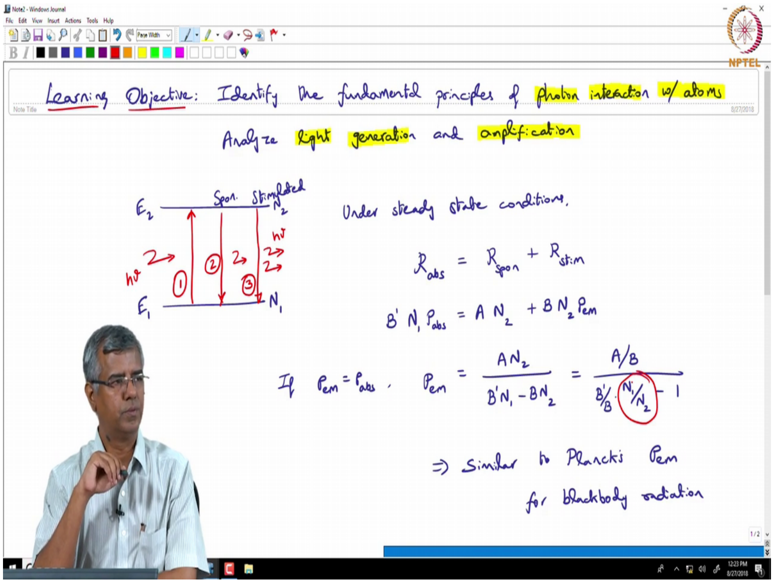

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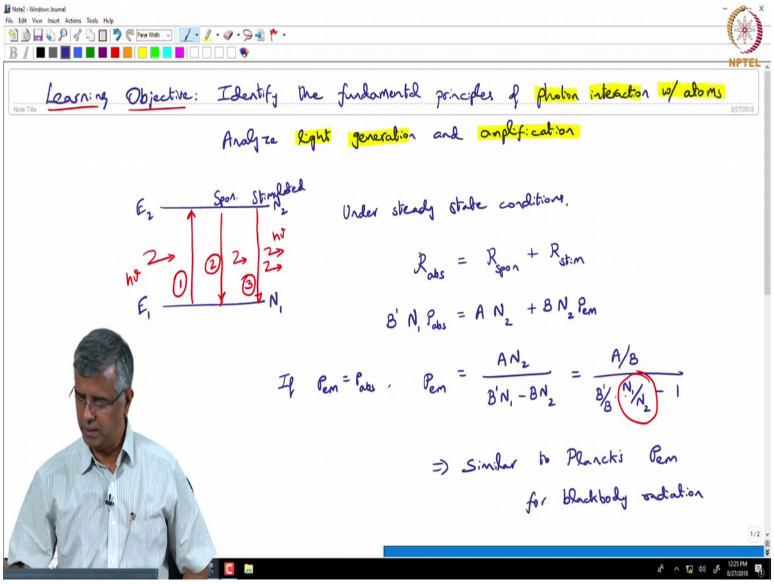
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So let us see what that actually gives us right, so I have this atomic system and at this point it also helps to understand what blackbody radiation is all about, so as you know blackbody radiation is something that happens when you heat up a material to high-temperature okay and to explain what happens to the material at high temperatures okay, let us just look this picture right, so when you consider any material it is going to have multiple energy levels, let me just draw this, so we talked about only two energy levels but in general any arbitrary material may have multiple energy levels associated with this.

Once again these are all higher orbitals that are possible for a given atom, the question is what is the probability that an electron may exist in one of the higher energy levels okay under the specific case of thermal excitation, why thermal excitation because that is what blackbody radiation is about right, so it is essentially thermal excitation and to explain that will have to essentially go to you know what Boltzmann came up with where he said that under thermal equilibrium conditions, so what is thermal equilibrium conditions mean that you have elevated the atoms to a particular temperature, you subject them to a particular temperature and you are holding them their okay.

So things are not changing rapidly around that point okay, under thermal equilibrium conditions the probability that you will find an electron at an energy level EM, let us basically say this corresponds to E1, E2, E3, E4, E5 even all that right and some arbitrary energy level EM, the probability that you will find in electron occupying and energy level EM, you are saying that, that would be proportional to exponential of minus EM over KBT.

Where KB is what, it is the Boltzmann constant right, so and KBT now corresponds to some energy term right, so that is essentially your thermal energy, so if you examine this closely what it says is higher thermal energy more is the probability that an electron will occupy in one of the higher energy levels, in another words if T is zero, T is absolute zero, thermal energy is zero all my electrons are going to be only in my valence orbital right, they are not going to, you will not find any electron in any of the higher energy levels right.

So but we are considering blackbody radiation where we are exciting this body or subjecting that body to high-temperature and in that case higher the temperature, higher will be the energy, the probability that you will find in electron at a higher energy level right and so we can draw that here, so what it says is if I am plotting the probability of finding an electron at energy level EM, where M could be anything from one to whatever, that probability would be an exponential okay.

So you have probability being higher for lower energy levels than higher energy levels okay, so in this case if we have, if there are N number of atoms then what we can say is NM which corresponds to the number of atoms at energy level EM, NM over N will correspond to the probability that you have an electron in that exerted state EM okay and this implies that if you are looking at the relative numbers between two energy levels, why are we interested in that because here we have a factor N1 over N2 right.

So you want to see under thermal equilibrium conditions will be N1 over N2, in this case you can write it as N2 over N1 is going to be equal to exponential of E2 minus E1 divided by KBT, now let us look at that for a second what it says is that if E2 minus E1, let us say is a

100 m electron volt right, if that is the energy difference, if I am at room temperature where KBT would correspond to about 25.6 m electron volt okay.

If I am at room temperature then N2 will be far less than N1 on the other hand if I heat up that material okay to a elevated temperature may be 100º C their you may actually have energy corresponding to 100 electron volt in which case you have, you know equal number of atoms in the exhorted level compared to the level okay, so that is what this means, now and of course I can write this simply as explanation of minus delta E over KBT right.

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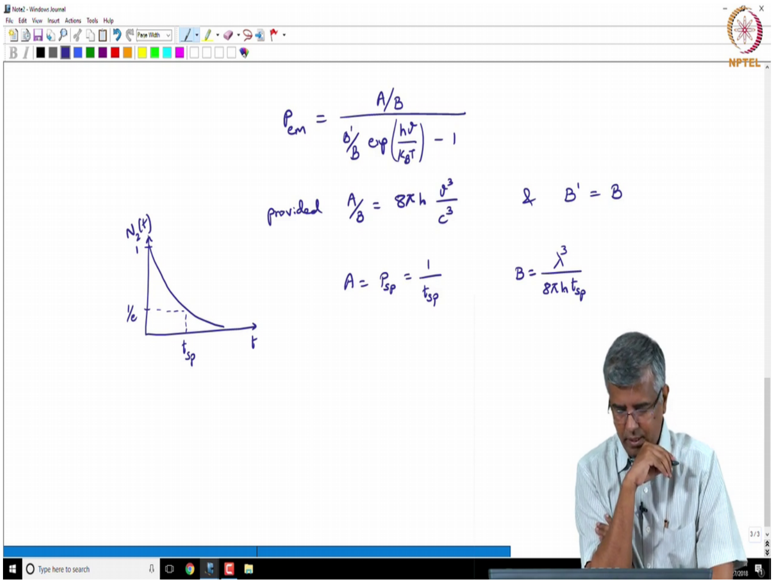

$$P_{em} = \frac{A/B}{B'/B \exp\left(\frac{h\nu}{K_BT}\right) - 1}$$

provided $A/B = 8\pi h \frac{\nu^3}{c^3}$ & $B' = B$

$A = P_{sp} = \frac{1}{t_{sp}}$ $B = \frac{\lambda^3}{8\pi h t_{sp}}$

Now I can plug this back into what Einstein has observed right, I would then get the expression that the emission probability density is going to be equal to A over B divided by B prime over B potential of delta E over KBT but from the perspective of emission what does delta E corresponds to the energy of the photon right, so the photons that emitted will correspond to H new, so I can write this as H new over KBT -1 okay and Einstein just observe that this is, once again very close to, very similar to what Plank has observed provided A over B can be substituted by what Plank has derived as this factor 8 by multiplied by the planks constant multiplied by mu cube over C tube which you can choose to write us 1 over lambda cube.

And B prime equal to B right or B prime by B equal to 1 which is essentially be B prime here equal to B, so if you substitute this constants A, B and B prime by these factors, this is exactly what plank had observed okay, so then he said okay then all these are consistent with planks observation, so only thing that Einstein is adding here over what Plank was observing, plank was observing only radiation as spontaneously emitted radiation, whereas now Einstein observed that there could be actually stimulated emission as well okay and further if we, what he was observing was this constant A is actually providing the probability density of the spontaneous emission and that is actually inversely proportional to the spontaneous lifetime.

Okay and what is spontaneous lifetime? We discuss that last week if you are tracking the number density that exerted state and two has a function of T the spontaneous emission is going to happen in such a way that it comes down as potentially okay, if you say this is normalized to 1 where it goes to 1 over E, that corresponding time is what we call as the spontaneous lifetime.

So once again like I was mentioning last week when people think about spontaneous lifetime they think that okay this atom get excited, goes to that higher energy level and that specific electron which goes to the higher energy level spends this specific time and then it get emitted, no that is the wrong picture right, it is not like, it spends that much time it gets emitted all along right, that is what is it means, it is getting emitted all along, N2 is reducing the function of time that means this spontaneous emission is happening all along, it is just that when it goes to 1 over E that is what we characterize as a this spontaneous lifetime okay.

So it gives you some ideas to the timescale over which most of the emission happens but it is not like a delay, a deterministic delay that you know some people may think of okay and given this we defined B as or you just substitute since A over B is already given by this expression you substitute in that and you can get expression for B which is lambda cube over 8 pie H TSP okay.

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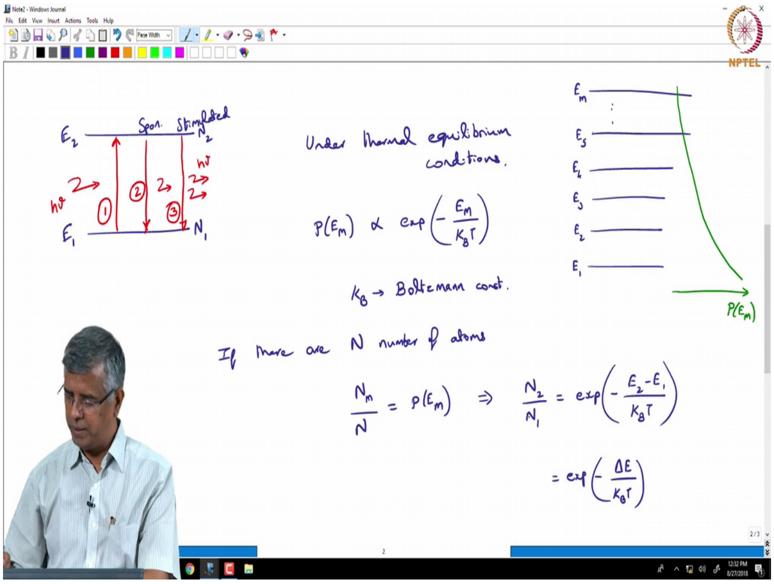


Under thermal equilibrium conditions.

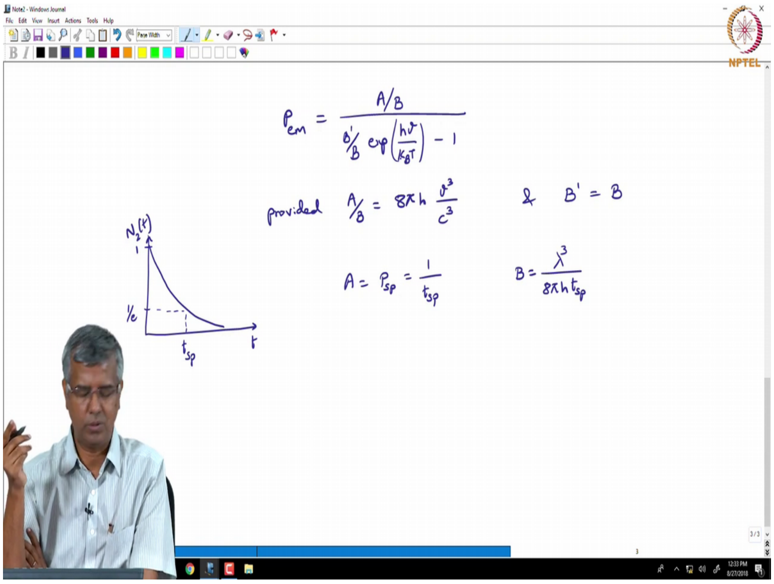
$$p(E_m) \propto \exp\left(-\frac{E_m}{k_BT}\right)$$

$k_B \to$ Boltzmann const.

If there are N number of atoms

$$\frac{N_m}{N} = p(E_m) \Rightarrow \frac{N_2}{N_1} = \exp\left(-\frac{E_2-E_1}{k_BT}\right)$$

$$= \exp\left(-\frac{\Delta E}{k_BT}\right)$$



$$\rho_{em} = \frac{A/B}{\frac{B'}{B}\exp\left(\frac{h\nu}{k_BT}\right) - 1}$$

provided $A/B = 8\pi h \frac{\nu^3}{c^3}$ & $B' = B$

$A = R_{sp} = \frac{1}{t_{sp}}$ $B = \frac{\lambda^3}{8\pi h t_{sp}}$

We started with just two-level system we track what all the possible evens that can happen, you know absorptions, spontaneous emission, stimulated emission, we were tracking the rates of all of this and through that we were actually getting an expression for the probability density for the emission process and like we said it you know Einstein said that it is similar to what we have for the planks expression for the blackbody radiation and to give a background of that we went back and looked at thermal equilibrium conditions, what Boltzmann law says is that probability of occupying a higher energy level depends on the thermal energy right.

So you have a higher probability for more thermal energy and so we got is expression for N1 over N2, we found N2 over N1 but you invert that you get N1 over N2 which is what we plugged it back into the expression for probability density of emission and we said okay it is all consistent with planks back blackbody radiation theory okay, at this point I want to bring your attention to a collection of atoms okay this was the question about that earlier, So in reality we are typically dealing with a collection of atoms.

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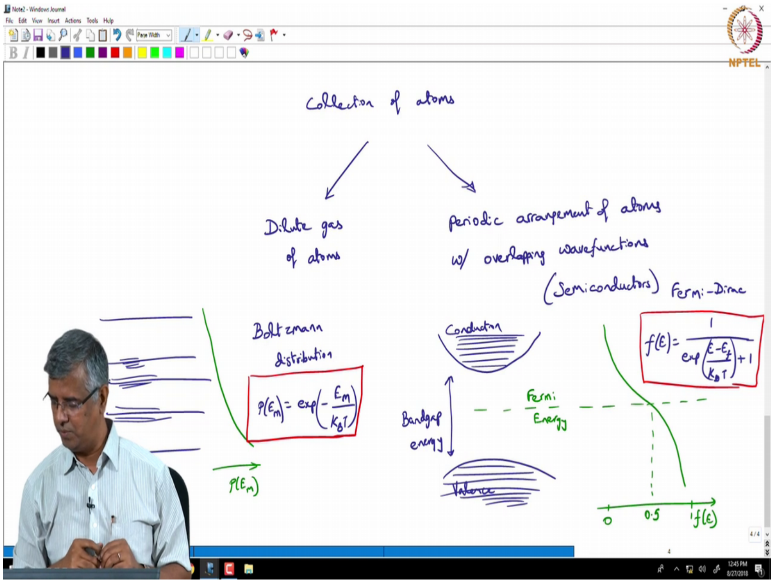

So when the deal with a collection of atoms what happens in the corresponding energy levels or the probability that you will find an electron in a particular energy level is actually different for two different sets of, two different classes of systems, one is you can consider this as a dilutive gas of atoms okay it is just a dilutive gas of atoms that are dispersed in this host medium which of course holds good for gases, holds good for relatively low concentration solutions that, this kind of solutions that you were dealing with, rhodamine 6G in methanol something like that.

So that is or even in a solid, in a glass metrics you are doping it with some, let us say some rarathion okay, the typical doping density is that you are able to, you know accomplish, you can very much considers that as dilutive gas of atoms, now this is different from another system where you have periodic arrangement of atoms okay, specifically with overlapping wave functions, does that remind you all something, periodic potential wells whose wave functions are overlapping each other, is that remind you all something.

That is what you find in semiconductors right, so in semiconductors you would typically have is periodic arrangement of atoms with overlapping wave functions, what is the effect of that? Because of this what do you have in semiconductors, yes, a particular characteristics or semiconductors, bandgap right, you have this concept of bandgap come into the picture based on this pre-audacity which gives you the potential wells are located and you have this overlapping wave functions.

So when you draw the, what is called the EK diagram energy versus momentum diagram right, you finds that you have something called a conduction band and valence band and you have certain energy levels between that which are forbidden, which you call as bandgap energy, so it turns also out that when you look at the probability of electron occupying one of these energy levels right, this cannot be described by Boltzmann strategic, here in the dilutive gas of atoms we said okay you have energy levels like this right and here we said okay we can describe the probability that you have in electron and in one of the higher energy levels is exponentially distributed right and so this is P of EM right.

So this lower probability as you go to higher energy levels but here it is not the same, what you have here is another distribution which hinges around an energy, a characteristic energy called the Fermi energy and if you are tracking F of E, let us say that is the energy that the probability that an electron is occupying one of these energy levels that actually is a value from 0 to 1 and has a value of 0.5 at the Fermi energy right that is how you define the Fermi energy.

And in this case when we define that F of E that actually follows what is called a Fermi Dirac distribution okay, which is given by F of E 1 over its potential of E minus EF over KBT +1 right, so in dilutive gas you have Boltzmann distribution that describes the number of items in an exerted state right, whereas if you are looking at semiconductors you essentially have to, you know take into account this Fermi Dirac distribution just for completeness let basically say this occupying an energy EM is given by exponential of minus EM over KBT.

Why we talking about all this? Because as we move on to understanding absorption and emission we will realise that absorption and emission depends on the kinds of atomic system that you are dealing with okay it is different for these two atomic systems right and we will look at traditional atomic systems where you model this as a dilutive gas of atoms and you will have, you will follow Boltzmann distribution and then you go on to looking at semiconductor light sources right and then you say that case you will have to follow Fermi Dirac distribution.

So you need to understand what is the kind of material that you are dealing with? How can you characterize that material okay and not of the emission characteristics depend on these aspects but luckily we just do this two classification and we are done with it right, so it is either this or that and of course we will see examples of you know this system versus that system, how the emission process is, emission and absorption process as well.

For example I mean just thinking about the absorption process you can say that absorption in this can correspond to multiple specific frequencies or maybe there are bands of frequencies, you know because these things are typically split around their center, so you have bags of frequencies but you have only specific bands of frequencies where you have absorption.

Whereas here if you are frequency of the photon, you knows is that the energy of that photon is less than that bandgap energy what happens? If is less than the bandgap energy it just goes straight through, there is no probability of absorption because there is no higher energy level that you can go to, it is forbidden right, so you need energy at least that bandgap energy before you can actually have that absorption happening but once you achieve that you see that all these energy levels are relatively, closely packed then it is almost like a continuum of absorption that is happening, beyond a particular frequency right absorption will be almost continuous.

Whereas here it is only in specific bands that you have this absorption happening, so this is actually very nice from our spectroscope prospective because you can turn this around and say I can detect specific gases let say right, I can detect specific gases based on where they absorb right, so each you say carbon dioxide, you say oxygen, nitrogen whatever right, each one of them have a very specific energy level structure, so they absorb only its specific frequencies, so when you are looking at their absorption that is like a fingerprint okay.

If you see this fingerprint then that corresponds to one element, if you see this fingerprint that corresponds to another element and so on right, based on it you can do absorption spectroscopy and identify different, you know gas species or different species in general chemicals species in general because each one of them have their own unique fingerprint corresponding to these specific absorption that is happening between the energy levels.

Whereas here it is, yes you can say that okay I am doing the absorption spectroscopy and only beyond a certain frequency it is start absorbing, so then you say okay I can figure out my bandgap energy based on that but beyond that you cannot tell what exactly that material is okay, I wanted to go to one more concept maybe I just give you a hint about this and will continue this in the next lecture right.

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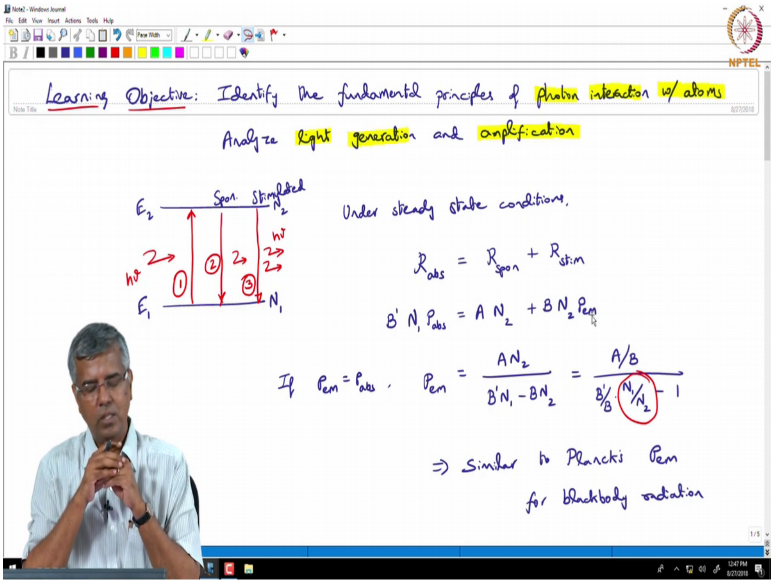

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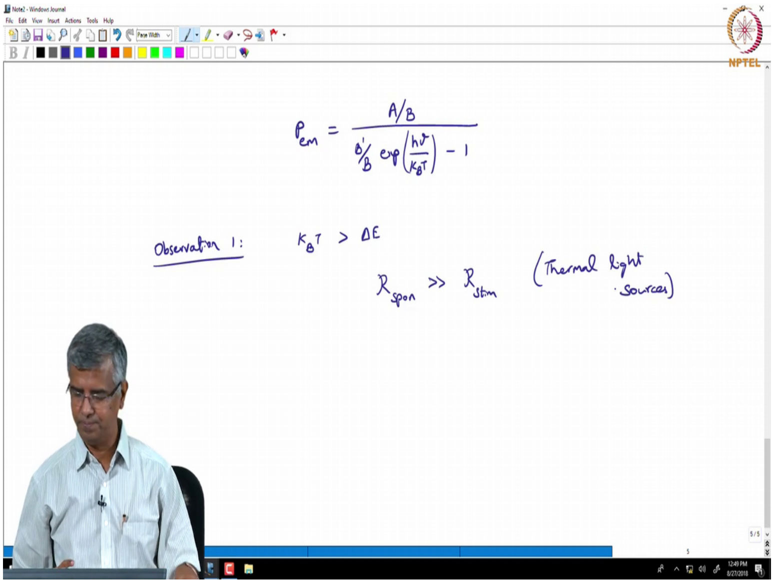

$$\rho_{em} = \frac{A/B}{B'/B \exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

Observation 1: $k_B T > \Delta E$

$R_{spon} >> R_{stim}$ (Thermal light sources)

So I am looking at this expression and I am starting to read into it from prospective of what is a relative value of KBT and H new because based on those relative values the value of the probability density of emission changes quite rapidly and if that changes, if you go back and look at this could actually way that comes into the picture is determining which is more probable, spontaneous emission or stimulated emission okay, so if row EM or the probability density of emission is highly and you can say stimulated emission would be the dominant mechanism, which is what you may want when you are amplifying something right or you are building a laser right.

On the other hand if that quantity, that probability density very small, if it is a small number then spontaneous emission is the dominant mechanism which is why, you know we have this light emitting diodes and all others primary spontaneously emitted sources okay, so we want to look at different possibilities, so specifically one observation would be at KBT is greater than delta E, it is greater than, you know the energy difference between this two energy levels that we are considering.

So what do we have in this case? In this case there is actually equal probability may be or that in an atom can exist or an electron can be in this energy level or that energy level okay, so the energy level themselves their broaden and overlap and in this case what we will see is the spontaneous emission rate or in general row EM or the emission probability density is very low and the spontaneous emission rate is far greater than the stimulated emission rate, which is what we have for thermal light sources, what is an example of thermal light sources? This tungsten filament lamp right that candle sent lamp.

So you know you have this and then you come up with this other observation at to have stimulated emission one of the necessary condition is that your delta E is far greater than KBT okay but then you still need fairly large value of row EM, so in which case you will find that you need to somehow get atoms to go to a higher energy level and maintain atoms at are higher energy level with respect to the lower energy level only when you have that inversion of population that is when you will get stimulated emission okay, we will look at all those conditions in the next lecture okay. Thank you.