Introduction to Photonics
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Optical Amplification

Morning, welcome to introduction to Photonics so this week we being looking at how photons interact with matter and specifically we have being trying to light analyze light generation and amplification so we will continue the discussion with little bit of practical example. So you know will look at the example of why we need amplification right so till the last lecture we were looking at how amplification, what are the conditions required to achieve light amplification. Today we will look at why we need amplification will take the specific example of optical communication right so just one of the you know really widespread application of optics in today's world.

So we will look at the that specific example and will look at why we may need an optical amplifier in that case and we will try to go ahead and build an optical amplifier today ok so that is what we going to try to do today.

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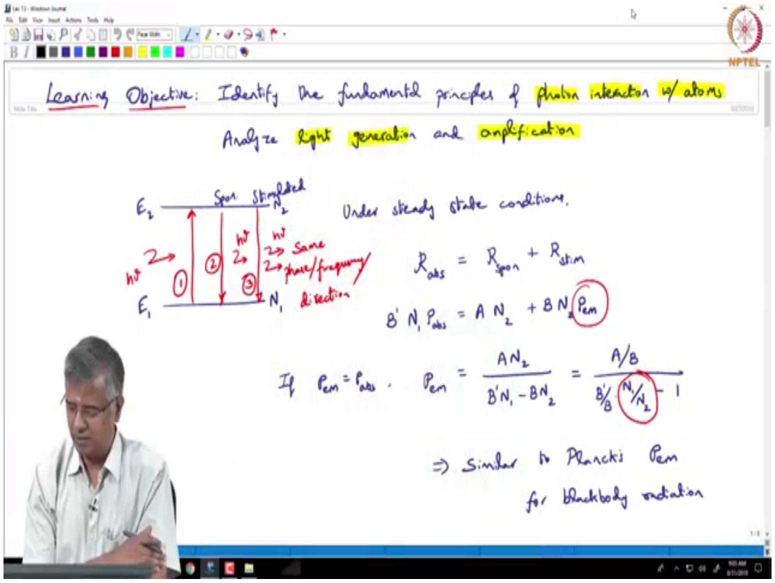

Learning Objective: Identify the fundamental principles of photon interaction w/ atoms

Analyze light generation and amplification

Under steady state conditions,

$$R_{abs} = R_{spon} + R_{stim}$$

$$B' N_1 \rho_{abs} = A N_2 + B N_2 \rho_{em}$$

If $\rho_{em} = \rho_{abs}$. $\rho_{em} = \frac{A N_2}{B' N_1 - B N_2} = \frac{A/B}{\frac{B'}{B} \frac{N_1}{N_2} - 1}$

$\Rightarrow$ Similar to Planck's ρ_{em} for blackbody radiation

Just to give a quick recap we started with looking at an elementary system consisting of two energy levels where we tracking all the different process between those two energy levels absorption, spontaneous emission, stimulated emission and then we were trying to quantify this emission priority density which actually it is representing the stimulated emission terms so may think of it as the stimulated emission priority density and the probability that stimulation stimulated emission should happen and we came up with this expression and then we were trying to be cut specific cases.

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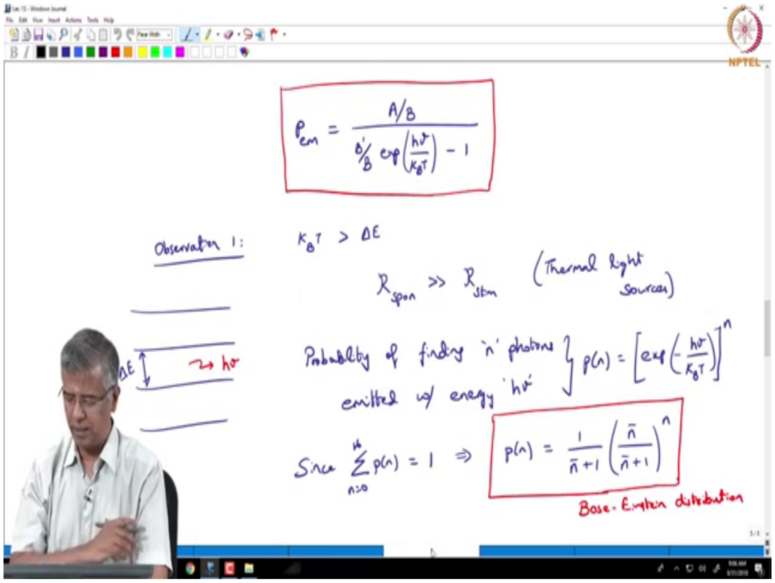

$$\boxed{\rho_{em} = \frac{A/B}{\frac{B'}{B}\exp\left(\frac{h\nu}{K_BT}\right) - 1}}$$

Observation 1:

$K_BT > \Delta E$

$R_{spon} >> R_{stim}$ (Thermal light sources)

Probability of finding 'n' photons emitted w/ energy 'hν' } $p(n) = \left[\exp\left(-\frac{h\nu}{K_BT}\right)\right]^n$

Since $\sum_{n=0}^{\infty} p(n) = 1 \Rightarrow \boxed{p(n) = \frac{1}{\bar{n}+1}\left(\frac{\bar{n}}{\bar{n}+1}\right)^n}$

Bose-Einstein distribution

Specific we are trying to make some specific observation and one of the first observations we made was what if you are looking at a source where you have atoms going to the (())(02:53) stage just because of thermal energy and specifically the thermal energy is greater than the energy difference between the two atomic energy levels and in that case we said we are predominantly getting spontaneous emission and then there was discussion about what this graph means.

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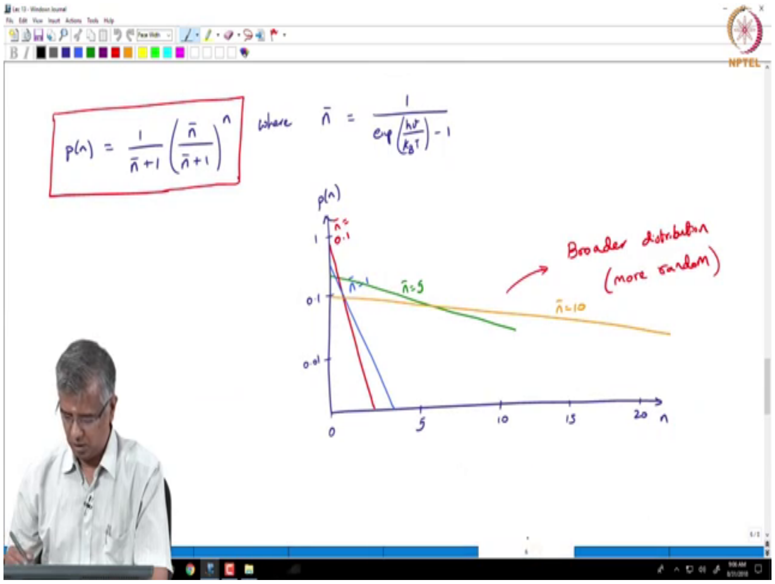

$$\boxed{p(n) = \frac{1}{\bar{n}+1}\left(\frac{\bar{n}}{\bar{n}+1}\right)^{n}} \quad \text{where} \quad \bar{n} = \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

You know so what we are saying is for thermal light source the probability of finding n photons is something that could be quite broad right so which means that it is almost if you have an average number of photons (())(03:44) if you really go to a very-very small time scale right lets say nanosecond time scale. So question is how small is small? You have a characteristic lifetime for emission right in atomic system we talked about the lifetime being maybe in the order of milliseconds, microseconds or nanoseconds for example (())(04:22) sample that you guys worked with earlier in a task something a lifetime in the order of 10's of nanoseconds so if you go to a fraction of that ok then you may actually see that even though you have average number as a certain number the probability that you find much lower number of photons within that time scale there is a finite probability for that. There is a discussion at the end of yesterday's lecture as to what this all means you know so what so why do we even then talk about average number being 10 or 20 or whatever right.

Just think of it like this I saying that if you go to a really-really small time scale you will find this priority distribution but what if you try to average over multiple times slots (where we) if you don't look at nanosecond level you look at microsecond level or millisecond level in which case your averaging a number of photons that you find over that entire time scale. What happens to the distribution? What do you think the distribution would look like? This entire distribution when would essentially collapse, collapse to that in a specific value that we are looking at now we say n bar equals to 10 right the mean number of photons is 10.

If you go to a really-really small time scale you might find 0 or 1 or 2 or 10 or 20 or 30 even right but then if you what is within in this time scale? Now if you average over this much time you have already seen in a all this different times slots having this you know different photons but when you average them together you basically say there is a mean number that you get right. So essentially if you do time averaging this entire thing is going to collapse to something closer to you know whatever you are suppose to see that n bar equal to 10 ok.

So that is what this priority density, priority distribution means so in other words why are we seeing I mean outside the thesis are pretty random source right this LED is spontaneously emitted source is a pretty random source but we are seeing a constant intensity from that, why is that? Because I is actually averaging everything over millisecond times scale over that millisecond times scale it is as if n bar equal to 10, right so that constitutes if you do time averaging this constitutes something like this right that entire distribution collapses and that is essentially what we see, we see a time averaged you know number ok.

You understand this point? Any questions about this? Ok, so even a thermal source even like a incandescent lamp we don't get to see in a time slots of nanoseconds level our eye cannot pick up (())(08:55) integrating over this entire times scale right and when you integrate over all the time scale it actually looks as if it is a constant intensity light source ok.

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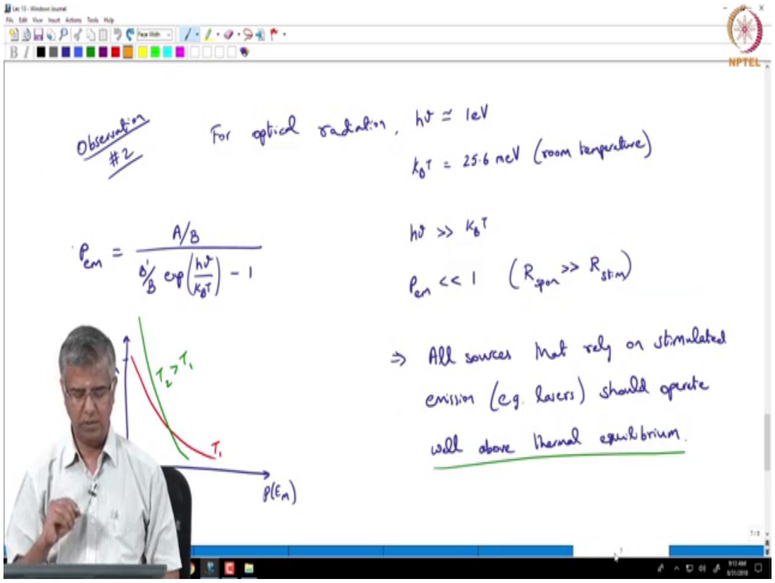

Observation #2

For optical radiation, $h\nu \simeq 1\,\text{eV}$

$k_BT = 25.6\,\text{meV}$ (room temperature)

$$P_{em} = \frac{A/B}{\frac{B'}{B}\exp\left(\frac{h\nu}{k_BT}\right) - 1}$$

$h\nu \gg k_BT$

$P_{em} \ll 1 \quad (R_{spon} \gg R_{stim})$

$\Rightarrow$ All sources that rely on stimulated emission (e.g. lasers) should operate well above thermal equilibrium.

So then the next observation we made was you know optical radiation if you are emitting optical radiation then optical radiation has energy that is much higher than thermal energy at least at room temperature. So if that is the case then the priority of emission priority of stimulated emission is very low so you lightly to get only spontaneous emission unless you are operating at a point that is well about thermal equilibrium right and thermal equilibrium we are defining as if there is only thermal energy given to that medium right and so everything is getting to steady state and we are looking at that we are actually going by Bolt’s man statistics.

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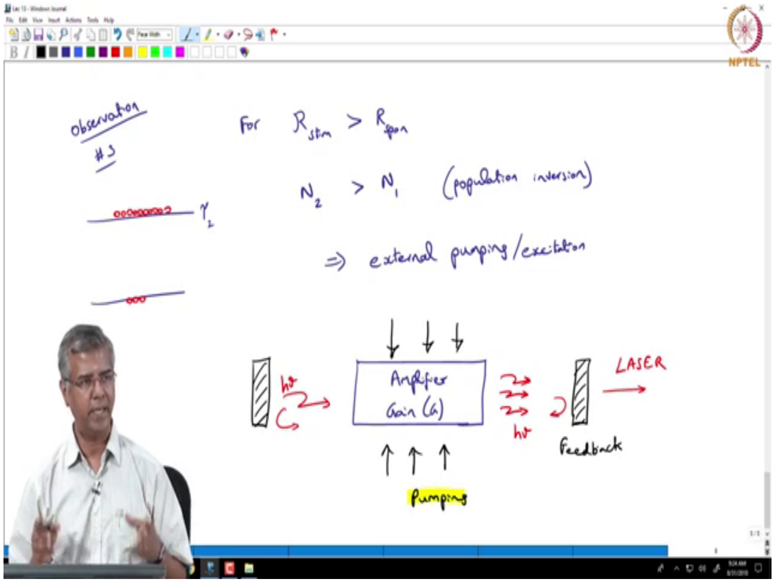

Observation #3

For $R_{stim} > R_{spon}$

$N_2 > N_1$ (population inversion)

$\Rightarrow$ external pumping/excitation

But you can get out of thermal equilibrium and that was our observation 3 where we said we can get out of thermal equilibrium if we use this external pumping right by using external pumping we can actually achieve this population inversion and once you achieve the population inversion then you could potentially have a large rate of simulated emission happening in compare to spontaneous emission and through that process we achieve amplification light amplification right that's way we left off things. So apparent when you look at this picture a is the fact that when we look at this atomic system this lifetime right lets say this corresponds to tau2.

What can we say both are lifetime? What do we need ideally for stimulated emission to happen? It should be a very long lifetime right, so it is not enough to just get this atoms to go to the higher energy level but it should stay there until when, until a photon comes by to stimulate the transition right. If it is a very short lifetime there, what will happen? It will immediately come down you know spontaneously emitting a photon right and you know that is not what we want as far as lifetime application because a spontaneously emitted photon in this picture it corresponding to the same frequency that we want to get but it is not the same phase as the photon that you want to amplify.

In other words a photon that you want to amplify has nothing to do with this emission process right, if it is a spontaneously emitted photon. So you don't want so in other words when you are trying to amplify this photons you may have some of this photons going through stimulated emission which is what we want us for amplification concern and some photons maybe generated you know this atom is like tired of sitting there waiting for a photon to come by, nothing is coming by so I am just going to go down right. So it just jumps and then it spontaneously emits a photon and that photon is not in phase with the photon that is coming in right, so that is I having some arbitrary phase.

So what happens at the receiver? When you are capturing that photon you have certain photon that are coming in phase and certain photons that are like arbitrarily coming around right. So you can have this constructive destructive interference happening and that could essentially noise essentially so this spontaneously emitted photons as far as an amplification process concern would constitute noise and this probably a little early to talk about those concepts we will come back and look at this in much more detail but I just want to give you a feel for why this this is it is important to have lesser spontaneous emission and most stimulated emission as far as in amplification process is concern ok.

But it requires external pumping to create that inversion it requires a long lifetime for the higher excited energy level it should be relatively happy saying in that other orbital it is a no hurry to come down right if so if you have atomic systems where that excited state lifetime is long that is actually very good and so happens that (())(14:49)rare earth ion specifically erbium, eterbium, presidium, neodymium, terbium right all this all rare earth ions, rare earth ions means what? Why do they call it rare earth? You cannot you rarely find them in nature right so this rare earth ions actually have relatively long metastable lifetimes and so most of the light sources that you see other than semiconductor light sources will come back and look at that later.

Other than semiconductor light sources most of the other light sources that you see especially in solid state light sources. So you can obviously have this amplification happening in all this different states of matter right solid, liquid, gas right so what is preferable? Well if you have a gas right so you have to put into a tube and you have to maintain the purity so it has to be a vacuum seal tube and you need to keep the tube you know protected from external elements which it can say that it is going to be relatively fragile so you need to make sure you package it very nicely in order that so there is lot of handling issues with that and saying with liquid you know state amplifiers once again you could have what you dealt with once again (())(16:48) that is a amplifying medium right and that is in the liquid state but you have to handle that liquid carefully.

You had it in a (())(16:59) very careful handling the (())(17:01) you didn't want to like drop it accidently and spill the liquid in all which would be a quite a bit of mess. So from that perspective you say solid state lasers this rare earth ions doped in glass it is you know that is actually a highly preferred medium for this amplifiers. So you know push for solid state lasers traditionally you know things have evolved from gas and so there used to be a lot of gas lasers like argon ion lasers, helium ion lasers we still have a helium ion laser in the lab you might have seen that that is still around but that is about the only gas laser that I can think which is around in most of the labs, most of the other lasers have other gas lasers have been taken out or been replaced by solid state lasers and similarly there are die lasers, die amplifiers and those have also been most of them have been replaced by solid state lasers ok.

So you need a solid state laser or solid state amplifier and you need in that material you need a fairly long lifetime I have been going back and forth with amplifier and laser can you imagine how you would build a laser using an amplifier? General laser is like an oscillator ok does that give you a clue how do you take an amplifier and make an oscillator? You might have done this in electronics circuits provide feedback right. So where do you provide feedback in this? You put a mirror here, you put a mirror here, so what is that feedback? It basically feedback this photons right this photons which enhance stimulated emission because every time they come around they going to stimulate this transitions so you going to have (you know) stimulated emission and through that you have a laser.

A laser is an acronym for light amplification by stimulated emission of radiation right so this feedback that you provide through you know a set of mirrors is one configuration is what can take an amplifier and you can make a laser out of that ok will come back to some of those specifics but lets go on to take an example now ok.

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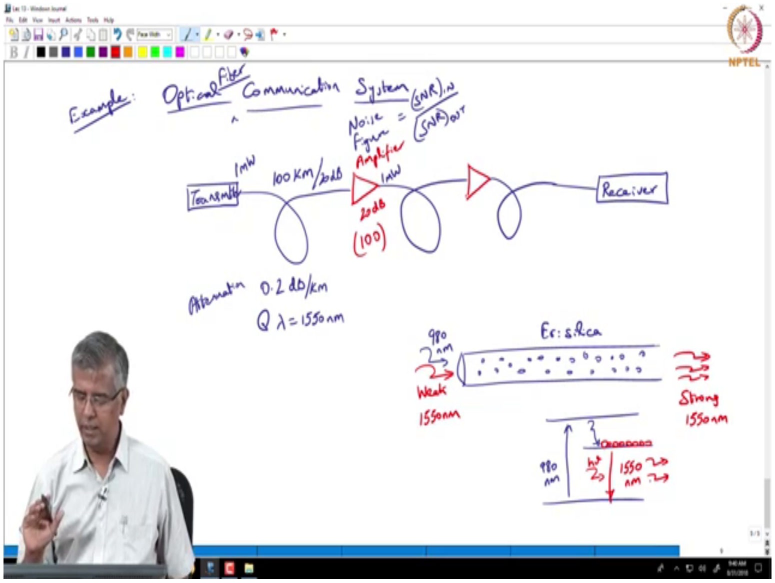

I will take the example where we need an optical amplifier right, what is the use of an optical amplifier? Ofcourse we went ahead and saw the optical amplifier is a basic building block for a laser so you may need a laser for multiple applications but you know will defer that discussion but lets just see where we need an optical amplifier and it turns out that optical communication system is one such example where we need an optical amplifier and what exactly do we need?

Well in an optical communication you have transmitter right and then you may have a receiver ok and distances between the transmitter and receiver could be the order of hundreds of kilometers thousands of kilometers right we talking about going across the ocean Atlantic ocean, Pacific ocean so right now you know the entire world is wired with optical amplifier right several dozens of strands of optical amplifier I mean optical fiber (())(21:56) so clearly in an I should have said maybe optical fiber communication system ok and clearly when we are using an optical fiber it go through of long length of optical fiber that fiber has some inherent attenuation ok. What causes this attenuation you know there may be some scattering happening in the fiber something called Rayleigh scattering because the optical fiber is an amorphous is made of this silica glass but it is amorphous in nature and there is lot of density variations in the glass ok and because of those density variations your light tends to get scattered and that is the process that we call as Rayleigh scattering because the density variations are much smaller to the wavelength of light.

So that works out to be one of the primary causes and the other cause it could be that there are some absorption in the fiber right and because of all that you have 0.2 dB per kilometer loss so this is the attenuation that you see in a fiber but at specific wavelength of 1550 nanometer right that is where you get to see this sort of attenuation ok. So you go through this fiber and lets say you going through 100 kilometers of this fiber right over 100 kilometers how much loss would you have? 20 dB right so over 100 kilometers you are loosing 20 dB of the light so lets say we have transmitted 1 milli watt of power (from where) from the transmitter.

So you have 20 dB loss ok and it is still far away from the receiver right so you need to make up compensate for the loss. So what do you need? You need an amplifier right so you need an amplifier with the gain of 20 dB so that it can boost up the signal again to the original level and then it can go another 100 kilometers and there you put another amplifier and so on right so you can probably you know can have a series of amplifiers followed by fibers and you are covered several 100 kilometers with this ok.

So this is the typical scenario in an optical fiber communication system and what we are saying is we need an amplifier which provide a gain of 20 dB ok, 20 dB is what? A factor of 20 dB in power right it is 10 log is what you take when you do power related measurements so 20 dB would correspond to 10 power 2 right so that is 100 factor of 100. So we want to achieve a factor of 100 gains so (())(26:11) we are going to do that right before I do that thing you had a question.

Student: (())(26:15)

Sure, so that is a very good question, question is the amplifier just as we discuss a little while ago might also have spontaneous emission and that spontaneous emission could get amplified that is the problem spontaneously emitted photons by themselves are not much of an issue they are getting amplified right so they are also multiplying in number along with the signal photons that you want to amplify and those are all you know just constituting noise right so this is what we teach at the end of an advance course on fiber optic communications system but I will let out the secret now so when you concatenate so many amplifiers you are accumulating all the spontaneously emitted photons.

So this amplified spontaneous emission A C is what we call that is accumulating along the link so it is degrading the signal to noise ratio as you go further and further ok. So you think that (you know) I had 1 milli watt I came down to 10 micro watt after 100 kilometers but I put an amplifier and at the end of the amplifier I got my 1 milli watt back. So my signal is back to where it was. So it is as if it is new (no) that signal also has a fraction of amplify spontaneous emission and that fraction keeps growing as you go deeper into the link as you go to concatenate multiple amplifiers so you know overall signal to noise ratio keeps reducing.

This is not just in optical amplifier is even other electronic amplifier is you will see that before you get to do that amplification is the highest signal to noise ratio that you have. You say ok but through an amplifier you are amplifying the signal but you are also amplifying the noise whatever noise was present at the input and that amplifier adds its own noise so net effect is the signal to noise ratio is only going to degrade. So all these amplifiers would have a something called noise figure. So what is noise figure? SNIR at the input divided by (sorry) SNR at the input divided by SNR at the output ok.

So they add their own noise to it so you would see that the (signal) SNR the signal to noise ratio is degrading as we go further down that system ok. Now that you have learned that you going not take that course, I am just joking. No so lets come back to what we wanted to do so we want to achieve this amplification. Now how do you build an amplifier? Ok it turns out that since we are dealing with optical fibers we don't want to like take light out of an optical fiber you know how difficult it is taking light out of an optical fiber, amplifying through some external medium and then coupling that light back into the fiber right you incur losses at all this stages right.

So you would preferably want to have amplification happening within the fiber itself ok and guess what, we are lucky because actually there is this professor in University of Southampton professor David Pane would happens to be one of our collaborators also, so he had this invention where you could dope this rare earth ions. Remember I mentioned this rare earth ions are very good that they could they have a fairly long lifetime so they qualify for optical amplification. So you doped rare earth ion specifically one ion by named erbium right and when you dope erbium into silica, so essentially what we are looking at is lets just say we are looking at the core of the optical fiber so you have this erbium ions distributed in the core of the optical fiber.

We won't go to the details of how erbium got in there, there is some solution doping (())(31:49) when go to the details of that if you impregnate this glass with erbium so the erbium ions are sitting inside then you could in an erbium specifically has an energy level like this so you are pumping external energy at 980 nanometers it goes to a higher energy level an erbium is such that it actually comes down to a lower energy level through non-radiative relaxation and from here this transition corresponds to 1550 ok this is actually god given specific rare earth ions having you know specific energy level structure right and so that clearly if you want to get amplification at 1550 that delta E between two energy levels has to be 15 corresponding to 1550 right.

So you cannot you know manufacture ion two have that particular transition we have not there yet ok so you look for which rare earth ion actually has this transition to his credit he found that erbium has this transition at 1550 so he doped erbium into silica so this is basically erbium doped silica optical fiber and then he found that this if you couple week radiation at 1550 nanometer you can go out with strong radiation or much many more number of photons, photon are you know counted in terms of their flux density so the input flux density is very low but the output flux density is very high and where did we get that extra energy? Through the pump right so we have supplied somehow we have supplied this 980 nanometer photons also into this medium ok and so we got this amplification.

So the question is if we are feeding in 980 according to this picture you already have 1550 nanometers right so why are we feeding in 1550? Turn around the question 1550 is what we want to amplify right so we want to amplify 1550 and to amplify 1550 you need a large number of this erbium ions sitting at that higher energy level right to have that many ions at the higher energy level you need to excite all this erbium ions and to excite the erbium ions we are feeding in 980 right. So hope that is clear so this is h mu at 1550 and then we are getting this amplification happening through stimulated emission.

Ok so let us go into some specifics lets actually take an elemental system I don’t want to go into the details of erbium yet let me just take an elemental system and try to quantify the gain of that system ok. So lets (())(36:30) try do that and to do that I would consider.

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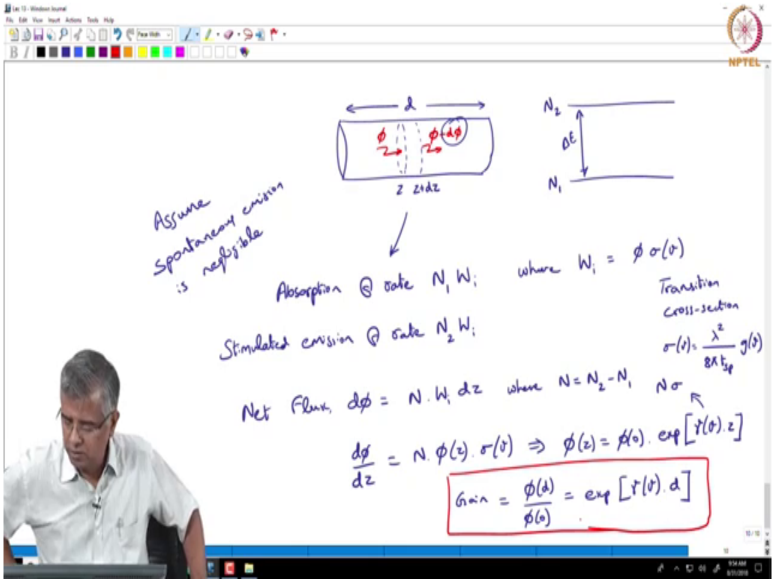

Assume spontaneous emission is negligible

Absorption @ rate $N_1 W_i$ where $W_i = \phi\,\sigma(\nu)$

Stimulated emission @ rate $N_2 W_i$

Transition cross-section

$\sigma(\nu) = \frac{\lambda^2}{8\pi t_{sp}} g(\nu)$

Net Flux, $d\phi = N \cdot W_i\, dz$ where $N = N_2 - N_1$ $\quad N\sigma$

$$\frac{d\phi}{dz} = N \cdot \phi(z) \cdot \sigma(\nu) \Rightarrow \phi(z) = \phi(0) \cdot \exp[\gamma(\nu) \cdot z]$$

$$\boxed{\text{Gain} = \frac{\phi(d)}{\phi(0)} = \exp[\gamma(\nu) \cdot d]}$$

Let’s say a gain medium right and lets specifically consider a small slice of that so the overall length of this gain medium is d but I am considering a slice between z and z plus d z ok and I am also considering certain flux density going into this region and of course you have a certain flux density coming out and we want to quantify that the d phi corresponds to the amplification that we have achieved right.

So this is the picture we want to look at and of course it correspond to lets say this gain medium is an atomic system it is two level atomic system with n1 corresponding to the number density in the lower energy level and n2 corresponding to the number density in the higher energy level and delta E corresponding to the energy difference between the two levels ok. So lets track what has happening over here, so what is happening is you have absorption happening which takes an atom from lower energy level to a higher energy level is it ok that I say from here on just atom whenever I am saying an atom is going from lower energy level to a higher energy level what I mean is an electron is taking from one orbital to another orbital right.

So let just say that the atom is excited and it is going to that excited higher energy level so absorption is happening at a rate that is given by ofcourse absorption is going to depend on how many atoms are at that lower energy level so n1 and it is going to the (())(39:32) state with rate we call as W I ok where W I clearly depends on the photon flux density right the number of photons that number density of photons that come per unit time and there is a you know let me talk about all of this are probabilistic events right so it is not like every photon you know that comes around gets absorbed by a an atom ok.

So you have a certain probability that a photon will get absorbed by a particular atom and that probability density we were looking at before but in the last lecture we said that probability density we will express in terms of sigma which is known as the transition cross section right, sigma is the corresponds to the transition cross section right and it is specifically quantifying sigma I think we did this yesterday is lambda square over 8 pie multiplied by T s p which we are calling as the spontaneous lifetime. Now in general we say sigma is actually a function of (()) (41:27) what is (())(41:48) frequency right. So what we are implying is that sigma is different for different frequencies.

In this case we have only one transition so how many frequencies would it correspond to? It is a very specific frequency that we are talking about but in reality we would have a bunch of you know energy levels that are possible in a collection of atoms in material you will have multiple energy levels that are (())(42:10) corresponding to a ground state and multiple energies corresponding the excited state and there are multiple transitions that are possible between the ground state and the excited state and each of those transitions have a different (possibility) different probability ok. So sigma mu can be different for different values of frequency and to account for that we typically use a quantity which we call G of mu is called the line shape function ofcourse G of mu equals to 1 for this particular case because we are just looking at just unitary transition but G of mu can take it can be normalized to 1 and can take different fractional values for different frequencies in general.

Will comeback look at them specific examples on that later on but key thing is there is some transition cross section this determines this rate at which absorption takes place and similarly you can say that stimulated emission will also happen at a rate that will be given by that will depend on number of atoms (())(43:46) state so n2 times W I right is W I going be different for absorption is this emission, if you are considering one single transition right single pair of energy levels that is not going be different. So if this is the same W I that you have for absorption the same W I you have for emission also ok and we will assume that spontaneous emission is negligible in this case ok.

We will do the more regress you know treatment later on but at this point we will make it fairly simple ok. So if you are tracking what we are interested in is this value d phi, so the net flux d phi would correspond to what? The difference in the two rates right because we are saying stimulated emission can happen but the same emitted photon can also be absorbed right because that is also probable in this particular thing so the net flux is what we are trying to track and that will correspond to n multiplied by W I and that is integrated over length d z ok where what did n corresponds to? Net emission n2 minus n1 that is the n corresponds to the inversion that we have in the system and that inversion is created by in practical sense that inversion could be created by an external source we are not considering that we just considering the transition between this two.

So you re-arrange terms and you can say d phi over d z will correspond to n times W I but now W I will actually write out W I corresponds to d phi or d z? It correspond to phi of z multiplied by sigma of mu ok and the solution for this will be phi of z is given by phi of zero what is phi of zero here? We say that, that is the incoming photon flux density right phi of zero multiplied by exponential of gamma of mu multiplied by z in this case right. So where gamma of mu is representing n multiplied by sigma right so gamma of mu is representing that inversion multiplied by the transition cross section ok.

So if we were to look at the gain in the system we are saying is the total gain of the system which corresponds to the flux density at the output of this entire length d divided by phi of 0 that will be what? Exponential of gamma of mu multiplied by d right. So that is the gain that we get from that medium the gamma of mu corresponds to the inversion right multiplied by the transition cross section. The transition cross section is given for a given system you are not going to be able to change that ok but the inversion you can change through the pumping which we have not considered here but going moving forward we will actually look at you know the traditional systems like you can take example erbium and see how the pumping happens how we get inversion and through that inversion how do we get gain ok. So this is just a simple model but like I said will go into more details as we move on ok thank you for your attention will meet again in the next lecture.