**Introduction to Photonics**
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**Four Level Systems**

Alright welcome back to another session of introduction to Photonics, so what we have been discussing lately is light generation and amplification. We have been specifically looking at how photons interact with matter through which you could have absorption of those photons and subsequently you could have emission of photon either as spontaneous emission or stimulated emission and we took this example of an optical amplifier which is part of an optical fibre communication system.

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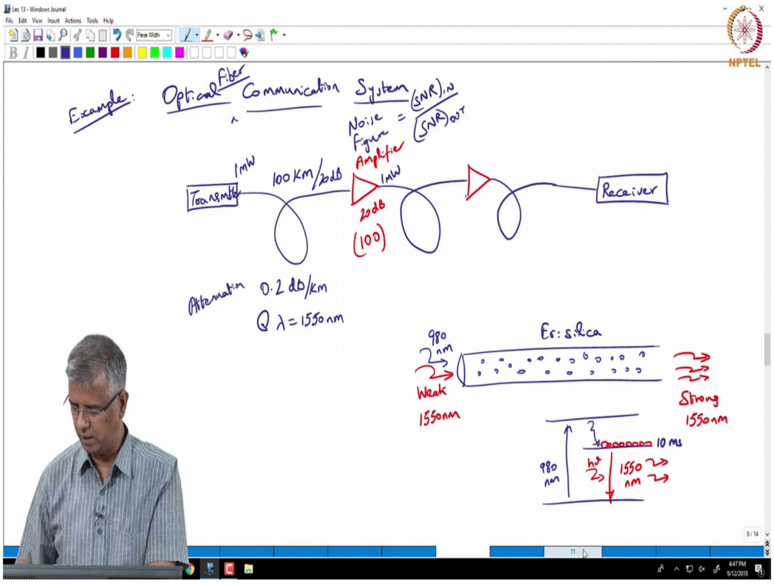


So as we know now erbium doped silica fibres are used in this optical amplifiers and erbium doped silica essentially consist of 3 energy levels, so you are able to go from the ground state, the balance orbital of this atom erbium atom to higher energy level with the absorption of 980 nanometre photon and then we said the excited state is relatively short lived so and then the next energy level is relatively close, when I say relatively close what I mean by that is that energy difference is corresponding to the phonon energy of the glass okay phonon energy is the vibrational energy that you have in the glass.

So when you have 2 energy levels that are very close to each other that is within the phonon energy of the glass you lose that energy through just non-radiative relaxation okay, so that is what is happening in erbium it goes to that highest energy level and then goes down to one step below where that energy level is relatively long-lived, so this energy level is in the order of… It has got a lifetime order of 10 milli second.

So whatever you pump to the higher energy level goes and stays or accumulates in this particular energy level which has 10 millisecond lifetime and through that through enough pumping you create population inversion and once you have population inversion you have the stimulated emission happening wherein weak photon or a weak radiation consists of few number of photons at 1550 nanometre goes through this amplifier and gets amplified into a much larger number of protons. So we had been looking at expression for this inversion as a function of the pumping rate and we got to that expression in the last lecture but before we go to that I want to make just one point about this.

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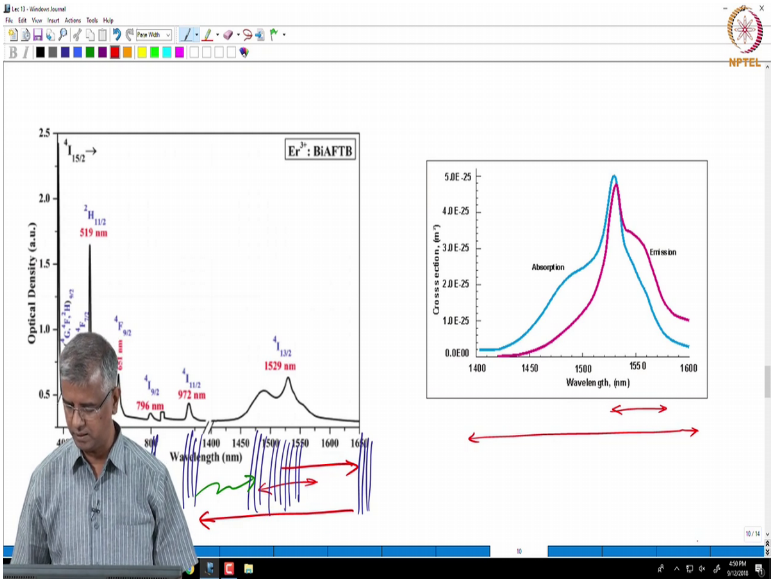


Remember we were looking at this erbium absorption spectrum, erbium in this other glass which is bismuth-based glass we said this was the absorption spectrum. One quick point to note as far as this is concerned is that this absorption spectrum corresponds to essentially absorption between different energy level, so I can basically say that my ground state is somewhere over here I have some excited state over here, another bunch of energy levels over here, another bunch over here, another bunch over here and so on okay.

So what we are doing here if we were to visualise this is you are going up in energy, right? So whenever you see an absorption spectrum like this you can say that your ground level is the lowest energy level is on the rightmost side, right? And from there you are making these transitions to the higher energy level, so each of these absorption peaks corresponds to one of those transitions, do you understand what I am saying?

Is there a question on this? Right so this is one way of visualising the absorption spectrum of any given element okay but we were considering erbium as a 3 level system wherein we are pumping into this energy level, we do work non-radiative relaxation to this band of energy levels and from there we are having stimulated emission when this transition happens okay, so that is what we were talking about we just turned around the energy level energy band diagram when we were considering this in their previous lectures okay.

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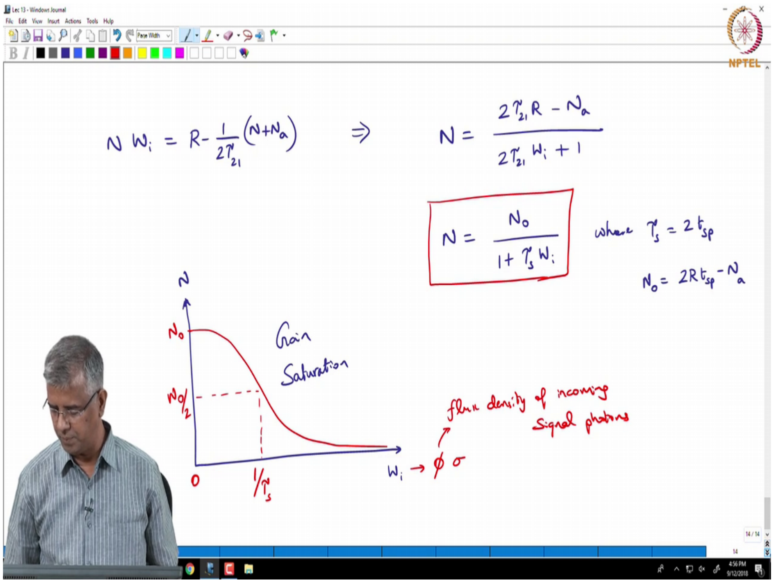


So we stopped at the point where we looked at the inversion that you could create for a 3 level system and erbium and silica is actually a good example of this and the inversion we said is equal to N naught over 1 plus tau s Wi where N naught what does that correspond to? That is the inversion that you get there is a 0 photons, 0 signal photons that are coming through, right? So to get this inversion obviously you are pumping the system, right? So you are supplying pump photons to take these erbium ions from the ground state to an excited state, right? But assuming you do not have any signal photons coming through you would be maintaining a certain inversion, right?

That inversion is what we called as N naught and then tau s we got a expression for that tau s is correspond to twice the spontaneous emission lifetime that is the lifetime of your excited energy level N2 energy level correspond with corresponding number density N2 and then we have got an expression for N naught as well right, so we stopped at this point where we derived an expression for N naught which depends on R where R is what, what is R correspond to? The rate at which we are pumping, right?

So the pump photons which are taking the erbium atom from ground state to an excited state and Na, what is Na correspond to? So that is the total number density of erbium atoms in that fibre the gain medium, right? And one thing I did not mention explicitly is Wi. Wi correspond to the rate at which the stimulated emission or the stimulated absorption happens and Wi corresponds to the flux density multiplied by the cross-section there is the emission or absorption cross-section okay. Now where we stopped is what this graph is showing as is as we increase…so initially when we have 0 photons coming in signal photons coming in, so this flux density is corresponding to 0.

You have a certain inversion for a given pump rate and then as you increase the signal photon flux density, right as more and more signal photons come through we see that the inversion that is available to you is not as much as in the case of 0 signal photons, so the inversion decreases and there is one particular point at which the inversion becomes half of what it was when there was no signal photons that is what we are denoting by N naught over 2 and the corresponding rate is given by one over tau s okay.

So this corresponds to certain characteristics rate right and the time corresponds to some characteristics time which we will come back and define as the saturation lifetime of the system okay and what happens as you go to higher and higher values of flux density, signal flux density is that your stimulated absorption and stimulated emission terms dominates, so essentially they have equal probability right, so the stimulated emission and the stimulated absorption typically have equal probability in which case the gain essentially goes to 0 okay, so when Wi is very large value what we are seeing is that the gain actually goes down to 0 okay. So let us take an example well we will continue with the same example of erbium silica gain medium but let us start quantifying few things.

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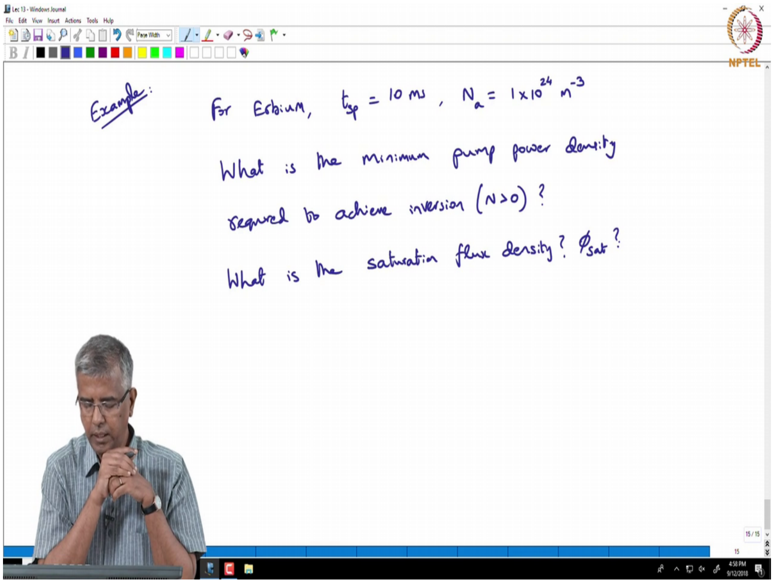


So just workout an example problem right, for erbium it is given that the spontaneous lifetime TSP is 10 millisecond and you are asked to consider the total number density of erbium in this medium as 1 10 to the power of 24 per metre cube, right you are asked to figure out what is the minimum pump power density required to achieve inversion, so inversion essentially means that your N2 minus N1 is greater than 0 or N is greater than 0 right, so what is the minimum pump power density required to achieve inversion and you are also asked to figure out what is the saturation flux density? Okay suppose you are asked to figure out these 2 quantities, the saturated flux density you would denote as phi sat, right so what is that value is what is asked okay.

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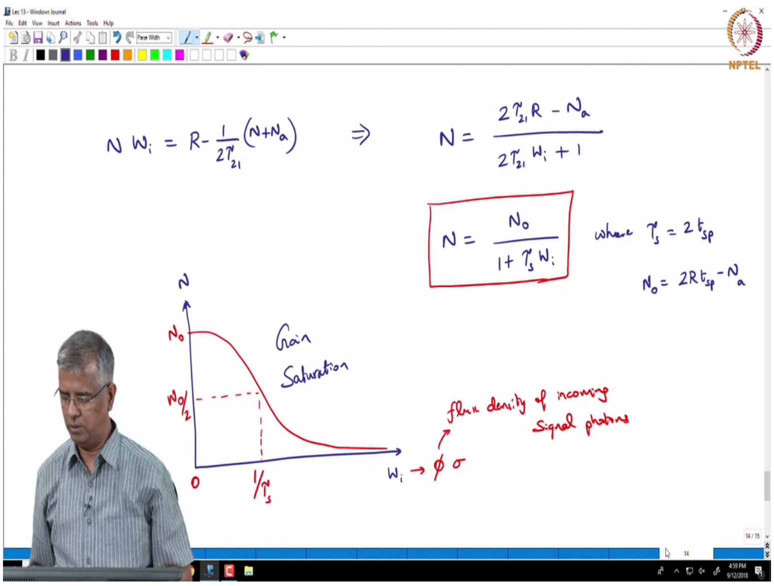

So what we know is this that your inversion is given by N naught over 1 plus tau s Wi where N naught is given by 2 times R t sp minus Na, so we minimum pump power that you need to achieve inversion that is N naught is greater than 0 right, so to do that to achieve that what condition do you have to satisfy?

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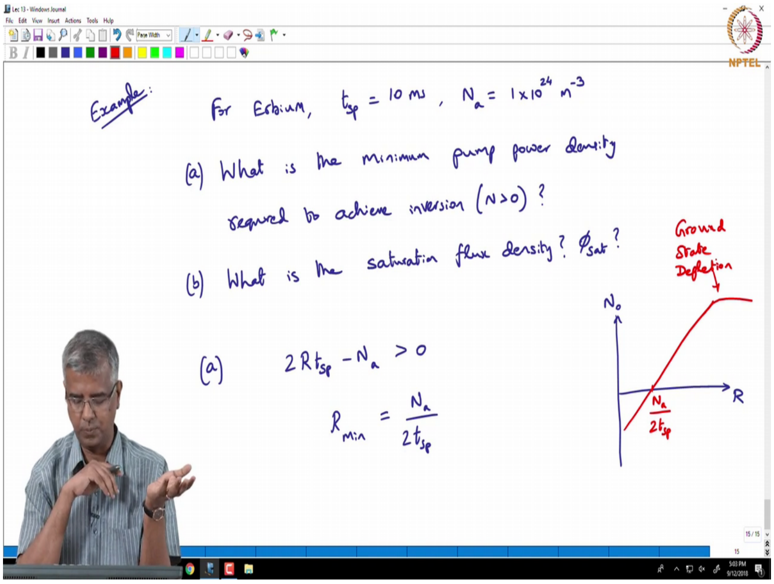


So let us say this is a and this is the b part, the a part what we are saying is that you have a 2 times R tsp minus Na has to be greater than 0 right, so for N naught to be greater than 0 that is the condition you have, so your pump rate, the minimum pump rate has to be equal to Na over 2 times tsp right so that is the minimum pump rate that you require to achieve inversion and at this point it is probably helpful to understand how the inversion changes or how the inversion depends on the pump rate right, so if you were to plot the inversion N as a function of the pumping rate, how is that going to look?

Let us say we specifically looking at N naught as a function of the pump rate, what sort of dependence is that? That is a linear dependence, right? So you basically going to have a linear dependence like this and what that tells you is for small values of R you have negative inversion, what is negative inversion mean? More ions sitting in the ground state compared to the excited state and then you get to a certain value of R where that inversion is equal to 0 which means there is an equal number of you know excited atoms to the ground state atoms and that value is now given by Na over 2 times tsp right and then beyond that you have higher inversion that you can build with higher pump rate and that does not extend forever, it will saturate at certain point, why does it saturate?

What is happening beyond a certain pump rate? No more ions in the ground state, so you have a finite number of these erbium atoms in the matrix and if you flood the gain medium with so many pump photons that all the erbium atoms are going to the excited state there is no more erbium atoms in the ground state, right so there is no more inversion that could be created, right? So this is basically due to what is called ground state depletion, right?

So every gain medium you can say is going to have a finite number of these atoms and beyond a certain pump power level you are going to deplete the ground state and you are going to saturate the inversion and because your inversion is saturated your gain is also going to get saturated. Now this saturation that we are talking about is different from the other saturation we discussed previously, right?

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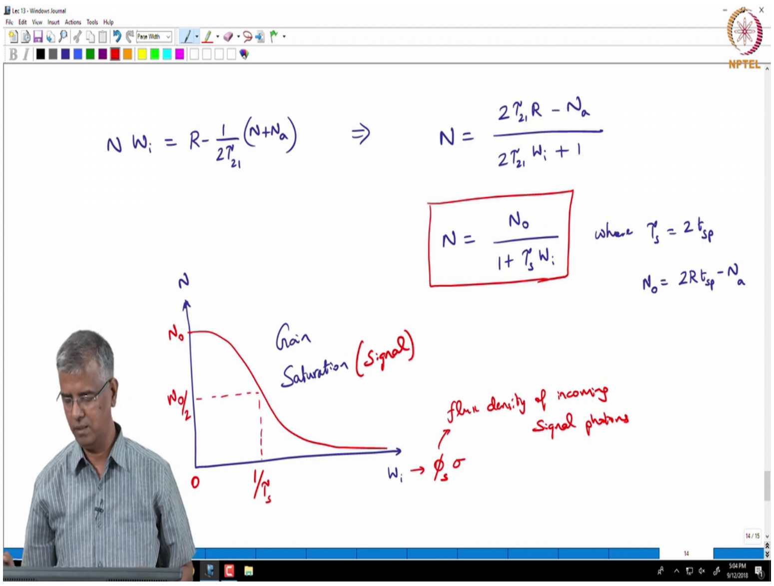


The other saturation here, what is this due to? Here when we talk about gain saturation, what are we talking about?

Student: (())(19:46)

Prof: We are basically saying that the gain is reduced as you increase what?

Student: (())(19:56)

Prof: Input signal flux density, so it is all with respect to the signal photons as you increase more and more number of signal photons your gain is reducing, right? Because as you have more signal photons more stimulated emission happens, which takes away the inversion which reduces the inversion okay, so that is…

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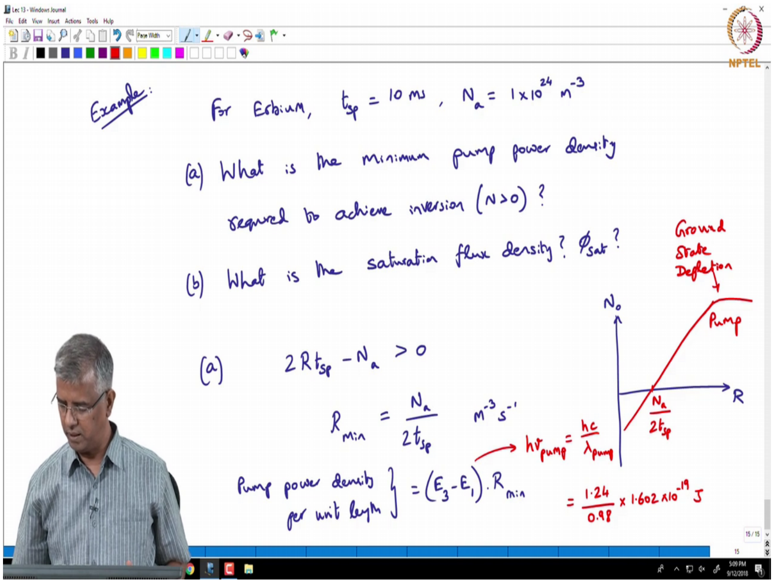


You know this again saturation is due to signal whereas this again saturation is due to pump, right? As you go to a very high level of pumping you start having saturation of the gain and that is due to ground state depletion. You see the difference between the 2 cases? In fact this is what you are going to be experimenting this week you are going to build an erbium doped fiber amplifier where you will look at the gain as a function of pump power as well as function of signal power.

Okay but what we are asked to figure out is what is the minimum pump power density required to achieve inversion, so what we have is a rate, right the rate which has a unit of per metre cube per second right that is what you have and you need to actually figure out the pump power density, so how would you proceed from here? What do you have to multiply this rate with to get power? So if I were to say what is the pump power density per unit length, what quantity…

If you have a rate what quantity do you to multiply that to get power? Energy, right? Energy multiplied by… or energy divided by time is power right, so what energy? Energy corresponding to the pump photons right, so in this case energy corresponding to the difference between the 2 energy level right E3 minus E1 multiplied by R, right? So that would give you the…in this case we are looking at the minimum pump power density, so we will say E3 minus E1 multiplied by R times minimum. How do you find E3 minus E1?

Student: (())(23:35)

Prof: That is the energy corresponding to what?

Student: (())(23:42)

Prof: Energy corresponding to pump photons right, so it is basically h nu pump, do we know this energy? Yes we are told that it corresponds to 980 nanometre photon, right so h nu pump you can write it as hc over lambda where lambda or the pump corresponds to 980 nanometres, right so you will find that this is basically 1.24 EV divided by lambda of pump which is 0.98 lambda pump express in micron so it is 0.98 and then this is in EV, so what do you do to convert to joules?

Student: (())(24:41)

Prof: So multiply this by 1.602 into 10 power minus 19 and that will be joules, right and do we have enough data to calculate R main? Yes because NA is giving you know to tsp so you can find out R main, so I can go ahead and write it all out okay.

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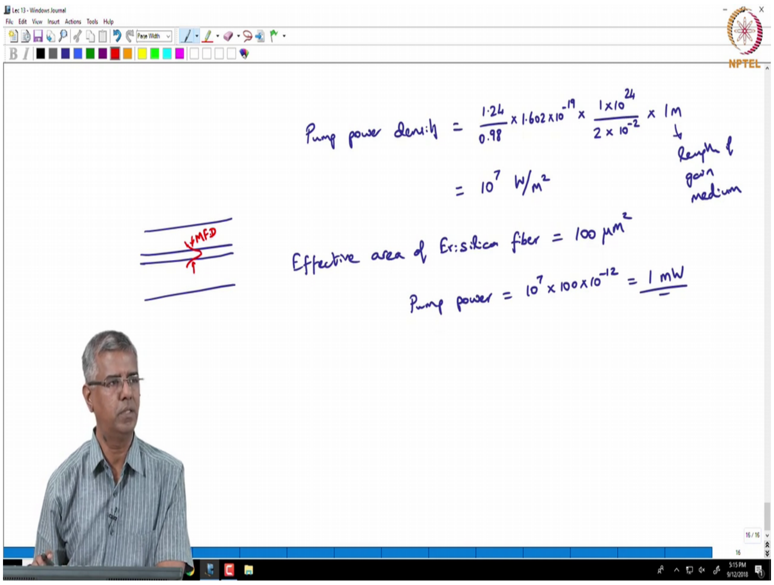


So the pump power density is given by this H nu pump which is 1.24 divided by 0.98 multiplied by 1.602 into 10 power minus 19 multiplied by your Na which is 1 into 10 power 24 divided by your 2 times tsp and tsp is given as 10 milliseconds, so 2 times 10 power minus 2 seconds okay. So if we do all of that and mind you when we calculate this, this comes out as watt per metre cube okay, so this corresponds to a volume, right it as per unit volume is what you have but typically pump power density is a cross-section, so you have essentially say okay if your gain medium is this long you are pumping from the site and your gain medium is this long right that length you multiply over that, right and then you say okay so what you get is in terms of metre square okay.

So you have cross-section if we assume let us say this is 1 metre this is the length of gain medium, right then you get final value in terms of per metre square, so this would work out to be 10 power 7 watt per metre square okay, so that as 10 megawatt per metre square that seems like a very large value right, and 10 megawatts I mean you look at the room lights here this is in the order of few milli watts even this bright LED lights that we have here is only in the order of few milli watts whereas we are talking about megawatts, so is this even possible?

Well it turns out for the example that we consider we were talking about erbium of fibre and erbium of fibre is actually a single mode fibre, so the effective area of the erbium silica fibre is in the order of 100 microns square right, so 100 into 10 power minus 12 meter square, so if you multiply a power density with the area you get pump power just 10 power 7 multiplied by 10 power 100 into 10 power minus 12 which works out to be 1 milli watt bright.

So although the pump power density seems like a very large value if you are focusing all that energy on to this tiny core which is what you have to do to make this amplifier you find that you do not need a lot of power right, so in this case you just have one milli watt of power and then you are able to achieve inversion of 0, so anything more than milli watt is actually building up your version, so you can start having gain be on that.

Student: (())(30:11)

Prof: So effective area is the area of your fiber so we talked about…sorry a core and cladding of single mode fibre and we said that light is actually confined in the core region primarily a little bit of that is spilling over the cladding region, right and we actually said that this corresponds to a certain mode field radius right and certain mode field diameter, so from that mode field diameter you can calculate the effective area okay, so that is the 1st part any questions about this? Yes.

Student: (())( 31:10)

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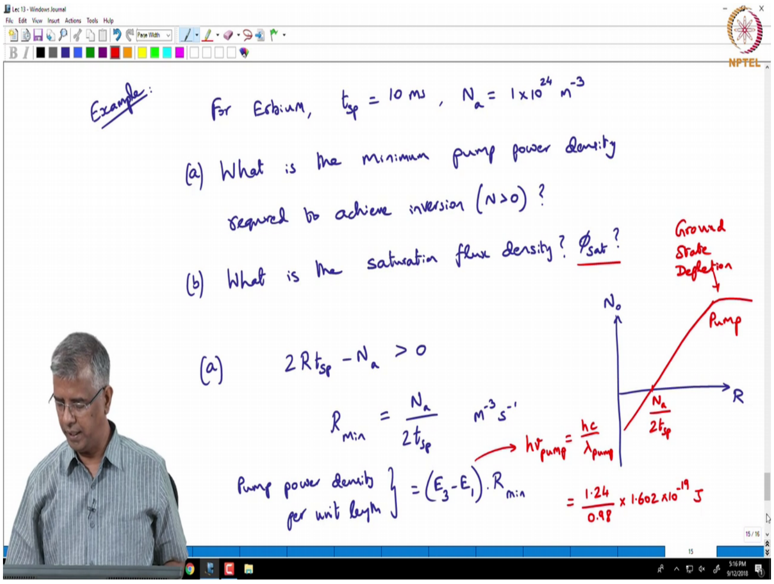


Prof: Yes so right because it is actually per metre cube quantity that you get because our number density is in terms of unit volume but typically when we talk about pump power density we talk about you know power over area not volume, right so I wanted to take that length dependence out and that is what we did by assuming the length to be 1 metre. Then the question is what about this guy?

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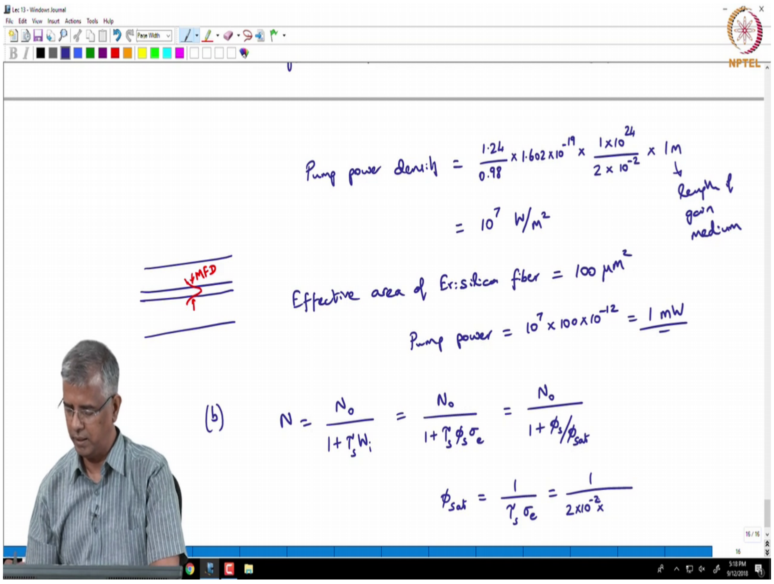


How do we calculate the saturation flux density and for that we need to once again examine our expression for inversion of little more closely, so the expression for inversion is N equal to N naught divided by 1 plus tau s Wi, right?

That is what we derived previously but Wi can be written in terms of the flux density multiplied by the transition cross-section, so I can write it as N naught divided by 1 plus tau s phi s multiplied by the transition cross-section (())(32:43) the emission cross-section okay and then for a given material tau s is a constant, Sigma e is a constant, so I can choose to write this as N naught divided by 1 plus phi s over phi sat, phi s is a variable, phi s corresponds to the signal flux density whereas phi sat is a constant for a given material right which depends on the spontaneous lifetime as well as the emission cross-section. So in this case phi sat is given by 1 over tau s multiplied by sigma e so tau s is 2 times tsp so that is basically 2 into 10 power minus 2 seconds and sigma e I think we considered for this example earlier right.

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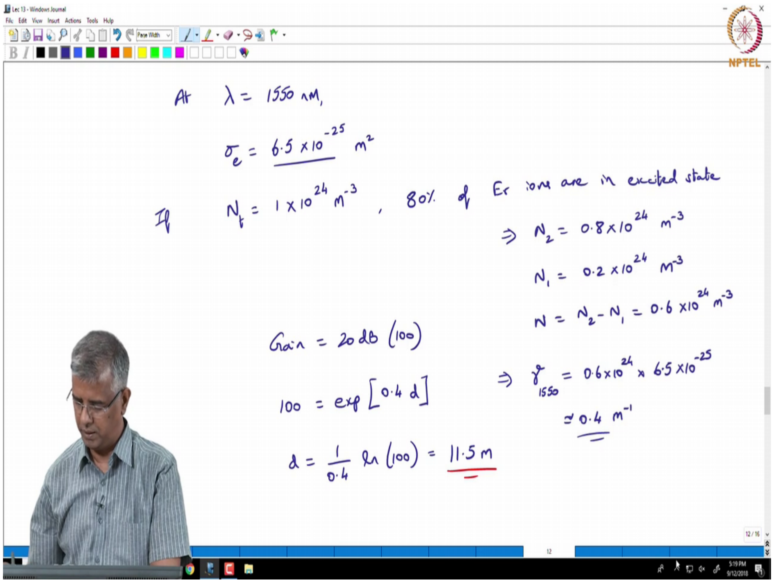


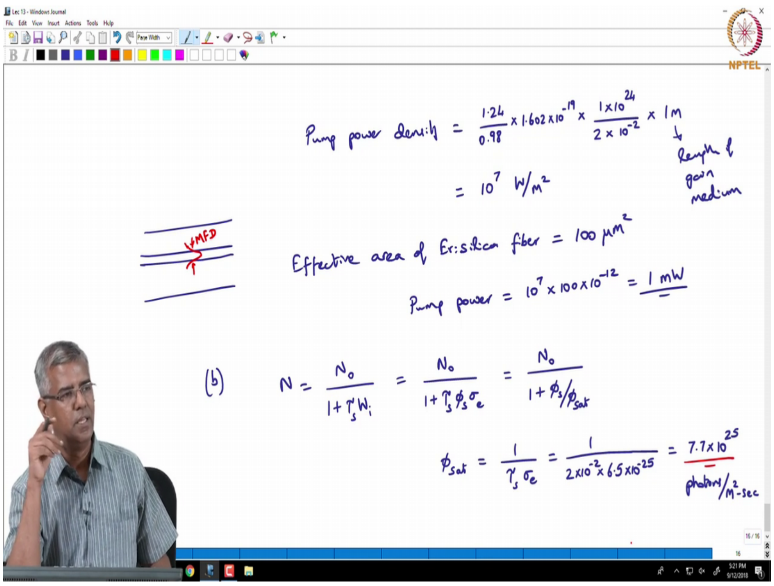

So sigma e we say it is corresponding to 6.5 into 10 power minus 25, so mind you sigma e when you write it down it is actually a different number for different wavelengths okay because different wavelengths or different frequencies corresponds to different transitions and each transition may have a slightly different transition probability. So that emission cross-section is actually a function of wavelength but we are picking one particular wavelength and calculating it and this is what it come out to be and if you calculate that it comes out to 7.7 multiplied by 10 power 25 photons per metre square second, right that is the flux density, the saturation flux density. So the other way of defining your gain saturation is such that phi s equal to phi sat, when phi s signal flux density equals to phi sat, what happens? They inversion goes down by factor of 2 okay and that corresponding phi s is what we are calling as the saturation flux density.

No this phi sat corresponds to your saturation of the gain due to the inversion reducing, the inversion is reducing as you have more and more signal photons coming in.

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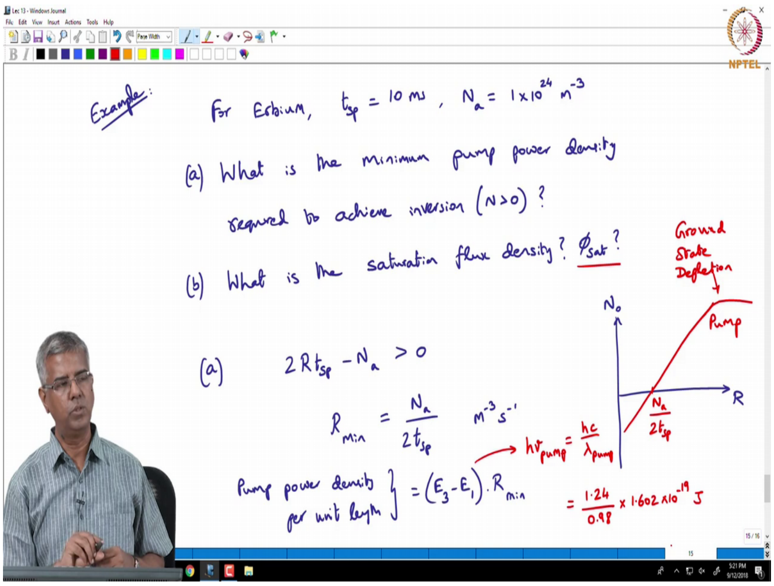

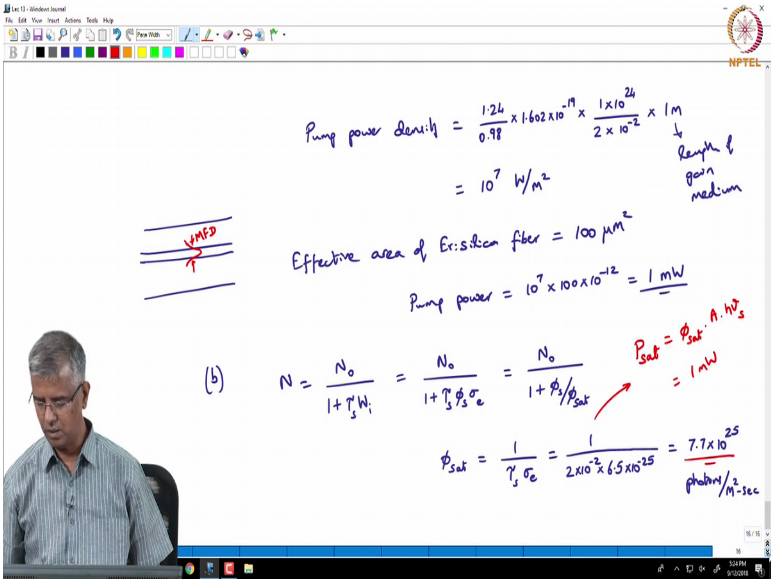


So in this case we looked at the ground state depletion we looked at pumping very hard, right? Such that all your atoms go to the excited state so very few atoms on the ground state and because of that the gain is limited K whereas here we are talking about there are so many signal photons coming in which are depopulating the excited state and as a result of that you are inversion is reducing your gain is decreasing, so those are 2 different one is with respect to signal photons other is with respect to pump photons and of course you can do this exercise this is actually saturation flux density, right?

The saturation flux density, if you want to find the corresponding saturation power phi sat, how do you go from flux density to power? What do you have to multiply this by? Area for sure right this is the area cross-section and what else, so that gives you flux from flux to power what do you need to use? Energy, right so energy over time is power right, so energy of what? With respect to signal transition we are looking at so this is the energy corresponding to the signal photons right? Do you understand that?

And if we do the same for like the area is 100 micron square in this case and the signal photons corresponds to let us say 1530 nanometres if you calculate this, this would work out to be something in the order of milli watt also okay, so the saturation power phi sat you know can be picked up and that also would correspond to (())(38:52) few milli watts. So that is all we want to talk about as far as 3 level systems are concerned okay and let us have a brief discussion on slightly different system which is very important, we will find out why in a few minutes which corresponds to a 4 level system okay.

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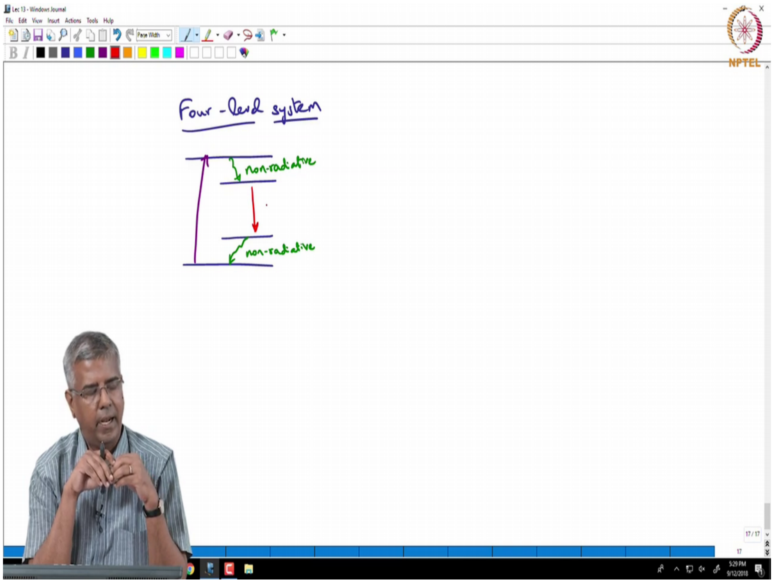


So what is the structure corresponding to a typical 4 level system you have once again a ground state, you have an excited state and then you have one energy level which is relatively close to the excited state, right? Relatively close in terms of you know being within the phonon energy of the glass okay, so because of that you can say that this transition is going to be non-radiative in nature okay.

So you have pumping taking an atom from the lower energy level to higher energy level and then non-radiative relaxation taking it to one level below this which is very similar to what we had in the 3 level system except that you have one more energy level over here okay and that energy level let us say is located such that it is within phonon energy compared to the ground state, so this is also non-radiative in nature okay.

So that is a typical 4 level system and the key thing that we are looking for is transition between these 2 energy levels which give us gain. You know those 2 energy levels are much larger than the thermal energy and all so much larger than the phonon energy of the glass that is very important because as long as you have you know the energy levels very close comparable to the thermal energy or the phonon energy, the non-radiative relaxation is more probable than a radiative relaxation okay.

So those sort of systems are very difficult to get gain from, so one typical example is you know we have been talking about getting gain around near infrared wavelengths whereas for a lot of sensing applications you need sources in mid infrared region 3 microns or above 3 to 5 microns typically is very interesting but when you go to 3 to 5 microns the energy corresponding to that transition what can you say it will be like when you go to longer wavelength?

Energy will be very small right and energy will start approaching the phonon energy or the glass in which case it is very hard to get these radiative relaxation, so it is very hard to get gain from a regular silica glass for transition at 3 to 5 micron region okay, so what you have to do? If you want to get gain for those sort of transition is to find a glass which has relatively low phonon energy very low vibrational energy that essentially means that you have to get to a glass whose molecules are relatively whose atoms are relatively heavy, if you have heavy atoms they do not to move around very much the vibration energy is relatively low okay, so in fact I have done part of my research in making lasers at around 3 micron region and 3 micron believe it and not is what you get you know when you deal with the erbium this transition over here and let us see if I can highlight.

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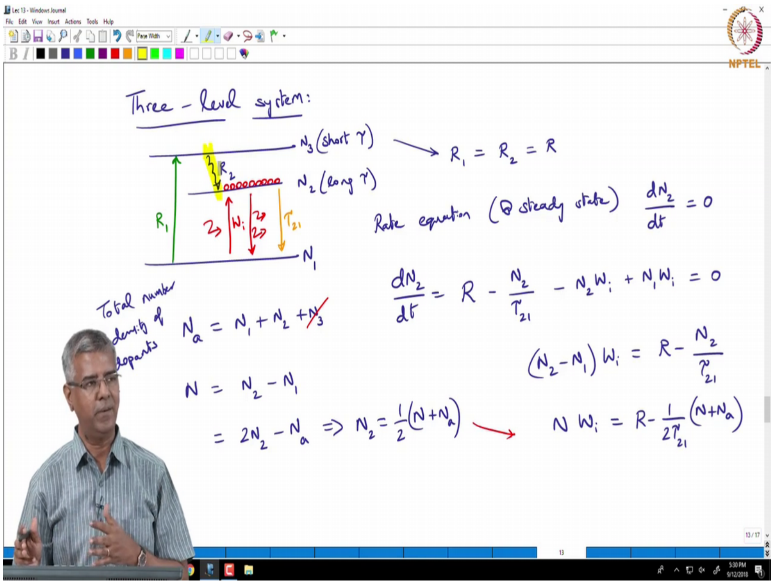


So this transition over here actually corresponds to 3 micron radiation but you do not see that it is actually a non-radiative relaxation as far as silica host is concern, so what we had to do in our work to get that a radiative relaxation is choose a glass host which had very low phonon energy, so we went to glass host called zeeblan it was basically a zirconium based glass and in that glass the phonon energy was so low that we go to get this as a radiative relaxation and we could demonstrate you know lazing at 3 micron so at that point we demonstrated what level output power from a 3 micron laser which was a world record. I am talking about almost 20 years ago but that just giving you an idea of where you get a radiative relaxation, this is a non-radiative relaxation.

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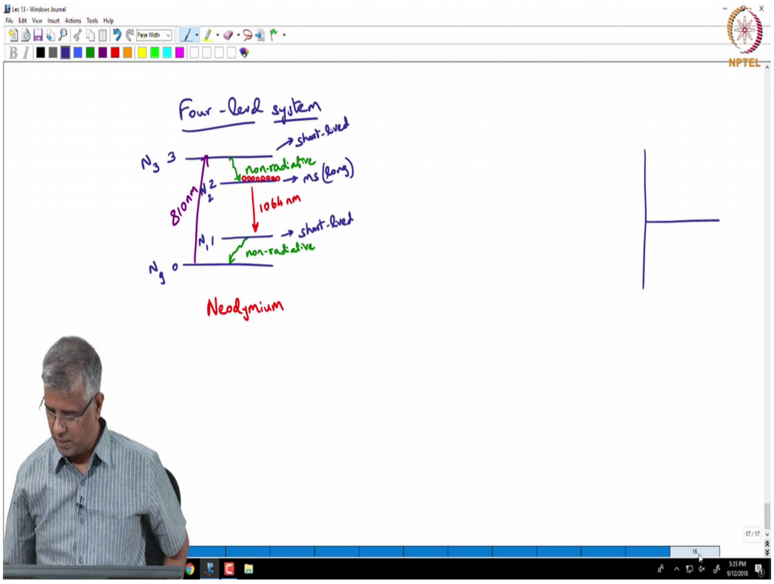


If you get non-radiative relaxation you do not get gain right, only for transition where you can have a high probability of a radiative relaxation that is where you can get gain. Nevertheless coming back so if we have a system like this and we are not talking about some imaginary system this is actually what you have in neodymium okay neodymium is another rarer element. In neodymium you have a structure like this where this pumping typically happens at relatively low shot wavelengths at 810 nanometres and this transition corresponds to 1064 nanometre okay.

So you get especially gain at 1064 nanometre okay it is a very popular laser because it is neodymium which is doped in yag crystals yttrium aluminium garnet crystals can give you a very good again and you can get watts 10s of 100s of watts power from such systems right, so they have been very popular but they are characterised by 2 things one is that this… If I call this level 0, 1, 2 and 3 okay and I will say that Ng, I will say is you know the number density in the ground state N1 is number density in the State and N2 is number density in this excited state and N3 is a number density corresponding to the highest energy level there.

This is very short lived meaning the lifetime of that level is very short okay whereas this is in the order of milliseconds relatively long right and similarly this is also short lived, so can you see what is going to happen? When you pump at 810 right the atoms going to go to level 3, it is going to quickly come down to level 2 typically non-radiatively and all your population is going to accumulate over here and what do you have, so what can you say about the conversion? Inversion is N2 minus N1, what can you say about N1? N1 is typically 0, why?

Because anything that comes to N1 is immediately going to the ground, so N1 is practically 0 most of the time right, so any single atom that goes to the energy level to immediately see is available as inversion, so what will say about…you know we talked about and we are jumping forward but that is okay in this case or may be I will just go back to this previous thing that we used here.

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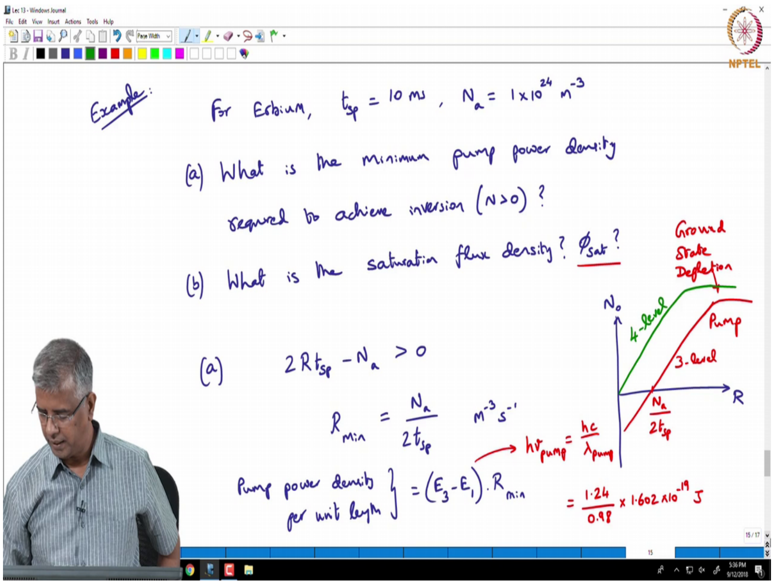


So this is for 3 level system now how is this graph going to look for a 4 level system? It is going to look like this the key point is with very little pump energy with very little pump photon you can immediately get gain, can immediately get inversion and through that you can get gain right? So that is fairly…that corresponds to 4 level system, so 4 level system is actually very good from that perspective that you do not need to have a lot of pump photons to achieve gain from 4 level system, right? So in 3 levels you need certain pump power to achieve transparency meaning your inversion goes to 0 and beyond that only you can get gain whereas in 4 level systems you can get gain rather easily.

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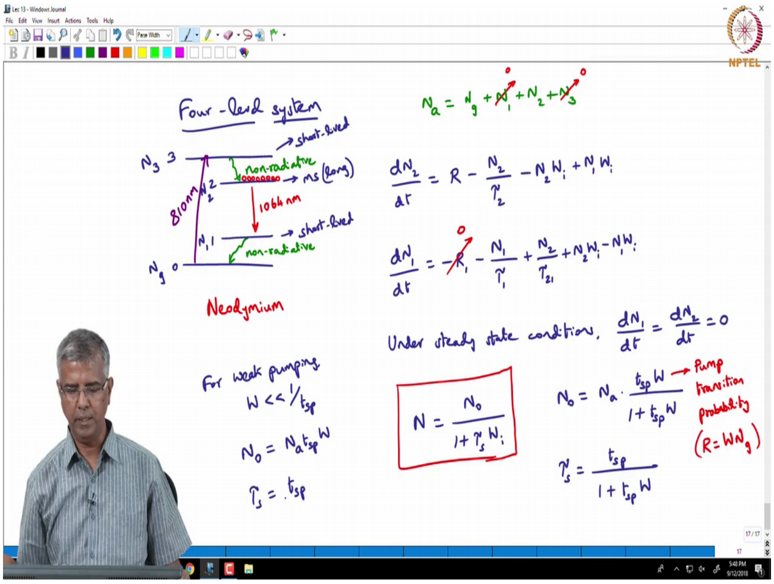


But having done that I can go back here and I can do the same thing that I did before, so I would say that Na is going to be given by Ng plus N1 plus N2 plus N3 but N3 is going to 0, N1 is going to 0 because the corresponding energy levels have very short lifetime right something in the order of 10s of microseconds so you know those essentially go to 0 the (()) (51:54) population there so Na is approximately Ng plus N2, so I can go ahead and write an explanation for the rate equation for N2 and N1.

So dN 2 over dT is going to be depending upon what? That is going to depend on the rate at which…so N2 increases as you increase your pump rate, right and but it will decrease due to spontaneous emission from energy level okay and then it will increase or rather let us talk about this first we can also decrease because of what, what else can happen from energy level to? Stimulated emission right but it will increase because of stimulated absorption sorry which depends on N1.

Student: (())(53:29)

Prof: We assume N1 to be 0 then yes you do not have the term at all but what we are trying to get an explanation for is that inversion, so at this point we have to keep N1 because N2 minus N1 is what we are trying to get but of course that is what I was saying I jumped the gun a little bit and saying that you get inversion at very low pumping rates but nevertheless that is what we get to at the end. Similarly dN1 over dt if you were writing an expression for that, that would be minus of R1 that is the radiative relaxation back to the ground state minus N1 sorry R1 corresponds to or relaxation to any other state okay.

In this case what we do is we will say R1 will go to 0 because there is no other state involved here but this is actually a generic derivation that is given in your book but this one will happen that you could have rated which N1 can depopulated but it will benefit from any spontaneous emission from energy level 2 and stimulated emission from energy level 2 and then you have N1 Wi corresponding to stimulated absorption and like I said maybe I should not even written it explicitly this is going to 0 but because there is no other energy level that it can go to, so that term we need not even consider okay.

So we do, what do we do next? What did we do in the 3 level system next? What condition did we assume? We are interested in steady state condition, so we will go ahead and say under steady state conditions what do we have? dN 1 over dt equal to dN 2 over dt will be equal to 0, so we assume that right and then we go through the same sort of analysis that we did it for the previous case and finally we come up with an expression for the inversion which will find is actually of the same form as what we did for the 3 level system accept this N naught and tau s expressions are going to change because of the constraints that we have shown here for the 4 level system.

So tsp multiplied by Wi where Wi sorry multiplied by W where W corresponds to the pump transition probability divided by 1 plus tsp W and similarly for tau s is going to be given by a tsp divided by 1 plus tsp W right, where I should make a note saying that W corresponds to the pump transition probability. W is representation of R we previously considered R is nothing but W multiplied by Ng okay, so I can just write it here such that R equals to W multiplied by Ng okay.

So yes we could have just written it in terms of R as well just to be consistent with the previous case but once again what we are saying yes and 4 level systems also you have gain saturation and to get a relatively simpler peak about what this gain saturation dependent on if you take a specific case, so from here if you take a specific case for weak pumping, so weak pumping is a case where W is (())(60:02) lesser than 1 over tsp okay. If you consider that case then basically in the denominator 1 plus tsp W will become 1, right under this assumption, so in this case you say that N naught is going to be multiplied by Na tsp W right and that time constant tau s is can be approximated as the spontaneous lifetime itself okay.

So the important thing to note here is you know it depends on the transition probability, the pump transition probability but it does not depend on the number of pump photons, the inversion that you get and this is what we were saying that because N1 is typically N1 is close to 0 is negligible you do not need a lot of pump photons to achieve inversion okay. So the pump there is not a whole lot of pump power dependence as far as achieving inversion is concerned.

So that is the advantage of 4 level systems, how we use this advantage we will actually have to consider the case of a laser before we realize the true impact of this sort of an advantage, so you know that will happen and we will discuss that in the next few lectures but let me stop here and I also want to mention that I jumped a little bit from the rate equation to the final expression. 2 reasons, 1 is it is similar to what we have already done further 3 level systems and also that this is coming directly from (())(62:40) so if you want to go look at the details you can refer to our textbook. Okay, so let me stop there. Thank you.