

Introduction to Photonics
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Semiconductor Detectors – 3

Welcome back to introduction to Photonics, we will be talking about semiconductor live detectors and specifically we will talk about avalanche photodiodes. So the last lecture we were talking about the gain for an avalanche photodiode, so the gain is denoted by this multiplication factor M which we say is the total electron current density after the multiplication region to be divided by the current density that you had before the electrons entered the multiplication region. So we said we can go through some great equation and then we could solve it and then finally get to an expression that is given like this, the bottom right portion of your screen where M is given in terms of Alpha E and Alpha H.

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Example: APD

Gain, $M = \frac{J_e(W_m)}{J_e(0)}$

$\frac{dJ_e}{dx} = \alpha_e J_e(x) + \alpha_h J_h(x)$

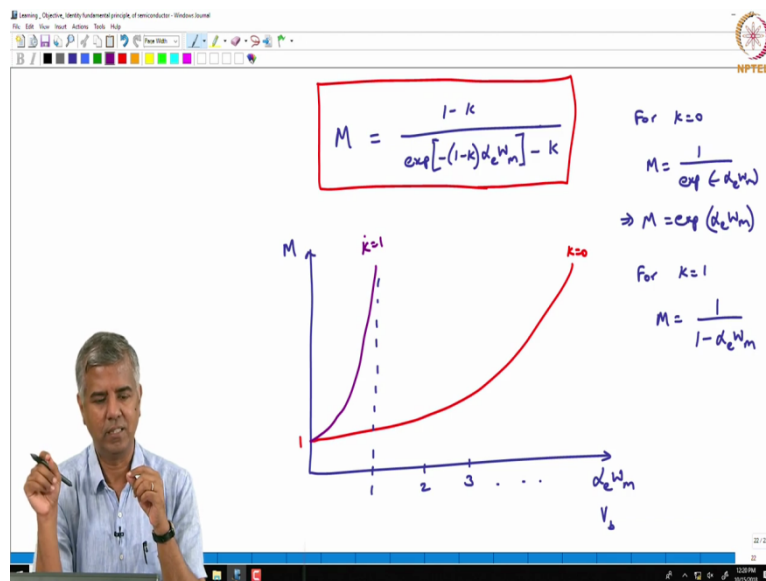
Charge neutrality $\frac{dJ_e}{dx} = -\frac{dJ_h}{dx}$ $J_e(x) + J_h(x) = \text{const.} = J_e(W_m)$

Ionization ratio, $K = \frac{\alpha_h}{\alpha_e}$ $M = \frac{\alpha_e - \alpha_h}{\alpha_e \exp[-(\alpha_e - \alpha_h)W_m] - \alpha_h}$

So you can actually define at this point quantity known as ionisation ratio let us call that K such that this is given by over Alpha H over Alpha E ok. So what does that mean? It is the ratio of the impact ionisation coefficient for holes divided by impact ionisation coefficient for electrons. So for materials like silicon and gallium indium gallium arsenide, what can you say about value of K? Is K much less than 1, is K equal to 1 or K much greater than 1? Ha? K is much less than 1 because Alpha H is lesser than Alpha E. Just to give you an idea K can be greater than 1 in certain material like Indium phosphate is a very good example where the in indium phosphate the impact ionisation coefficient for holes is greater than impact ionisation coefficient for electrons.

So K can be greater than 1, but for indium gallium arsenide we have K being approximately equal to 0.5, of course it can change based on the material that we are using and also the specific configuration that you are using for your avalanche photodiode, but in silicon K is about 0.1 right. And once again by designing things appropriately K can even be less than 0.1 for silicon, 0.006 as been demonstrated okay. So now if you define this then what you can do is this in this expression for M, both in the numerator and the denominator you can divide by Alpha E okay then you can get an expression in terms of K, let us do that.

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If you do that what you get is am will be equal to 1 minus K divided by exponential of minus of 1 minus K Alpha E W M minus K right. All I have done is I have divided both the numerator and denominator by Alpha E so you get 1 minus K over here and you will get factor of 1 here and within this you also have 1 minus K and Alpha E goes out right and then the last factor is K okay so that is what we have written here. So if I plot this for different values of K okay, I can plot M for different values of K M as a function of Alpha E W M, so let us do that.

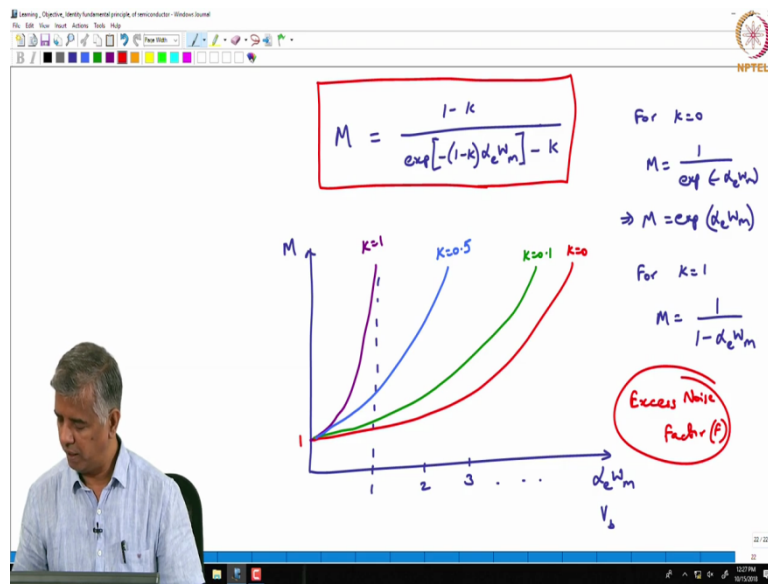
Let us look at M as a function of Alpha E W M, now W M is a constant once you have your design for your avalanche photodiode, W M is not going to vary. Can you vary Alpha E? Alpha E corresponds to the impact ionisation coefficient, what does that depend on? The electric field right, the electric field existing in that region and that electric field is dependent on the applied voltage reverse biased voltage. So as I increase my reverse bias voltage I can increase Alpha E so I can as well plot this is my function of my bias voltage also right so if I plot this function what do I get?

Now for K equal to 0 right, impact ionisation coefficient for electrons is far more greater than α_H , if I have that case then K is approximately can be approximately 0. If K is 0 then you get M is equal to 1 over exponential of minus $\alpha E W M$ right, which you can write as so M equals to exponential of $\alpha E W M$ right. So how does that look over here so that I am going to an exponential here, this is for K is equal to 0 starting from M equal to 1 right so it is just an exponential dependence. Now the other extreme is for K equal to 1 right, for K equal to 1 which means that impact ionisation coefficient is the same for electrons and holes okay.

In that case what we get; $1 - 1$ is 0 so and the denominator is also 0 so $0/0$ (7:52) so you go for (7:55) rule right, you take the derivative of the numerator and denominator and you look at that, and if you do that what you get is M equals to 1 over $1 - \alpha E W M$ is what you get ok. So how do you plot that here, let us say this is 1, 2, 3 and so on, so this is asymptotic with respect to $\alpha E W M$ equal to 1 so in this case it will be something like this okay. So question is what do you prefer, we prefer K equal to one or K equal to 0? So K equal to 1 is even at very low voltages I can get very high levels of gain so you normally say yes that is preferable right, I do not have to go very high voltages to get my gain.

I mean you will not be completely wrong with that except that you think about it K equal to 1 so both your electrons as well as your holes are equally capable of generating this avalanche okay, electrons will keep drifting this way and creating havoc you know creating this avalanche in this direction, holes is actually drifting this way and that is all creating lot of electrons secondary electron-hole pair that is also creating avalanche. But the problem is, every time we talk about creating an avalanche, but the problem is every time we talk about creating an avalanche we are talking about a statistical process.

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It is not like one electron is going to go hit with atom and that is going to hit another atom that is going to hit another atom, so for one electron M is equal to 10, it is not always M equal to 10. M now is varying, it is sort of random write so M can be 7 or M can be 20 also in some extreme case, very low level of probability but in principle it can be whatever value right. So it actually generates noise okay in the multiplication, anything that is random we can say that is going to be noisy process right. So even when we do it with electron it is already noisy, now you should do it with holes it becomes even more noisier because there was two different two independent processes in terms of creating that avalanche ok.

So you have much more noise much more chaos essentially happening when you have both of your carriers generating large independently, you have a question yeah... That is right so yes physically the question is, the holes are we say you know spaces are occupied by left behind by electrons, but in a highly charged electric field if you have a hole it is going to be replaced by another electron right, so you are great to have these holes filled by another electron and in that process you essentially have the holes moving this way. So although physically you cannot think of this hole generating an avalanche process by itself, the holes does create a perturbation which essentially generates secondary electrons that are left behind right.

So this high electric we essentially what we are saying is the holes propagates, how does the will propagate because the neighbouring electron will occupy this holes so the holes goes this way and then that neighbouring electron from there is going to come here so accepts the hole

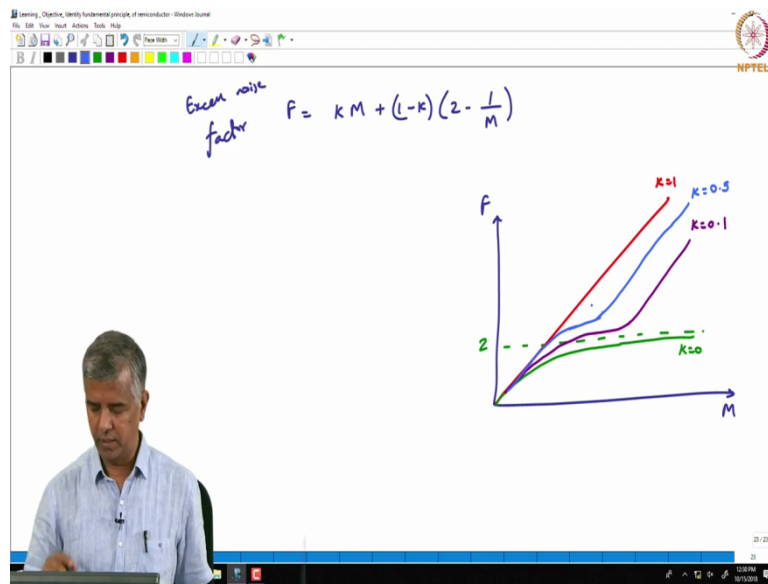
is propagating. But in that process in the process of electron jumping over it can actually I have nice and so you say holes are generating avalanche as well that is a very good question but whenever there is a holes there is also transport of electron that is happening so that is actually as such the hole is going that way.

Okay so you do not want K is equal to 1 from that perspective because that that essentially means if I go back and look at this discussion we had in this impact ionisation, the dotted and the solid lines are overlapping each other so for the same electric field both of them happen right, both of them have the same threshold so both of them happen and that that creates more uncertainty overall in your secondary electron-hole pair generation. So that would be an issue here okay and to put things in perspective if you are talking about silicon, silicon has K is equal to 0.1 so that would be somewhere over here and for indium gallium arsenide K is equal to 0.5 so that would be something like this ok.

So yes indium gallium arsenide you can say has got higher impact ionisation coefficient so even for low voltages it gives you much gain, but beyond a certain point you start triggering your holes also to provide the avalanche multiplication so that that can become very noisy. So in other words we cannot associate this picture without seeing the corresponding noise picture right, so that noise picture is represented by what is called an Excess noise factor right? So we need to consider this Excess noise factor which we can represent as F ok.

Why do we call it as Excess noise factor? That is to say that inherently in the photodiode you are going to have some noise okay, we will come back and look at what is the basis of that in a few minute but beyond that noise process you may have extra noise because of this avalanche process that is why it is called as excess noise factor. Now the excess noise factor I do not have derivation for this because the derivation is actually very long derivation.

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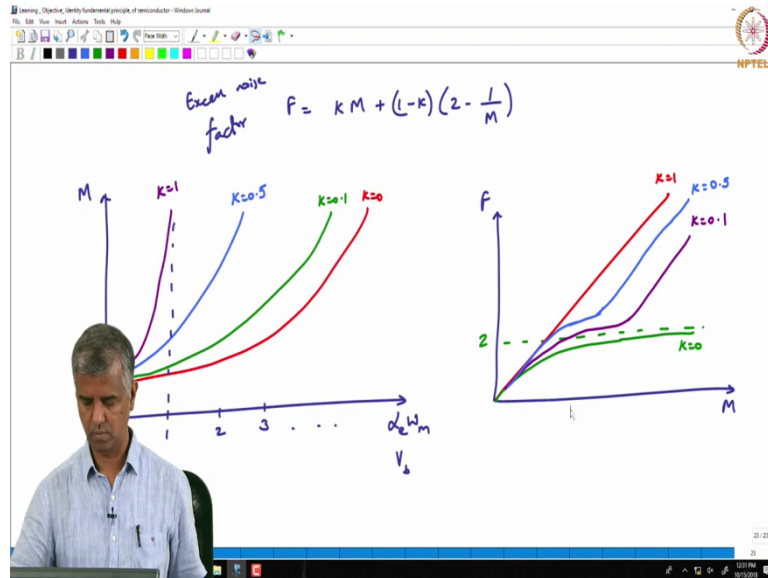
I can give you appropriate reference if you are interested but this excess noise factor is given by this expression F is equal to $K M$ plus 1 minus K multiplied by $2 - 1$ over M ok. So if you plot this, I will plot F as a function of M which is actually the function of the bias voltage so you can also say that F is being plotted as a function of bias voltage. So you have one case where you know K is equal to 1 , if you put K is equal to 1 in this expression the 2^{nd} factor goes away so you are just having a straight line K is equal to 1 right. Whereas, if you look at K is equal to 0 right is K is equal to 0 the 1^{st} factor goes away, is the 2^{nd} vector becomes $2 - 1$ over M and that is just something that is asymptotic with a value of 2 over here so it will go somewhere like this, this is for K is equal to 0 .

So which one would you prefer now? Here you are getting gain but you are also accumulating right noise as rapidly as you get gain so overall you are not improving your signal to noise ratio right. Whereas, in the other case when T is equal to 0 it essentially you know gets it levels of beyond initial values of M , so beyond certain value of M all you have is gain and not so much of loss because you do incur extra noise just because the electron is going to create that variability in terms of achieving gain, the gain is not a fixed value but you do not get any more you know noise because of holes kicking in.

And in between if you look at for silicon, it will be something like I am sorry I do not think I am following the same colours as the other graphs, but for silicon it will be something like this, at low values of M the 2^{nd} factor dominates, for large values of M the 1^{st} factor dominates right. So this is for K is equal to 0.1 and for indium gallium arsenide it is

something like this K is equal to 0.5 and just to give you a perspective let me cut and paste this also ok right.

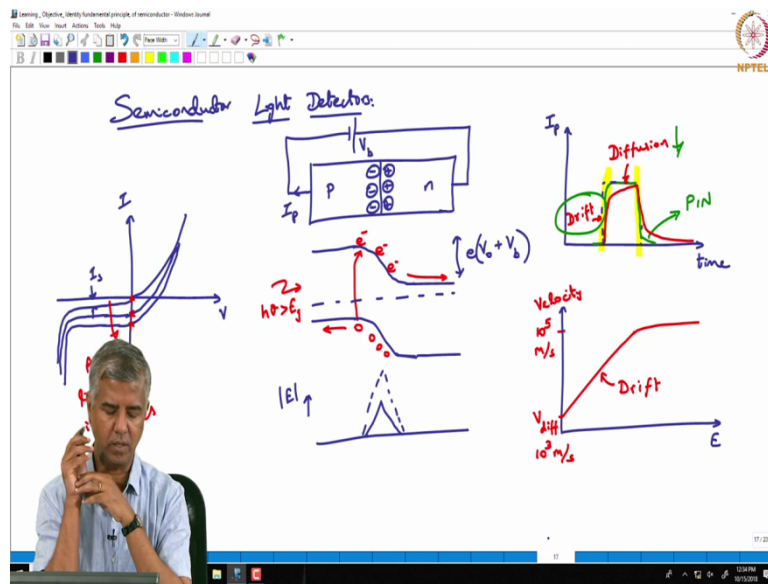
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Like I said I am not using the same colours for both the things but bear with me on that but you get the idea that it is not just you know the gain has an absolute number what you get, it is the quality of gain that you get right so the quality of gain is defined by how much noise you are accumulating with respect to the gain. And from that perspective there are some interesting observations you can do, if you were to if you have indium gallium arsenide and if you were to choose the best gain at which you can get the maximum signal-to-noise ratio where would that point be?

We are looking at the blue curve on the on the right graph right, and we are trying to figure out where would I want to operate right, I would want to operate somewhere over here the knee point because until that point I have not accumulated a holes lot of noise, I have fairly good gain but beyond that point if I am trying to achieve more gain I am also accumulating noise right, so that would be my operating point for indium gallium arsenide. And for silicon somewhere over there right, so these will be my optimum operating point right. So it tells you that of course, you know as you keep increasing your bias voltage, at some point you are going to get into breakdown right we talked about the IV characteristics right somewhere over there, trying to figure out where it was we discussed that.

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So we talked about IV characteristics and we said, beyond a certain voltage it is going to punch through it is going to break down right so you will have a similar breakdown voltage limiting this, so beyond a certain voltage you are not going to be able to get any more gain because the device is punch through. But what is picture is showing you should not even wait until then until that breakdown voltage, there is an optimum point before that where the signal-to-noise ratio you are not getting any improvement okay, so you need to stop at that point ok.

And we will look into this in little more detail as we start discussing about noise but I just wanted to give you that picture and it also tells you that the amount of gain that you can extract from your indium gallium arsenide the optimum gain is going to be lesser than that you extract from silicon just because of the fact that indium gallium arsenide has a larger K value, K value close to 1 then what you have for silicon ok so that is just a material property by itself, you understand that ok.

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For InGaAs ($K=0.5$), $M_{opt} \approx 10-20$, $V_b = 10-50$ V, $W_m = 0.1 \mu\text{m}$
 Si ($K=0.1$), $M_{opt} \approx 100-300$, $V_b = 100-500$ V, $W_m = 0.5 \mu\text{m}$
 Multiplication time, $\tau_m \approx \frac{MKW_m}{V_{dr}} \Rightarrow \tau_m = 5$ ps (InGaAs)
 50 ps (Si)

So let us actually throw some numbers into the picture so what we are talking about here is for indium gallium arsenide I will just go to the next slide and talk about that. For indium gallium arsenide what you will have is you can get the optimum gain to be in the order of 10 to 20 right before your noise starts picking up, but that you can get with V_B the bias voltage in the order of 10 to 50 volts itself because of the fact that your impact ionisation coefficient for indium gallium arsenide is higher than as for silicon.

So if you look at silicon, that M_{opt} can be in the order of 100 to 300 even in certain case for K is equal to 0.006 as somebody as understated, M_{opt} can even up to 300 and V_B also has to be in that level, it is typically 100 to 500 volts is the operating voltage to get again but you are able to finally get much higher gain with your silicon than with your indium gallium arsenide.

That is one picture; the other picture is with respect to the multiplication time. Remember we talked about multiplication time we said that it is actually an excess time that slows down the response of your photodiode, this multiplication time can be I am not deriving this because again it is a fairly long derivation but it can be approximated as M multiplied by K multiplied by W_m over V_{DR} . Now W_m over V_{DR} we know that is just corresponding to the transit time across that multiplication region, but we are multiplying by M because there are M events and it is also weighted by the impact ionisation ratio K .

So if you calculate this, for these above values we know that indium gallium arsenide K is equal to 0.5 is what we are taking an silicon K is equal to 0.1 right. If you substitute these values let us say in this case you do not need a lot of $W M$ so $W M$ can be as small as 0.1 micron, where as you are the impact ionisation coefficient is not as high so $W M$ needs to be more, it has to have a larger multiplication region. And if you do this what you will find is if you substitute in here what you find is τM equals to 5 picoseconds for indium gallium arsenide, whereas it is in the order of 50 picoseconds for silicon.

That is understandable because in silicon you have much many more multiplicative events and also you have to transverse longer distance with respect to your multiplication region because the impact ionisation coefficient is lower we have to traverse longer distance across the multiplication region ok. So essentially saying is that okay your silicon APD is going to have lower bandwidth compared to indium gallium arsenide APDs, which is good to know but it really does not matter because silicon is going to be used for very different applications than Indian gallium arsenide. Indium gallium arsenide you, where would you going to use indium gallium arsenide photodiodes? What spectral region does it address?

1.55 so communications so for your communication applications you use indium gallium arsenide, where as silicon is more for spectroscopy and other free-space applications so there is no direct comparison between the two. Any questions after this point? Did you understand this picture? Why there is actually an optimum value for gain? Yeah. Okay let us actually juggle on and let us look into another important topic which is noise in photodetectors **detectors** and the corresponding photo receivers. So when we talk about noise, there are primarily 2 types of noises associated with photo receivers, one is called photoelectron noise. So what is photoelectron noise? You have photons **going** being absorbed in the medium and you are generating electron-hole pair right.

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For InGaAs ($K=0.5$), $M_{opt} \approx 10-20$, $V_b = 10-50$ V, $W_m = 0.1 \mu m$

Si ($K=0.1$), $M_{opt} \approx 100-300$, $V_b = 100-300$ V, $W_m = 0.5 \mu m$

Multiplication time, $\tau_m = \frac{MKW_m}{V_{dr}} \Rightarrow \tau_m = 5$ ps (InGaAs)
 50 ps (Si)

Noise in photodetectors/receivers

- Photoelectron/shot noise (random arrival of photons/electron generation)
- Thermal/Johnson Noise $\sigma_s^2 = 2e I_p B$
- Noise $\sigma_T^2 = 4 \frac{k_B T}{R_L} B$

But we actually quantify something a parameter called Zeta which was the photoelectron conversion efficiency, so that says that not every photon is converted to an electron hole pairs, it is like only on the average so many photons are converted to electron hole pairs. When we say the average, in real time it is actually some random process that is happening so there is actually you have photons 1st of all that are arriving in a random fashion because we talk about all the light sources those all the light sources are characterised by certain randomness. When we talk about some particular level of intensity we are always talking about a certain mean number of photons coming through, but if you slice it into small enough time slots may be in the order of nanoseconds then you find that the mean number then actually instantaneous number is actually fluctuating right.

If you say 10 photons in the mean number, within a very small time slice that number can be anything that could be on the average it is 10 so there is a good chance that it will be 10 but there is also a chance will be like 8, it will be like 7, it will be like 15 and so on. So what does that tells you, as from my receivers perspective you have photons that are coming not so uniformly and then there are actually the photoelectron conversion process that does not that also is impacted by that that does not happen so uniformly. So you have this noise which is also called shot noise that is happening right, this is primarily because of random arrival of photons and their subsequent conversion to electron hole pairs ok to understand that so that is one type of noise.

The other type of noise does not really have anything to do with photodiode itself but nevertheless when you are trying to extract the current from the photodiode, you have to slowly through an external circuit and convert that current to voltage because voltages what we typically recognise with all our electronics right. So in that process you have something called thermal noise or it is also called Johnson noise that happens. Thermal noise happens whenever you send a photo current through a resistor for example, that resistor is going to have a particular value and it says on the average it allows this much current to pass through or it allows a certain rate of electrons going through.

But that actually depends on you know the number of collisions that it has while it is propagating through that and that is actually random process by itself ok. So that current that noise because of the random propagation of random flow of current through a resistor is what you call as thermal noise okay or Johnson noise so there are 2 different noise sources. And if we quantify this noise, the noise variance which you can say is corresponding to σ^2 , σ corresponds to the RMS noise, σ^2 corresponds to the noise variance and the noise variance is going to be given by $2 \times E \times I_P$ multiplied by B .

So $E \times I_P$ is actually the noise variance across the spectrum, but it is actually like white noise it is flat it is distributed flat across the spectrum, but what our receiver captures is only the components under receiver bandwidth. The receiver bandwidth is what we are denoting as B over here and it is actually a two-sided spectrum that that is where the backdrop to also comes into the picture. So it is integrated over you know to $2 \times B$ and you have $2 \times E \times I_P$ times the photo current multiplied by B . Essentially it says that it is proportional to the photo current, more the photo current that you have which would correspond to more number of photons that is falling on the photodiode, more will be the variance.

This is some concept which you were discussing much earlier when we were talking about photon and statistics itself, we were saying more number of photons will give you more variance. Remember we were talking about Poisson's statistics for the arrival of photon and that is where Poisson's statistics were proportional to the number of photons that that variance was proportional to the number of photons that are falling on that are being collected.

So here we are talking about that corresponding to the photo current that we are picking up. Whereas, if you look at the thermal noise component that is once again white noise which is corresponding to the thermal energy $k_B T$ over the resistance over which that you know

current is passing through is a factor of 4 over here because it is $2 k_B T$ over $R L$ is the noise variance but it is integrated over 2 times B , once again it is double sided spectrum so it is integrated over 2 times B so $4 k_B T$ over $R L$ multiplied by bandwidth of your receiver that is the thermal noise. What we want to proceed is to quantify this shot noise, this is for an APD you can write it like this but for a PIN can write it like this, but for an APD it also depends upon the gain of the APD.

So we will go to the expression for the gain of the APD which will also incorporate that access noise that we talked about and then we will see you know under what conditions do you use an APD, when do you use a PIN and when to use an APD ok, let us let us try to figure that out in the next lecture okay thank you.