# Multirate Digital Signal Processing

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## Lecture - 10
## Properties of Upsampler and Downsampler

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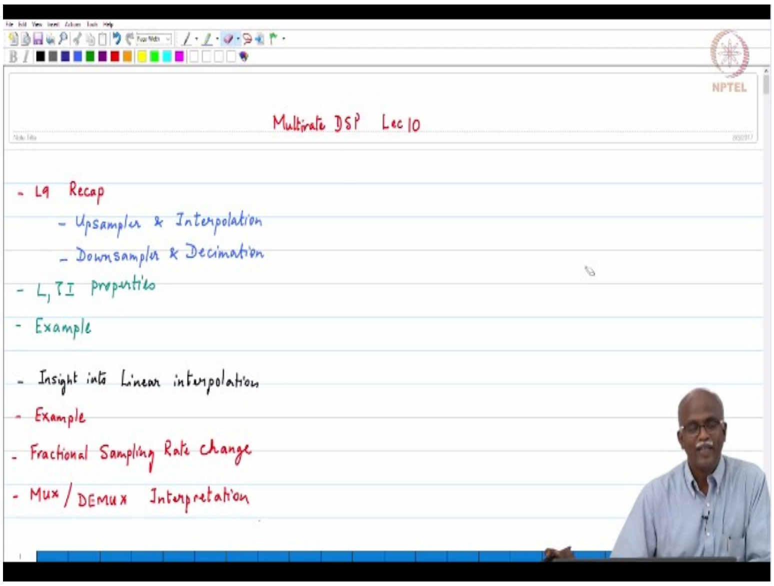


Good morning and welcome to lecture 10 of the multirate DSP course. As we saw in the last lecture, we have been covering the blocks multirate blocks of the upsampler and the downsampler. We will begin by reviewing those principles. Upsampler also leading to the operation of interpolation, the downsampler or the decimation where we are reducing the sampling rate.

So we will be looking at both of these blocks. First, we would like to begin by studying the properties of linearity and time invariance and then look at some examples and also apply the concepts of the upsampling in the filtering aspects. So that is the goal of today's lecture. Let me begin by reviewing the concepts of upsampling and downsampling.

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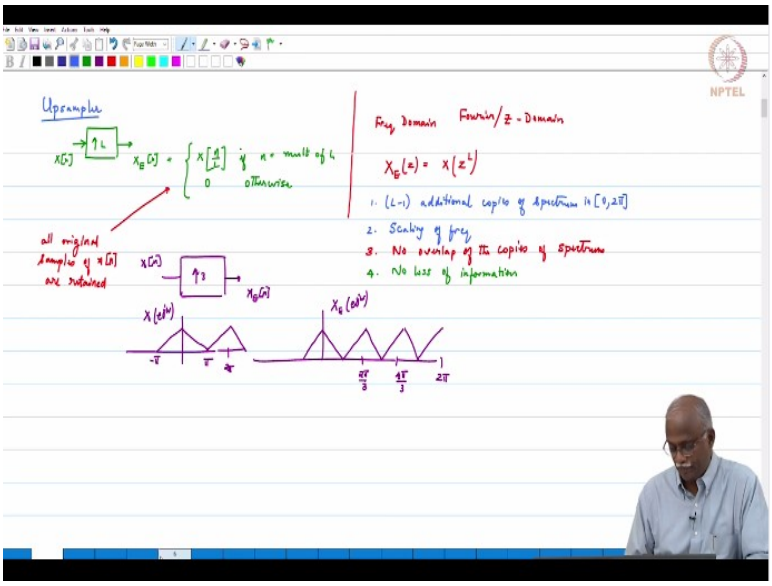


So we begin by looking at the upsampler. The upsampler block, upsampler by a factor of L, an integer factor of L. If the input is X of n, the output is XE of n and in the time domain we describe this as a sequence which is nonzero if the index n is a multiple of L, if n is equal to a multiple of L and is 0 otherwise. This is something that we have seen and this is the description of the upsampler in the time domain.

Now let us write down side-by-side the frequency domain. Frequency domain could be the Fourier domain or the z-domain. So what is the corresponding expression in the transform domain? So we have derived the following result XE of z the z domain is=X of z power L and we made the following observations about the spectral properties of the expanded signal. The expanded signal the first and foremost has got L-1 additional copies of the spectrum.

Additional copies of the spectrum between 0 and 2 pi, copies of the spectrum in the frequency region of interest between 0 and 2 pi, additional copies of the spectrum okay. So there is a scaling of frequency. So inherently there is a scaling of frequency and we also note the additional aspects that these multiple copies do not overlap in frequency, no overlap of the copies of the spectrum.

So the spectral copies do not overlap with each other, copies of the spectrum. So what we mean by this is if I were to upsample by a factor of 3 and I have a input spectrum which is from –pi to pi. The output spectrum this is X of n, XE of n. If this is XE of j omega then XE of j omega okay, the spectrum of XE of j omega, the expanded signal has got 3 copies of the spectrum okay.

So this is 2 pi, similarly for this one we will see the copy of the signal at 2 pi. Here we have copies of the signal at 2 pi/3 and at 4 pi/3 okay. So we have the additional copies but notice that there is no overlap of the spectrum, this is what we mean by that and as a result of this we can also state that there is no loss of information. No loss of information which can be also deduced from the time-domain equation.

Because we have not lost any of the original samples, all the original samples are retained. So in this block all original samples of the signal X of n, all original samples of X of n are retained. So there is no loss of information and this is an important difference between the upsampler and the downsampler. So this is the comprehensive summary of the upsampler. So the upsampler, a single-page description of the complete system.

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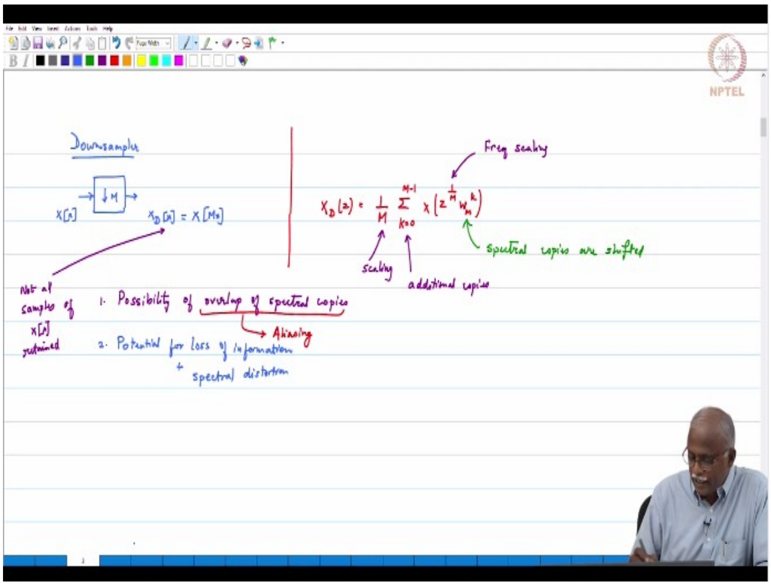


Now let us move on to the downsampler. We have spent the last few lectures understanding the process of downsampling and the both the time domain as well as the frequency domain interpretations. So let us write down the block diagram first. The block diagram is a downward arrow with the integer factor by which the downsampling is going to happen. So if this is X of n, the downsample signal XD of n is given by X of Mn okay.

So we are retaining one out of every M samples. So that is an important element. Now if we were to describe in the frequency domain or the in the transform domain, we have derived the expression that XD of z has got copies of the spectrum, shifted copies of the spectrum with the scaling 1/M k=0 through M-1 X of z power 1/M WM k okay and we showed that there is the following elements.

The element of a scaling factor 1/M scaling factor, the fact that there are additional copies, additional copies of the spectrum. There is a frequency scaling, there is a frequency scaling that is present and these copies are shifted. The spectral copies are shifted. Spectral copies are shifted okay. So from this combination of the time domain and the transform domain, we can write down the following aspects.

The first important point to note is that there is a possibility of the overlap of the spectral copies. It was not, there was no overlap in the case of the upsampling but in this case because of these frequency scaling and the shifting, there is a possibility. So possibility of overlap of the spectral copies and this is what we identified as aliasing. Previously, we identified overlap of this is the distortion that we refer to as the aliasing distortion.

And as in the unlike the upsampler case, here some samples are not being retained, so we can make a not all samples of X of n are retained and as a direct consequence of that we make the observation that there is a potential for loss of information and also for signal distortion, spectral distortion, so plus spectral distortion. This is a possibility we can do downsampling without these errors under certain conditions.

So in general there is a potential for the overlap of the spectral copies which will lead to aliasing and this in turn leads to loss of information and spectral distortion okay. So this is overview of the upsampler and the downsampler. Now in the last few lectures whenever we have talked, after we introduce the notion of the upsampler and the downsampler, we also introduced appropriate filtering.

So what I would like to do now is these multirate blocks in conjunction with their corresponding filtering operations.

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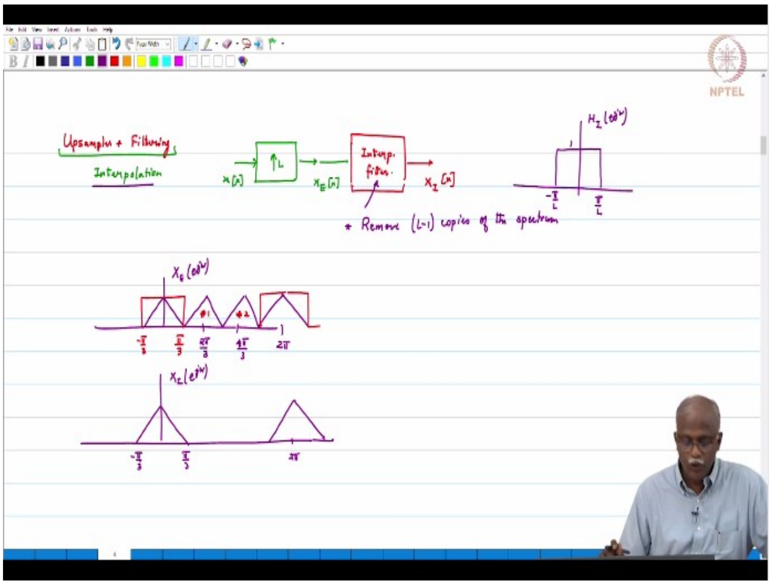

So let us look at the upsampler. So we will look at the upsampler plus the filtering okay. So the combination of these two is what we call as interpolation. Interpolation means that you now have samples at a higher rate, so the process of upsampling by a factor of L, this is X of n, XE of n the upsampled or the expanded signal followed by an appropriate filter. This is what we call as the interpolation filter.

This will then give us the output which can be written as XI of n okay. The role of the interpolation filter is to remove the unwanted images that are produced by the upsampling process. Additional L-1 copies were produced; remove the L-1 copies of the spectrum that were obtained in the process of upsampling okay.

So what the interpolation filter looks like, the interpolation filter, ideal interpolation filter is a low pass filter which goes from –pi/L to pi/L. This is the interpolation filter HI e of j omega, the interpolation filter and the role of this filter is to remove all those unwanted images. So if we go back and look at the spectrum that we obtained, when we interpolated by a factor of three let us see how the interpolation will look in this case.

So we have the upsampling by a factor of 3, the role of the interpolation filter is to remove the unwanted images, this corresponds to the image number 1, this is image number 2. So it is upsampling by a factor of 3. So the interpolation filter is an ideal low-pass filter when we do upsampling by a factor of 3 from –pi/3 to pi/3 and then the spectrum will be periodic with respect to 2 pi and so on.

So basically this is what is retained. So the output of the XI of the interpolated signal XI e of j omega. Now is a signal which has got the first spectral copy between –pi/3 to pi/3 and then we have a sampling rate which is 3 times higher than the previous sampling rate. So this is 2 pi and then you then see the copies of the signal. Now this is the process of interpolation. So this is the process we call that as interpolation, the upsampling followed by the interpolation filter.

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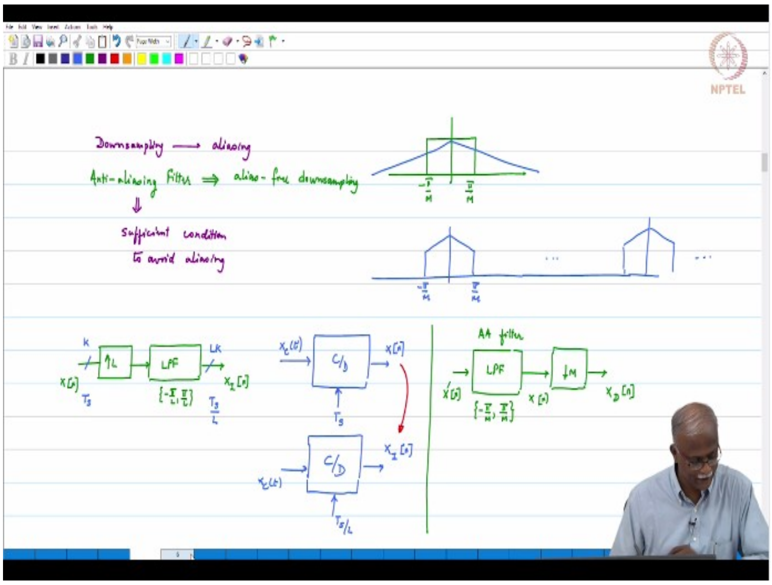


Now the corresponding filtering operation in the context of the downsampling which we saw, so we have the downsampling and in order for the downsampling, the downsampling could potentially introduce aliasing. The role of the anti-aliasing filter that is the filtering that you do in downsampling case, anti-aliasing filter will ensure alias-free downsampling and as we saw in the last lecture, the downsampling process produces a stretching of the spectrum.

So the aliasing, anti-aliasing filter is a low-pass filter which will be band limited to pi/M from –pi/M to pi/M. Now the purpose of this is to band limit the signal, the original signal may have significant component outside of the band limiting filter but in order for us to do the downsampling without aliasing, the part that can be retained is this –pi/M to pi/M and then at 2 pi correspondingly another copy of the signal.

Now this signal can be downsampled without aliasing. So that is the key, so the anti-aliasing filter can be considered as a sufficient condition for avoiding aliasing okay. So that is an important element. It is a sufficient condition to avoid aliasing and we will be coming back to reviewing this multiple times, the aspects of the anti-aliasing, sufficient condition to avoid aliasing. Now you can easily work out the stretching of the spectrum that is to be done for the down sampled signal.

You can also look at the shifted versions of the signal and then you can do the stretching of the spectral components and you can find that there will not be any aliasing that is present in the signal. So this is a very important aspect, the aspect of upsampling and downsampling. So this is upsampling, the downsampling part, the upsampling with the filtering which leads to interpolation which gives us a higher sampling rate.

And the alias-free downsampling which gives us a reduction in the sampling rate. So the general block diagram that we should be looking at whenever we are doing the upsampling, doing the interpolation will be upsampling by a factor of L and followed by a low-pass filter okay. The low-pass filter will be from –pi/L to pi/L and this is the signal. If this is X of n, we will get a signal XI of n.

If this is at a sampling rate K, this will be at a sampling rate L times K, K samples per second, this will be at LK samples per second. So this is as if you were sampling, if the original sampling period was TS, this corresponds to TS/L. So if you remember we talked about a C to D block, C to D block in the case of we have a continuous-time signal and we obtained X of n and this used a sampling rate of TS.

Now the interpolated signal can be understood as the original continuous time signal sampled at a rate that is TS/L, a much faster sampling rate and this is what we will produce by XI of n. Now what we have done is produced XI of n from X of n directly in the discrete time domain but however underlying the process and you know the visualization of what the change in the sampling rate can be seen in this picture.

Now correspondingly if we had wanted to capture the downsampling part, then what we would have to draw is the anti-aliasing filter. This is the AA filter; anti-aliasing filter is also a low-pass filter with cutoff pi/M. So basically it has a gain of 1 in the range –pi/M to pi/M. This followed by the downsampler and this is the alias-free down sampling okay. So if this is X of n then this is a band limited signal, band limited to the pi/M.

So let us call this as X prime of n and this as X of n. If this is band limited, then we can do alias-free downsampling which is given by XD of n. So this is the complete picture of the upsampling and the downsampling and I hope you are able to relate the concepts with the information that we have discussed. Now we move on to a very important assessment of the processes that we have just introduced.

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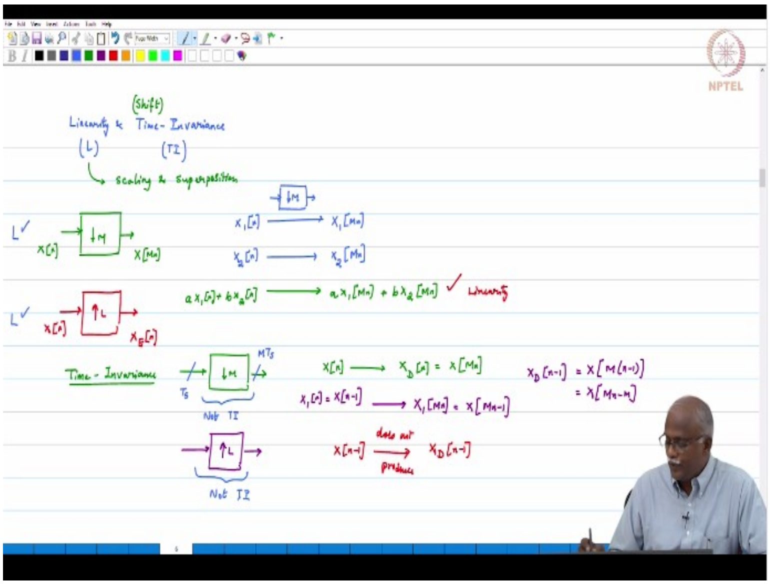


This is the understanding of the properties of linearity and time or shift invariance okay. So time invariance that is what we refer to in DSP, LTI. If a system is, so this is denoted by the acronym L, this is by TI, sometimes it is also called shift invariance maybe I can just write that down. So this can also be called shift time or shift invariance. So if you shift the input by a certain amount in terms of the index, does the output shift by the same amount?

So that is an important question that we ask and that we try to answer okay. So let us first address the issue of linearity. Linearity requires us to satisfy two properties, the properties of scaling and superposition. Let us take the downsampler, downsampler by a factor of M integer factor M, input is X of n, output will be X of Mn okay. Now if I had an input X1 of n fed to my downsampler, so I will just put the box here to indicate the operation that is being carried out okay and what I will get here is X1 of Mn.

If I have an input X2 of n, I will get X2 of Mn. Now notice, very important, if I scale the input by a factor of a it does not affect because the downsampler does not change the amplitude, all it does is retain some samples and throw away some samples. So if I feed in a times X1 of n+b times X2 of n, then what we will get at the output will be a times X1 of Mn+b times X2 of Mn.

So this basically satisfies the property of linearity. Exactly, the basic concepts validated and it is also true for the upsampler as well. So both of these are going to satisfy the property of the linearity, X of n, XE of n and both of these will satisfy the linearity property okay. Now the question is does it also satisfy the time invariance property or the shift invariance? Let us take a closer look.

Always good for us to examine the properties of time invariance using a specific example okay. So we will look at the downsampler again, downsampler by a factor of M, now we know that if I feed in X of n, I am going to get the output which is XD of n which is equal to X of Mn okay, retaining one out of M samples. Now the question is if we shift the input X1 of n by some unit, some number of units of time, let us take the simplest case.

I am shifting by 1 unit of time. Now the question is does the output also get shifted by 1 unit of time? So first let us answer the question. What happens when you shift the output by 1 unit of time? XD of n-1, if XD of n is=X of Mn, this is equal to X of M*n-1 that is going to be X of Mn-M okay. So this is an important observation that we make. So shifting the output by 1 unit of time gives us this particular expression.

Now what happens when we shift the input by 1 unit of time? So if I pass my input X1 of n to the downsampler, it is going to produce for me X1 of Mn okay. If X of n is=X of n-1, X1 of Mn is going to be equal to X of Mn-1 okay. So the input passed through a downsampler produces X of Mn-1 but the output shifted by 1 unit of time is given by X of Mn-M. So we can conclude that X of n-1 does not produce XD of n-1, it does not produce.

So the shift in the input is different when I shift by 1 unit of time is different from shifting the output of the original output by 1 unit of time. So this is a very important result. So we find that the downsampler is not time invariant and it is very probably intuitive to see that because we are dealing with different sampling rates at the input and the output, so a shift by 1 unit of time at the input corresponds to a different shift at the output.

In fact, it is sort of intuitive to see that if you look at the sampling rate here, sampling period, this is TS downsampled by a factor of M. That means the sample spacing has now increased, this has become M times TS. So 1 unit of shift in the input will produce a very different shift corresponding to a shift of the output because a shift of the output corresponds to M units of the input time because one shift in the output corresponds to M times TS right.

If you change TS to TS+1, so you will basically get a factor of M corresponding to which is more which is different from what the shift will be on the input side. So the downsampler is not time invariant. By using a similar reasoning, we can also show that the upsampler is also not time invariant okay. This is also not time invariant.

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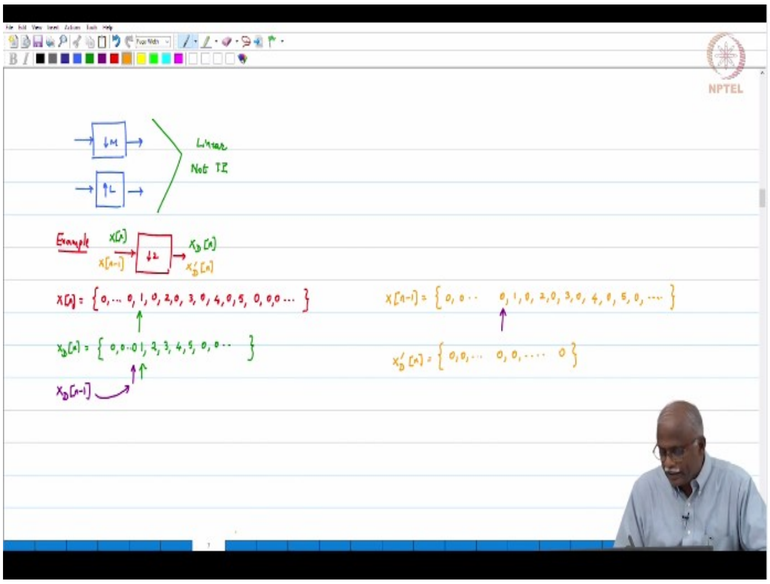


So we basically are dealing with two blocks, the upsampler and the downsampler, downsampling by a factor of M, upsampling by a factor of L okay, both of these are linear, both of these are not time invariant okay. So the notions of time invariance which are so important in conventional DSP, keep in mind a very important element that we have a situation that the upsampler and the downsampler will not permit us to have the time invariant property to be retained.

Now may be an example is very helpful for us to understand the difference between the time invariance property. So let us take the following example. Take an example; we will do a downsampling by a factor of 2 okay, very simple, a downsampling by a factor of 2 means that you retain every alternate sample okay. So the input sequence X of n is a sequence which has got number of 0s.

Then, we have a non-zero segment which is 1, 0, 2, 0, 3, 0, 4, 0, 5 and then again it has all 0s okay. There is a large sequence of 0s, in between there is and as we had indicated the origin will be denoted by an arrow. So the origin corresponds to the sample which has got the value equal to 1. Now if I were to ask you, if this input was fed into a downsampler, what would be XD of n, the downsample by a factor of 2?

Now XD of n will be 1, the sample value at origin, then you skip over 1 sample and you take the next sample which is 2, 3, 4, 5 and then 0s before and after okay. So this is my origin and this is my downsampled signal, now the downsampled signal delayed by 1 unit of time. If I wanted XD of n-1 what should I do? I should look at the original sequence and shift the origin that is all that is needed.

So now this corresponds to XD of n right, the origin got shifted from the original point to a sample to the left, one index to the left that becomes the new. So this is the sequence. On the other hand, if I had fed to this particular system X of n-1 okay and I obtained XD prime of n fed in the shifted sequence. Now what is X of n-1? It is nothing but the original sequence, all the 0s, we have 1, 0, 2, 0, 3, 0, 4, 0, 5, 0 dot dot dot, all 0s beyond that point.

The shift by 1 unit of time says that the origin is now shifted 1 unit to the left okay. Now if I pass this through the downsampler, I get XD prime of n, the downsampled which retains the sample number 0 and then every other sample after that and please note the output of this system is going to be 0, 0, 0, 0 0, 0s everywhere okay. What did we do? In one case, we took the output and shifted it by 1 unit of time.

In the other case, we shifted the input by 1 unit of time and then passed it through the downsampler. So in this case, clearly we can see that the two are not the same and therefore this is just an illustrative example. So to indicate or to validate the claim or the statement that the sequence, multirate blocks downsampler and the upsampler are not time invariant, they are time varying blocks and therefore we will have to handle them appropriately okay.

So if the aspects of time invariance are nicely captured, what we can then move on to look at a little bit more in terms of the downsampling process.

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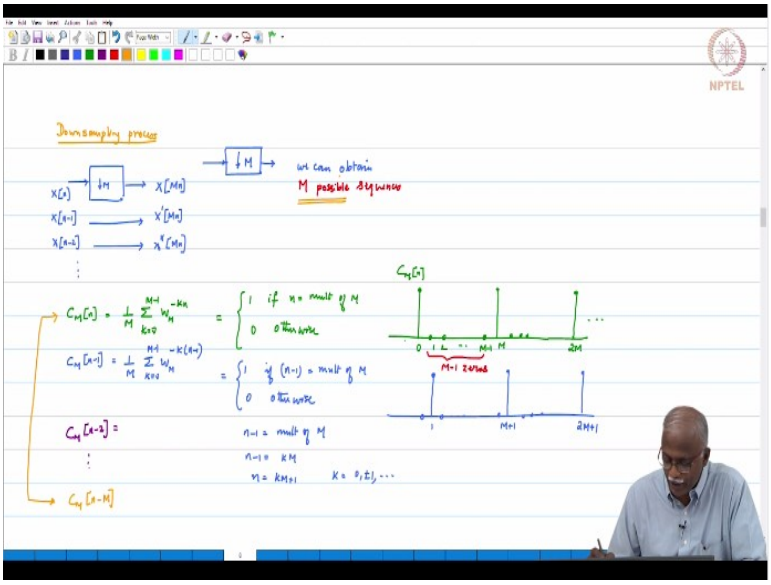


There is a very important or a very intuitive simple result but something that is important for us to recognize and to apply. So when we do the downsampling, downsampling by a factor of M, we can get a sequence which is given by X of Mn. Now remember the downsampling operation is not time invariant, so therefore if I apply X of n-1, I am going to get a different downsampled signal.

If I apply X of n-2, I will get another different signal X double prime of Mn and so on. So typically when we have a downsampling by a factor of M, we can get multiple downsample sequences. We can obtain multiple and how many such downsample sequences distinct, downsample sequences that we can get, there is a total of M, M possible sequences, downsample sequences okay.

That is a very important result and this can also be viewed and understood in the mathematical context. If you recall the way we obtained the downsampling process okay, let us just review that part. So the downsampling process, we introduced a comb sequence, CM of n and this has a value 1/M summation K=0 to M-1 WM –KM and this is equal to 1 if n is equal to a multiple of M=0 otherwise okay.

So this is a very important result because this produced for us a sequence which had a 1 at origin, then it had 0s at 1, 2 and so on all the way to M-1 and then again had a value of 1, again 0s all the way until you reach to M and so on okay. So this is the sequence, this is CM of n. So basically there are series of 1s followed by interspersed with M-1 0s. So there are M-1 0s between these nonzero values.

Now when we multiply the input sequence by this comb sequence, what it does is it removes M-1, nulls out these M-1 samples and then we retain only the nonzero samples. So basically this is what produces for us the downsampled sequence. Now a very interesting result is what happens if you look at CM of n-1 okay? The expression will now become 1/M summation K=0 to M-1 WM -K times n-1.

So basically the same result, this would be equal to 1 if n-1 equal to a multiple of M, equal to 0 otherwise okay. Now n-1=multiple of M means okay sorry if n-1 is a multiple of M, so n is equal to sorry n-1 is equal to some integer multiple K times M and n is=K times M+1 where K=0, +-1 and so on. So if you were to now look at which are the indices that will be nonzero, you will find that a very interesting observation.

This will have a 0 at 1 and will have a 0 at M+1, so this is 1, M+1, 0s everywhere else and then at 2M+1 and so on okay. So this is the other sequence. Now of course we can then go on to looking at what CM n-2 will look like and so on. So notice that we can get different subsequences from the process of downsampling but if you go beyond the for example if you go until CM of n-M, if you have reached a shift of M, you will find that this is once again related to the original downsampling comb sequence.

So what we get is M possible sequences when we do downsampling by a factor of 2. So may be an example would be helpful for us to visualize.

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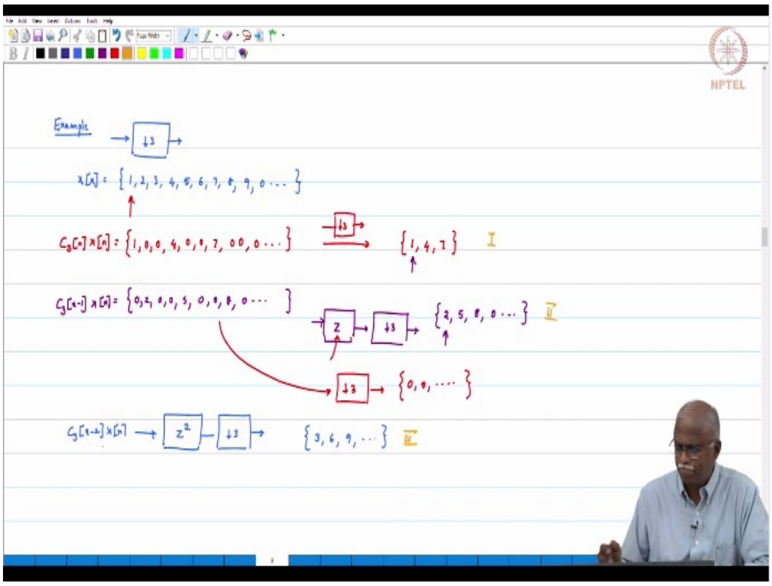

Let us take the following sequence example. We are going to look at downsampling by a factor of 3 and the input sequence X of n is given by 1, 2, 3, 4, 5, 6, 7, 8, 9 and then 0s after that okay and the origin is indicated here okay. Now when we do the conventional if you multiply by C3 of n times X of n, this sequence will be 1, 0, 0, 4, 0, 0, 7, 0, 0, 9 right 5, 6, 7 no, the last one is actually 0 yeah 0 dot dot dot okay.

Now if we downsample this, downsample by a factor of 3, the sequence that we will now get will be 1, 4, 7 with the origin being here okay. Now let us look at one more sequence, C3 of n-1 times X of n, this will be 0, 2, 0, 0, 5, 0, 0, 8, 0, 0, 0 okay. Now if I pass this through an advanced operator and then downsampled by a factor of 3 okay what will we get? We will get the sequence 2, 5, 8, 0, 0, 0 with this being the origin.

Now why did we use the advanced operator? If we did not use the advanced operator and if you are just downsample this by a factor of 3, then so if you have taken this sequence and downsampled it by a factor of 3, the output would have been 0, 0, 0, 0 everywhere, basically you would have only gotten all the zero valued samples, so the advanced operator sort of pulls in.

Similarly, if you have C3 of n-2 times X of n, you obtain this. Then, do an advanced operator of 2 units of time, downsampled by a factor of 3, this sequence will be 3, 6, 9 and so on okay. So you can see that when we did the downsampling by 3 from the original sequence, we got a sequence number 1, sequence number 1 is obtained here, sequence number two was obtained here, sequence number 3 is obtained here.

And any further shifts would have produced the, will go back to producing the one of the sequences already obtained. Now what we would like to do is build on this understanding and develop it into a concept that we are very comfortable with. So let us take a quick look at what we can do with this particular concept and then develop it into a system that we can leverage fully okay.

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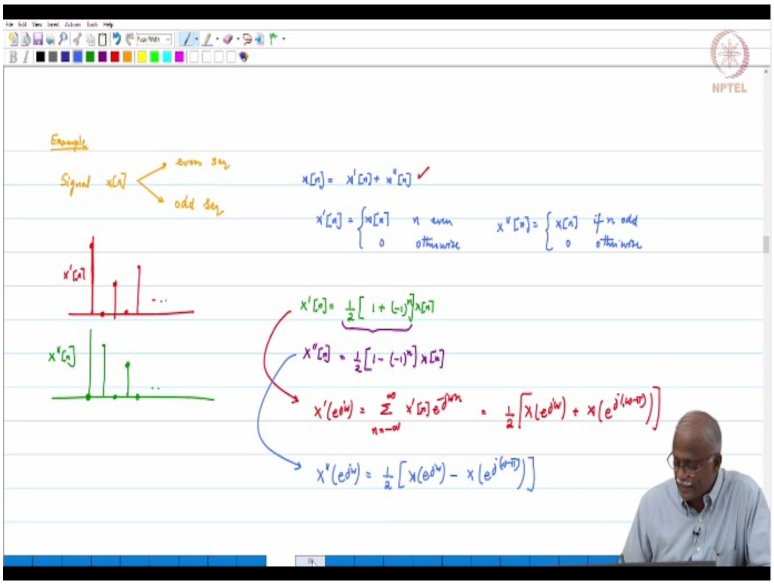


So this is an example where we will split a system into even and odd signals. So basically our input is going to be split into two parts to even and odd and then we will develop that okay. So an input signal split into even and odd signals okay. So a signal X of n which we will split into an even sequence and an odd sequence okay. Basically, through the process of downsampling, one of them will retain all the odd values, other one will retain all the even values okay.

So let us see if we can capture this in the following manner. X of n is=X prime of n+X double prime of n where X prime of n has all the even samples, X prime of n is equal to X of n if n is even, equal to 0 otherwise okay. This is you can apply the appropriate comb sequence and obtain this particular and similarly X double prime of n is equal to X of n if n is odd 0 otherwise okay. By virtue of their definitions, we can easily see that X of n, this condition will be satisfied okay.

This condition is satisfied, so the process is that we split the signal into two parts okay, all the even valued samples are retained in X prime of n, the odd samples are retained in X double prime okay. So basically that is what we are capturing here okay. So this is X double prime of n. Now how would we get an expression for X prime of n? X prime of n can be written as 1/2 of 1+ -1 power n times X of n.

So this is the comb sequence that will produce for us the only the even sequences, this is like a comb sequence. Similarly, X double prime of n can be written as 1/2 of 1- -1 power n X of n, this is also another comb sequence which has got all the odd values okay. Now if you were to write down the corresponding Fourier transforms, discrete time Fourier transforms, you can verify, please do check it up.

X prime of e of j omega, this is equal to summation from n=- infinity to infinity X prime of n e power -j omega n. Please substitute for X prime of n and verify that what we will get here is 1/2 of X e of j omega+X e of j omega-pi okay. Similarly, if you were to get the Fourier transform of X double prime of n, X double prime e of j omega can be written as 1/2 of X e of j omega-X e of j omega-pi. Please verify.

The purpose of this particular exercise is twofold; one is to refresh your memory of these simple relationships that are present for us in the context of DSP okay. So how do we implement obtaining the odd and even sequences as a multirate process okay?

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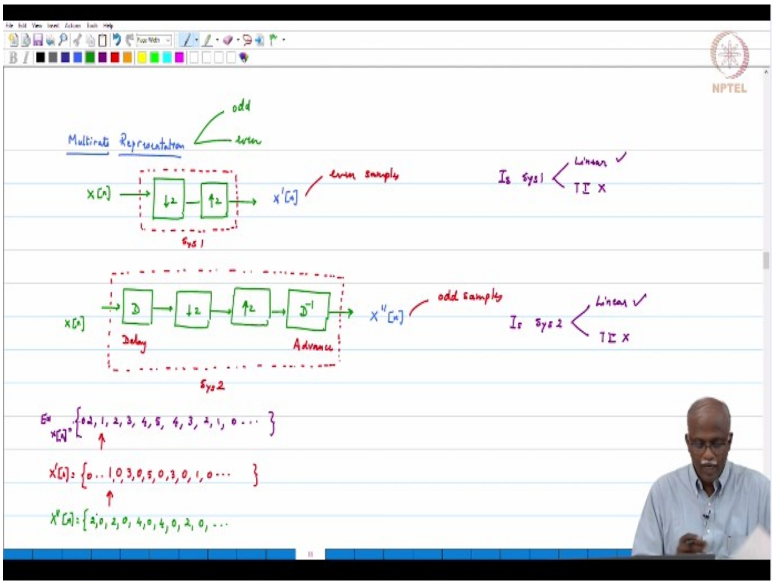


So the multirate interpretation or multirate representation, the multirate representation of obtaining the odd and even sequences, representation for the odd sequence and the even sequence okay. So the first one, if I take X of n downsampled by a factor of 2 followed by upsampling by a factor of 2, then the claim is that this will produce X prime of n. What I would like you to do is to verify that this indeed is correct.

So where X prime of n has all the even-numbered samples, all the even-numbered samples okay. This box around this we will call this as system 1. System 1 takes an input sequence and produces an output sequence in which only the even samples are nonzero, the odd samples have become 0 okay. Now how can we generate the odd samples using a multirate operation? The following will work, please verify.

Pass the input signal through a delay 1 unit of time, then apply system 1, downsample by a factor of 2, upsample by a factor of 2 and then apply an advanced operator okay. So I am going to call it as D-1. So this is a delay operator, delay by 1 unit of time, this is an advance operator by 1 unit of time. So basically this is what we have put down. Now the claim is that if you consider this as a box including the delay and the advance operator, those are very important.

If you do not have those, this claim is not true okay. Now if you have this as the system 2, the claim is that the output that we obtain here is nothing but X double prime of n which retains all the odd-numbered samples, odd samples okay. So here is a very interesting observation. Now if I were to ask you, is system 1 linear? The answer would be yes. Is it time invariant? The answer is no.

The reason would be because there are time variant blocks inside. The downsampler and upsampler are not time invariant blocks. Similarly, is system 2 linear? Obviously, linear. Time invariant? No. Once again, the answer is it does not produce okay. So now if I were to give you a sequence as an example, X of n=2, 1, 2, 3, 4, 5, 4, 3, 2, 1 and then 0s after that okay and the origin is this point.

Now X prime of n will be equal to it will have 0s to begin with, then it will have the sample number 1, it will have a 0, sample number 3, 0, sample number 5, 0, sample number 3, 0, sample number 1 and so on. The origin is located here okay. So I am going to indicate that there are 0s preceding okay and the nonzero portion is what we have captured. Now similarly if you were to write down X double prime of n, X double prime of n will capture for us all the odd samples.

There will be 2, 0, 2, 0, 4, 0, 4, 0, 2, 0 dot dot dot okay. X double prime of n and this is the one that captures for us the odd samples. So what we find is that basically it is a time varying operation but the interesting thing is the following.

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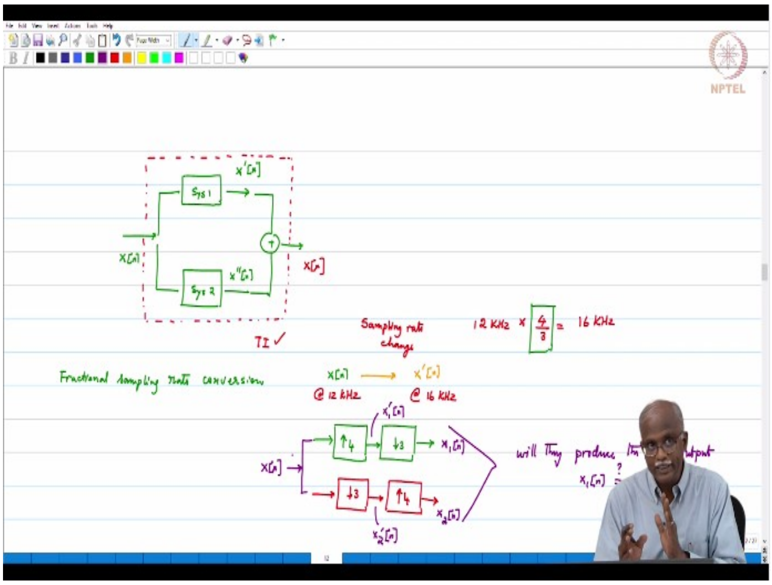


If you can combine these two operations, take X of n, pass it through system 1, take X of n, pass it through system 2, system 1 and system 2 are defined in the previous slide. So this has X prime of n, this has X double prime of n and we add this together. You can please verify that the output will be equal to X of n.

So in other words, this block combination of system 1 and system 2 actually has produced for us a time invariant box, sort of a trivial time invariant box because it has produced the input at the output. So this is a time invariant block, so a very key observation. Now in the process of downsampling and upsampling, we find that these blocks are time variant. Now the combination of these time variant blocks can sometimes produce for us time invariant blocks.

So this is something that we would like to build upon. Now another key element that we will introduce in today's lecture but we will continue in the next lecture is what we call as fractional sampling rate conversion okay. Now supposing I want to do the following, supposing I have a signal X of n that is sampled at let me call this as sampled at 12 kilohertz okay. I want to convert this into a signal X prime of n at 16 kilohertz okay.

The sampling rates are it is not an integer multiple, so the sampling rate by which a sampling rate conversion, sampling rate change is not an integer. So in our case, it will be 12 kilohertz sampling rate*4/3, this will be equal to 16 kilohertz. So the sampling rate conversion that we need to achieve is not an integer multiple. So we cannot use only an upsampler or only a downsampler, what we find is that we would have to use an upsampler and a downsampler.

So obviously when you want to implement this, I can implement the upsampler first, upsampling by a factor of 4 followed by downsampling by a factor of 3. Now this is one option that is possible. A second option that is possible is when I have to do the downsampling first. Do the downsampling by a factor of 3 and upsampling by a factor of 4. Now are these two the same? Will they produce the same output? Important question.

What I would like you to do is work through what will be the intermediate signal in each of the two cases and what is the final result. So in both cases, we are going to apply X of n and let us say that we are going to get X1 of n as the output of the first branch, in this case it is going to be X2 of n. The intermediate signal let us call that as X1 prime of n and this is going to be X2 prime of n.

Now please write down expressions for X1 prime of n, X2 prime of n, X1 of n and X2 of n. Will they produce the same output okay? Please look at it mathematically because it is very hard to visualize it you know diagrammatically. Write down the expressions, time-domain and then we can analyze. Now in other words is X1 of n the same as X2 of n okay? Now this is a very important question that arises.

Because the question that is being asked is are these multirate blocks interchangeable okay? When we saw LTI systems, we noted that LTI systems can be interchanged without affecting the output. These are time variant blocks; can we interchange them? So the answer will be a very interesting result which we will obtain in the next class.

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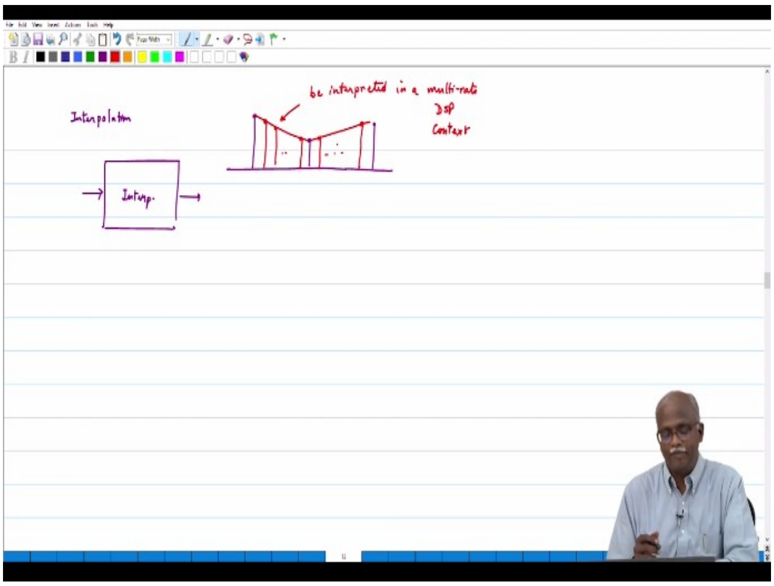

The other aspect that we would like to also understand which we will again pick it up in the next lecture is the notion of interpolation in the context of mathematics okay. Now the context of interpolation is as follows. Now we assume that I have two samples and I asked you to interpolate the signal or reconstruct the signal and very often we will do a linear interpolation.

Basically, we will say that these are the samples and we are going to do a linear interpolation. Now can this interpolation be interpreted? Can this be interpreted in a multirate context, in a multirate DSP context and if so what is that context? Very important for us to understand because you know effectively what we are saying is produce a large number of intermediate samples right.

Even in the discrete-time that is what we are saying. Can you produce a large number of discrete-time samples through linear interpolation and that can it be interpreted or can it be expressed as a multirate operation? The answer turns out to be yes and we can also get a lot of insights into when linear interpolation is a good option for us. So with that we will conclude today's lecture and we will pick it up from here in the next lecture which will be a lecture 11. Thank you very much.