Multirate Digital Signal Processing
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Lecture - 21
Study of Two–Channel Filter Bank

Good morning let us begin with a quick summary of lecture 20 and move into the main topic of todays lecture which will be the two channel filter bank, again we will look at the complete development from in the context of the mathematical analysis, intuition as to what happens when aliasing occurs where does aliasing occur and what are the forms in which the aliasing takes place and then look at it.

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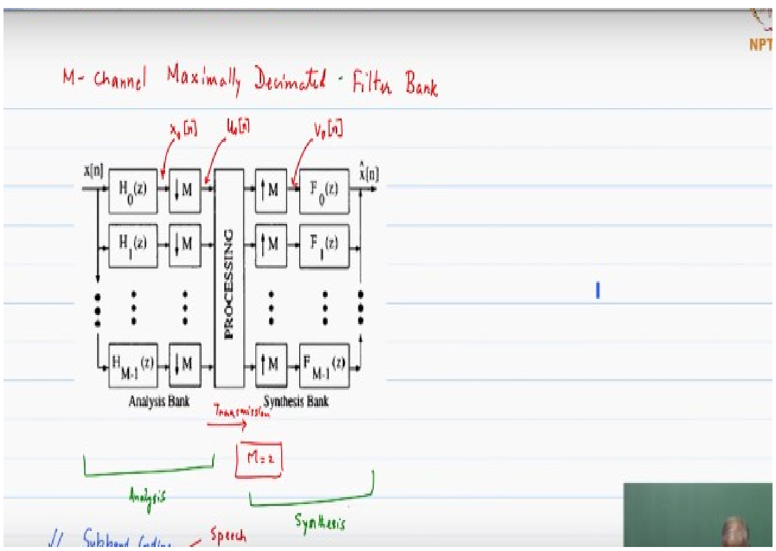

So, the review of yesterday's lecture, the key points that would like to mention.

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So, lecture 20 review just 2 key points wanted to highlight, the first one is that we talked about the OFDM transmitter and we have linked OFDM transmitter is in the context of an OFDM modem the transmit section we showed that this is identically equal in terms of functionality to a trans multiplexer which does the conversion from time division multiplexing to frequency division multiplexing and this is from a filter bank perspective.

One point to mention when we talk about TDM in the context of OFDM. TDM has got many channels each of them you can treat it as a narrow band channel so multiple narrow band channels typically in the time division multiplex system these narrow band channels are interleaved in time, so multiple narrow band channels on the other hand, we have a single wide band signal.

Okay so what you have done is you have taken a number of a narrow band signals and place them in the frequency at different centre frequencies, so you get a single wide band signal, and this is a method that can be used to generate any wide band signal so that is one of the strengths of OFDM that you can generate very a wide band signals; 20 megahertz 50 megahertz 100 megahertz you can generate without increase in significant increase in complexity.

Because what you are dealing with are primarily narrow band channels which are then getting a suitably multiplex in the frequency domain. So, this is the key element from yesterday's lecture and then we looked at the 2 channel filter bank case and the result at which we stop let me just pick it up from there.

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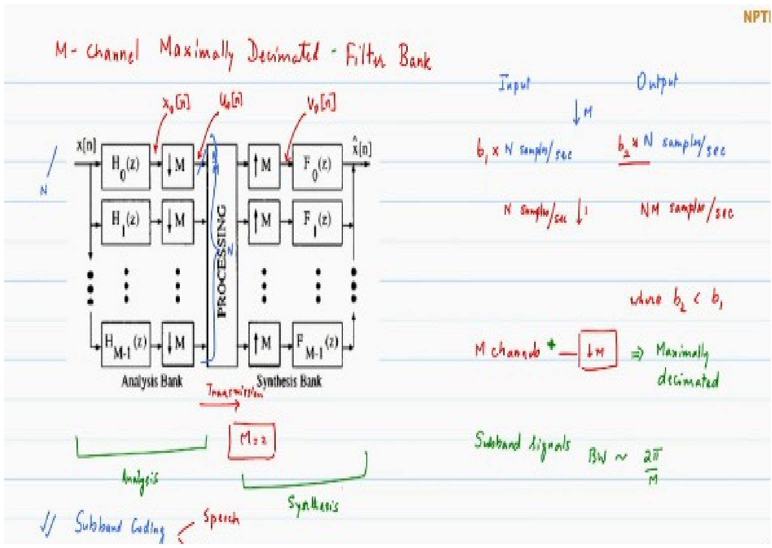

This is the M channel general setup as we mentioned yesterday is different from a trans multiplexer, the conventional approach is that in a filter bank typically the analysis filters come first followed by synthesis filter. In a trans multiplexer they are reversed in terms of position because you have the synthesis happening first but again from a view point you can study it as a trans multiplexer or as a conventional filter bank.

And we will take the filter bank of the sub band filter bank approach so the filter banks primary job in the analysis section is to take a wide band signal and to split it into sub band signals okay, so we have taken the case of M= 2, we said that this is a very widely used in speech and image processing using the 2 channel filter bank you can actually generate a tree structure. One of the key elements is a why are we decimating by a factor of M.

That was a question that was asked yesterday can we do > M? Can you do < M? And so the key points let me just sort of clarify that just by way of the sort of also covers the review as well. Now if you think of the sampling rate of X of n as some n samples per second and when you look at something that has been down sampled by a factor of M, this means this would be N/M samples per second.

And you have M such channels so which means that the collective data rate of this still is retained at N so now if you did not do any down sampling at all so if I do down sampling by a factor of M in the each of the channels then input rate, input output, output would be the after the collection of all sub band signals, if you have n samples per second this is still n samples per second.

So, basically you have not allowed any additional redundancy to creep into the system, now on the other hand, if we had taken the case of M=1, so basically you did not do any down sampling so your down sample by M=1, so that it is not the number of channels, basically I just down sampling part alone has been set=1. So in which case if you have n samples per second may be this is not good notation.

Because M=1 may think that there is only one channel so basically your down sampling by 1, that means no down sampling at all, so you will get n times m samples per second the whole idea of doing sub band dealing with sub bands is to primarily for compression, now what have we done? If you do not down sample, you have actually expanded the signal so actually you are going in the wrong direction.

So, the generation of sub bands is really not beneficial at all so if you think of the primary motivation for a splitting a signal in sub bands one could be for spectral analysis but if you were also thinking of it in terms of compression you would be very much interested in the case where you are, so now can you go to less than n samples per second? That is a key question, now when it comes to compression it is not only the number of samples per second.

But the number of bits that you are using per so let us say that you were using B1 bits for each of the samples just B1*n and if you end up with something which is B2* n as your resultant where B2 is < B1, that is still compression so therefore you have effectively done what you achieved what you, so it is very important to note that one is you do not allow an increase in the sampling rate number of samples per second.

Then you also reduce the resolution at which you represent it, so compression can happen in each of those forms, now can I reduce it to less than n samples per second? If the original signal was sampled at Nyquist rate, that means you need n samples per second to represent the information if you go anything lower than that obviously there will be because it is a net rate so you will run into the problems of representations.

So, you will lose information, so without losing information if you want to represent and the original signal was Nyquist sampled. So, since we do not know what the input signal is, we assume it is Nyquist sampled so then we said okay the best that I can do at the sub band stage is to preserve the number of samples per second, not increase it or not decrease it will I may run into trouble not increase it.

And of course then a compression can happen at this stage and why we are down sampling by a factor of M? Now when I have M channels combined with a down sampling by M so with a + sign okay M channels where each of them have got a this operation this is what we refer to as maximally decimated because anything >M, will cause your greater than the number of channels.

But cause you to go below the sampling rate the input sampling rate which will cause problems so basically you are down sampling to the maximum extent possible. That is point number one, the second point is once you have fixed your down sampling by a factor of M what can you tell me about the spectrum of the signals going into the down sampling. It has to be around 2Pi/M because if it is much larger than 2pi/M you are going to have significant problems in aliasing, so that is also constraints, so your sub band signals the band width it should be of the order of 2pi/M that again is sort of linked to it, of course you do have applications where you do non- uniform this thing down sampling and you can one of those applications if you want to think about it.

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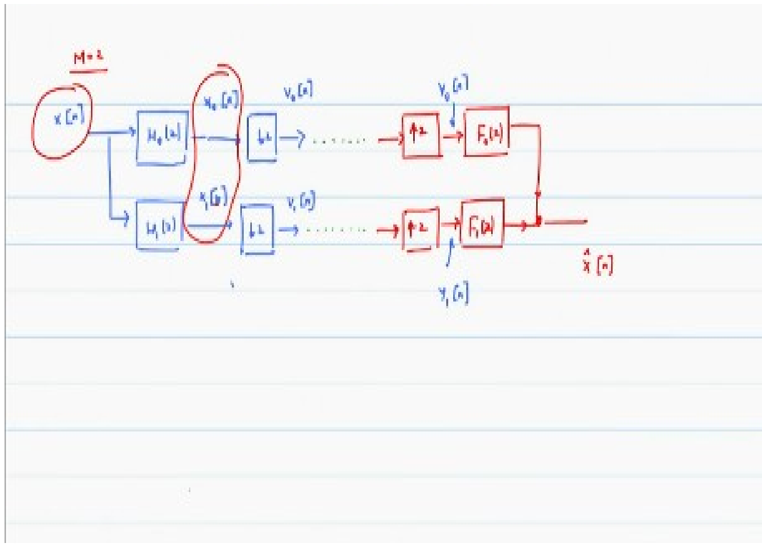

You visualize it, is it in the following manner?

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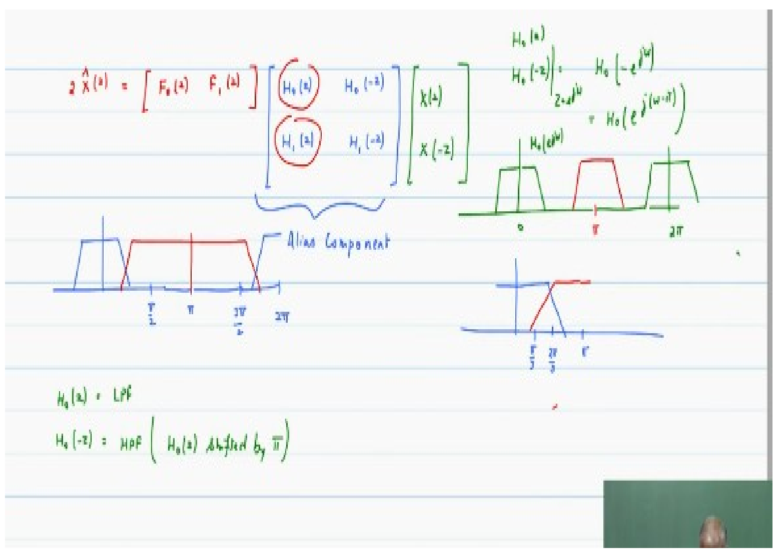

So, supposing I do a I do a 2 channel split okay, so I will write down all the filters there is a filter here and there is a down sampling by a factor of 2, okay now I split only this branch as one more stage down sampling by a factor of 2, and this is down sample by a factor of 2, so now if you look at it. looks like you have different down sampling in the different branches because you have a 4. in the upper two branches and two in the lower branch.

Now it looks like you have a non uniform system because the band width of the signals are different, but it has been a constructed in a very careful manner such that each of these upper branches have got spectrum approximately Pi 4 this is 2 Pi/4 is the bandwidth this one will have approximately Pi/2 as the bandwidth so again the non- uniform nature can come.

Because you are having more number of sub bands on one dimension but the key point to notice that when I am doing down sampling, I want to keep the bandwidth of the order of 2 Pi/M, the total bandwidth so that we can down sample it and then preserve the, and because these are practical filters if I insist that both of them are less a down sample by a factor of 2, I do not want aliasing at all then this is what will become.

The next filter what will happen, that will also require no aliasing so therefore this is what the second filter will be, now the problem will arise is that I do not know what happens in this information, signal is lost. So if I insist that my filters be such that there is no aliasing then I will be I will lose signal in a signal information is lost now this is not acceptable because I cannot really construct.

So, the only practical approach that we will encounter is that these signals this will be slightly wider than Pi/2 to capture all the signals and this one will also be wider than Pi/2 so this in overlap is inevitable so there is going to be some amount of aliasing so we kind of go into this sub band analysis the following aspects in mind, I want to down sample by the maximum sample down sampling rate possible which is M. Given practical filters, then each of these must have approximately bandwidth 2 Pi/M which means that there will be overlap between adjacent sub channels, sub band signals which is fine which may result in aliasing but that is where the challenge lies and that is what we are going to be looking at in todays class so that is sort of a quick review of where we are.

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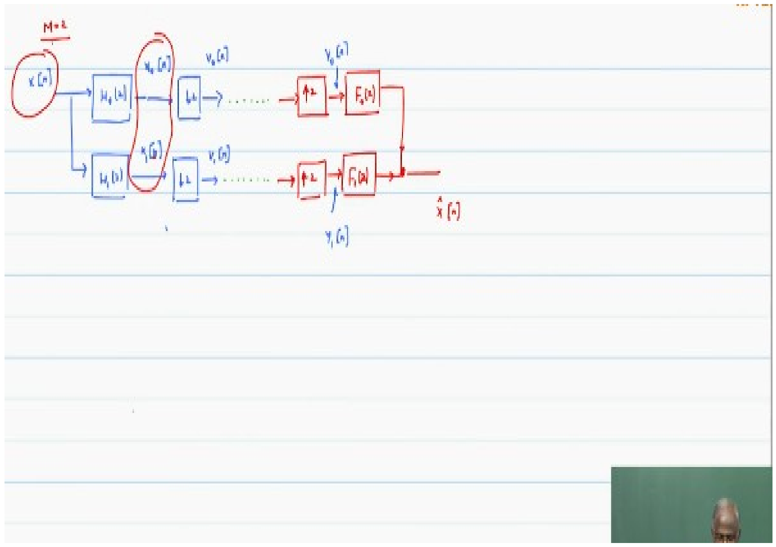

So, the 2 channel case this input signal X of n which we split in through a low pass and a high pass signal. We call that as X0 and X1 and down sample each of them called V0 V1 then you know there is processing but we said that in the analysis stage we are saying that we are not considering the processing, it is basically those signals are connected to the synthesis filters up sampling by a factor of 2 filter filtering through F0 and F1.

To preserve that portion of the because up sampling will produce two copies of the spectrum, preserved that portion of the spectrum that you want and then produce the output signal.

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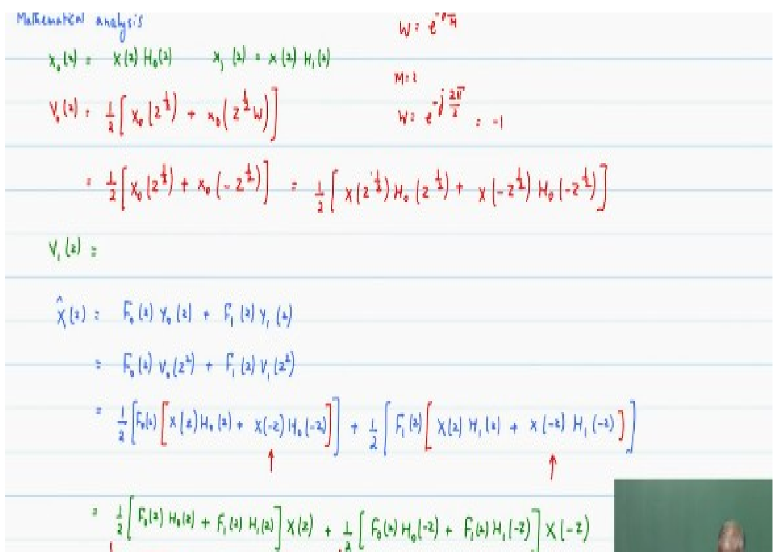

So this was the setup we went through the few steps basically a simple steps of down sampling, up sampling and filtering and we said that the input - output relationships is given in terms of a transfer function T of Z times X of Z that is the input signal and a second term which depends on X of minus Z that is a frequency shifted version of X, that is where the aliasing terms come in if that is present at output.

Then there will be a distortion because I will have input output depending on X of Z the input as well as X of – Z which is a shifted version, so we need to be careful with that, so this is pretty much where we stopped.

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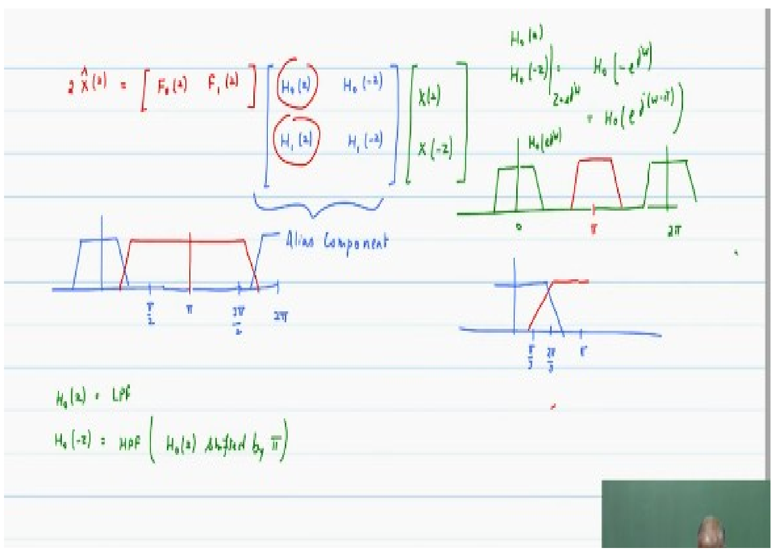

So, that we said we could write this as an as a matrix equation which has the following structure okay now, I just wanted to quickly just sort of refresh ourselves that if you talk about H0 of Z and H0 of – Z, this is if you look at it at Z= e of j omega on the unit circle this becomes H0 – e of j omega this can also be written as e of j omega – Pi, okay e power j Pi+ Omega - Omega does not matter e power j omega - Pi.

So, the reason we like Omega is that it means it shifted the spectrum got shifted from center frequency of 0 whatever the center frequency of 0 now shows up at Pi so if you were to look at a signal or a filter of this type, this is H0 e of j omega and this is what H, this would be the spectrum this is 2 Pi this is 0 now H0 e of j omega will be located at Pi, so basically it is if the

H0 is a low- pass filter H0 of – Z will be a high- pass filter, now will the 2 of these filters overlap.

Depends on the bandwidth of the signal, so if you had chosen your original filter to have bandwidth which is beyond Pi. Let us say you chose 2 Pi/3, so this is Pi/3 and this is Pi so which means that the filters will have significant amount of overlap, so suppose that is something like that so when you shift it you look at it simple, so the notion is that yes, we understand what H0 of – Z is.

Now at the input right at the sub band splitting stage we said that there is a constraint on H0 and H1, that between the two of them they must capture all of the information content in the signal so if you happen to choose H0 as a narrow filter and let us say this is Pi/2, Pi, 3Pi/2 to Pi and so basically you will get a copy of these of this filter there now it the in order for you to actually construct the sub band signal.

This filter has to be of this type right, it has to overlap, and it has to pick up all of the remaining spectrum okay so when I down sample this red signal by a factor of 2. I am going to run into severe amount of aliasing because this is a much wider than Pi/2 and therefore there is a penalty that I will have to pay but right now all we are saying is, between H0 and H1 they must capture all of the information that is all the point that is being made.

So, the key points to summarize are we typically want H0 of Z to be a low pass filter okay then H0 of - Z which appears in our equation is a high- pass filter, this is H0 of Z shifted by Pi the frequency response is shifted by Pi so that is a observation that we have made, and we now are interested in doing the aliasing cancellation.

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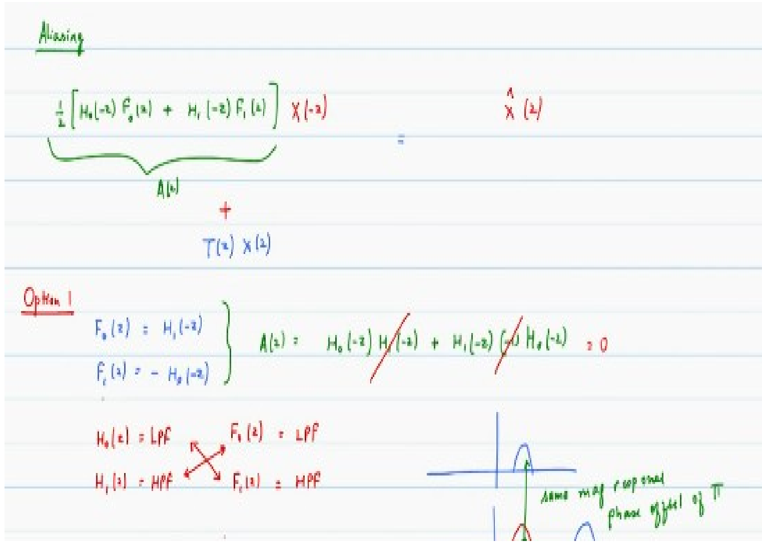

So the aliasing term, if you go back and look at the last equation that we wrote the aliasing term consists of H0 –Z F0 of Z + H1 of –Z, F1 of Z you can have a half or not does not matter may be just put a half also there, this whole thing if we call that as A of Z there is some transfer function times X of - Z. This is the contribution of X of – Z to the output so basically this is what will show up at the output of X hat of Z.

It is not a transformation this is what shows up this is the contribution there is a T plus maybe to write it down +T of Z times X of Z then you actually have= sign okay so this is this is equal to, but we are focusing on the aliasing cancellation part and over a period of time a very clever intuitive you know people with insight came up with several ways of dealing with this problem of canceling the aliasing.

So, the first or the earliest one solution that had come up in terms of cancelling the aliasing, I am to call it as option 1 because there are several solutions that have come and again, I am sure that as you study this problem you will see okay yeah this is probably a very obvious thing once you look at it carefully, so the first one had the following a solution. The choice was choose F0 of Z to be= H1 of –Z and F1 of Z= -H0 of – Z.

So, using these two combinations, we would like to know verify that A of Z= H0 of – Z, F0 is H1 of –Z + H1 of –Z and H1 of Z is -1 times H0 of – Z, basically these two terms cancel giving you equal to 0 okay now what did this choice of filters actually imposed did it impose any awkward or you know unintuitive conditions so here is a quick summary, if H0 of Z is a low pass filter H1 of Z it has to be a high pass filter okay.

Now if you have split the upper branch as the low frequency components and the lower branches as the high frequency components on the synthesis side what should your filter F0 be picking out? It should pick out the low frequencies that it would make sense because the upper branch contains the low frequencies components so when you want to reconstruct a signal you may make the upper branch contribute the low frequency a portion of the signal.

The lower branch contribute, so which means that F0 has to be a low pass filter because when I do up sampling by a factor of 2, I will get multiple copies F0, now does that satisfy if H1 of Z is a high pass filter H1 of –Z will be a low pass filter so no problem at all, so F0 of Z will be a low pass filter. It is not connected to H0 it is connected to H1 that is and F1 of Z automatically turns out to be a high pass filter and it is connected to H0 of Z.

Basically it takes the low pass filter shifts it and also adds a face basically minus is phase term of Pi basically introduces that those two terms can cancel each other okay so this is also another very key element how do you cancel two signals you make sure that this have the same magnitude response and then introduce a phase of Pi. They will cancel each other so when you say that it has the same magnitude response, very important.

This is a observe, if you say that the spectrum of a signal is here, and I want to cancel it I cannot cancel it with a signal who spectrum is elsewhere I can only cancel when they are similar, so I cannot cancel these two on the other hand I can cancel these two, if I ensure that their magnitudes are the same and they have a phase so basically between these two if they have same magnitude response and a phase offset of Pi.

Then I can actually cancel the two spectra again very key insight that happens so that is what was in what was achieved in this process, you ensure that the transfer functions were the same and they had a phase shift of Pi and that is exactly how the, so the key point is that using this option we can cancel aliasing, so aliasing is now something that we have already handled now what about the rest of the system.

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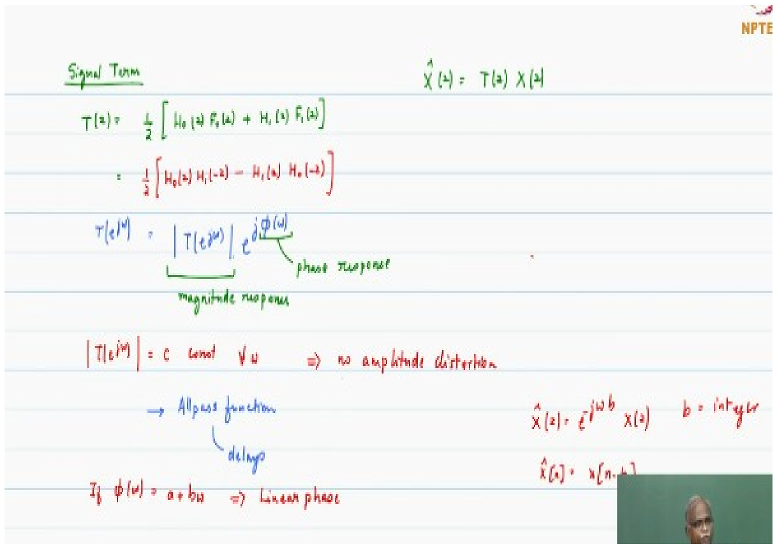

So here is the remaining portion of the transfer function so the signal term, aliasing term has vanished, so T of Z now comes out to be one half of H0 of Z F0 of Z +H1 of Z F1 of Z, now if you substitute the expressions then we get this to come out to be one half of H0 of Z H1 of –Z – H1 of Z H0 of –Z some very interesting symmetries are emerging something for you to think about and particularly all the concepts from signal processing.

Now sort of coming to play in basic DSP, however we are going to take a very general approach where we say that okay a good that we got T of Z and try to look at it in the frequency response on the unit circle, I can write it as magnitude of the e of j Omega Times a phase term, so this is nothing but the magnitude response or what we would refer to as the magnitude response of the transfer function.

I can talk about a transfer function because it is now an LTI system because aliasing has been removed so I have, so the relationship is that X hat of Z=T of Z times X of Z so basically the transfer function is T of Z and I am relating it, and Phi of omega is referred to as the phase response of the transfer function, so we have the magnitude of response and we have the phase response.

Okay now comes a couple of a very key observations very simple but very important the first one says, if my T e of j omega if the magnitude response is=c a constant for all values of omega then I do not have any magnitude distortion in my reconstructed signal because it will be X hat of Z, T of magnitude T of Z you can set it equal to a constant time so basically this means that there is no amplitude distortion.

Distortion in the reconstructed signal a very important one, aliasing has been removed now there is a transfer function. The transfer function says that there is a magnitude part and a free phase part now if there is something funny with the magnitude you will have magnitude distortion if the phase is has got something does some can produce some distortion then you will have a phase distortion.

So, you have aliasing, magnitude, phase three types of distortions one of them has been removed the second one can be removed only if you have that transfer function=1 but this transfer function setting it equal to this basically means that it is an all pass function but all pass function typically the most common all pass functions that we know of our delays right of course there are filters that have a response which is very close to one.

But they are IIR filters, all pass IIR filters but now I have a FIR system, now the question is can this FIR system be all pass can an FIR system can be an all pass but not a delay, you understood the question? All pass filters are IIR. The only known case of FIR all pass filters are delays so again there is a question mark you know can this FIR system actually turn out to be a delay again the answer is yes, but we do not know the steps yet.

So, we are basically we are following the chronological development at this point the question we ask is T of Z has to be and all pass function, to the best of our knowledge only delays are possible and right now that may not even be an option so what about the phase? So, phase response if Phi of omega is of the form a + b omega that means it is linear in omega, so this is the general form of what we refer to as a transfer function with linear phase.

Linear phase transfer function is perfectly acceptable to us. Why? Because that tells us that X hat of Z= e power – j omega if you look at this one just setting a=0 for now because a is non zero is just as complex scale factor e power j omega b times X of Z, opposite right so what does this actually indicate in terms of the relationship assuming b was a integer then it says that X hat of n= X of n – b, b= integer so why linear phase? This is the key.

Because if the output is a delayed version of the input then you will get linear phase so linear phase you may say I want zero phase means it is a non causal system that is not something that we may be able to accept so linear phase is the best that we can do in terms of phase distortion.

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So, to summarize the key points are that we have three types of distortions in a two channel filter bank, the first one is aliasing distortion that is represented by A of Z only if A of Z is 0 then we can talk about the other one then the transfer function becomes T of Z if A of Z= 0 then the transfer function now is T of Z, which can have magnitude distortion you want to eliminate this you need to have an all pass function.

So, to eliminate you need to have an all pass function this is basically says, make sure T of Z somehow is an all pass function and if you want the phase distortion that will be eliminated if T of Z has got linear phase so this, so the thought process is a very systematic, get rid of the aliasing thing whatever transfer function you have, can you make it look like an all pass, a best case get linear phase.

So, if you can show that T of Z somehow becomes a delay, then perfect because it is a all pass and it also gives the linear interface, but the question is it is a actually when your design H of Z you design it to be a low pass filter and then H1 has to be designed as an all pass filter high pass filter. Now taking them and constructing this T of Z has no control over whether the output will come out to be a delay or not.

So, right now this is where we are so let us now spend a few minutes trying to analyze what has happened in the 2 channel Filter bank, so the mathematical framework is now in place, we now want to get the insights.

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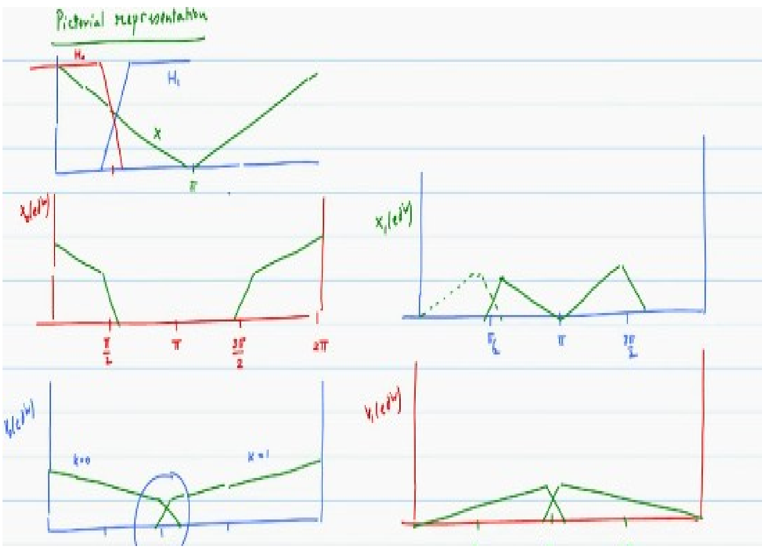

So, the pictorial representation this is usually the best way to get the insights and all we are going to invoke here at this point is our ability to down sample a signal down sample a signal, up sample a signal, so here is a starting point this stakes a little bit of time to draw but it is worth the effort. I am going to have 2 filters H0 and H1 which are approximately of same bandwidth, so which means that around somewhere around Pi/2, they should cross over, but there is no other constraint if my input signal is Nyquist sampled then I have a information content all the way from 0 to Pi, that is the input signal. Now want to look at the first filter, crosses over like this second filter crosses over there, somewhere around the halfway points so the portion of the signal that has been picked up by the low pass filter that is X0 e of j omega, so this is H0 use the same colors, red is H0 and blue is H1.

Green is X, that is the input signal, so the first part that we want to pick up is what does the signal X0 look like? X0 e of j omega, this is the portion Pi notice that there is a part of the signal which is captured between the pass band without any distortion and then the transition band occurs. So, which means from that point on it will drop, that is the portion of the input that is captured by a filter H0 and it will have a copy on the other side at 2 Pi.

So, if you have a this is as 2Pi then this is Pi, there is a copy of the signal which is mirror image to this, so make sure your draw it symmetrical you can do it better than me, so the other one is the portion of the signal that is picked up by H1, again it may seem like we are doing something which is fairly basic but very important that we are able to do it because this actually gives us the insight that we need to actually design these systems. 3 Pi/2 and 2Pi so please draw the spectrum of X1. X1 has the following structure Pi by 2 3 Pi/2 it is a portion of the signal that is picked off by H1. H1 also has got a part that is without any basically preserves the input signal because its= 1 and then there is a transition band so in the transition band you will see a sloping park basically and then the rest of the portion where you get signal get cut off.

So, here again it will be symmetric, so I have two portions the two filters have picked off the respective portions of the signal and have and these are of the order of 2 Pi, the total bandwidth that we are talking about is of the order of so that we can actually sorry of the order of Pi and so we can do the down sampling by a factor of 2 so again a sufficient condition is that the net bandwidth is 2 Pi/M, M= 2 so that the total bandwidth.

The first one is got band width of Pi and the second one has got the bandwidth of Pi, so I can do the down sampling, so first step is the down sampling of X0 down sampled we labeled it as V0. V0 e of j omega so please sketch that, V0 e of j omega. First of all says that there is a scale factor of one half, so we have to take that into account so basically the signal will go down in terms of spectrum.

And it will get stretched all the way to Pi so it is something of this type it is down and then stretched, there is a shifted version of it this is also present, so this is the k=0 version of the spectrum and this is k=1. So, basically you have, so this is the down sample signal has got this overlap and therefore there is some aliasing happening at this point we do not know yet we need to analyze, but for now this is so can you draw the corresponding this signal down sampled.

So, this is V1 e of j omega and this is X1 e of j omega, so if I down sample it again I get a scaling factor of half and there is a stretching and of course there is a shifted version also so very important that we are able to draw it correctly and then be able to draw the scaled version as well, so if you go ahead and do the careful the drawing of this, basically go through the stretching process and the shifting process.

What you would find is that we have the following signals present, that is Pi Pi/2 Pi 3Pi/2 and that is a stretching by a factor of 2 and also the shifted version so basically you stretch by a factor of 2 and apply the shifted version what you find is that you get a version of the signal, scaled version crossing over it so basically there is a scale factor of half be very careful how we do that and basically generate if you have to remember how we did the down sampling.

What did we do? We first replicated but shifted versions of it and then stretched it so if you do the shifted version you will get a copy of the signal here, so you will get a copy of the signal which is here for the shifted version and then you stretch it that means that that is when you get this portion so be careful when you do the down sampling of a band pass signal you have to be make sure that you get the correct scaling of the frequencies.

The next step is a straightforward from V0 and V1, get the up sampled signals.

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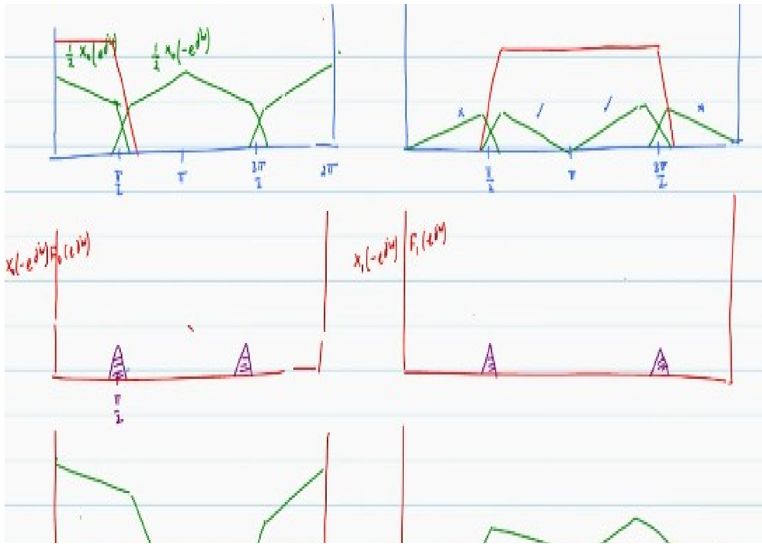

That is fairly straightforward. Let me just quickly sketch that so the spectrum of V0 replicated twice scale factor of 1 half is still present so this is Pi/2 Pi 3Pi/2 and 2Pi, so the shapes are again you can probably draw it much better than I can. I will not look for perfection at least the representation is reasonably close, so this is this portion of it is 1 half X0 e of j omega, this is 1 half X0 of –e of j omega.

The shifted version so basically this is what gets presented when we have the up sampled signal onto this, I am applying F0 of Z, so F0 of Z is a low pass filter which will have some response of that type, so the first copy of the signal goes through. That is what we want it also picks up a portion of X of –Z, that is where the problem occurs so the part that we need to eliminate is the portion of the spectrum which is contributed by the following term: X0 – e of j omega, multiplied by F0 e of j omega what is that? Basically it is the intersection of this red filter with the green line but the shifted version so what we get is something of the form, it is around Pi/2 I will get some sort of a bump. This is the unwanted portion that is coming through the upper branch and of course there is a similar copy of the filter on the other side this is also the unwanted portion.

Now a very quickly let us do the other signal just for completeness I am sure you will be able to do it on your own but please do try it out on your own as well, this is just for completeness Pi/2 3Pi/2 basically we said that the shape of the signal is of this type. The shifted version is going to be of this type, now to this I have to apply the filter F1 it is a high pass filter, so the high pass filter will have a response which basically goes from this direction.

And there is a desired portion that is captured I want to look at what is the undesired part that is captured by the lower branch the undesired portion that is captured by the lower branch is represented by X1- e of j omega*F1 e of j omega and that turns out to be also centered around Pi/2, a shape that looks like this and another 1 that also is in the frequency content now why does the signal go to zero elsewhere.

On one side because the filter itself goes to zero, on the other side also the unwanted portion of the spectrum so basically the unwanted portion is what we are looking at so that this is the wanted portion, so this is the I will put a tick mark for the wanted portion, this is the wanted portion, this is the unwanted part. The unwanted part filtered is what is here now notice that when we achieve aliasing cancellation what is actually happening?

These two unwanted portions are canceling each other now what is passing through from the upper branch. What is passing through is the following signal, is a signal that is of this form and something which is that is what is passing through from the upper branch from the lower branch the signal passing through is something that looks like this, and the combination of these two is the resultant reconstructed signal.

So, it has all of the frequency components but the combination of these 2 may have magnitude distortion, we do not know, may have phase distortion, again that is the part that we would have to we would have to deal with, now we have to take the next step so at this point we look at the problem and we say okay what are the options that are left to us? Can I do something with the flitter design H0 and H1.

Because obviously I am trying to get a the T of Z to become a delay so yes, I would like to explore the option of the T of Z so when you start going in that direction, I have to solve a magnitude problem I have to solve a phase problem distortion so to eliminate one of them I will try to see whether I can restrain myself to linear phase filters, H of Z H0 of Z must be a linear phase filter.

If I say that if my resultant comes out to be linear phase that T of Z turns out to be linear phase, then I am in good shape. The answer is it may be sort of obvious that that is the right way to go because if H0 and H1 are constrained to be linear phase, the product of them will be linear phase and therefore I am kind of moving in the right direction, so I will explore that option as one thing The other option that we want to explore is we are multi rate signal processing.

We are done poly phase components, so well you know can I do something in the poly phase component domain split H0 of Z into poly phase components into 2 poly phase components and see if there is a better insight, that is possible too right? We can say that you know may be rather than dealing with the filters since anyway down sampling is going to happen, I may want to look at it in the poly phase domain, that is a second option.

So, we will look at it both from the poly phase approach and from the linear phase approach so quickly brush up your you know there are 4 types of linear phase filters we will be looking at type 2 linear phase filters and we will justify why, so in case you are not forgotten that just look at what type 2 linear phase looks like the other option is can you redraw this figure the 2 channel filter bank case.

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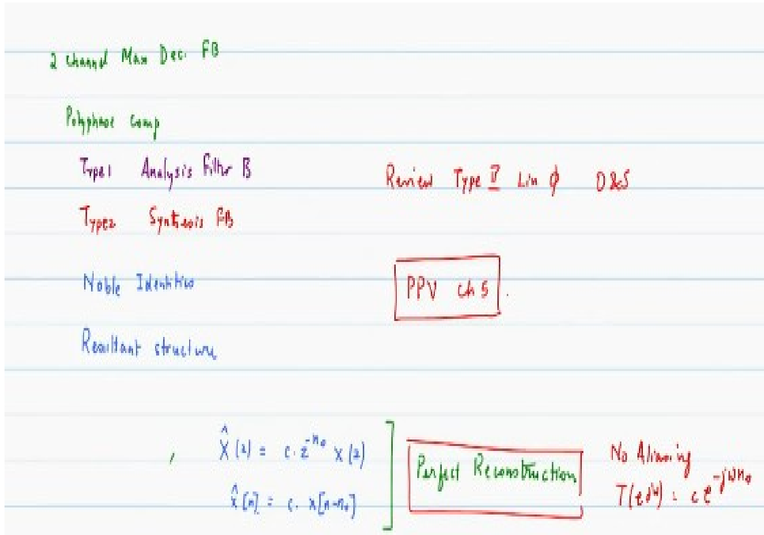

Can you redraw this in terms of the poly phase components with the following constraint? So this is the task for you to do, the task is the 2 channel maximally decimated filter bank you do poly phase components with the following constraint do type 1 for the analysis filters bank you do type 2 for the synthesis filter bank and then do apply the noble identities because you will, it will lend itself to applying the noble identities and obtain the simplified structure.

Resultant structure so if you can come prepared to sort of with the resultant structure one step the poly phase component part, we can address a resultant structure. The other one is, come with a expression for type 2 linear phase filters. We plug in we see what is the condition that we get so the key question is how do we solve it? Do either of these options give us a solution where we can satisfy X hat of Z= c times Z - n0, X of z or in other words X hat of n= c times X of n – n0.

Why is this? Because this is what we refer to as perfect reconstruction. you are reconstructing it to within a scale factor and a delay perfect reconstruction. So, ideally, we would like to achieve perfect reconstruction when there is no signal compression and other things just the synthesis filters and so this is our ultimate goal no aliasing, T e of j omega= c times e power - j omega n,0 it is a scale factor a linear phase and then we achieve the result that were looking for.

Okay so we stop here but please do try the poly phase implementation as well as a review of the type 2 linear phase filters that will be very helpful for us in the next class. Type 2 linear phase in

Oppenheim and Schafer is present ,the portion that we have done today is in P.P. Vaidyanathan's book Chapter 5 again definitely I encourage you to read because that will sort of set the stage for everything that were going to discuss in the next class. Thank you.